

Abstract

Time Diversification

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A year ago, a prominent finance scholar argued to his audience that, "There is no such thing as time diversification." We conceded that there is indeed no such thing for the buy-and-hold investor, but focused instead on an investor who makes the trades necessary to sever next year's bet from last year's outcome.

For the latter investor, wealth at the end of this career can be viewed as a portfolio whose value is the (appropriately weighted, algebraic) sum of the gains and losses in individual years. Unlike individual stocks in cross-sectional diversification, the individual years are virtually uncorrelated.

But when the market level doubles, this investor has to sell roughly half his stocks, etc. What he needs is a diversified asset that, unlike the stock market, maintains constant dollar risk when the market level fluctuates. Such an asset exists in theory; fortunately, the theoretical asset can be approximated with assets an investor can actually own.

Investors' recognition of the importance of diversification is reflected in the popularity of mutual funds. Yet it has probably never occurred to most investors that diversification across time is potentially more useful than diversification across stocks.

Consider a 25-year-old investor concerned with his wealth at age 75. Each of the intervening 50 years adds to or subtracts from his "terminal" wealth. At year end, portfolio value reflects the (appropriately weighted, algebraic) sum of gains and losses for his stocks. At the end of the investor's career, terminal wealth reflects the (appropriately weighted, algebraic) sum of the dollar gain and losses from the individual years. Isn't there a valid analogy?

The investor who buys into the market at the beginning of his career and holds until the end bases the size of next year's bet on last year's outcome. We have seen the value of his terminal wealth will be the product of the market's results (measured as one plus the rate of return) in the individual years. For the buy-and-hold investor there is consequently no such thing as time diversification. But if the investor makes whatever trades are necessary to sever next year's bet from last year's outcome, then his gains and losses from each individual year add rather than multiply. And impacts from the individual years on his terminal wealth are

- 1) cross sectionally diversified, so that all his risk bearing is fully compensated (under the CAPM)
- 2) mutually uncorrelated

Unless the investor is rash enough to predict that the prospects for next year are better than the prospects for last year, he should be making roughly the same dollar bet on both years. In order to do so, however, he will need to sell every time the market rises and buy every time it falls. He will need to do a lot of buying and selling.

On one hand, the potential for time diversification is there, even if the buy-and-hold investor can't realize it. On the other, the cost of trading back to a constant level of dollar risk every time the stock market rises or falls may be daunting. Is this why hardly anyone has tried time diversification?

Risk and Reward

Time diversification is about assuring that equal intervals have equal impact on the investor's terminal wealth. The terminal impact of the dollar gains and losses of particular years depends on the riskless interest rate. In The Millennium Book, Dimson, Marsh and Staunton estimate the real riskless rate at 1.2 percent for the century 1900-2000. It's curious that this number is in the range of what many mutual funds charge shareholders. The effective rate for the time-diversifying investor should also allow for trading costs and taxes. But we defer further speculation of the riskless rate to our discussion of inflation—except to say that we will avoid a lot of circumlocution by not spelling out these complexities over and over.

Consider one year's rate of return on the US stock market. It has a certain distribution, with a certain standard deviation and a certain mean. Even if that distribution is indeed roughly stationary across time, we can measure only the actual rates of return for past years. The investor's probability of terminal loss—of arriving at the end of his career with less wealth than he started out with--depends on both the market risk and the market premium, the expected reward for taking this risk. Because its error can be reduced by subdividing the time sample more finely, estimating the standard deviation isn't a problem. For real annual rates of return on the market, Dimson and his co-authors estimate it at 20.3 percent and 20.1 percent for the US and UK respectively.

But sample error is a potential problem for estimates of the mean. Take the authors' 100 year sample: the standard deviation of the sample mean is

$$\frac{0.20}{\sqrt{100}} = \frac{0.20}{10} = 0.02 .$$

The Central Limit Theorem also applies to the dispersion of means of randomly drawn samples. There is roughly one chance in three that when a Normally distributed sample mean is 0.06 (6 percent), the true universe mean is less than 0.04 or more than 0.08. Although he can benefit greatly from reflecting on Dimson's numbers, we think the investor has to make his own judgment about the market premium. Accordingly, we include in the following table a range of market premiums, as well as a range of possible career lengths.

Terminal Dollar

Unless an investor's riskless rate is zero, a current dollar corresponds to a different number of terminal dollars, depending on his age. But if he is time diversifying, then he wants his potential gains and losses at different ages to have the same terminal impact. So it's useful for him to measure his current risk in terms of what it represents for his terminal wealth—to measure his current risk in terminal dollars. Then he can time diversify by maintaining a constant number of terminal dollars worth of risk from the beginning of his career to the end, i.e., by maintaining a fixed number of “terminal dollars” worth of current risk. The following table is expressed in terms of a lifetime risk level equivalent to one terminal dollar's worth of market risk.

The first two panels in the table, which are consequently net of interest, management fees and trading costs, sum up market premium and market risk across investment careers varying from 16 to 64 years. Then the third panel computes ratios of terminal reward to terminal risk. The panels do this for a range of assumptions about the hard-to-measure market premium.

The risk that an investor will be worse off at the end of his career for having taken stock market risk depends on this ratio. If terminal risks are Normally distributed, for example, that probability is 0.0036—three chances in 1000—for the most favorable case (a 64-year career length and a 7 percent risk premium).

Terminal Reward vs. Terminal Risk**Expected dollar gain over career for a lifetime risk equivalent to one "terminal dollar"**

	-----	Market Premium per year		-----
Career length	.04	.05	.06	.07
16	0.64	0.80	0.96	1.12
25	1.00	1.25	1.50	1.75
36	1.44	1.80	2.16	2.52
49	1.96	2.45	2.94	3.43
64	2.56	3.20	3.84	4.48

Standard Deviation of Terminal Wealth

	.04	.05	.06	.07
16	0.80	0.80	0.80	1.08
25	1.00	1.00	1.00	1.00
36	1.20	1.20	1.20	1.20
49	1.40	1.40	1.40	1.40
64	1.60	1.60	1.60	1.60

Expected career gain/ Standard Deviation of Terminal Risk

Career length	.04	.05	.06	.07
16	0.80	1.00	1.20	1.40
25	1.00	1.25	1.50	1.75
36	1.20	1.50	1.80	2.10
49	1.40	1.75	2.10	2.45
64	1.60	2.00	2.40	2.80

Constant Dollar Risk

Is there such a thing as a financial asset with constant dollar risk? Such an asset would permit the investor who owned it to achieve time diversification without trading.

All commercial risk measurement services focus on relative risk—surprise in an asset's value, divided by its beginning value. The only justification for such commercial measures is that the probability distribution of the ratio is stationary. (See our evidence below.) But then dispersion of the asset's dollar risk—surprise in its dollar value—fluctuates with fluctuations in the asset's value.

These comments apply to both individual common stocks and portfolios, including portfolios intended to proxy the value of the whole stock market. Let the stock market level—the value of the market portfolio—be x and the value of an asset with constant absolute risk be y , and let dx and dy represent dollar surprise in x and y respectively. If both assets are completely diversified, then the market level x determines the value of y . Let the relation between the two values be

$$y = f(x).$$

We ask: What functional dependence of y on x translates the constant relative risk of x into the desired constant dollar risk of y ?

When the functional relation between y and x is such that, for all years,

$$dy = \frac{dx}{x},$$

the left-hand side and the right-hand side, being equal, will necessarily have the same probability distribution. In particular, if the right-hand side—the relative return on the stock market—is stationary across time, the left-hand side will also be stationary. But whereas the right-hand side is relative change in x — dx divided by the level x —the left-hand side dy is dollar change in y . So if, as the market level x fluctuates, its relative risk is truly stationary, then the dollar risk in y is also stationary.

If we take indefinite integrals of both sides, we have

$$y = \ln x + \ln k$$

$\ln k$ = constant of integration, or

$$y = \ln kx.$$

The asset with constant dollar risk is the asset whose value varies with the logarithm of the market level.

Inflation

We don't have the option of investing in the real market level. The values of the market and our log approximation are nominal values. But the risk we want to maintain constant over time—as the price level changes—is the real risk. If, as we have argued, the risk in nominal market return is stationary, then the risk of nominal dollar gains and losses in the log portfolio is also stationary. But this means that if, for example, the price level is rising, then the risk of real dollar gains and losses is falling.

Let x be the nominal market level and y be the nominal value of a portfolio that varies with the logarithm of the market level. And let the respective real values be x' and y' , where the price level is p . We have

$$x' = \frac{x}{p}, \quad y' = \frac{y}{p}.$$

For investment surprises we have

$$dx' = \frac{dx}{p}, \quad dy' = \frac{dy}{y}.$$

The logarithmic portfolio is defined by a relation between nominals

$$dy = \frac{dx}{x}.$$

Substituting, we have

$$p dy' = \frac{p dx'}{p x'} = \frac{dx'}{x'}.$$

We see that, if surprise in the real market level is stationary, surprise in the nominal level will also be stationary.¹ But if surprise in the nominal value of the logarithmic portfolio is stationary, surprise in its real value

¹ Past inflation has the same effect on the units of measure for the numerator and denominator. Current inflation adds algebraically to both market gains and losses, but affects the mean of these numbers rather than the dispersion.

$$dy' = \frac{dy}{p} .$$

won't be. This means that if, for example, the price level is rising over the investor's career, the real risk in his logarithmic portfolio is falling.

Consider first the case where the real riskless rate of interest is zero. To offset the effect of inflation, investment positions in recent years in his career should be rescaled relative to early years, with the rescaling from year to year equaling that year's inflation rate.

Then consider the case where inflation is not a problem but the riskless interest rate is positive rather than zero. Then investment positions in recent years should be rescaled relative to early years, with the rescaling from year to year equal to the riskless interest rate.

Taken together, these considerations argue for rescaling from year to year by an amount equal to the nominal riskless interest rate.

An Approximation

The logarithmic asset is probably not available in today's security markets. But it can readily be approximated using assets that are. Consider the following Taylor Series expansion of the logarithmic function, where a is greater than zero:

$$\ln \frac{x}{a} = \frac{x - a}{a} - \frac{1}{2} \left(\frac{x - a}{a} \right)^2 + \frac{1}{3} \left(\frac{x - a}{a} \right)^3 - \dots .$$

Although the accuracy of the approximation increases with the number of terms retained in the series;² we retain only the first two. Expanding these terms we have

$$\ln \frac{x}{a} \approx \frac{x - a}{a} - \frac{1}{2} \left(\frac{x - a}{a} \right)^2 .$$

The investor who seeks time diversification is actually concerned with the corresponding risks. How well does the risk of the right-hand side approximate the risk of the left-hand side? The dollar risk on both sides depends on a product. One factor in

² There are other power series approximations—even other Taylor Series approximations—to the logarithmic function.

the product is the rate of change with respect to the market level x . We have for the respective sides

$$\frac{d}{dx} \ln \frac{x}{a} = \frac{1}{a} \frac{1}{x/a} = \frac{1}{x} = \frac{1}{a} \left(\frac{x}{a} \right)^{-1}.$$

The other factor in both products is the dollar risk in x . But if $\frac{dx}{x}$ is stationary then the dollar risk in x is proportional to the (known, non-risky) value of x .

If we invest in the approximation portfolio when x equals a , then the above rate of change is $\frac{1}{a}$ for both the logarithmic portfolio and the approximation. But the risk in the approximation drifts away from the log portfolio as the market level x moves away from a .

The Role of Beta

Beta is a measure of how much an asset's value changes when the general market level changes. Specifically, it is the ratio of two rates of return. Let x be the market level, for example, let y be the (highly diversified) asset's value and let beta be represented by the Greek letter β . Then we have

$$\frac{\frac{dy}{y}}{\frac{dx}{x}} = \beta,$$

$$\frac{dy}{y} = \beta \frac{dx}{x}.$$

Taking the indefinite integral, we have

$$\ln y = \beta \ln x + \ln k$$

where $\ln k$ is a constant of integration. Taking antilogs we have

$$y = k x^{\beta}.$$

We see that a diversified asset's value is linked to the market level by a power that equals its beta. Our truncated Taylor Series approximation to the logarithmic function of the market level contains two powers of the market level x . Evidently the terms

containing these powers correspond to investment positions in diversified assets with betas of one and two.

Accuracy of the Approximation

How bad are the errors in the approximation portfolio? Let:

- a = beginning market level
- x = market level at the end of the year
- dx = change in market level
- σ_{dx} = standard deviation of change
- y = value of approximation portfolio
- dy = change in value of approximation
- σ_{dy} = standard deviation of change

As noted, its dollar risk is the product of its rate of change with respect to the market and the dollar risk in the market. The first column in the table displays a range of possible ratios of the ending market level x to the beginning market level a. The second column shows the resulting new market levels. The third column shows the standard deviation of the market's absolute risk for the following year—assuming its relative risk, the standard deviation of its rate of return, is still 20 percent.

The fourth column shows the rate of change of the approximation portfolio with respect to change in the stock market level. The fifth column is the product of the third and fourth columns. Because the third column measures absolute risk in the market level, and the fourth column measures its rate of change with respect to that level, the fifth column measures absolute risk in the approximation portfolio.

The absolute risk in the ideal, logarithmic portfolio is 20 percent of the initial market level a, no matter what the subsequent change in market level. But the approximation is imperfect. The fifth column shows how its absolute risk drifts progressively farther from the correct, constant value as the new market level x moves away from the beginning level a. (It may be worth noting, however, that the absolute risk of the approximation portfolio is always less than or equal to the correct value.) The sixth column expresses the errors as percentages of the correct absolute risk.

Approximation Errors

x/a	x	Δdx	$\frac{dx}{dy}$	Δdy	% error
1.30	1.30a	0.26a	0.70/a	0.1820	9.00
1.25	1.25a	0.25a	0.75/a	0.1875	6.25
1.20	1.20a	0.24a	0.80/a	0.1920	4.00
1.15	1.15a	0.23a	0.85/a	0.1955	2.25
1.10	1.10a	0.22a	0.90/a	0.1980	1.00
1.05	1.05a	0.21a	0.95/a	0.1995	0.25
1.00	1.00a	0.20a	1.00/a	0.2000	0.00
0.95	0.95a	0.19a	1.05/a	0.1995	0.25
0.90	0.90a	0.18a	1.10/a	0.1980	1.00
0.85	0.85a	0.17a	1.15/a	0.1955	2.25
0.80	0.80a	0.16a	1.20/a	0.1920	4.00
0.75	0.74a	0.15a	1.25/a	0.1875	6.25
0.70	0.70a	0.14a	1.30/a	0.1820	9.00

The table shows that a 20 percent move up or down in the market level changes the dollar risk in the approximation portfolio only four percent. To trade back to constant dollar risk every time his portfolio changed four percent, a conventional investor would have to trade

$$\left[\frac{0.20}{0.04} \right]^2 = 5^2 = 25$$

25 times as often. (If the dispersion of random fluctuations over a time interval varies with the square root of its length, the length of the time interval varies with the square of the stipulated dispersion.) Is this why conventional investors don't attempt to time diversify?

Rebalancing

We have seen that, when the market has moved up or down one standard deviation, or 20 percent, the new standard deviation for the approximation portfolio is no longer 20 percent of the original dollar investment, but merely 18.2 percent. (Roughly one year in three, the market moves more than 20 percent.) When the market level x moves away from the “beginning” level a , two things happen:

- 1) approximation breaks down as the risky positions’ 4:1 ratio breaks down
- 2) scale, or magnitude, of net risk moves away from beginning net risk.

There are many combinations of the two risky positions that will satisfy the 4:1 condition, hence restore the logarithmic character of the portfolio. And there are many combinations that will restore the original net risk. But one, and only one, combination of the two positions can satisfy both conditions. If the investor changes the “beginning” market level a in this ratio to the current market level x , the ratio reverts to its original value of one. But when the values of the risky positions were based on a ratio value of one, they

- 1) were in the accurate 4:1 ratio and
- 2) had the desired level of net dollar risk that the investor’s wanted to maintain over his lifetime.

What the new value of a doesn’t do is retain the same net investment in the two risky positions they had before we changed the ratio back to one. This is where the third, constant, “riskless” term in the Taylor Series formula comes in: when we are making the trades in the risky assets dictated by the change in the ratio, these trades absorb or free up cash, which then flows to or from the third, riskless, position. (Obviously, changes in the value of the riskless position don’t change the portfolio’s risk³ so if, after these trades, the risky positions have the correct risk, so has the portfolio.)

³ When we use year-end data for the market level, we restrict our opportunities for rebalancing back to an accurate approximation of the logarithmic asset. In practical application, changes in the market level can be followed and responded to almost continuously.

When increasing approximation error forces us to rebalance back to our original investment positions, these positions should be scaled up from those of the previous rebalancing by a factor reflecting appreciation at the nominal riskless rate over the interval

In Table 1, the beginning market level is arbitrarily set at 1000. Then the long position is

$$2 (1000) = 2000,$$

and the short position

$$1/2 (1000) = 500.$$

So, the net value of the two risky positions (the “risky equity”) is then

$$2000 - 500 = 1500.$$

Each rebalancing returns the risky equity to 1500. But offsetting transfers to or from the riskless asset preserve the investor’s total equity.

Table 1

End of Year Values for Rebalancing Years (x/a)

Year	US mkt index	
1977	169	
1979	179	178/169 = 1.0592, 1.0592 ² = 1.1218 2(1059) – 1/2 (1122) 2118 – 561 = 1557, 1557 – 1500 = 57
1980	210	210/169 2(1.243) – 1/2 (1.544) 2486 – 772 = 1714, 1714 – 1500 = 214
1981	225	225/210 = 1.0714, 1.0714 ² = 1.1479 2(1.071)– 1/2(1.148) 2142 – 574 = 1568, 1568 +214 – 1500 = 282
1982	208	208/210 2 (0.990) – 1/2 (0.981) 1980 – 491 = 1489, 1489 + 214 – 1500 = 203
1983	281	281/210 2 (1338) – 1/2 (1790) 2676 – 895 = 1781 + 214 – 1500 = 495 = 1.3381, 1.3381 ² = 1.7905
1984	283	283/281 2 (1007) – 1/2 (1014) 2014 – 507 = 1507, 1507 + 495 – 1500 = 502
1985	324	324/281 2 (1153) – 1/2 (1329) 2306 – 665 = 1641, 1641 + 495 – 1500 = 636
1986	409	409/281 2 (1456) – 1/2 (2119) 2912–1059=1853, 1853 + 495 – 1500 = 848
1987	516	516/409

between rebalancings. (If the price level is inflating very rapidly, rescaling doesn’t have to wait for the next rebalancing. Then, however, the investor incurs additional trading costs.)

		$2(1262) - 1/2 (1592)$ $2523-796 = 1727, 1727 + 848 - 1500 = 1075$ $= 1.2616, 1.2616^2 = 1.5917$
1988	478	$478/409$ $2(1169) - 1/2 (1366)$ $2338 - 683 = 1655, 1655+848-1500 +1003$
1989	577	$577/409$ $2(1411 - 1/2 (1990))$ $2822 - 995 + 1827, 1827 + 848-1500=1175$
1990	609	$609/577$ $2 (1.055) - 1/2 (1114)$ $2110-557=1553, 1553+1175-1500=1228$ $= 1.0554, 1.0554^2 = 1.1140$
1991	695	$695/609$ $2 (1441) - 1/2 (1302)$ $2282-651=1631, 1631+1228-1500=1359$
1992	765	$765/695$ $2(1101) - 1/2 (1212)$ $2202-606=1596, 1596+1359-1500=1455$ $= 1.1007, 1.1007^2 = 1.2116$
1993	806	$806/695$ $2(1160) - 1/2 (1345)$ $2320-672=1648, 1648+1359-1500=1455$
1994	841	$841/806$ $2(1043) = 1/2 (1088)$ $2086-544=1542, 1542+1507-1500=1549$ $= 1.0434, 1.0434^2 = 1.0887$
1995	1000	$1000/841$ $2 (1189) - 1/2(1414)$ $2378-707=1671, 1671+1549-1500=1720$
1996	1235	$1235/1000$ $2(1235) - 1/2 (1525)$ $2470-763=1707, 1707+1720-1500=1927$ $= 1.2350, 1.2350^2 = 1.5252$
1997	1595	$1593/1235$ $2(1290) - 1/2 (1664)$ $2580-832=1748, 1748+1927-1500=2175$
1998	1987	$1987/1593$ $2(1247) - 1/2 (1556)$ $2494-778=1716, 1716+2175-1500=2391$ $= 1.2473, 1.2473^2 = 1.5558$
1999	2513	$2513/1987$ $2(1265) - 1/2 (1600)$ $2530-800=1730, 1730+2391-1500=2621$ $= 1.2647, 1.2647^2 = 1.5995$
2000	2728	$2728/2513$ $2 (1086) - 1/2 (1178)$ $2172-589=1583+2621-1500=2704$ $= 1.0856, 1.0856^2 = 1.1784$

Table 1 shows how the approximation portfolio would have functioned using actual US stock market data for end-of-year levels from 1977 to 2000. Although, given the limited data, rebalancings couldn't be triggered by daily market closes, there were eleven rebalancings during this period. (during the bull market at the end of the period, the approximation was rebalancing almost every year.)

The exhibit devotes three lines of calculation to each year (except 1978). For the current value of a , the first line calculates the ratios $\frac{x}{a}$ and $\frac{x}{a}$. The second line applies the coefficients in the approximation formula to the respective ratios. The third line calculates the new risky equity, and the change since the last balancing.

For rebalancing years, the third line is more complicated. Since each rebalancing wipes out the difference between the current risky equity and the original investment (in this example, 1500). The third line also calculates the new value of the riskless asset, reflecting the cash freed up or absorbed in returning the risky positions to their original values.

The value of the approximation portfolio to the investor includes the net value of both his risky positions and the remaining sum of these (algebraic) additions to the riskless asset. Thus the three-line entry for a rebalancing year reflects both the effect of rebalancing, which takes place at the beginning of that year, and the effect on the value of the risky positions of the subsequent change in market level between the beginning and the end.

Question: if rebalancing restores the original dollar risky positions at rebalancing, why isn't this evident in JLT's 22 year example using actual US stock market data?
Answer: Whereas rebalancing occurs at the beginning of the year, the worksheet numbers are based on market level at the end.

The Evidence

The last three decades of the century included several painful market collapses as well as a celebrated bull market. The nominal market level increased 16 times, the real level four. Surely this period is a worthy test of whether

- 1) the risk in the markets' rate of return is really stationary;

2) the dollar risk in the logarithmic portfolio is really stationary.

In order to test whether risks were stationary, we need to be able to measure ex ante risk ex post. Actuaries use a special kind of graph paper called “probability paper” to do this. Its vertical axis is conventional, with horizontal lines equally spaced. But its horizontal axis is variously compressed and stretched so that, when drawings from a Normal sample are ranked from lowest to highest and then accorded equal probability increments (rather than equal distances) on that axis, they plot as a straight line. Depending on the chosen scale of the conventional vertical axis, the slope of that line reflects the sample’s dispersion.

The point, of course, is that if the sample is drawn from a universe with different dispersions—if, across time, the risk isn’t stationary—then the sample can’t plot as a straight line.

Were the two risks really stationary over the sample period? Figure 1 displays the data for the market’s rate of return. Figure 2 displays the data for the year-to-year change in the dollar value of the logarithmic portfolio.

Did the approximation portfolio really track the logarithmic portfolio? Figure 3 displays the data for the dollar values. Figure 4 displays the data for the year-to-year changes in dollar value of the two portfolios—i.e., for their risks.

Implementing the Approximation Portfolio

As the market level goes up, the value of the short position increases, even as the value of the long position increases. Rebalancing entails reducing the long and short positions after the stock market has gone up and increasing the long and short positions after the stock market has gone down. In the first case, proceeds from selling part of the long position are used to cover part of the short position. In the second case, profits from the short position are used to extend the long position.

The broker who borrows the stock the investor sells short will demand “margin”—valuable assets to protect them in case he is unable to cover if the market has risen too much. If he deposits his long position with the broker, his margin doesn’t start to shrink until market level has doubled (five standard deviations). It doesn’t run out until market level has quadrupled ($3 \times 5 = 15$ standard deviations or annual stock

market return). But in the meantime he has rebalanced to less risky positions, over and over.

On the other hand, when the market falls the investor loses margin. But he doesn't lose all of it until market level reaches zero. In the meantime, when the market falls, he uses the profits from the short position to increase the value of the long positions.

The 4:1 target ratio assures that the long position will always provide more margin for the short position than even the most timid broker would require.

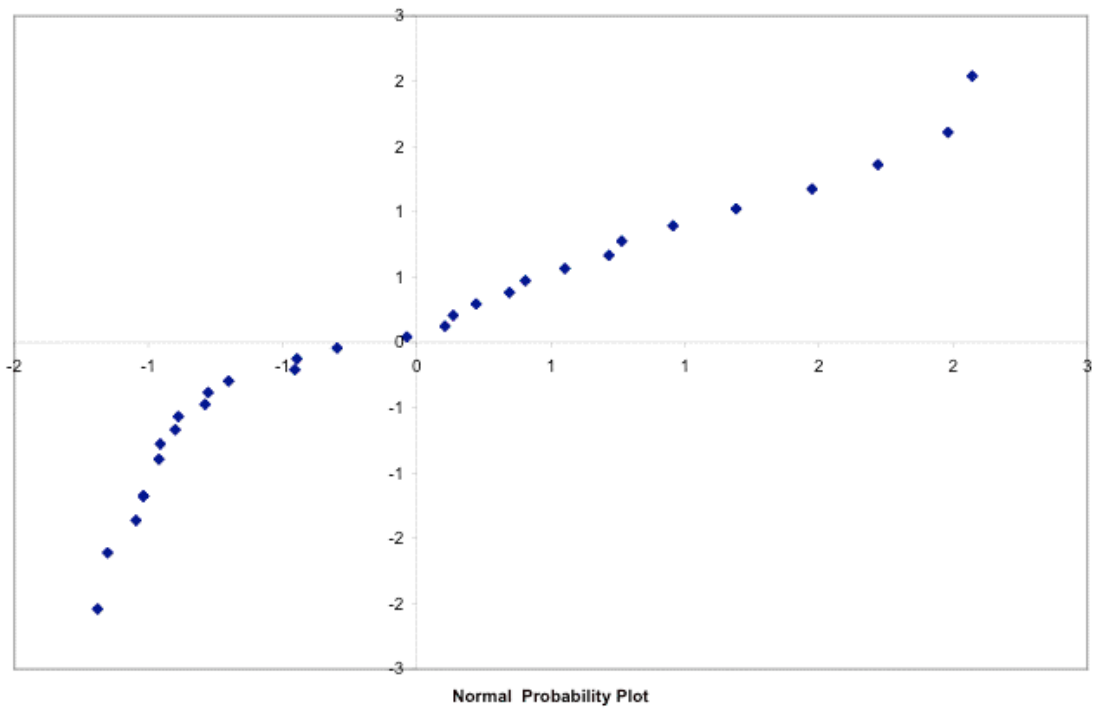
Should Risk Decline with Age?

We have argued that, if his real riskless rate is zero—or just big enough to offset trading and other costs—an investor who wants to time diversify will take the same dollar risk in the last year of his investment career as he takes in the first. Doesn't this prescription conflict with the intuition that an old investor should take less risk than a young investor?

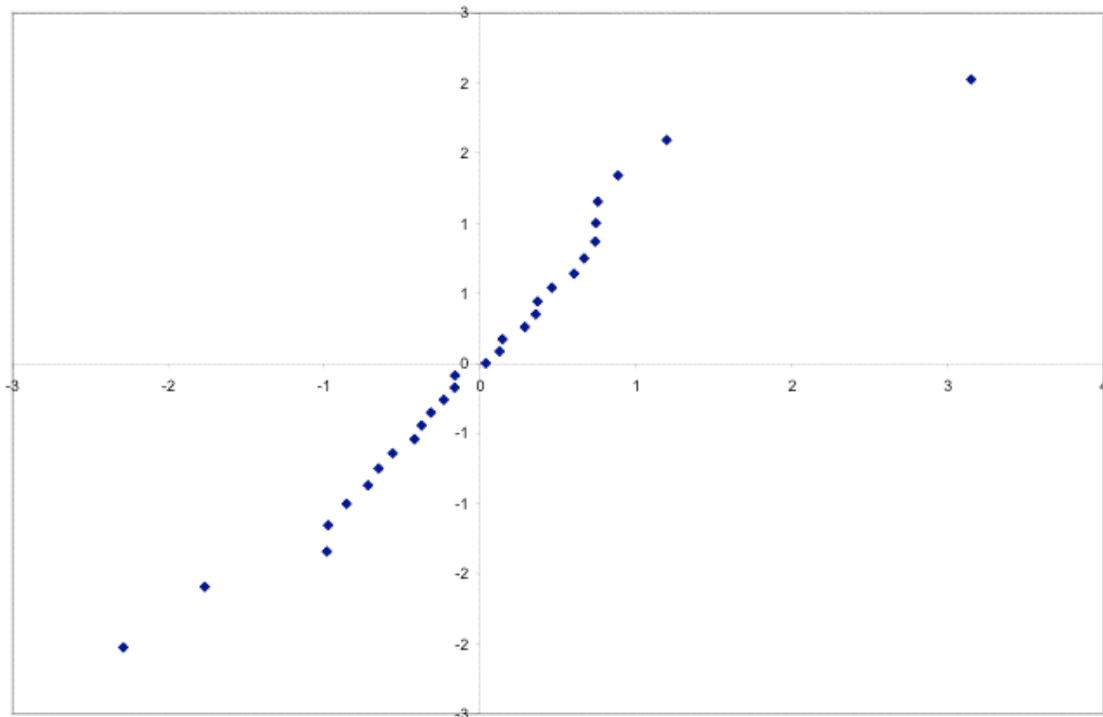
We have seen that, if he has time diversified, the investor approaching the end of his career is likely to be richer than when he began. But then the same dollar risk at the end of his career represents a smaller relative risk. And relative risk is the way most investors—especially conventional investors—think about risk.

Is constant dollar risk just an unfamiliar way of expressing a familiar intuition?

Graph 1 Rate of Return on U.S. Market '71 - '00



Graph 2 Year to Year Changes in the Dollar Value of a Portfolio That Varies with the Logarithm of the US Market (1972 - 2000)



Normal Probability Plot

