The Cross-section of Managerial Ability and Risk Preferences

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Measuring managerial ability

- Mutual fund alphas from a performance regression using style benchmarks

\[ R_{it}^A - r_f = \alpha_i + \beta_i \left( R_{t}^B - r_f \right) + \varepsilon_{it} \]
Measuring managerial ability

- Mutual fund alphas from a performance regression using style benchmarks
  \[ R_{it}^A - r_f = \alpha_i + \beta_i \left( R_t^B - r_f \right) + \epsilon_{it} \]

- Reduced-form approach ignores that fund returns are the outcome of a portfolio-choice problem
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- Reduced-form approach ignores that fund returns are the outcome of a portfolio-choice problem

- Often leads to dynamic strategies that could induce to misspecifications
New approach: Portfolio choice theory

- Consider an active portfolio manager’s problem
- Manager dynamically selects portfolio to maximize utility
New approach: Portfolio choice theory

- Consider an active portfolio manager’s problem
- Manager dynamically selects portfolio to maximize utility
- Two basic components:
  1. **Managerial ability** \((\lambda_Ai)\): shapes the investment opportunity set
  2. **Risk preferences** \((\gamma_i)\): determine which portfolio is selected along this set
Consider an active portfolio manager’s problem

Manager dynamically selects portfolio to maximize utility

Two basic components:

1. **Managerial ability** ($\lambda_{Ai}$): shapes the investment opportunity set

2. **Risk preferences** ($\gamma_i$): determine which portfolio is selected along this set

Main idea: Use restrictions from structural portfolio management models to estimate the cross-section of managerial ability and risk preferences

**Analogy**: Use household’s Euler condition to estimate preference parameters

Hansen and Singleton (1983), Vissing-Jorgensen and Attanasio (2003), and Gomes and Michaelides (2005)
Main economic questions

- Which economic restrictions follow from portfolio choice theory
- What can we learn about the dynamics of mutual fund strategies?
- Does heterogeneity matter?
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  1. Which economic restrictions follow from portfolio choice theory
  2. What can we learn about the dynamics of mutual fund strategies?
  3. Does heterogeneity matter?

- Main answers:
  1. Economic restrictions can be used to disentangle both attributes
  2. Fund alphas reflect both ability and risk preferences
  3. Second moments of fund returns contain information about the manager’s attributes
  4. Structural model captures important dynamics of fund strategies
  5. Heterogeneity matters: utility costs up to 4% per annum by ignoring heterogeneity
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1. Which economic restrictions follow from portfolio choice theory
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4. Structural model captures important dynamics of fund strategies
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Main methodological contribution:

- Develop econometric framework to enable likelihood-based inference in continuous-time, dynamic optimization models
Modeling managerial preferences

- **Model I**: preferences for assets under management
  

- Model features managerial incentives:
  
  1. Fund flows that depend on past performance
  2. Promotion/demotion risk that depends on past performance
Modeling managerial preferences

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- **Model II**: preferences for returns relative to the benchmark

  Advantage: Derive cross-equation restriction analytically
Modeling managerial preferences

**Model I**: preferences for assets under management


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1. Fund flows that depend on past performance
2. Promotion/demotion risk that depends on past performance

**Model II**: preferences for returns relative to the benchmark


**Advantage**: Derive cross-equation restriction analytically

**Unfortunately**, cross-equation restriction for fund alphas strongly rejected

**Analogy**: CRRA preferences cannot match consumption and asset pricing data → Requires a generalization of preferences

Hansen and Singleton (1983), Vissing-Jorgensen and Attanasio (2003), and Gomes and Michaelides (2005)
Modeling managerial preferences

- Model points to a desire for **underdiversification**: managers overinvest in the active portfolio.

- Generalize the manager's preferences: **quest for status** as a motive for underdiversification.

- The manager has preferences for:
  1. Assets under management
  2. Fund status: relative position in cross-sectional asset distribution
Modeling managerial preferences

- Model points to a desire for **underdiversification**: managers overinvest in the active portfolio.

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- The manager has preferences for:
  1. Assets under management
  2. Fund status: relative position in cross-sectional asset distribution

- Different curvature parameters for:
  1. Assets under management: controls passive risk taking
  2. Fund status: controls active risk taking

- Standard models nested
Conventional approach to measure ability

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Conventional approach to measure ability

- Mutual fund alphas from a performance regression using style benchmarks

\[ R_{it}^A - r_f = \alpha_i + \beta_i \left( R_B^t - r_f \right) + \epsilon_{it} \]

- Cross-sectional distribution displays heterogeneity and estimation error

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Economic restrictions and efficiency

- The impact of imposing the economic restrictions

- The variance of alphas is **three times smaller**
Managerial ability and risk aversion are highly positively correlated.
Outline

- Data
- Financial market and preferences
- Cross-equation restrictions
- Status model
- Novel econometric approach to estimate dynamic models of delegated portfolio management by maximum likelihood
- Main empirical results
- Economic costs of heterogeneity
Data

- Manager-level database based on CRSP data from 1992.1 to 2006.12
- Assign each manager-fund combination to one of nine styles reflecting size and value orientation
Manager-level database based on CRSP data from 1992.1 to 2006.12

Assign each manager-fund combination to one of nine styles reflecting size and value orientation

3,694 unique manager-benchmark combinations consisting of 3,163 different managers and 1,932 different funds
Manager-level database based on CRSP data from 1992.1 to 2006.12

Assign each manager-fund combination to one of nine styles reflecting size and value orientation

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Construct returns before fees and expenses
Manager-level database based on CRSP data from 1992.1 to 2006.12

Assign each manager-fund combination to one of nine styles reflecting size and value orientation

3,694 unique manager-benchmark combinations consisting of 3,163 different managers and 1,932 different funds

Style composition (R. = Russell)

<table>
<thead>
<tr>
<th>Mutual fund style</th>
<th>Selected benchmark</th>
<th>Fraction of observations (%)</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large/blend</td>
<td>S&amp;P 500</td>
<td>20.1</td>
<td>714</td>
</tr>
<tr>
<td>Large/value</td>
<td>R. 1000 Value</td>
<td>11.7</td>
<td>427</td>
</tr>
<tr>
<td>Large/growth</td>
<td>R. 1000 Growth</td>
<td>11.6</td>
<td>448</td>
</tr>
<tr>
<td>Mid/blend</td>
<td>R. Mid-cap</td>
<td>10.2</td>
<td>383</td>
</tr>
<tr>
<td>Mid/value</td>
<td>R. Mid-cap Value</td>
<td>6.3</td>
<td>228</td>
</tr>
<tr>
<td>Mid/growth</td>
<td>R. Mid-cap Growth</td>
<td>13.7</td>
<td>526</td>
</tr>
<tr>
<td>Small/blend</td>
<td>R. 2000</td>
<td>7.8</td>
<td>291</td>
</tr>
<tr>
<td>Small/value</td>
<td>R. 2000 Value</td>
<td>6.2</td>
<td>200</td>
</tr>
<tr>
<td>Small/growth</td>
<td>R. 2000 Growth</td>
<td>12.4</td>
<td>477</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>100.0</strong></td>
<td><strong>3,694</strong></td>
</tr>
</tbody>
</table>
The manager can trade 3 assets:
Financial market

- The manager can trade 3 assets:

  - Cash account:
    \[ dS_t^0 = S_t^0 r_f dt \]
The manager can trade 3 assets:

1. Cash account:
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2. Style benchmark portfolio:
   \[ dS_t^B = S_t^B (r_f + \sigma_B \lambda_B) dt + S_t^B \sigma_B dZ_t^B \]
The manager can trade 3 assets:

1. **Cash account:**
   \[ dS^0_t = S^0_t r_f dt \]

2. **Style benchmark portfolio:**
   \[ dS^B_t = S^B_t (r_f + \sigma_B \lambda_B) dt + S^B_t \sigma_B dZ^B_t \]

3. **Idiosyncratic technology of the manager (Active portfolio):**
   \[ dS^A_{it} = S^A_{it} (r_f + \sigma_{Ai} \lambda_{Ai}) dt + S^A_{it} \sigma_{Ai} dZ^A_{it}, \]
   where \( \lambda_{Ai} \) measures managerial ability, with \([Z^B, Z^A_i]_t = 0\)
Standard model of preferences

- Preferences for returns relative to the benchmark:

$$\max_{(x_{it})_{t \in [0, T]}} E_t \left[ \frac{1}{1 - \gamma_i} \left( \frac{R_{iT}^A}{R_{iT}^B} \right)^{1 - \gamma_i} \right]$$

- $x_{it} = (x_{it}^B, x_{it}^A)'$: fractions invested in benchmark and active portfolio

- Optimization subject to the dynamic budget constraint
Standard model of preferences

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\[
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\]

- \( x_{it} = (x_{it}^B, x_{it}^A)' \): fractions invested in benchmark and active portfolio

- Optimization subject to the dynamic budget constraint

- Optimal strategy:

\[
x_i = \frac{1}{\gamma_i} \Sigma_i^{-1} \Lambda_i + \left( 1 - \frac{1}{\gamma_i} \right) e_1,
\]

with \( \Sigma_i = \text{diag}(\sigma_P, \sigma_{Ai}) \), \( \Lambda_i = (\lambda_P, \lambda_{Ai})' \), and \( e_1 = (1, 0)' \)
Implications of the cross-equation restriction

- Asset dynamics:

\[
\frac{dA_{it}}{A_{it}} - r_f dt = \left( x_{it}^A \sigma_{Ai} \lambda_A + x_{it}^B \sigma_{Bi} \lambda_B \right) dt + x_{it}^B \sigma_{Bi} dZ_t^B + x_{it}^A \sigma_{Ai} dZ_t^A
\]
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\]

- Substitute the optimal strategy:

\[
\frac{dA_{it}}{A_{it}} - r_f dt = \frac{\lambda_i^2}{\gamma_i} \alpha_i dt + \left( \frac{\lambda_B}{\gamma_i \sigma_B} + \frac{\gamma_i - 1}{\gamma_i} \right) \left( \frac{dS^B_t}{S^B_t} - r_f dt \right) + \frac{\lambda_i}{\gamma_i} \sigma_{\epsilon_i} dZ^A_{it}
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\]

- \(\lambda_{Ai}\) and \(\gamma_i\) follow from:

\[
\beta_i = \frac{\lambda_B}{\gamma_i \sigma_B} + \frac{\gamma_i - 1}{\gamma_i}
\]

\[
\sigma_{\varepsilon_i} = \frac{\lambda_{Ai}}{\gamma_i}
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\]

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\[
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\]

\[
\sigma_{\epsilon_i} = \frac{\lambda_A}{\gamma_i}
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- The cross-equation restriction on the fund’s alpha, \(\alpha_i\):

\[
\alpha_i = \frac{\lambda_A^2}{\gamma_i} = \sigma_{\epsilon_i}^2 \left( \frac{\lambda_B / \sigma_B - 1}{\beta_i - 1} \right)
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Implications of the cross-equation restriction

- Substitute the optimal strategy:

\[
\frac{dA_{it}}{A_{it}} - r_f dt = \frac{\lambda^2 A_i}{\gamma_i} dt + \left( \frac{\lambda_B}{\gamma_i \sigma_B} + \frac{\gamma_i - 1}{\gamma_i} \right) \left( \frac{dS^B_t}{S^B_t} - r_f dt \right) + \frac{\lambda A_i}{\gamma_i} dZ_{it}^A
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- **Main conclusion**: Fund alphas

  1. Reflect ability and risk preferences
  2. Can be estimated from information in second moments

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Empirical results: Preferences for returns rel. to benchmark

<table>
<thead>
<tr>
<th>S&amp;P 500</th>
<th>Model-implied</th>
<th>Performance regr.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_i$</td>
<td>$\lambda_{A_i}$</td>
</tr>
<tr>
<td>Mean</td>
<td>46.08</td>
<td>1.36</td>
</tr>
<tr>
<td>St.dev.</td>
<td>108.15</td>
<td>0.34</td>
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$$\beta_i = \frac{\lambda_B}{\gamma_i \sigma_B} + \left(1 - \frac{1}{\gamma_i}\right)$$

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<th>Mean</th>
<th>46.08</th>
<th>1.36</th>
<th>6.27%</th>
<th>1.10</th>
<th>4.48%</th>
<th>0.82%</th>
<th>0.96</th>
<th>4.10%</th>
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</thead>
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<tr>
<td></td>
<td>St.dev.</td>
<td>108.15</td>
<td>0.34</td>
<td>3.51%</td>
<td>0.05</td>
<td>2.02%</td>
<td>2.98%</td>
<td>0.11</td>
<td>1.97%</td>
</tr>
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\[ \beta_i = \frac{\lambda_B}{\gamma_i \sigma_B} + \left(1 - \frac{1}{\gamma_i}\right) \]

\[ \sigma_{\epsilon_i} = \frac{\lambda_{Ai}}{\gamma_i} \]

\[ \alpha_i = \frac{\lambda_{Ai}^2}{\gamma_i} \]

- It requires **underdiversification** to match the moments of fund returns
Managerial preferences: The status model

- Quest for status as a motive for underdiversification
Managerial preferences: The status model

- **Quest for status** as a motive for underdiversification

- Motivation status concerns
  - Hard-wired: Larger funds more visible, higher in ratings, ...  
  - Evolutionary forces  
  - Strategic interaction among fund managers

- Large literature in economics argues that status concerns are important for financial decision making
Managerial preferences: The status model

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- Large literature in economics argues that status concerns are important for financial decision making

- Modeling fund status:
  - Total mass of managers normalized to unity, with measure $\mu(\cdot)$
  - Status measured by the percentile rank:
    \[
    \varrho_t(a) = \mu \left( i \left| \frac{A_{it}}{\bar{A}_t} \leq a \right. \right),
    \]
    where $\bar{A}_T$ is median fund size
Managerial preferences: The status model

- Manager’s objective:

  $$\max_{(x_{it})_{t\in[0,T]}} E_0 \left[ \eta \frac{A^{1-\gamma_1}_{iT}}{1-\gamma_1} + (1-\eta) S (1-\gamma_2 i) \bar{A}^{1-\gamma_1}_T q_T \left( \frac{A_{iT}}{\bar{A}_T} \right)^{1-\gamma_2} \right],$$

  where:
  - $q_T(\cdot)$: maps relative fund size to fund status
  - $S(\cdot)$: sign function
  - Restrictions: $\eta \in [0, 1]$, $\gamma_1 i > 1$, and $q'_T(\cdot) \geq 0$
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where:

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- $S(\cdot)$: sign function
- Restrictions: $\eta \in [0, 1]$, $\gamma_1i > 1$, and $q_T'(\cdot) \geq 0$

Comments:

- $\gamma_2i$ can be negative
- CDF captures the opportunities to improve status
- Nests standard model of preferences
Fund status and risk taking

For most funds, risk aversion and fund size are positively correlated.

- $\gamma_{1i}$ controls passive risk taking, $\gamma_{2i}$ active risk taking.
Estimation strategy

- Define $r_{t+h}^B = \log S_{t+h}^B - \log S_t^B$ and $r^T = \{r_h, \ldots, r_T\}$
Estimation strategy

- Define \( r^B_{t+h} = \log S^B_{t+h} - \log S^B_t \) and \( r^T = \{r_h, \ldots, r_T\} \)

- Two-step maximum-likelihood estimation procedure:
  1. Estimate \( \Theta_B = \{\lambda_B, \sigma_B\} \) using \( \mathcal{L}(r^{BT}; \Theta_B) \)
  2. Estimate \( \Theta_{Ai} = \{\lambda_{Ai}, \gamma_{1i}, \gamma_{2i}\} \) using \( \mathcal{L}(A^T_i \mid r^{BT}, A_{i0}; \Theta_{Ai}, \hat{\Theta}_B) \)
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- Main complication: computing $\mathcal{L}(A_i^T | r^{BT}, A_{i0}; \Theta_A, \hat{\Theta}_B)$
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- Main complication: computing $\mathcal{L}(A_i^T \mid r^{BT}, A_{i0}; \Theta_A, \hat{\Theta}_B)$

- Density of $A_{t+h}$ given $A_t$ unknown:
  
  $$
  dA_t = A_t \left( r + x_t^*(A_t)' \Sigma \Lambda \right) dt + A_t x_t^*(A_t)' \Sigma dZ_t
  $$
Using the martingale approach in estimation

- I develop a new approach based on martingale techniques of Cox Huang (1989)
Using the martingale approach in estimation

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- Main steps of the martingale method:
Using the martingale approach in estimation

- I develop a new approach based on martingale techniques of Cox Huang (1989)

- Main steps of the martingale method:
  
  1. **Choose optimal year-end asset level** \((A_T^*)\) that solves:

  \[
  \max_{A_T \geq 0} E_0 [u (A_T)] \\
  \text{s.t. } E_0 [\psi T A_T] \leq A_0
  \]

  Solution: \(A_T^* = (u')^{-1} (\xi \psi_T)\)
Using the martingale approach in estimation

I develop a new approach based on martingale techniques of Cox Huang (1989)

Main steps of the martingale method:

1. Choose optimal year-end asset level \((A^*_T)\) that solves:

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\max_{A_T \geq 0} E_0 \left[ u (A_T) \right] \\
\text{s.t. } E_0 \left[ \varphi_T A_T \right] \leq A_0
\]

Solution: \(A^*_T = (u')^{-1} (\xi \varphi_T)\)

2. By no-arbitrage, time-\(t\) assets under management \((A^*_t)\):

\[
A^*_t = E_t \left[ (u')^{-1} (\xi \varphi_T) \frac{\varphi_T}{\varphi_t} \right] = f(\varphi_t),
\]

with \(f(\cdot)\) invertible under mild conditions
Using the martingale approach in estimation

1. I develop a new approach based on martingale techniques of Cox Huang (1989)

2. Main steps of the martingale method:
   - Choose optimal year-end asset level \( A_T^* \) that solves:
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     \]
     \[
     \text{s.t. } E_0 [\phi_T A_T] \leq A_0
     \]

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     A_t^* = E_t \left[ (u')^{-1} (\xi \phi_T) \frac{\phi_T}{\phi_t} \right] = f(\varphi_t),
     \]
     with \( f(\cdot) \) invertible under mild conditions

3. Key insight: transition density \( (\varphi_t) \) known exactly
Novel econometric approach using martingale techniques

- Estimation procedure:
  1. Map assets under management ($A^T$) to the state-price density ($\varphi^T$)
  2. Change-of-variables (Jacobian) formula for random variables

\[
\ell\left(A_t \mid r_t^B, \varphi_{t-h}; \Theta_A, \Theta_B\right) = \ell\left(\varphi_t \mid r_t^B, \varphi_{t-h}; \Theta_A, \Theta_B\right) + \log \left| \left( \frac{\partial A^*_t}{\partial \varphi_t} \right)^{-1} \right|
\]

- Exact likelihood up to one expectation computed using Gaussian quadrature

- If $u(\cdot)$ is locally convex, apply concavification techniques

- Enables likelihood-based estimation of a large class of dynamic models
Summary statistics ability and risk aversion

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$RRA$</th>
<th>$\lambda_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>4.05</td>
<td>9.50</td>
<td>5.16</td>
<td>0.28</td>
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<tr>
<td><strong>St.dev.</strong></td>
<td>2.41</td>
<td>24.57</td>
<td>7.69</td>
<td>0.38</td>
</tr>
<tr>
<td><strong>Coeff. of variation</strong></td>
<td>0.60</td>
<td>2.59</td>
<td>1.49</td>
<td>1.36</td>
</tr>
</tbody>
</table>

If anything, dispersion in risk aversion higher than in ability
Reduced-form $\alpha$ estimates are very noisy

- Compare implied estimates from structural model to reduced-form performance regression:

$$\hat{\beta}_i^{\text{Reduced-form}} = -0.00 + 1.00 \hat{\beta}_i^{\text{Structural}} + u_i, \quad R^2 = 97.67\% \quad (1)$$

$$\hat{\sigma}_{\varepsilon,i}^{\text{Reduced-form}} = -0.00 + 1.04 \hat{\sigma}_{\varepsilon,i}^{\text{Structural}} + u_i, \quad R^2 = 98.69\% \quad (2)$$

$$\hat{\alpha}_i^{\text{Reduced-form}} = -0.00 + 0.99 \hat{\alpha}_i^{\text{Structural}} + u_i, \quad R^2 = 35.11\% \quad (3)$$

- To match the unconditional moments: intercept equals zero and slope equals one

- Low R-squared in (3) reflects estimation error in reduced-form $\alpha$ estimates

- Variance in fund alphas three times smaller
Model specification test

- Specification test:
  - $H_0$: Performance regression with the same distributional assumptions
  
  $\frac{dA_{it}}{A_{it}} - r_f dt = \alpha_i dt + \beta_i \left( \frac{dS_t^B}{S_t^B} - r_f dt \right) + \sigma_{\epsilon i} dZ_t^A$

  - $H_1$: Status model

- Likelihood ratio test for (non-)nested models to test hypotheses
  Vuong (1989)

- Perform test at manager's level; reject if rejection rate exceeds 5%

- **Rejection rate:** 10.3%

- Status model captures **important dynamics** of fund strategies
Forecasting ability

- Cross-sectional stability (rank correlation):
  - Risk aversion: 65.0%
  - Ability: 32.9%
Forecasting ability

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- Time-series predictability: Two ways to estimate ability over a 3-year period
  1. Appraisal ratio using a performance regression
  2. Structural estimation using the status model

- Estimate appraisal ratio over the consecutive year (works against the structural model)

- Compute the RMSE: $\sqrt{E \left[ \left( \lambda_{i,t+1}^{A} - \hat{\lambda}_{it}^{A} \right)^2 \right]}$
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- Using performance regression: RMSE = 0.6628
- Using status model: RMSE = 0.3881
Are managers really skilled?

- Fraction of alphas that recovers their expense ratio:
  - Reduced-form approach: 46%
  - Structural: 31%

- Fraction of alphas that *significantly* exceed their expense ratio:
  - Reduced-form approach: 9%
  - Structural: 13%

- Structural approach leads to a more positive view on managerial talent
Why are ability and risk aversion positively correlated?

- Managerial ability and risk aversion are highly positively correlated

- This is consistent with selection effects or reflects career concerns

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Why are ability and risk aversion positively correlated?

- Choose between mutual fund industry and savings bank
- The bank provides a known and constant income $O_T$ at $t = T$
- Value function mutual fund industry
  \[ J^{MF} = \frac{1}{1 - \gamma} \exp \left( (1 - \gamma)r + \frac{1 - \gamma}{2\gamma} \left( \lambda_A^2 + \lambda_B^2 \right) \right) \]
- Value function bank
  \[ J^{OO} = \frac{1}{1 - \gamma} O_T^{1 - \gamma} \]
- The indifference locus reads
  \[ \bar{\lambda}_A (\gamma) = \sqrt{(\log O_T - r)2\gamma - \lambda_B^2} \]
- Fund managers will opt into the industry only if $\lambda_A \geq \bar{\lambda}_A (\gamma)$
Heterogeneity in ability and risk aversion

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Ability (log((\lambda_A)))</th>
<th>Risk aversion (log((RRA)))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>T-statistic</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-8.87%</td>
<td>-2.55</td>
</tr>
<tr>
<td>Tenure</td>
<td>7.27%</td>
<td>2.19</td>
</tr>
<tr>
<td>Turnover</td>
<td>6.36%</td>
<td>2.01</td>
</tr>
<tr>
<td>Log(Expenses)</td>
<td>5.04%</td>
<td>1.16</td>
</tr>
<tr>
<td>Stock holdings</td>
<td>-6.37%</td>
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</tr>
<tr>
<td>Loads</td>
<td>-3.41%</td>
<td>-1.00</td>
</tr>
<tr>
<td>12B-1 fees</td>
<td>0.04%</td>
<td>0.01</td>
</tr>
<tr>
<td>Log(Family TNA)</td>
<td>0.10%</td>
<td>0.03</td>
</tr>
<tr>
<td>Fund age</td>
<td>3.53%</td>
<td>1.10</td>
</tr>
<tr>
<td>R-squared</td>
<td>13.0%</td>
<td>6.6%</td>
</tr>
</tbody>
</table>

- Managers of large funds tend to be less skilled, but more aggressive
- Skilled managers are more experienced and have higher turnover
- Aggressive managers charge higher expense ratios and hold less cash
- Substantial unobserved heterogeneity
Large/value managers are on average more conservative than small/growth managers.

Larger fraction of small/growth managers is skilled.
Does heterogeneity matter?

- Investor allocates capital to cash, benchmark, and actively-managed funds
- Three ways to account for heterogeneity:
  1. Use performance regressions to estimate cross-sectional distribution
  2. Ignore heterogeneity: use average values
  3. Use status model to estimate cross-sectional distribution

![Graph showing utility costs (bp) against coefficient of relative risk aversion of the individual investor. The graph compares utility costs when ignoring heterogeneity and when using performance regressions. The graph has a blue line for ignoring heterogeneity and a red line for using performance regressions.](image)
Variation in risk aversion and expected returns

- The status model endogenously generates time variation in risk aversion

- Time series of expected returns from Binsbergen and Koijen (2007)

- The correlation is 62%

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