# The Implied Equity Premium

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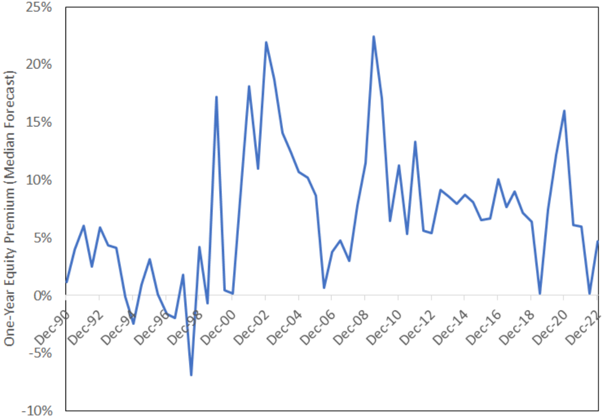
# The Equity Risk Premium

- Equity premium: expected return on stocks versus risk-free Treasuries
  - Historical US premium since 1871: average $\approx 5\%$; volatility $\approx 18\%$
  - Compensates for higher risk of stocks relative to gov't bonds
- The equity premium is important for the real economy and markets
  - Affects wealth and consumption (Di Maggio et al. (2020))
  - Affects real investment (Baker et al. (2003))
- The equity premium seems to vary a lot Intuition
  - Economic risk and investor risk aversion vary
  - Behavioral biases also could drive variation

# Estimating the Equity Premium: Surveys Models

- Do economists know and agree on the (one-year) equity premium?
  - Dispersion of 6.2% in estimates in Dec 2021 (Livingston survey)
  - Average value of 6.8% (std dev of 6.0%) from 1990 to 2022



# A New Approach to Finding Risk Premiums

- Learn as much as we can from market prices
  - Observe prices of stock index, options, and bond
  - Prices reveal how much investors dislike risk
  - Stock index return variance reveals the amount of risk
- Analyze well-developed (e.g., US) markets with informative prices
  - Frictionless: no arbitrage in stock index, options, and bond
  - Complete: securities enable trading any stock index event
  - S&P 500 index and options markets are the natural setting

# Risk Premiums in Frictionless Markets

$$\mathbb{E}_t \tilde{R}_T = \mathbb{E}_t \left[ \frac{M_T}{\mathbb{E}_t M_T} \frac{\mathbb{E}_t M_T}{M_T} \tilde{R}_T \right] = R_{f,t}^{-1} \mathbb{E}_t^* \left[ M_T^{-1} \tilde{R}_T \right]$$

$$= R_{f,t}^{-1} \mathbb{E}_t^* \left[ \left[ R_{f,t} + \sum_{k=1}^{\infty} w_{k,t} \left( \tilde{R}_{m,T}^k - \mathbb{E}_t^* \tilde{R}_{m,T}^k \right) \right] \tilde{R}_T \right].$$

- $\mathbb{E}_t^*$: risk-neutral expectation at time $t$
- $\tilde{R}_{m,T}$: excess market return at time $T > t$
- $M_T > 0$: pricing kernel in frictionless markets
- Use growth-optimal portfolio return for $M_T^{-1}$ in last equation

# Preview of Main Theoretical Results

Risk premiums are weighted sums of risk prices (from index options)

Stock Wt ↓ $w_1$ ; Var Deriv Wt ↓ $w_2$

$$\text{EqPrem} = w_1 \ \text{PrcVar} + w_2 \ \text{PrcSkew} + \ldots$$

$$\text{VarPrem} = w_1 \ \text{PrcSkew} + w_2 \ \text{PrcKurt} + \ldots$$

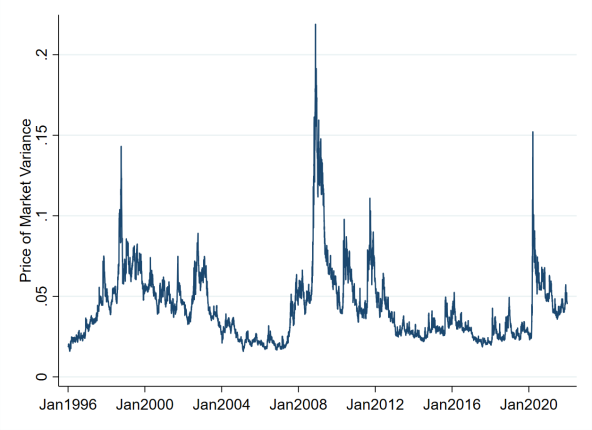
- Weights, $w_k$, are those in a growth-optimal portfolio
    - Optimal for a price-taking, rational, log utility investor
    - Coefficients in regression of variance premium on risk prices
- Behavioral investors hold the rest of asset supply (not $w_k$)
- Equity premium varies with shifts in risks, preferences, and beliefs

# Preview of Main Empirical Results

- Growth-optimal weights are not $w_1 = 1$, and $w_k = 0$ for $k \geq 2$
  - Stock weight is $w_1 > 1$; and it increases after 2008
  - Variance and skew weights are negative: $w_2 < 0, w_3 < 0$
- Equity premium predicts realized excess stock returns well
  - Annual $R^2 > 9\%$ vs. $< 1\%$ for representative log utility
  - Gain of 13% in expected returns for a log utility investor
- Predicted variance premium tracks expected variance premium
- Predictions work best in frictionless markets—e.g., not Sept '08

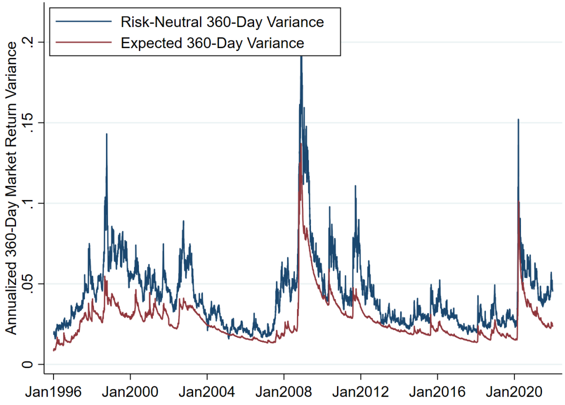
# Price of Market Variance Risk in Complete Markets

- Price of squared returns reveals the price of market variance
- It is roughly the square of the popular volatility index (VIX)
- Ranges from 0.015 to 0.21, implying price of volatility is 12% to 46%



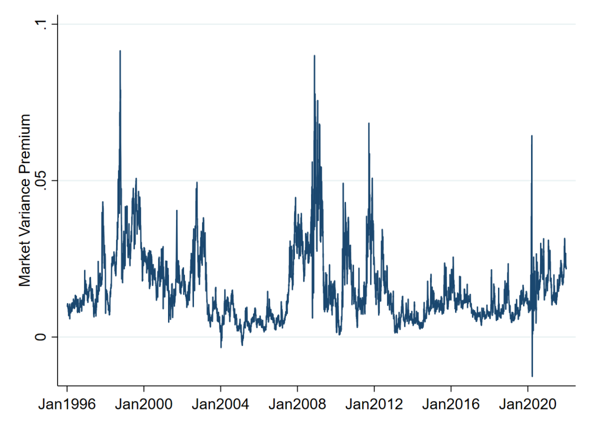
# The Price of Variance versus Expected Variance

- Estimate long-term variance from daily realized variance 𝔼Var
- Compare to price of variance: difference is variance premium
- Expected variance is correlated 0.9 with price of variance



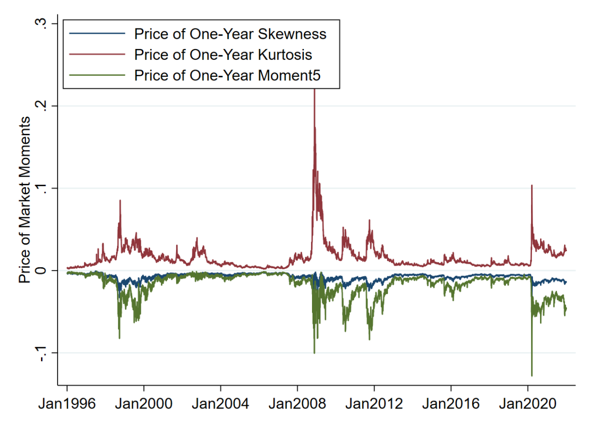
# The Expected Variance Premium Is Positive

- Investors usually pay 1% or 2% to hedge variance risk
- Increased variance is associated with negative market returns
- Variance hedging insures against market declines



# Options Price Skewness, Kurtosis, and More

- Odd (even) moments are negative (positive)
- All moments increase in magnitude in crises
- Investors pay to hedge market skewness (crashes)

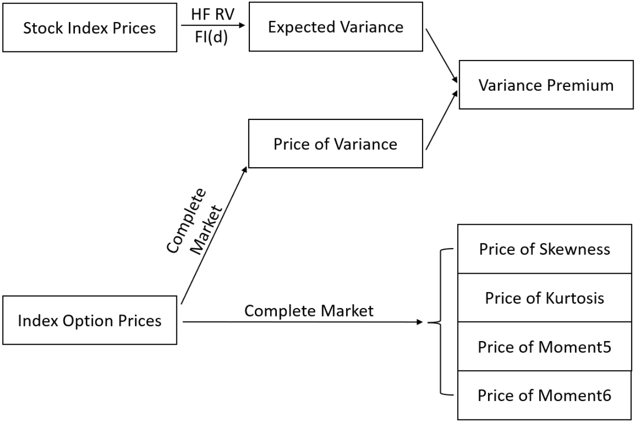


# Summary: What We Learn from Prices

- Options price all stock market risks in complete markets
  - Variance, skewness, kurtosis, and more
  - They enable investors to trade all market risks
- High-frequency returns reveal stock market variance
- Price of variance is at a premium over actual variance

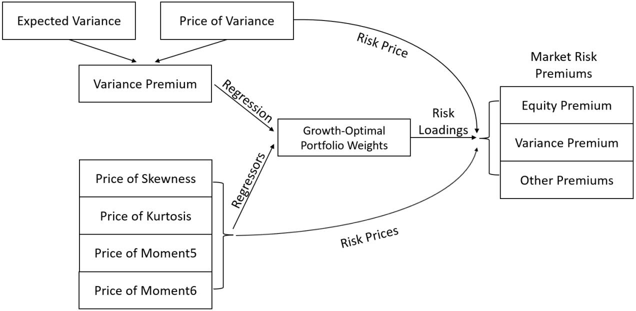
# Complete Markets and Information from Prices

- Prices are highly informative in complete markets
  - They reveal all moment prices and the variance premium



# Frictionless Markets Simplify Risk Premiums

- In frictionless markets, the reciprocal of the growth-optimal portfolio return prices all assets (Long (1990))
  - Pricing kernel comes from FOC of hypothetical log utility investor



# Theory: Pricing Market-Related Securities

- In complete, frictionless markets, the pricing kernel is the reciprocal of the growth-optimal portfolio return:

$$M_T = \left[ R_{f,t} + \sum_{k=1}^{\infty} w_{k,t} \left( \tilde{R}^k_{m,T} - \mathbb{E}^*_t \tilde{R}^k_{m,T} \right) \right]^{-1}$$

- Apply kernel to excess returns, $\tilde{R}^k_{m,T} - \mathbb{E}^*_t \tilde{R}^k_{m,T}$, to find risk premiums
  - Apply to stocks ($k = 1$) to get the equity premium:

$$\mathbb{E}_t \tilde{R}_{m,T} = R^{-1}_{f,t} \sum_{k=1}^{\infty} w_{k,t} \mathbb{E}^*_t \tilde{R}^{k+1}_{m,T}$$

  - Apply to variance derivatives ($k = 2$) to get the variance premium:

$$\mathbb{E}^*_t \tilde{R}^2_{m,T} - \mathbb{E}_t \tilde{R}^2_{m,T} = -R^{-1}_{f,t} \sum_{k=1}^{\infty} w_{k,t} \left( \mathbb{E}^*_t \tilde{R}^{k+2}_{m,T} - \mathbb{E}^*_t \tilde{R}^k_{m,T} \mathbb{E}^*_t \tilde{R}^2_{m,T} \right)$$

Premium Intuition or Interpretation

# Estimating Growth-Optimal Weights

- Approximate the pricing kernel as the reciprocal of the growth-optimal portfolio return based on $K = 4$ securities:

$$M_T \approx \left[ R_{f,t} + \sum_{k=1}^{4} w_{k,t} \left( \tilde{R}_{m,T}^{k} - \mathbb{E}_t^* \tilde{R}_{m,T}^{k} \right) \right]^{-1}$$

- The variance premium depends on four risk-neutral moments

$$\mathbb{E}_t^* \tilde{R}_{m,T}^2 - \mathbb{E}_t \tilde{R}_{m,T}^2 = -R_{f,t}^{-1} \sum_{k=1}^{4} w_{k,t} \left( \mathbb{E}_t^* \tilde{R}_{m,T}^{k+2} - \mathbb{E}_t^* \tilde{R}_{m,T}^{k} \mathbb{E}_t^* \tilde{R}_{m,T}^2 \right)$$

- Estimate $w_1, w_2, w_3, w_4$ using rolling linear regressions

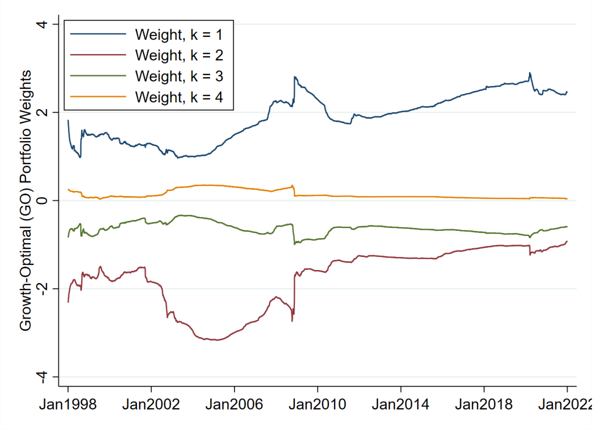
# Growth-Optimal Weights as Regression Coefficients

$$\text{VarPrem} = \underbrace{w_1}_{\text{Stock Wt}} \text{PrcSkew} + \underbrace{w_2}_{\text{Var Deriv Wt}} \text{PrcKurt} + w_3 \text{PrcMom5} + w_4 \text{PrcMom6} + \epsilon$$

- Time-series regression of variance premium on prices of moments
  - Coefficients are growth-optimal weights on stocks, variance, etc.
- Empirical regressions encounter practical issues Reg Eqn
  - Rolling: Allow for time-varying optimal weights
  - Approximation: Use only four prices of moments: $k + 2 = 3, 4, 5, 6$
  - Heteroskedasticity: Use precision weights

# Empirical Growth-Optimal Portfolio Weights

- Stock weight: positive (usually $> 1$) to earn equity premium
- Variance weight: negative to earn variance premium
- Skewness weight: negative to hedge against crashes

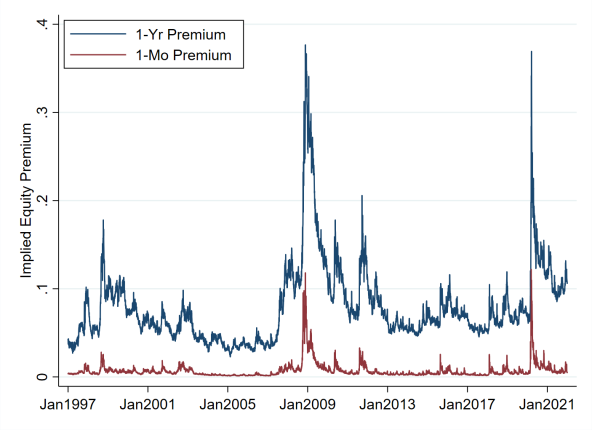


# Equilibrium Interpretation

- Rational agents hold the growth-optimal portfolio: $w_{k,t}, k = 1, 2, 3, 4$
- Rational agents must hold assets that behavioral agents won't
  - Markets clear in equilibrium
- Rational agents buy assets that behavioral agents sell
  - If behavioral agents sell stocks, $w_{1,t} \uparrow$ and equity premium$\uparrow$
  - If they increase variance hedging, $w_{2,t} \downarrow$ and equity premium$\uparrow$
  - Price impact depends on prices of variance and skewness risks

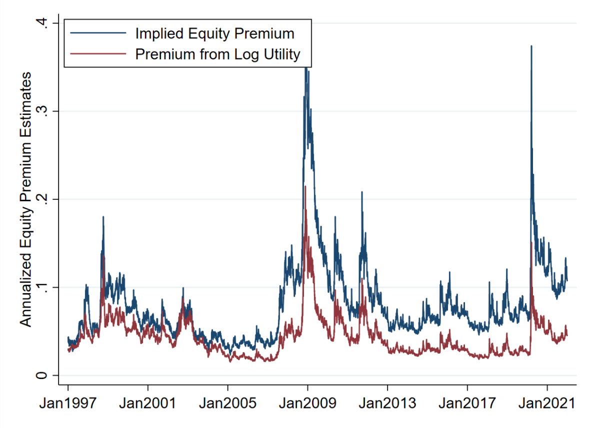
# Implied Equity Premium Over Time

- The equity premium is highly countercyclical
- It's also persistently higher after the 2008 crisis
- 0.85 correlation between monthly and yearly premiums



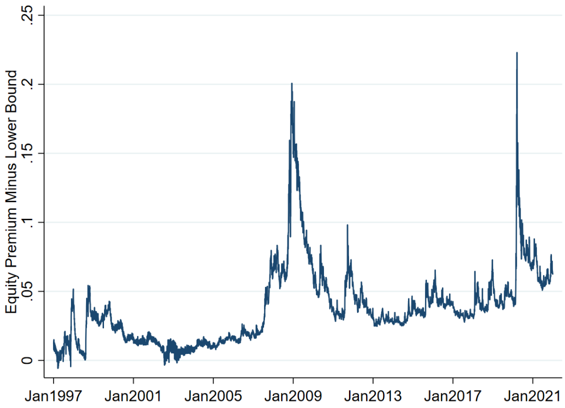
# Implied Equity Premium vs. Lower Bound on Premium

- 0.82 correlation between implied equity premium and lower bound on premium from log utility (Martin (2017))
- On 99.6% of days, implied equity premium > lower bound



# Difference between Premium and Lower Bound

- Difference ranges from 0% to 22% per year
- Difference is much higher after the 2008 crisis



# Main Empirical Results

- The implied equity premium forecasts returns well
  - Annual out-of-sample $R^2 \approx 9\%$ vs. $< 1\%$ for benchmark
  - Expected returns increase by 13% per year for log utility investor

- The model forecasts the expected variance premium very well
  - Annual out-of-sample $R^2 \approx 70\%$ vs. 5% for benchmark

# The Implied Equity Premium Predicts Market Returns

- Evaluate predictions at horizons from 30 to 360 days
- Annual out-of-sample $R^2$ values imply $\mathbb{E}R \uparrow$ by 13% per year
- Monthly $R^2$ value implies $\mathbb{E}R \uparrow$ by 14% per year

Out-of-sample $R^2$ Values

| Horizon | Benchmark Premium | Implied Premium | Obs |
|---|---|---|---|
| 30 | 0.58 | 1.12 | 303 |
| 60 | 1.00 | 2.23 | 151 |
| 90 | 0.99 | 3.04 | 100 |
| 180 | 1.81 | 8.74 | 50 |
| 360 | -1.29 | 9.15 | 24 |

# Return Prediction Regressions for 180-Day Horizon

- Regress equity premium (EP) on implied equity premium (IEP)
  - Controls: variance premium (VP) and rep log util eq prem (RLUEP)
- "No Arb" sample excludes 61 days during 2008 crisis

| | $EP_{180}$ | $EP_{180}$ | $EP_{180}$ | $EP_{180}$ | $EP_{180}$ | $EP_{180}$ |
|---|---|---|---|---|---|---|
| $IEP_{180}$ | 1.03** | 1.25 | 1.43** | 1.46*** | 1.54 | 2.00*** |
| | (0.48) | (1.43) | (0.58) | (0.42) | (1.38) | (0.48) |
| $RLUEP_{180}$ | | -0.54 | | | -0.21 | |
| | | (2.81) | | | (2.84) | |
| $VP_{180}$ | | | -1.18 | | | -1.53 |
| | | | (1.26) | | | (1.33) |
| Observations | 6,167 | 6,167 | 6,167 | 6,106 | 6,106 | 6,106 |
| Sample | All | All | All | No Arb | No Arb | No Arb |
| $R^2_{adj}$ | 0.080 | 0.081 | 0.089 | 0.122 | 0.122 | 0.136 |

# Variance Premium Predictions

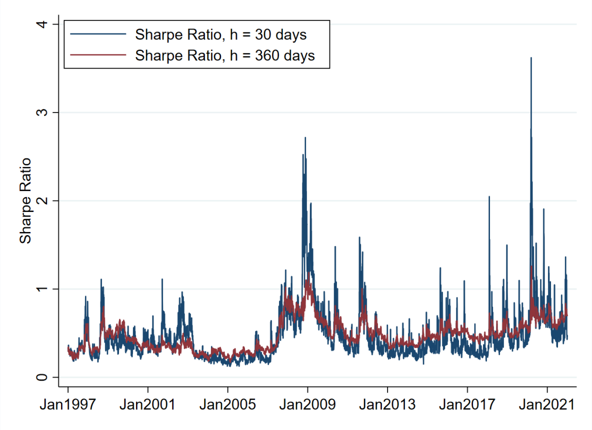
- Model with growth-optimal weights beats log utility weights
  - IVP: Implied variance premium
  - RLUVP: Rep log util variance premium

Out-of-sample $R^2$ Values

| Horizon | RLUVP | IVP | Days |
|---|---|---|---|
| 30 | 6.0 | 62.3 | 6293 |
| 60 | 6.6 | 70.0 | 6293 |
| 90 | 3.3 | 70.5 | 6293 |
| 180 | -7.4 | 70.1 | 6293 |
| 360 | -25.9 | 55.5 | 6293 |

# Sharpe Ratio Is Countercylical

- Highly countercyclical Sharpe ratio at all horizons
- Modest range from 1st to 99th percentile: 0.22 to 0.97



# A New Framework for Estimating Risk Premiums

- Estimate "implied" equity premium imposing minimal structure
  - Data: option prices and high-frequency stock returns
  - Assumptions: frictionless, complete index and index option markets and approximation of true growth-optimal portfolio
  - Estimation: accurate variance forecasts and short-run stability of growth-optimal portfolio weights

- What is not assumed?
  - Anything else about the economy—e.g., wealth, constraints, etc.
  - Anything else about investor preferences, beliefs, or heterogeneity
  - Stationarity and long-run model stability

# Concluding Remarks

- The implied risk premium framework is useful
  - Predicts stock market returns well
  - Offers new insights into the economy and markets
- Improved cost of capital estimates have myriad applications
  - Improving business and policy decisions
  - Predicting economic growth
- Ongoing work: stock price impact of behavioral biases
- Ongoing work: apply this framework to individual stocks

# Extra Slides

# The Equity Premium and Stock Prices

- Suppose a stock will soon pay a random dividend: $D$
  - Log dividend is normal with volatility $\sigma$
  - A safe bond pays a return of $r_f \geq 0$
- Investors value the stock at a price of $\mathbb{E}D/\left(1 + r_f + \gamma\sigma^2\right)$
  - Investors like higher mean, $\mathbb{E}D$, but dislike variance, $\sigma^2$
  - Risk aversion, $\gamma$, is how much they dislike variance
- The equity premium over the risk-free rate, $r_f$, is $\gamma\sigma^2$
  - Risk aversion and amount of risk determine the equity premium
  - A higher equity premium implies lower stock prices

Back to Intro

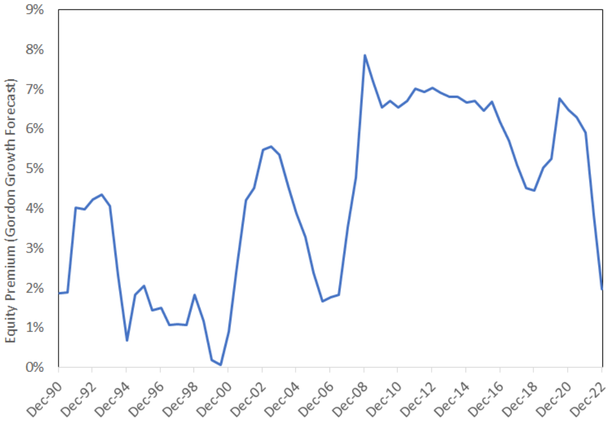
# Estimating the Equity Premium: Models

- Economists estimate the premium under strong assumptions History
  - Stylized economies: Constant dividend growth
  - Stationarity and model stability: Predictive regressions and VARs
  - Rational stock analyst beliefs: Implied cost of capital
  - Preference assumptions: Risk aversion known, fixed, or bounded
- But these models don't forecast stock returns well
  - Most models' forecasts flunk "out of sample" tests
    - Goyal and Welch (2008, 2021)
  - A few simple models outperform constant premium benchmark
    - Campbell and Thompson (2008)

Back to Surveys

# Stylized Long-Run Equity Premium Estimates

- Long-run equity premium with constant (5%) dividend growth
  - Premium = Div Growth + Div Yield − Risk-free Rate
  - Average 4.2% (Std Dev 2.2%) from 1990 to 2022 Back to Surveys



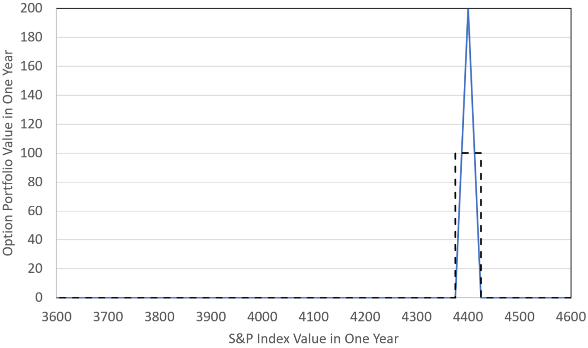
# Estimating the Equity Premium: History

- Historical average stock return is an unbiased estimate
  - Average dividend is $\mathbb{E}D$, so average premium is $\gamma\sigma^2$ ($\approx$ 5%)
  - But standard error is large because returns are very volatile ($\approx$ 18%)
- The true equity premium varies—e.g., because $\gamma$ (preferences) vary
  - So we can't tell whether and when historical returns are relevant

Back to Equity Premium Models

# Complete Market: Option Prices Reveal State Prices

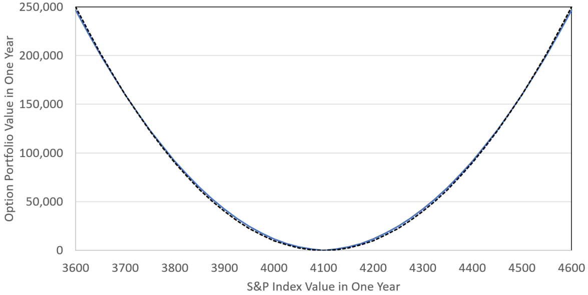
- Butterfly spread prices market payoffs in a range (e.g., 4375–4425)
  - Complete market: options with all strikes, $K$, are available
  - Spread with strikes $K^-, K, K^+$ reveals risk-neutral probability of $K$




Price of Mkt Var

# Option Portfolios Reveal Prices of Return Moments

- Option portfolios span any function of stock market returns
  - E.g., squared stock payoff ≈ sum of OTM puts and calls
  - Piecewise linear approximation is accurate with tradable options




Price of Mkt Var

# Expected Long-Term Variance

- Expected $T$-day variance comes from intraday stock return variance
- Estimated daily realized variance, $rv_t$, is the key input
  - $rv_t$ is sum of squared log SPY returns over 78 intervals
  - $rv_t$ is subsample averaged over 10 staggered sets of 78 intervals based on 790 SPY prices equally spaced in transaction time
  - Procedure follows Patton and Sheppard (2015) and predecessors
- Expected $T$-day variance is the real-time (rolling) forecast from $t+1$ to $t+T$ from an ARFIMA(0,$d$,0) model, where $0 \leq d \leq 0.5$:

$$\begin{aligned} (1-L)^d rv_t &= \epsilon_t, \\ rv_t &= (1-L)^{-d} \epsilon_t, \end{aligned}$$

Back to Variance Premium

# Central Theoretical Result

- All market-related risk premiums are weighted sums of the prices of higher-order market moments
  - Equity risk premium is a weighted sum of the prices of market variance, skewness, and more
  - Variance risk premium is a weighted sum of the prices of market skewness, kurtosis, and more

- The weights, $w_k$, are the same for all risk premiums
  - They are the **growth-optimal** portfolio allocations to stocks ($w_1$), variance ($w_2$), skewness ($w_3$), and so on ($w_4, \ldots$)
  - Representative log utility is a benchmark
    - Stock weight is $w_1$ (100%); all others are zero ($w_2, \ldots = 0$)

Back to Risk Prem Eqn

# Intuition for the Implied Equity Premium

- Frictionless, complete markets for the index and its options are key
- The growth-optimal kernel prices the index and all its options
  - So the equity and variance risk premiums exhibit analogous dependence on the prices of all market risks
- The relation between the (observable) variance premium and prices of market risks reveals weights in the growth-optimal kernel
  - Apply these weights to prices of risk to find the equity premium
  - This equity premium is **implied** by index and option prices

Back to Risk Prem Eqn