The Cross-section of Managerial Ability and Risk Preferences

Ralph S.J. Koijen

University of Chicago GSB

July 8, 2008

Abstract

I use structural portfolio management models to study the joint cross-sectional distribution of managerial ability and risk preferences using manager-level data. The economic restrictions following from theory imply that (i) fund alphas reflect the manager’s ability and risk preferences and that (ii) information in second moments of fund returns can be used to estimate both attributes. The estimation relies on a novel framework to empirically analyze dynamic portfolio-choice models. The findings are twofold. First, the restrictions implied by recently-proposed models of managerial preferences are strongly rejected in the data. Second, introducing relative-size concerns into the manager’s objective delivers plausible estimates and is formally favored over the standard models and reduced-form performance regressions. Based on this model, I document large and skewed heterogeneity in risk preferences, but less dispersion in ability. Risk aversion and managerial ability are highly positively correlated. Finally, ignoring heterogeneity can lead to large welfare losses for an individual investor who allocates capital to actively-managed mutual funds.
An investor’s decision to allocate capital to actively-managed funds relies on the premise that mutual fund managers are endowed with skills that enable them to outperform a passive investment strategy. This premise has spurred a large literature aimed at measuring managerial ability and at characterizing the cross-sectional distribution of managerial talent.\(^1\) The recent view contends that there is a small fraction of managers who are able to significantly recuperate fees and expenses.\(^2\) However, ever since Jensen (1968), the typical approach is to rely on performance regressions to measure skill. In such performance regressions, mutual fund returns in excess of the short rate are regressed on a constant and a set of excess benchmarks returns. The intercept of the performance regression, the manager’s alpha, is then taken as a measure of ability.\(^3\) This reduced-form approach ignores that fund returns are the outcome of a portfolio management problem. In structural portfolio management models, the manager’s ability shapes her investment opportunity set, while the manager’s preferences determine which portfolio is chosen along this investment opportunity set. The fact that a manager chooses along the investment opportunity set that depends on her ability points to two important dimensions of heterogeneity: managerial ability and risk preferences. It turns out that the standard alpha reflects both managerial ability, as defined by the price of risk on the active portfolio,\(^4\) and risk preferences. I show that the restrictions implied by structural portfolio management models can be used to disentangle both attributes. As such, this paper is the first to impose the economic restrictions following from theory to recover the joint cross-sectional distribution of managerial ability and risk preferences.

The controversy surrounding the existence of managerial ability is largely the result of inefficient inference.\(^5\) It is well known that averaging (risk-adjusted) returns over short time spans leads to noisy estimates (Merton (1980)). Hence, the estimated cross-sectional distribution of managerial ability reflects not only true heterogeneity, but also, and perhaps predominantly, estimation error. The restrictions implied by structural portfolio management models lead to much sharper estimates of managerial ability and risk aversion because they exploit information in the volatility of fund returns and in the covariance of fund returns with benchmark returns. By combining these estimates, I recover the cross-

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\(^1\)Jensen (1968), Gruber (1996), and Carhart (1997).
\(^3\)Alternatively, risk adjustments are performed by comparing fund returns to portfolios with matched characteristics (Daniel, Grinblatt, Titman, and Wermers (1997)). Also in this case, risk-adjusted returns are averaged to gauge the manager’s skill.
\(^4\)DeTemple, Garcia, and Rindisbacher (2003) and Munk and Sorensen (2004) show that the manager’s investment opportunity set can be summarized by the instantaneous short rate and prices of risk in a continuous-time economy. Nielsen and Vassalou (2004) show that if an investor has to select one fund, then she will prefer the one that has the highest price of risk. This makes the price of risk on the active portfolio a natural measure of ability motivated by portfolio-choice theory.
\(^5\)Pastor and Stambaugh (2002b), Lynch and Wachter (2007a), and Lynch and Wachter (2007b) propose to use longer samples of benchmark returns to sharpen the estimates. Managerial ability is still estimated by averaging risk-adjusted returns over short periods.
sectional distribution of the standard performance measure, alpha. Figure 1 displays the cross-sectional distribution of fund alphas following from performance regressions (top panels) and structural estimation (bottom panels), both before (left panels) and after fees and expenses (right panels).

The distribution following from the structural model displays considerably less dispersion; its variance is three times smaller than in the case of standard performance regressions. For 13% of the managers, I find that their alpha is significantly higher than fees and expenses at the 5% significance level while this number drops to 9% if I would rely on standard performance regressions.

As a point of departure, I analyze two existing models of managerial preferences. In the first model, the manager derives utility from mutual fund returns in excess of a benchmark. The manager's ability and her risk preferences can be recovered from the fund’s beta (passive risk) and the amount of residual risk (active risk), and together imply its alpha. The resulting estimates of ability are precisely measured, but implausibly high. The implied distribution for alpha ranges from 6% to 12% on an annual basis. In the second model, the manager derives utility from assets under management, which is motivated by her compensation scheme (Elton, Gruber, and Blake (2003)). Assets under management fluctuate because of internal growth (mutual fund returns) and external growth (performance-sensitive fund flows). In addition, stellar (below-par) performance may trigger promotion (demotion) to a larger (smaller) fund. It turns out that such incentives have little impact on the manager’s optimal strategies in the relevant range of risk aversion. The manager therefore acts as if she cares only about internal growth. The resulting model has its own problems: the manager’s risk aversion is mechanically centered around the Sharpe ratio of the benchmark return divided by its volatility. This has the undesirable consequence that the risk aversion of a given manager who controls multiple funds in different styles changes in a predictable manner across styles. As such, the resulting estimates no longer reflect risk aversion. The first important finding of this paper is therefore that the cross-equation restrictions implied by standard models of delegated management lead to economically implausible estimates of either managerial ability or risk aversion.

I propose an alternative model of managerial preferences, which imputes a concern for the relative position in the cross-sectional asset distribution into the preferences of

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6This model has been studied in Roll (1992), Becker, Ferson, Myers, and Schill (1999), Chen and Pennacchi (2007), and Binsbergen, Brandt, and Koijen (2007).

the manager. I call this position the “fund’s status.” Managers are concerned about
the amount of assets they have under management relative to their peers, and derive
additional utility when they control larger funds. The status model nests the two standard
models. I allow for different curvature parameters over assets under management and over
fund status. The former preference parameter can be interpreted as risk aversion over
passive risk while the latter measures risk aversion over active risk. The idea of different
risk aversion parameters resonates with the views of Bob Litterman:

Some authors observing this perplexing use of active risk have concluded that
investors actually have two separate risk aversions, one with respect to market
risk, and another, much higher risk aversion, to active risk. . . . Alternatively, one
might explain the active risk puzzle to be the result of an aversion on the part of
fund managers to taking career (sometimes called “peer”) risk that would arise if
they were to create the opportunity for too much underperformance relative to their
strategic benchmark. In this case, two different risk aversions make sense because
they apply to very different risks faced by different individuals.8

The two curvature parameters and the fund’s status together imply an estimate for the
manager’s coefficient of relative risk aversion. Unlike the vast majority of models of
delegated portfolio management, the status model is not homogenous in assets under
management. It therefore predicts different risk-taking behavior for small and large
funds. The two key consequences are that larger funds take on less active risk and that
fund alphas are negatively related to fund size. Both are stylized facts documented in
the empirical mutual fund literature, and neither can be explained by existing models.9
This lends further credibility to the utility specification. The status model also makes
contact with models where households derive utility from their position in the wealth or
consumption distribution.10 If status concerns matter at all, the mutual fund industry
is likely to be the environment where such concerns are the most powerful and therefore
easiest to identify.11

The status model leads to a plausible cross-sectional distribution of managerial ability
and risk aversion as summarized in Figure 2. The horizontal axis displays the implied
coefficient of relative risk aversion, and the vertical axis shows the price of risk on the
active portfolio, which is my measure of ability. The median coefficient of relative risk
aversion equals 2.51, its mean 5.16, and its standard deviation 7.69. The median price of

8 “The Active Risk Puzzle: Implications for the Asset Management Industry,” Goldman Sachs Asset
9 Chevalier and Ellison (1999b) document the relation between fund size and risk-taking and Chen,
10 See Robson (1992), Zou (1994), Zou (1995), Bakshi and Chen (1996), Carroll (2000), Becker, Murphy,
and Werning (2005), and Roussanov (2007). These models are referred to as “spirit-of-capitalism” models.
It also relates to the external habit model of Campbell and Cochrane (1999) and Shore and White (2006)
and to the matching models with status concerns of Cole, Mailath, and Postlewaite (2001). Goel and
Thakor (2005) study the implications of status concerns for corporate investment decisions.
11 One hypothesis is that status concerns are hard-wired in the manager’s preferences. Alternatively,
rank concerns may arise due to strategic interaction among fund managers (Basak and Makarov (2007)).
Sirri and Tufano (1998) discuss the importance of mutual fund rankings for fund flows.
risk on the active portfolio (to be read as a Sharpe ratio) equals .14, the mean is .28, and the standard deviation .38. Both distributions are right-skewed. In addition, managerial ability and risk aversion are highly positively correlated; their unconditional correlation is about 80%. Skilled managers are likely to be conservative. I show that this correlation is consistent with selection effects that arise when managers have a riskless outside option. Less talented managers only opt into the actively-managed mutual fund industry if they are sufficiently aggressive.

[Figure 2 about here.]

There are interesting differences in the joint distribution across the various investment styles. Given the attention asset pricing devotes to market capitalization and book-to-market ratios, it is interesting to compare the large/value and small/growth styles. The left panel of Figure 3 displays the estimated cross-sectional density of the coefficient of relative risk aversion for both investment styles. The median growth manager is more aggressive (median risk aversion equals 1.49) than the median value manager (median equals 3.95). The density of growth managers has much fatter tails: a substantial share of growth managers display considerably higher risk aversion than value managers. For the same groups of managers, the right panel of Figure 3 depicts the cross-sectional density of managerial ability. The average growth manager is more skilled, and this ordering prevails for higher-ranked managers in the tails. More generally, I analyze how cross-sectional variation in risk aversion and ability relates to observable characteristics. I find that ability is negatively related to fund size\textsuperscript{12} and stock holdings, and positively related to the manager’s tenure and asset turnover. Risk aversion is negatively related to fund size, expenses,\textsuperscript{13} and stock holdings. In both cases, observables only account for a limited fraction of cross-sectional variation, leaving considerable unobserved heterogeneity.

[Figure 3 about here.]

The estimates of managerial ability and risk aversion follow from joint assumptions about the financial market and the manager’s preferences. I formally show that the status model significantly improves upon the two standard models of delegated portfolio management. Perhaps more importantly, the status model is favored over performance regressions. This implies that the conditional distribution of the status model provides a better description of fund returns than performance regressions for which the conditional and unconditional distribution coincide. Therefore, the status model is able to capture important dynamics of mutual fund strategies that performance regressions cannot.

\textsuperscript{12}Chen, Hong, Huang, and Kubik (2004) document this negative relation for fund alphas, which are a non-linear function of preferences and ability. I show that even after correcting for heterogeneity in preferences, managerial ability relates negatively to fund size.

\textsuperscript{13}The relation between expenses and both managerial ability and risk aversion is consistent with the equilibrium model of Berk and Green (2004) because the implied fund alphas relate positively to ability and negatively to risk aversion.
The average coefficient of relative risk aversion across managers varies over time due to variation in the amount of assets under management and variation in the cross-sectional asset distribution. Given the link that exists between risk aversion and the equity risk premium in equilibrium asset pricing models (Campbell and Cochrane (1999)), it is interesting to relate both time series. I measure the equity risk premium using the apparatus developed in Binsbergen and Koijen (2007). The time-series variation in risk aversion that I estimate from the universe of mutual fund managers tracks the equity risk premium; their correlation is 62% (see Figure 4).

[Figure 4 about here.]

In conclusion, the second important finding of this paper is that introducing relative-size concerns into the manager’s objective delivers plausible estimates of managerial ability and risk aversion, and is formally favored over the standard models and over reduced-form performance regressions.

I quantify the economic importance of heterogeneity by solving for the optimal allocation to the style benchmark and actively-managed mutual funds. An investor who ignores heterogeneity incurs a loss in certainty-equivalent wealth of up to 4% per annum, depending on the investor’s risk preferences. An investor who relies on reduced-form performance regressions to quantify heterogeneity still experiences a loss that can exceed 1% per annum. This underscores the economic importance of accounting for heterogeneity in ability and risk preferences using efficient inference methods.

I develop a novel econometric approach to bring the models to the data. The finance literature typically restricts attention to infinite-horizon models, but they are inappropriate for the problem at hand. Because the optimal policies in dynamic finite-horizon models are often unknown in closed-form, one has to rely on numerical dynamic programming. Estimating structural parameters in combination with a finite-horizon dynamic programming method is computationally (nearly) infeasible. The problem gets worse with multiple assets. The main technical contribution of this paper is to develop an estimation method that relies on the martingale method for continuous-time models in complete markets (Cox and Huang (1989)). The estimation method provides a powerful tool to formulate the likelihood and enables me for the first time to estimate dynamic finite-horizon models. One additional advantage is that the computational burden is independent of the number of assets. Koijen (2007) explains the method in a simple model and illustrates its accuracy. The method may well prove useful to estimate (i) the cross-sectional distribution of managerial ability and risk preferences in the hedge fund industry (Panageas and Westerfield (2007)), (ii) dynamic games (Basak and Makarov (2007)), and (iii) corporate finance models that can be solved by martingale methods. Since the

14Admati and Pfleiderer (1997), Vayanos (2003), and Binsbergen, Brandt, and Koijen (2007) consider the case in which only the risk preferences of the manager are unknown.
method is likelihood-based, it can be used with both classical or Bayesian estimation procedures.\footnote{Baks, Metrick, and Wachter (2001), Pastor and Stambaugh (2002a), and Avramov and Wermers (2006) use Bayesian methods to derive the optimal allocation to actively-managed funds. The approach developed in this paper can extend these studies in at least two directions. First, the investor forms prior beliefs over the coefficients of a performance regression, ignoring the cross-equation restrictions. The economic restrictions can discipline the set of viable priors. Second, the investor learns about ability from first moments, which is inefficient. By taking a structural approach, the investor can learn more efficiently. This increases the discrepancy between the predictive density and the investor’s prior views.} Finally, to estimate the models, I construct a manager-level database that covers the period 1992.1 to 2006.12.\footnote{Other studies that construct a manager-level database are Baks (2006), Evans (2007), and Kacperczyk and Seru (2007).} Manager-fund combinations are allocated to one of nine investment styles that differ by size and book-to-market orientation.

While the finance literature has been focusing primarily on quantifying managerial ability, recovering risk preferences is an important question in the economics literature. Most estimates follow from game shows (Gertner (1993), Metrick (1995), and Assem, Baltussen, Post, and Thaler (2007)), horse races (Jullien and Salanié (2000)), car insurance markets (Cohen and Einav (2007)), labor supply decisions (Chetty (2006)), hypothetical income gambles (Kimball, Sahm, and Shapiro (2007)), or experiments.\footnote{Andersen, Harrison, Lau, and Rutstrom (2005) discuss the applicability of results obtained from experiments to real-life settings.} Most closely related is the consumption-based asset pricing literature, which uses the household’s Euler condition to estimate preference parameters. I propose to use the first-order conditions of fund managers to estimate ability and risk preferences. The mutual fund industry provides a great laboratory wherein to recover risk preferences for at least two important reasons.\footnote{Becker, Ferson, Myers, and Schill (1999) estimate a structural model of delegated management, but they impose the restrictions only on the benchmark allocation.} First, fund managers routinely take decisions under uncertainty. This implies that I observe a series of decisions by the same manager to estimate risk preferences. Second, the decisions made by the manager involve non-trivial sums of money and have non-trivial implications for the manager’s career. In addition, as argued by Cohen and Einav (2007), it is important to estimate risk preferences in the environment in which they will be applied. Estimates of risk aversion are of considerable practical relevance in the context of an investor’s decision to allocate money to actively-managed funds and in the context of mutual fund valuation.\footnote{Boudoukh, Richardson, Stanton, and Whitelaw (2004), Huberman (2007), and Dangl, Wu, and Zechn (2007) develop models of mutual fund valuation. These models can be extended with the preference specifications in this paper to study how heterogeneity in managerial ability and risk preferences impact fund valuation.}

The paper proceeds as follows. Section 1 describes the data. I provide details on the financial market model in Section 2. Section 3 introduces two standard models of delegated portfolio management and derives the cross-equation restrictions that are implied by theory. Next, Section 4 discusses the econometric approach and Section 5 provides the empirical results for the benchmark models. Section 6 introduces the status
model and Section 7 contains the empirical results for this model. I study the economic importance of heterogeneity for asset allocation decisions in Section 8. Finally, Section 9 concludes.

1 Data

Data sources I combine data from three sources. First, monthly mutual-fund returns come from the Center for Research in Securities Prices (CRSP) Survivor Bias Free Mutual Fund Database. The CRSP database is organized by fund rather than by manager, but contains manager’s names starting in 1992. I use the identity of the manager to construct a manager-level database. The sample consists of monthly data over the period from January 1992 to December 2006. Data on the Fama and French size (SMB) and book-to-market (HML) portfolios, Carhart (1997)’s momentum factor, and the short-rate data also come from CRSP. Second, manager-fund combinations are allocated to investment styles. I consider different approaches for robustness (discussed below). In one approach, I use the benchmark mapping from Cremers and Petajisto (2007).\textsuperscript{20} Third, benchmark returns are obtained from Datastream.

Sample selection I apply several screens to the mutual-fund data to obtain a sample of active domestic-equity portfolio managers. I first classify the funds by the investment objectives “Small company growth,” “Other aggressive growth,” “Growth,” “Growth and income,” and “Maximum capital gains” using the Wiesenberger, ICDI, or Strategic Insight Codes (Pastor and Stambaugh (2002b)). All funds that cannot be classified are omitted from the sample.\textsuperscript{21} I drop funds that have an average total equity position (common plus preferred stock) smaller than 80% in order to focus on all-equity funds. I also drop fund years for which the total net assets are smaller than $10 Million. I omit observations for which the manager’s name is missing and the years for which no information on returns or total net assets is available. I only include fund years for which the fund is “Active” in the nomenclature of Cremers and Petajisto (2007).

Several screens are specific to the manager-level database. First, manager names in the CRSP database can take three forms: a manager/management team is (i) fully identified, (ii) partly identified, or (iii) fully anonymous. For the partly identified or anonymous

\textsuperscript{20}I am grateful to Martijn Cremers and Antti Petajisto for sharing the data on the benchmark mapping. If $\omega_{it}$ denotes the weight at time $t$ in stock $i$, and $\omega_{it}^b$ the corresponding weight in benchmark $b$, then the active share relative to benchmark $b$ is defined as: $AS_t(b) = \frac{1}{2} \sum_{i=1}^{N} |\omega_{it} - \omega_{it}^b|$. In addition, they define a fund to be active when the active share exceeds 30% and when the name does not contain “Index” or “Idx.”

\textsuperscript{21}This selection excludes international funds, bond funds, money market funds, sector funds, and funds that do not hold the majority of their securities in US equity.
management teams, I consider separately each team that manages a different fund. This presumably overstates the number of anonymous management teams in the mutual fund industry, but there is no alternative way to match such (partly) unidentified teams. I focus for most of the analysis on funds for which the manager/management team is fully identified. Managers are matched on the basis of their names. Names in the CRSP database are, however, often misspelled and abbreviated in different ways. I first use a computer algorithm that detects commonly made errors. I then manually check all funds carefully and code them consistently. Further, the manager’s starting date in the CRSP database is subject to substantial measurement error (Baks (2006)). I remove a fund from a manager’s career profile when the starting date contains inconsistencies (Baks (2006) and Kacperczyk and Seru (2007)).

US mutual funds typically have multiple share classes associated with different fee structures. Consistent with the literature, I merge different share classes: I construct value-weighted returns, loads, expense ratios, and 12B-1 fees, fraction in stocks, and cash using the total net assets of the different share classes to construct the weights. I select the other variables from the share class that has the highest total net assets (Cremers and Petajisto (2007)). Finally, several managers manage multiple funds at the same time. For funds that are compared to the same benchmark, I merge the return using the fund’s total net assets to weigh them. When funds operate in different styles, I keep them as separate observations.

**Mutual fund costs** CRSP mutual fund returns are net fees and expenses, but computed before back- and front-end loads. To focus on true managerial ability, I compute gross returns by adding back expense ratios in line with Wermers, Yao, and Zhao (2007). I sum the annual expense ratio divided by 12 to each monthly return in a particular year.

**Benchmark selection** The prime motivation for benchmarking is to disentangle managerial skill and effort from the reward of following passive strategies. Benchmark selection is notoriously difficult, regardless of whether one relies on regression techniques, matched characteristics, or self-reported benchmarks. I employ two procedures to identify the benchmark for each manager-fund combination; one is regression-based and the other is holdings-based. In the first approach, I regress mutual fund returns on

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22Massa, Reuter, and Zitzewitz (2007) study the role of anonymous teams in delegated asset management.

23Nanda, Wang, and Zheng (2007) study the share-class structure of mutual funds.

2412B-1 fees cover expenses related to selling and marketing shares, see Barber, Odean, and Zheng (2005).


benchmark returns, both in excess of the short rate, and select the benchmark that maximizes the R-squared. Alternatively, I use the method of Cremers and Petajisto (2007), which selects the benchmark that minimizes the active share of the fund. This approach leads to a benchmark that has the highest overlap with a fund’s holdings. In this paper, I report all results for the regression-based approach. The main results are insensitive to the benchmark selection methodology.

I consider a set of nine benchmarks that are distinguished by their size and value orientation. For large-cap stocks, I use the S&P 500, Russell 1000 Value, and Russell 1000 Growth; for mid-cap stocks, I take the Russell Midcap, Russell Midcap Value, and Russell Midcap Growth; for small-cap stocks, I select the Russell 2000, Russell Value, and Russell 2000 Growth. The style indexes are taken from Russell, in line with Chan, Chen, and Lakonishok (2002) and Chan, Dimmock, and Lakonishok (2006).

**Summary statistics** The sample consists of 3,694 unique manager-fund combinations of 3,163 different managers who manage 1,932 different mutual funds. For 1,273 manager-fund combinations I have more than three years of data available. I impose a minimum data requirement of three years to estimate all models so that performance regressions deliver reasonably accurate estimates. The left panel of Table 1 displays the allocation of manager-fund combinations to the nine styles for the full sample, the right panel for manager-fund combinations for which at least 3 years of data is available. One fifth of the managers are compared to the S&P 500. The benchmarks are relatively equally divided across the size dimension, with a small tilt towards large-cap benchmarks. The majority of the large-cap funds are neutral in the value-dimension, but the medium- and small-cap funds are predominantly growth-oriented.

[Table 1 about here.]

Table 2 provides summary statistics for the total net assets under management (TNA), total net assets of the fund family (as defined in Chen, Hong, Huang, and Kubik (2004)), family size (the number of funds that belong to the fund family), expense ratio, 12B-1 fees, the total load (the sum of maximum front-end load fees and maximum deferred and rear-end load fees), cash holdings as reported by the fund, stock holdings as reported by the fund (the sum of common and preferred stock), manager’s tenure, fund age, and annual turnover. The summary statistics are broadly consistent with prior studies.

[Table 2 about here.]

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27 The correlation with the corresponding style index from Standard and Poor’s is in all cases higher than 96.5%, measured over the full sample period.

28 Elton, Gruber, and Blake (2003) find that managers that have explicit incentives in their compensation schemes are compared to the S&P 500 in 44% of the cases. This suggests that this benchmark is either more popular with managers who receive incentive compensation, or that managers deviate from their stated objectives. Sensoy (2007) provides evidence that managers deviate from the benchmarks reported in the prospecti of these funds.
2 Financial market

The manager’s asset menu contains three assets. The first asset is a cash account that trades at price $S^0_t$. The cash account earns a constant interest rate $r$ and its dynamics satisfy:

$$\text{d}S^0_t = S^0_t r \text{d}t.$$  \hfill (1)

The second asset is the benchmark portfolio with price $S^B_t$:

$$\text{d}S^B_t = S^B_t (r + \sigma_B \lambda_B) \text{d}t + S^B_t \sigma_B\text{d}Z^B_t,$$  \hfill (2)

where $\lambda_B$ is the price of risk, $\sigma_B$ the standard deviation of the benchmark portfolio, and $Z^B_t$ a standard Brownian motion. The coefficients are assumed to be constant. Third, manager $i$ can trade a manager-specific active portfolio, or idiosyncratic technology, with price $S^A_{it}$. Without loss of generality, I assume that the active asset does not carry any systematic risk. The economic interpretation is that the manager forms an overlay portfolio on top of the benchmark and this overlay strategy is neutral is purged from any systematic risk. The dynamics of the active portfolio read:

$$\text{d}S^A_{it} = S^A_{it} (r + \sigma_{Ai} \lambda_{Ai}) \text{d}t + S^A_{it} \sigma_{Ai}\text{d}Z^A_{it}.$$  \hfill (3)

I take the price of risk on the active portfolio, $\lambda_{Ai}$, as the measure of managerial ability.\textsuperscript{29} $\sigma_{Ai}$ denotes the volatility of the active portfolio.

It would be straightforward to extend the model with a set of passive portfolios that can easily be replicated by managers, like momentum, which are typically not considered to reflect skill.

Benchmark portfolio, assets under management, and state-price density The benchmark portfolio is given by the two-dimensional vector $V = (v, 0)'$ of portfolio weights. The remainder, $1 - v$, is allocated to the cash account.\textsuperscript{30} The value of the

\textsuperscript{29}The model can easily be set up by allowing the manager to trade $J$ stocks with different prices of risk. However, in all models I consider, the manager will perfectly diversify the active portfolio leading to a single active portfolio (see Chen and Pennacchi (2007) and Basak, Pavlova, and Shapiro (2007b)). The formation of the active portfolio becomes important in models of costly information acquisition (Van Nieuwerburgh and Veldkamp (2007)). In this case, the active portfolio will not be perfectly diversified, as costly learning capacity is allocated to a few stocks only.

\textsuperscript{30}I assume fixed benchmark weights. Binsbergen, Brandt, and Kojen (2007) show that benchmarks with constant weights can alleviate most efficiency losses that arise in a decentralized investment management environment. Basak, Pavlova, and Shapiro (2007a) provide a similar result for a manager that shifts risk in response to incentives. Both studies suggest there is little need for dynamic benchmarks.
benchmark at time $t$ is denoted by $B_t$. The benchmark dynamics read:

$$dB_t = B_t (r + v \sigma_B \lambda_B) dt + B_t v \sigma_B dZ_t^B. \quad (4)$$

Assets under management at time $t$, $A_{it}$, evolve according to:

$$dA_{it} = A_{it} (r + x_{it}^B \sigma_B \lambda_B + x_{it}^A \sigma_A \lambda_A) dt + A_{it} x_{it}^B \sigma_B dZ_t^B + A_{it} x_{it}^A \sigma_A dZ_t^A$$

$$= A_{it} (r + x_{it}' \Sigma_i \Lambda_i) dt + A_{it} x_{it}' \Sigma_i dZ_{it}, \quad (5)$$

where $x_{it}^B$ and $x_{it}^A$ are the fractions invested in the benchmark and active portfolio, $x_{it} \equiv (x_{it}^B, x_{it}^A)'$, $\Sigma_i \equiv \text{diag}(\sigma_B, \sigma_A)'$, $\Lambda_i \equiv (\lambda_B, \lambda_A)'$, and $Z_{it} \equiv (Z_t^B, Z_t^A)'$. The asset dynamics excludes fund flows from outside investors, which I will discuss in detail in Section 3.2.

The state-price density at time $t$ of manager $i$ is denoted by $\varphi_{it}$. The state-price density plays a key role in the econometric approach and its dynamics satisfy:

$$d\varphi_{it} = -\varphi_{it} r dt - \varphi_{it} A_i' \Sigma_i dZ_{it}, \quad \varphi_{0i} = 1. \quad (6)$$

I will omit the subscripts $i$ for the remainder of the paper for notational convenience. However, note that $\lambda_A, \sigma_A, \text{and } Z_t^A$, and correspondingly $S_t^A, x_t, \text{and } \varphi_t$, are manager-specific. The remaining parameters are common across all managers in a particular style.

# 3 Standard models of delegated management

As a point of departure, I consider two standard models of delegated portfolio management that have been suggested in the literature in Section 3.1 and 3.2. Section 3.3 derives the cross-equation restrictions that are implied by theory.

## 3.1 Relative-return preferences

The first model assumes that the manager derives utility from assets under management relative to the value of the benchmark:

$$\max_{(x_s)_{s \in [0,T]}} E_0 \left[ \frac{1}{1 - \gamma} \left( \frac{A_T}{B_T} \right)^{1-\gamma} \right] \quad (7)$$

where $\gamma$ is the coefficient of relative risk aversion. This model captures, in a reduced-form, that the manager’s performance and ultimately her compensation is relative to a benchmark.\(^{31}\) The optimal strategy is a constant-proportions strategy (Binsbergen, Brandt, and Koijen (2007) and Chen and Pennacchi (2007) provide some further motivation for these preferences.)
Brandt, and Koijen (2007)):

\[ x^* = \frac{1}{\gamma} (\Sigma \Sigma')^{-1} \Sigma \Lambda + \left(1 - \frac{1}{\gamma}\right) V. \]  

(8)

It combines the mean-variance portfolio and the benchmark portfolio. The two portfolios are weighted by the coefficient of relative risk aversion. Consistent with the standard interpretation in the investment industry, infinitely risk-averse agents (that is, \( \gamma \to \infty \)) hold the benchmark \( (x^* = V) \).

3.2 Preferences for assets under management

The standard model The second model assumes that the manager derives utility from assets under management:

\[ \max_{(x_s)_{s \in [0,T]}} E_0 \left[ \frac{1}{1 - \gamma A_T^{1-\gamma}} \right], \]  

(9)

where \( \gamma \) denotes the coefficient of relative risk aversion. These preferences are motivated by the observation that most managers are compensated by a fraction of the assets under management (Deli (2002)). The optimal strategy reads:

\[ x^* = \frac{1}{\gamma} (\Sigma \Sigma')^{-1} \Sigma \Lambda, \]  

(10)

which is also of the constant-proportions type.

Preferences, career concerns, and fund flows Assets under management fluctuate due to investment returns (internal growth), but also due to fund flows and promotion or demotion of the fund manager (external growth). Both performance-sensitive fund flows and career concerns may motivate the manager to deviate from the optimal strategy in (10). It is well known from empirical studies that new capital flows disproportionally to funds with stellar performance,\(^{32}\) which results in an increasing and convex flow-performance relationship. In addition, exceptional (below-par) performance can lead to promotion (demotion) to a larger (smaller) fund. I analyze the importance of these incentives using the calibration of Chapman, Evans, and Xu (2007). They calibrate promotion/demotion probabilities to observed career events and estimate the flow-performance relationship.\(^{33}\)

Appendix B uses this model to study the interaction between


\(^{33}\)Hu, Hall, and Harvey (2000) and Baks (2006) also estimate promotion and demotion probabilities. I use the calibration of Chapman, Evans, and Xu (2007) because their calibration covers most closely the sample period I study (1994 to 2006).
incentives and risk aversion. I show that in the relevant range of risk aversion, managerial incentives are not powerful enough to distort the optimal strategy. I therefore abstract from such incentives in the main text.

3.3 Cross-equation restrictions implied by structural models

The bulk of the performance literature averages risk-adjusted returns to obtain an estimate of managerial ability. This means that a few years of data are used to estimate a mean return, a notoriously noisy approach. The key difference in this paper is to use the optimality conditions of the manager’s portfolio problem to uncover managerial ability. In consumption-based asset pricing, it is common to use the household’s Euler condition to estimate preference parameters. The cross-equation restrictions implied by the standard models show that both ability and risk aversion can be estimated from second moments of fund returns. I use the first model to illustrate the restrictions.

I start from a standard performance regression, formulated in continuous time:

\[
\frac{dA_t}{A_t} - rdt = \alpha dt + \beta \left( \frac{dS^B_t}{S^B_t} - rdt \right) + \sigma_\varepsilon dZ^A_t, 
\]

which, using (2), is equivalent to:

\[
\frac{dA_t}{A_t} = (r + \alpha + \beta \sigma_B \lambda_B) dt + \beta \sigma_B dZ^B_t + \sigma_\varepsilon dZ^A_t. 
\]

The parameters \( \alpha, \beta, \text{ and } \sigma_\varepsilon \) are manager-specific; \( \lambda_B \) and \( \sigma_B \) are common to all managers.

The relative-return preferences in (7) lead to the optimal portfolio in (8), which I substitute into (5) to obtain the optimal asset dynamics:

\[
\frac{dA_t}{A_t} = \left( r + \frac{\lambda_A^2}{\gamma} + \left(1 - \frac{1}{\gamma}\right)v \sigma_B \lambda_B + \frac{\lambda_A^2}{\gamma} \right) dt + \left( \frac{\lambda_B}{\gamma} + \left(1 - \frac{1}{\gamma}\right)v \sigma_B \right) dZ^B_t + \frac{\lambda_A}{\gamma} dZ^A_t, 
\]

where \( \lambda_A \) and \( \gamma \) are manager-specific, and \( v \) is common to all managers. The cross-equation restrictions implied by the structural model follow from matching the drift and

\[34\text{This noise has motivated researchers to form portfolios based on observable characteristics to identify quality managers. These include the portfolios’ active share (Cremers and Petajisto (2007)), similarities in portfolio holdings (Cohen, Coval, and Pastor (2005)), measures of concentration in portfolio holdings (Kacperczyk, Sialm, and Zheng (2005)), or their reliance on public information (Kacperczyk and Seru (2007)). By pooling managers cross-sectionally, the precision of the estimates increases.}\]
diffusion terms in (12) and (13):

\[
\begin{align*}
\alpha &= \frac{\lambda_A^2}{\gamma}, \\
\beta &= \frac{\lambda_B}{\gamma \sigma_B} + \left(1 - \frac{1}{\gamma}\right) v, \\
\sigma_\varepsilon &= \frac{\lambda_A}{\gamma}.
\end{align*}
\] (14, 15, 16)

The right-hand side of (15) and (16) identifies the two manager-specific structural parameters, \(\lambda_A\) and \(\gamma\). They are identified off the fund’s beta:35

\[
\beta = \frac{\text{Cov}\left(\frac{\text{d}A_t}{A_t}, \frac{\text{d}S_B^t}{S_B^t}\right)}{\text{Var}\left(\frac{\text{d}S_B^t}{S_B^t}\right)},
\] (17)

and residual risk:

\[
\sigma_\varepsilon^2 = \text{Var}\left(\frac{\text{d}A_t}{A_t}\right) - \beta^2 \text{Var}\left(\frac{\text{d}S_B^t}{S_B^t}\right),
\] (18)

Equation (14) can be used to restrict \(\alpha\):36

\[
\alpha = \sigma_\varepsilon^2 \left(\frac{\lambda_A}{\lambda_B - v} - \frac{\sigma_B}{\beta - v}\right).
\] (19)

Recall that \(\beta\) and \(\sigma_\varepsilon\) are manager-specific, whereas \(\lambda_B, \sigma_B,\) and \(v\) are common to all managers. This results in an estimate of the manager’s alpha via (19) that relies solely on information in second moments.

The typical approach in the literature is to estimate \(\alpha, \beta,\) and \(\sigma_\varepsilon\) separately. The resulting estimate for \(\alpha\) is based on information in average (risk-adjusted) fund returns. As it turns out, this is the most inefficient moment to use. The likelihood-based estimation procedure in Section 4 efficiently combines information from the average and the volatility of fund returns as well as the covariance of fund returns with benchmark returns. Since likelihoods are typically much steeper in parameters that govern second moments, ability is effectively estimated using that information.

**Simulation exercise** The structural model implies that alpha is a performance measure that mixes information on ability (\(\lambda_A\)) and preferences (\(\gamma\)). In addition, it shows that second moments of mutual fund returns contain useful information on preferences and ability. To illustrate the benefits of imposing this cross-equation restriction, Table 3

---

35Formally, the covariance needs to be interpreted as the quadratic covariation, the variance as the quadratic variation, and df = 1.

36A similar restriction arises in the model of Section 3.2, \(\alpha = \sigma_\varepsilon^2 \lambda_B / (\sigma_B \beta)\), which coincides with (19) if \(v = 0\), that is, in case of a cash benchmark.
provides the results of a simple simulation experiment. I simulate 2,500 sets of three years
of monthly data from the model. The price of active risk takes values $\lambda_A \in \{.1, .2, .3\}$ and
the coefficient of relative risk aversion takes values $\gamma \in \{2, 5, 10\}$. The market parameters
correspond to the S&P 500 as the style benchmark. Panel A of Table 3 provides the
results for the maximum-likelihood estimators of $\lambda_A$ and $\gamma$. The resulting estimates are
unbiased and sharp. For $\lambda_A = .2$ and $\gamma = 5$, an 80%-confidence interval for $\lambda_A$ ranges
from .16 to .24; for $\gamma$ from 4.5 to 5.6. Panel B of Table 3 illustrates the efficiency gains
for fund alphas. I compare the model-implied alpha ($\alpha_{ML}$) to the one that follows from
a performance regression ($\alpha_{OLS}$). When $\lambda_A = .2$ and $\gamma = 5$, the true $\alpha = .8\%$. The
80%-confidence interval for $\alpha_{ML} = [.54\%, 1.05\%]$, whereas for $\alpha_{OLS} = [-.215\%, 3.93\%]$. In
this example, the standard deviation of $\alpha_{ML}$ is .78\%, whereas the standard deviation of
$\alpha_{OLS}$ is three times larger at 2.37\%. This implies that standard performance regressions
require nine times more data if the cross-equation restrictions are not imposed to deliver
the same accuracy in this model. It resonates with the empirical results in Figure 1 in the
introduction, which compares the implied estimates of alpha following from the model in
Section 6 to the estimates of performance regressions. This illustrates that imposing the
restrictions implied by theory significantly sharpens the implied estimates of $\alpha$.

[Table 3 about here.]

4 Econometric approach

In this section, I develop a general method to estimate the ability and preference
parameters of dynamic models of delegated portfolio management in a complete-markets
setting. The method is likelihood-based and can therefore be combined with both classical
and Bayesian estimation procedures. Appendix E contains further details and Koijen
(2007) discusses a simple example to illustrate the method and its accuracy. For the
models in Section 3, it is possible to construct estimators that are easier to implement
(Appendix E.1). However, because the estimates of ability and risk aversion following
from both standard models are economically implausible, I generalize the preferences in
Section 6. This model can no longer be estimated using standard techniques and requires
the novel approach given in this section.

The inference problem  Consider a manager who can trade the style benchmark, the
active portfolio, and cash. I estimate the model using information on benchmark returns,
$r^{BT}_t, r^B_t \equiv \log S^B_t - \log S^B_{t-h}$, and mutual funds returns, $r^A_t \equiv \log A_t - \log A_{t-h}$, with
$y^T \equiv \{y_h, \ldots, y_T\}$. I take $h = 1/12$ since the model is estimated using monthly data.
I set the short rate, $r$, equal to the average 30-day T-bill rate during the investment
period, which I take to be one year. The model parameters can be grouped into financial
market parameters that apply to all managers, $\Theta_B \equiv \{\lambda_B, \sigma_B\}$, and parameters that are manager-specific, $\Theta_A \equiv \{\lambda_A, \gamma\}$.\(^{37}\)

I adopt a two-step procedure to estimate the model. First, I estimate the financial market parameters that are common to all managers, $\Theta_B$. Because asset prices follow geometric Brownian motions conditional on the short rate, the log-likelihood of $r^{BT}$, $L(r^{BT}; \Theta_B)$, is trivial to construct. In the second step, I estimate the manager-specific parameters, $\Theta_A$, using the log-likelihood of fund returns conditional on the benchmark returns and the first-stage estimates, $L(r^{AT} \mid r^{BT}; \Theta_A, \hat{\Theta}_B)$. The main complication is to compute the second-stage likelihood.

While a single-step estimation would enhance the efficiency of the estimates, it would require modeling the cross-sectional correlation of active portfolio returns. The two-step procedure accommodates any cross-sectional dependence in active returns. It therefore requires less restrictive statistical assumptions, is not subject to misspecification of the correlation structure, and still results in consistent estimates. In addition, the two-step procedure saves substantially on computational time.

The conditional log-likelihood of mutual fund returns To appreciate why it is non-trivial to construct $L(r^{AT} \mid r^{BT}; \Theta_A, \hat{\Theta}_B)$, consider the dynamics of assets under management:

$$dA_t = A_t \left( r + x^*_t(A_t)'\Sigma \Lambda \right) dt + A_t x^*_t(A_t)'\Sigma dZ_t,$$

where $x^*_t(A_t)$ is the optimal investment strategy of the manager, which may depend on time and assets under management. There are two complications, which are related. First, the diffusion coefficient, $x^*_t(A_t)'\Sigma$, may be time varying if the manager implements a dynamic strategy. This is the case for the model I study in Section 6. This often implies that the exact discretization is unknown, which leads to a discretization bias. The typical approach in the literature is to stabilize the diffusion coefficient to mitigate the discretization bias. The likelihood is then constructed via simulations (Brandt and Santa-Clara (2002) and Durham and Gallant (2002)) or series expansions of the transition density (Ait-Sahalia (2002), Ait-Sahalia (2007), and Bakshi and Ju (2005)). Second, the optimal strategy, and therefore the diffusion coefficient, is in most cases not known analytically. This implies that standard stabilization methods cannot be implemented.

One solution would be to solve the dynamic problem numerically either in discrete or continuous time.\(^{38}\) This approach has, at least, two drawbacks. First, solving the

\(^{37}\)The volatility of the active portfolio cannot be identified from returns data only. This parameter is, however, unimportant because it does not enter the likelihood once evaluated at the optimal strategy.

\(^{38}\)See Balduzzi and Lynch (1999), Brandt, Goyal, Santa-Clara, and Stroud (2005), and Koijen, Nijman, and Werker (2007) for recent advances to solve such problems.
The manager’s problem and the martingale method  The econometric approach relies on the martingale method of Cox and Huang (1989). I first solve for the optimal terminal asset level, $A^*_T$:

\[
\max_{A_T \geq 0} E_0 \left[ u(A_T) \right],
\]

s.t. \[ E_0 \left[ \varphi_T A_T \right] \leq A_0, \] (22)

where (22) is the static representation of the dynamic budget constraint in (5). If the utility index is strictly concave, it holds that $A^*_T = I(\xi \varphi_T)$, where $\xi$ is the Lagrange multiplier corresponding to the budget constraint and $I(\cdot) \equiv (u'(\cdot))^{-1}(\cdot)$. By no-arbitrage, time-$t$ assets under management satisfy:

\[
A^*_t(\varphi_t) = E_t \left[ I(\xi \varphi_T) \frac{\varphi_T}{\varphi_t} \right],
\]

which is a function of $\varphi_t$ only because $(\varphi_t)_{t \geq 0}$ is Markovian. Since the utility index is strictly concave, $A^*_t(\varphi_t)$ is invertible (Koijen (2007)). This implies that observing assets under management, or fund returns, is equivalent to observing the time series of the state-price density, $\varphi^T$. I then apply the Jacobian formula:

\[
\ell \left( r_t^A, r_t^B, \varphi_{t-h}; \Theta_A, \Theta_B \right) = \ell \left( \varphi_t | r_t^A, \varphi_{t-h}; \Theta_A, \Theta_B \right) + \log \left| \left( \partial (\log A_t^* - \log A^*_{t-h}) / \partial \varphi_t \right)^{-1} \right|.(24)
\]

and note that $\varphi_{t-h}$ (or, equivalently, $A^*_{t-h}$) contains all time-($t - h$) information needed due to the Markov property. Both terms in (24) are straightforward to compute. Because $\varphi_t$ is log-normally distributed given $\varphi_{t-h}$ and $r_t^B$, this involves one-dimensional Gaussian quadrature. Koijen (2007) demonstrates its accuracy for a low number of quadrature points.

In Section 6, I develop a model in which the utility index is not globally concave. This implies that the martingale approach cannot be applied directly. The solution proposed in the literature is to replace the original utility index with the smallest concave function that dominates it (Carpenter (2000), Cuoco and Kaniel (2006), and Basak, Pavlova, and Shapiro (2007b)), and then use standard techniques. Appendix E.2 applies this approach to the model of Section 6. Further, I construct the standard errors using the
outer-product gradient estimator. Appendix F shows how to test hypotheses in dynamic models of delegated portfolio management. I use these tests to compare different nested and non-nested models.

This method results in the exact likelihood of fund returns, up to the computation of an expectation of a univariate random variable and its numerical derivative. The method is insensitive to endogenous state variables and the computational effort is independent of the number of assets in the manager’s menu. The only restriction on the method is that the market is dynamically complete.\footnote{It is theoretically possible to apply martingale techniques even in incomplete markets. See for instance He and Pearson (1991), Cvitanic and Karatzas (1992), and the application in Sangvinatsos and Wachter (2005).}

5 Empirical results for the benchmark models

Relative-return preferences Table 4 displays results for the model in Section 3.1. The benchmark weights are set to $V = (1, 0)'$ and $T$ is set to one year.\footnote{As an alternative, I also consider $\beta = (1 - \text{cash}, 0)$, with “cash” the average cash position in a particular year, and $\beta = (1 - \text{stock}, 0)$, with “stock” the fraction invested in common and preferred equity. The main conclusions are comparable for these alternative benchmark strategies.} The first two columns provide summary statistics for the estimates of ability and relative risk aversion for the nine investment styles. Columns three to five show the implied coefficients of a performance regression using Equation (14), and columns six to eight contain the results of standard performance regressions in a continuous-time framework (Appendix A).

The average coefficient of relative risk aversion is high and its distribution is right-skewed. The intuition is that mutual funds have a $\beta$ that is close to one. Since $\lambda_B/\sigma_B$ substantially exceeds one for all styles, Equation (15) implies that $\gamma$ needs to be high. However, to match the amount of active risk that managers take, $\sigma_\varepsilon$, the price of risk needs to be high to offset the high risk aversion estimate (see Equation (16)). The average price of risk ranges from .64 (small/growth) to 1.75 (midcap/value). Consequently, the implied alpha is implausibly high, a result of Equation (14). The average estimates for alpha are between 6.14% and 11.88% per annum. The average alpha is substantially higher than the alpha following from standard performance regressions for all investment styles. As such, this model is unable to simultaneously match the fund’s active and passive risk-taking and a low average risk-adjusted return.

[Table 4 about here.]

Preferences for assets under management Table 5 displays the results for the model in Section 3.2 and has the same structure as Table 4. This model almost perfectly replicates the distribution of active ($\sigma_\varepsilon$) and passive ($\beta$) risk-taking. The estimates for $\gamma$...
and $\lambda_A$ are considerably lower than for the model in Section 3.1. The estimates of alpha are correspondingly lower, and range from 86 basis points (bp) to 294bp. Despite the more reasonable estimates for managerial ability, the average coefficient of relative risk aversion tracks $\lambda_B/\sigma_B$ and displays little dispersion. The reason is that mutual funds have, on average, a beta of one with respect to the style benchmark. To generate a unit beta, $\gamma$ equals $\lambda_B/\sigma_B$ because the fund’s beta ($x_B$) is in this model given by $\lambda_B/(\gamma\sigma_B)$, see (8). This means that, by default, a value manager has a higher coefficient of risk aversion than a growth manager, on average, because the price of risk is higher and the volatility is lower for value stocks. Therefore, the estimated $\gamma$ does not reflect risk aversion. To make this point more clearly, I consider a sample of managers who manage multiple funds at the same point in time (not reported). Such managers should display stable risk preferences across styles. However, it turns out that the risk aversion estimates contain a “fixed effect,” captured by $\lambda_B/\sigma_B$. Finally, note also that the distribution is virtually symmetrical and displays very little dispersion (Table 5). This is at odds with Cohen and Einav (2007) and Kimball, Sahm, and Shapiro (2007), who find strong evidence in favor of right-skewed distributions.

In summary, this model generates estimates of risk aversion that are mechanically tied to that of the representative agent, leading to low dispersion in preference parameters and a “fixed effect” per asset class. That is, the average risk aversion moves in lock-step with $\lambda_B/\sigma_B$, which contaminates its interpretation as a coefficient of relative risk aversion.

**Robustness** I consider various extensions to ensure the robustness of these results. I allow for (i) time variation in risk premia that is governed by the short rate and the dividend yield (Ang and Bekaert (2007)),\(^{41}\) (ii) other passive portfolios such as momentum, (iii) cash positions in the benchmark, (iv) stochastic volatility, and (v) learning about managerial ability.\(^{42}\) These modifications do not alter the conclusions qualitatively. In conclusion, neither of the two standard models produces sound estimates of the joint distribution of managerial ability and risk preferences.

\(^{41}\)Becker, Ferson, Myers, and Schill (1999) discuss the importance of conditioning information in market timing models.

\(^{42}\)I extend the model in Section 3.1 to allow for the possibility that the manager does not know her ability as in Berk and Green (2004) and Dangl, Wu, and Zechner (2007). Instead, the manager starts off with a (Gaussian) prior on the price of risk and updates her views based on realized performance (Cvitanic, Lazrak, Martellini, and Zapatero (2006)). The estimation error that is taken into account increases the effective risk aversion. This implies that the manager’s prior mean needs to be even higher than the estimates in absence of parameter uncertainty to reconcile active risk taking. Consequently, the estimates for the prior mean are economically implausible or the prior is very tightly centered around the maximum likelihood estimates without learning, which shuts down the learning channel. Alternatively, I use the cross-sectional distribution of the mutual fund performance to form the prior instead of estimating the prior distribution for each manager separately. A formal specification test indicates that both learning models are strongly rejected in favor of the status model in Section 6. Technical details and further empirical results are available upon request.
6 Status model for delegated portfolio management

In this section, I develop and study the main implications of a new model of delegated investment management that features status concerns on the part of the manager. Section 7 presents the main empirical results.

Motivation Standard models of delegated portfolio managers postulate that the manager cares only about assets under management or about performance relative to a benchmark. However, a large literature in sociology and economics\textsuperscript{43} argues that status considerations may be important for economic behavior and financial decision-making. Given the numerous rankings of mutual funds and fund managers and their importance for fund flows (Sirri and Tufano (1998)), the mutual fund industry provides an economic environment where status concerns are clearly important. I generalize the manager’s preferences so that she derives utility from both assets under management and the position of the fund in the cross-sectional asset distribution. I call the latter the fund’s status. There are at least two ways to motivate the status-seeking behavior of fund managers. One hypothesis is that status concerns are hard-wired into the manager’s preferences as a result of evolutionary forces (Robson (2001)). Alternatively, relative performance concerns may arise endogenously from strategic interaction, as in Basak and Makarov (2007).\textsuperscript{44} The model that I develop is a parsimonious model of status concerns. An attractive feature of the model is that it nests both models from Section 3.

The model Each investment style in the mutual fund industry comprises of a continuum of mutual fund managers, in which each manager $i, i \in M$, manages a fund of size $A_{it}$ at time $t$. The total mass of managers is normalized to unity with a corresponding measure $\mu$. The percentile rank of a fund of relative size $a$ at time $t$ is defined by:

$$q_t(a) \equiv \mu \left( i \mid \frac{A_{it}}{\bar{A}_0 R_t^B} \leq a \right),$$

(25)

where fund size is scaled by the median of the initial cross-sectional asset distribution,\textsuperscript{45} $\bar{A}_0 \equiv \{ \bar{A} \mid \mu (i \mid A_{i0} \leq \bar{A}) = 0.5 \}$, multiplied by the benchmark return, $R_t^B \equiv B_t/B_0$. I update the initial median fund size, $\bar{A}_0$, with the benchmark return to account for overall


\textsuperscript{44}This relates to the models of De Marzo, Kaniel, and Kremer (2004) and De Marzo, Kaniel, and Kremer (2007) in which investors compete for a scarce good. In the mutual fund industry, the scarce goods are fund flows that outside investors allocate across different funds.

\textsuperscript{45}Alternatively, Roussanov (2007) normalizes by the mean of the, in his application, wealth distribution. The median is empirically more stable than the mean as it curbs the impact of outliers. The resulting cross-sectional asset distribution is more stable over time.
growth in assets under management during the year if the manager invests along with the pack. This implies that to improve status, the manager needs to deviate from the pack by increasing or decreasing passive risk, or by allocating capital to the active portfolio. I define $\bar{A}_t \equiv \bar{A}_0 R^B_t$. The manager’s preferences are represented by:

$$\max_{(x_t)_{t\in[0,T]}} E_0 \left[ \frac{\eta}{1 - \sigma_1} A_T^{1 - \sigma_1} + (1 - \eta) S \left(1 - \sigma_2\right) A_T^{1 - \sigma_1} \varrho_T \left(\frac{A_T}{\bar{A}_T}\right)^{1 - \sigma_2} \right],$$

(26)

with $\eta \in [0,1]$, $\sigma_1 > 1$, and $S(x)$ as a sign function: $S(x) = 1$ if $x \geq 0$ and $S(x) = -1$ otherwise. The manager’s utility is a weighted average of two terms with weights $\eta$ and $(1 - \eta)$, respectively. The first term summarizes the manager’s preferences for assets under management. The second term captures status concerns. The term $\varrho_T \left(\frac{A_T}{\bar{A}_T}\right)$ represents the manager’s position in the cross-sectional asset distribution. The curvature parameter $\sigma_1$ captures the manager’s aversion to fluctuations in assets under management; $\sigma_2$ controls aversion to variation in fund status.

Several aspects deserve further discussion. First, the distribution function $\varrho_T(\cdot)$ is by definition bounded between zero and one. $\sigma_2$ can therefore be negative without inducing global convexities that would render the portfolio-choice problem ill-defined. In economic terms, managers with a strong desire to improve their status are identified by low, possibly even negative, values of $\sigma_2$. Managers who are concerned about variation in fund status have high values of $\sigma_2$. The desire to move up in the asset distribution can justify high levels of active risk-taking despite a lack of skill. Second, I assume that the preferences are separable in assets under management and in status concerns. This allows an interpretation in which the first part represents the current year’s compensation and the second part captures the manager’s value function over her remaining career prospects. Such career prospects presumably become less pressing when the manager’s fund ranks higher in the cross-sectional asset distribution. In this interpretation, $\sigma_2$ measures the manager’s career concerns. Third, the fund’s rank is represented by the cumulative distribution function (CDF) of assets under management, $\varrho_T(\cdot)$, to simplify the interpretation. Theoretically, any increasing function of fund size can serve the same purpose, but the CDF captures the ease with which a manager can climb in the cross-sectional asset distribution. If the CDF is steep, a small increase in assets under management results in a substantial improvement in status. In contrast, a more dispersed asset distribution requires a more stellar performance to realize the same status improvement. Fourth, this model nests the models studied in Section 3. If $\eta = 0$, $\sigma_1 = 1$, and the asset distribution is uniform (that is, $\varrho_T(a) = a/C$, with $C$ the upper-bound of the asset distribution), I recover the preferences in Section 3.1 with a coefficient of relative risk aversion $\sigma_2$. If $\sigma_2 = 1$, the model reduces to the preferences in Section 3.2 with a coefficient of relative risk aversion $\sigma_1$. Fifth, the second term is pre-multiplied
by $\tilde{A}_T^{-\sigma_1}$. This implies that the preferences are invariant to changes in aggregate wealth (Roussanov (2007)). Sixth, I update the initial median fund size by the style benchmark return, $\tilde{A}_T = \tilde{A}_0 R_T^{SB}$. Alternatively, I could use the return on the median fund. Using the style benchmark return has two advantages. First, the benchmark return is easy for managers to track and seems like the most visible target to beat. Second, the definition of managerial ability gets obfuscated when the manager can trade the median fund.\footnote{One other alternative would be to update the median fund with the median fund return and restrict the asset menu to cash, the style benchmark, and the active portfolio. However, this renders the financial market to be dynamically incomplete.} If all managers are skilled, the median fund return inherits this skill. This would imply that I estimate the manager’s ability only insofar as it surpasses the skill present in the initial median fund return. By using the style benchmark return to update the median fund size, the definition of skill is consistent with the first part of the paper and the extant literature.

**Modeling the cross-sectional asset distribution** As a first step towards analyzing the model empirically, I model the cross-sectional asset distribution, $\phi_t(\cdot)$.\footnote{Note that the model endogenously generates a cross-sectional asset distribution, $\phi_T(\cdot)$, given $\phi_0(\cdot)$ and the cross-sectional distribution of managerial ability and risk preferences. For instance, Roussanov (2007) derives the stationary distribution that is consistent with the optimal policies of households in a life-cycle model. I am not merely interested in the stationary distribution, but also in the conditional distribution. In addition, I want to estimate ability and risk preferences for a large cross-section of managers. It is therefore computationally too intensive to impose the equilibrium condition as well. I therefore model the cross-sectional asset distribution directly.} First, I assume that the cross-sectional asset distribution is log-normal with mean $\mu_{\phi_t} \equiv E_t \left[ \log \left( A_{it} / \bar{A}_t \right) \right]$ and standard deviation $\sigma_{\phi_t} \equiv \text{Var}_t \left[ \log \left( A_{it} / \bar{A}_t \right) \right]^{1/2}$, $\phi_t(\cdot; \mu_{\phi_t}, \sigma_{\phi_t})$. Second, I assume that the asset distribution is stationary during the period $[0, T]$: $\mu_{\phi_T} = \mu_{\phi_0}$ and $\sigma_{\phi_T} = \sigma_{\phi_0}$. The first assumption is made for computational tractability. Because the main objective is to estimate the model for a large cross-section of managers, I need to impose some structure. The second assumption implies that the manager uses the asset distribution at the beginning of the year to make her assessment of status throughout the year. This assumption could be relaxed by allowing $\phi_T$ to be different from $\phi_0$, but the manager would need to be able to hedge the risk of a shifting distribution to preserve market completeness.

I estimate the coefficients of the log-normal distribution ($\mu_{\phi_0}$ and $\sigma_{\phi_0}$) for each style and each year using the cross-section of funds at the beginning of the year. To estimate the cross-sectional asset distribution, I use all mutual funds in the CRSP data set. Clearly, it would be inappropriate to use only those funds for which I can identify the manager or management team. I test the appropriateness of the distributional assumption using the Jarque-Bera test of normality. The average $p$--value across all years ranges from 10.2\% to 52.4\% for the nine investment styles, which supports the normality assumption.\footnote{There exists an interesting parallel between modeling the cross-sectional asset distribution of mutual}
Fund status, risk aversion, and risk-taking  
Fund size and the fund’s position in the cross-sectional asset distribution play a key role in explaining risk-taking behavior. First, I discuss the link between relative fund size and risk aversion. Second, I show that the parameters $\sigma_1$ and $\sigma_2$ determine whether the manager adjusts either active or passive risk if risk aversion changes. The former ($\sigma_1$) controls passive risk-taking; $\sigma_2$ determines active risk-taking.

I first relate relative fund size and risk aversion. The status model is not homogenous in assets under management. To understand the implications for risk-taking, I define the coefficient of relative risk aversion, $\text{RRA}(a_t)$:

$$
\text{RRA}(a_t) = -\frac{a J_{t,aa}}{J_{t,a}},
$$

where $J$ denotes the value function and subscripts partial derivatives. If $t = T$, I obtain the Arrow-Pratt measure of risk aversion (Appendix C provides further details). It is the weighted average of two terms, with $a_t \equiv A_t / \bar{A}_t$:

$$
\text{RRA}(a_T) = \omega(a_T)\sigma_1 + (1 - \omega(a_T)) \left[ \sigma_2 \frac{g'(a_T) a_T}{g(a_T)} - \frac{g''(a_T) a_T}{g'(a_T)} \right],
$$

with weight:

$$
\omega(a_T) \equiv \frac{\eta a_T^{-\sigma_1}}{\eta a_T^{-\sigma_1} + (1 - \sigma_2) (1 - \eta) S (1 - \sigma_2) g(a_T)^{-\sigma_2} g'(a_T)}.
$$

For most empirically plausible parameter combinations, $\omega(a_T)$ goes from one to zero if $a_T$ increases from zero to infinity. This implies that status concerns become more pressing if fund status increases. Equation (28) implies that the manager’s coefficient of relative risk aversion combines three measures of relative risk aversion: (i) $\sigma_1$, (ii) $\sigma_2 g'(a_T)a_T/g(a_T)$, and (iii) $-g''(a_T)a_T/g'(a_T)$. The third measure is the relative risk aversion if preferences are linear in status only ($\eta = 0$ and $\sigma_2 = 0$). Figure 5 displays the three components if $\sigma_1 = 4$, $\sigma_2 = .5$, and $\eta = .0005$. The asset distribution is calibrated to the S&P 500. The horizontal axis plots log($a_T$), the vertical axis the relative risk aversion. The first component ($\sigma_1$) is obviously invariant to size; the second component decreases in fund size, and the last component increases in fund size. For large funds, the third component

49I define the coefficient of relative risk aversion based on the value function, which is the relevant measure of relative risk aversion for decision-making at time $t$. In the empirical section, I use $\text{RRA}(a_0)$ as the measure of risk aversion.
always dominates the second component. The three components aggregate to the overall coefficient of relative risk aversion via the weight function \((\omega(a_T))\). For small funds, status concerns are irrelevant \((\omega(a_T) \simeq 1)\) and risk aversion is solely governed by \(\sigma_1 = 4\). By increasing the fund’s assets under management, status concerns gradually become more important \((\omega(a_T) < 1)\). As a result, the coefficient of relative risk aversion drops. In this region, the manager has a lot of scope to move up in the asset distribution by deviating from the pack. By moving further up in the cross-sectional asset distribution, the manager has little incentive to deviate from the pack for fear of losing her position in the distribution. Status concerns are key in this region \((\omega(a_T) \rightarrow 0)\). The minimum risk aversion is attained around the 25-th percentile of the asset distribution. This implies that risk aversion increases in fund size for most funds.

[Figure 5 about here.]

The parameters \(\sigma_1\) and \(\sigma_2\) are key to understanding whether the manager modifies active or passive risk-taking if the coefficient of relative risk aversion changes. I show in Appendix D that \(\sigma_1\) controls passive risk-taking. If the manager decides to use passive risk to deviate from the pack, she can choose to increase or decrease the fund’s beta. Either will lead to a tracking error relative to the average fund that has a unit beta. Appendix D shows that the manager chooses to increase passive risk if \(\sigma_1 < \lambda_B / \sigma_B\) and decreases passive risk if \(\sigma_1 > \lambda_B / \sigma_B\). For \(\sigma_1 = \lambda_B / \sigma_B\), the manager’s passive risk-taking is insensitive to changes in the coefficient of relative risk aversion. Unlike passive risk-taking, active risk-taking always increases when \(\sigma_2\) falls. This implies that \(\sigma_2\) controls active risk-taking. The main problem with the models in Section 3 is that both active and passive risk-taking are proportional to the coefficient of relative risk aversion. An increase in the fund’s beta goes hand-in-hand with an increase in active risk. The status model frees up this tight link.

I illustrate the role of \(\sigma_1\), \(\sigma_2\), and fund size by solving for the optimal initial allocation to the benchmark and the active portfolio. I set \(\eta = .0005\) and \(\lambda_A = .15\). Appendix E.2 discusses the solution method. The results are presented in Table 6. The first four columns show the impact of \(\sigma_2\). I set \(a_0 = 1\) and \(\sigma_1 = \lambda_B / \sigma_B\) so that \(x^B\) equals unity and is invariant to changes in \(\sigma_2\). The main observation is that \(x^A\) is inversely related to \(\sigma_2\), whereas relative risk aversion is positively related to \(\sigma_2\). If \(\sigma_2\) increases from \(-1\) to \(30\), the optimal allocation drops from \(x^A = 155\%\) to \(x^A = 5\%\).

Columns five to twelve illustrate the role of \(\sigma_1\) and the link between relative fund size and risk-taking. As before, I set \(\sigma_2 = .5\). I consider the optimal allocation for different initial fund sizes. Columns five to eight consider the case in which \(\sigma_1 = 3.75\) \((< \lambda_B / \sigma_B)\), whereas the last four columns correspond to \(\sigma_1 = 4.25\) \((> \lambda_B / \sigma_B)\). First, risk aversion is (inversely) hump shaped as in Figure 5. If \(\sigma_1 = 3.75\), the manager
increases passive risk \((x^B)\) as risk aversion drops, and decreases \(x^B\) if \(\sigma_1 = 4.25\) for the same change in fund status. Second, the manager always increases active risk if risk aversion decreases. Note that \(x^A\) is virtually unaffected by changing \(\sigma_1\), in particular for larger fund sizes. This implies that \(\sigma_1\) controls passive risk-taking and \(\sigma_2\) active risk-taking and provides a structural interpretation to the ideas of Litterman as iterated in the introduction. Risk aversion to passive risk translates into aversion to fluctuations in assets under management. Risk aversion to active risk translates into aversion to variation in fund status. In conclusion, fund status is for most funds positively related to risk aversion. How managers adjust risk-taking in response to a change in risk aversion is governed by \(\sigma_1\) (passive risk) and \(\sigma_2\) (active risk).

[Table 6 about here.]

A key implication of the model is that managers of small funds will behave markedly different from managers controlling large funds. Small funds have more room to grow and to improve their status, which provides an incentive to deviate from the pack. The opposite is true for managers of large funds. As such, large funds will take less active risk and produce smaller alphas, consistent with empirical evidence on risk-taking and performance in relation to fund size.

**Statistical identification** The model contains four manager-specific parameters, \(\Theta_A \equiv \{\sigma_1, \sigma_2, \eta, \lambda_A\}\). It turns out that \(\eta\) is weakly identified. I therefore calibrate \(\eta\) to a common value \(\eta = .0005\). The previous section shows that this model can generate a wide variety of risk-return distributions.

### 7 Main empirical results

This section presents the empirical results for the status model.

**The cross-sectional distribution of ability and risk aversion** Table 7 summarizes the main estimation results by investment style, with the overall results across all styles in the bottom panel. First, all parameters are right-skewed, in particular \(\sigma_2\). The coefficient of variation (the standard deviation normalized by the mean) is much larger for \(\sigma_2\) than for \(\sigma_1\) and \(\lambda_A\). This points to substantial heterogeneity in status concerns. The dispersion in \(\sigma_1\) is relatively small. This stems from the fact that \(\sigma_1\) controls passive risk-taking and the empirical result that mutual fund betas display little dispersion.

[Table 7 about here.]

The bottom panel shows that the average coefficient of relative risk aversion is estimated to be 5.16, with its median equal to 2.51 and a standard deviation of 7.69.
The average manager has therefore a risk aversion coefficient that is slightly lower than the average household’s risk aversion of 8.2 as estimated by Kimball, Sahm, and Shapiro (2007). It is appealing that mutual fund managers as a group are less conservative.

The average price of active risk is estimated to be .28, with a median equal to .14 and a standard deviation of .38. To put the estimates in perspective, I compare the model-implied estimates to the actual estimates of a performance regression (Appendix A). Recall that the standard models in Section 3 cannot easily reproduce the coefficients of standard performance regressions. The model-implied estimates are computed as the average $\alpha$, $\beta$, and $\sigma_\varepsilon$ sampled at a monthly frequency. To gauge the similarity, I perform the following cross-sectional regression for, for instance, $\alpha$:

$$\hat{\alpha}_{i,\text{Performance}} = \rho_0 + \rho_1 \hat{\alpha}_{i,\text{Status}} + u_i, \quad (30)$$

where $\hat{\alpha}_{i,\text{Performance}}$ is the estimate from a standard performance regression and $\hat{\alpha}_{i,\text{Status}}$ the estimate from the status model. The estimates following from the structural model are much sharper. I therefore use them as the right-hand side variables to mitigate the errors-in-variables bias due to estimation uncertainty. The resulting estimates read: for $\alpha$, $\hat{\rho}_0 = -0.00$ and $\hat{\rho}_1 = 0.99$ ($R^2 = 35.11\%$); for $\beta$, $\hat{\rho}_0 = -0.00$ and $\hat{\rho}_1 = 1.00$ ($R^2 = 97.67\%$); for $\sigma_\varepsilon$, $\hat{\rho}_0 = -0.00$ and $\hat{\rho}_1 = 1.04$ ($R^2 = 98.69\%$). In all cases it seems that the estimates are virtually unbiased ($\rho_0 = 0$ and $\rho_1 = 1$). The most striking result is the R-squared for the regression of mutual fund alphas. The estimates from the structural model are three times more accurate and do a good job of reproducing the average moments of performance regressions. This is also illustrated in Figure 1, where the top panels provide the results for a standard performance regression and the bottom panels for the structural model. The left panels display the fund alphas before fees and expenses, the right panels are net of all expenses. The distribution of fund alphas following from the structural model is much less dispersed. This implies that the cross-sectional distribution of fund alphas following from performance regressions reflects predominantly estimation error and not heterogeneity in managerial ability or risk preferences.

Figure 2 displays a scatter plot of risk aversion (horizontal axis) and managerial ability (vertical axis) to analyze their interaction. The correlation between ability and risk aversion is 80.2%. A second-order polynomial fitted through this cloud shows that managerial ability is increasing and concave in the coefficient of relative risk aversion. The last part of this section discusses potential mechanisms that can generate this positive relation.

\[50\] This implies that in the reverse regression of $\hat{\alpha}_{i,\text{Model}}$ on $\hat{\alpha}_{i,\text{Performance}}$, $\hat{\rho}_1$ would be downward biased and, correspondingly, $\hat{\rho}_0$ upward biased. Indeed, the reverse regression results in $\hat{\rho}_0 = .013$ and $\hat{\rho}_1 = .36$, which motivates the regression specification in (30).
There are also interesting differences across investment styles. I focus on large/value managers and small/growth managers. Figure 3 provides a standard kernel density estimate for risk aversion (left panel) and managerial ability (right panel). There are pronounced differences in the distribution of risk preferences for the two types of managers, despite the fact that the average risk aversion is very similar (5.66 for large/value and 5.49 for small/growth). Risk aversion is more evenly distributed for large/value managers, but it is more right-skewed for small/growth managers. The median risk aversion for the small/growth manager is 1.49, whereas the median large/value manager has a risk aversion of 3.95. Ability, by contrast, is considerably higher for small/growth managers on average, but their medians of .16 tie. This implies that there are more high-skilled managers in the small/growth investment style, which is reflected by the thicker tail of the distribution.

**Heterogeneity in risk aversion and ability** I relate the estimates of managerial ability and risk aversion to observable characteristics of managers and mutual funds using multiple cross-sectional regressions. The characteristics include total net assets, the manager’s tenure, turnover, expenses, investment in common and preferred stocks, loads, 12B-1 fees, and the total net assets of the family. The results are presented in Table 8. I include dummies to absorb style-fixed effects and use standard errors that are robust to heteroscedasticity.

[Table 8 about here.]

The dependent variables are expressed in logarithms and the independent variables are standardized. As such, the coefficients are to be interpreted as the percentage change for a one-standard deviation change in the characteristics. First, I find that skilled managers operate on smaller funds, consistent with Chen, Hong, Huang, and Kubik (2004), who document a negative relation between fund size and ability as measured by the fund’s alpha. My structural model implies that a one-standard deviation increase in fund size leads to almost a 9% decrease in the price of active risk.\(^{51}\) Second, managers with longer tenure periods are more skilled, which may be the outcome of selection based on skill or learning. A one-standard deviation increase in tenure increases the price of risk by 7%. Chevalier and Ellison (1999a) find the same sign for fund alphas as a measure for performance, but the effect is insignificant. Third, more skilled managers have higher levels of turnover\(^{52}\) and have smaller stock holdings. Fourth, skilled managers charge higher expense ratios, consistent with Berk and Green (2004), but the effect is insignificant. Fifth, I find that more conservative managers manage larger funds, have


\(^{52}\)Chen, Hong, Huang, and Kubik (2004) document a positive, but insignificant, relation between turnover and fund alphas. I find a significant relationship once corrected for heterogeneity in risk preferences.
smaller expense ratios, and allocate a smaller share of their capital to stocks. The relation between risk aversion and expenses is again consistent with Berk and Green (2004) because fund alphas and risk aversion are inversely related. Finally, note that there is considerable unobserved heterogeneity; the R-squared values are 13.0% for ability and 6.6% for risk aversion.

**Testing competing models** I study four models to describe mutual fund returns, of which three are structural (Section 3.1, 3.2, and 6) and one is reduced-form, namely performance regressions (Appendix A). A valid question is whether the status model statistically improves the other three models. Since the relative-return model of Section 3.1 is nested only if the asset distribution is uniform (which is inconsistent with the data) and the performance regressions are non-nested, I use the test developed in Vuong (1989) to compare non-nested models (Appendix F).

[Table 9 about here.]

I perform the tests at the manager level for significance levels of 5% and 10%. Table 9 reports the averages across all managers in a particular style. The status model is favored if the average number of rejections exceeds the 5% or 10% significance level. The test results provide a clear ranking of the models. First, all three competing models are rejected in favor of the status model. It is important to note that the status model is also favored over performance regressions. This implies that the conditional distribution of the status model provides a better description of fund returns than performance regressions for which the conditional and unconditional distributions coincide. Therefore, the status model is able to capture important dynamics of mutual fund strategies that performance regressions cannot. Second, the rejection rates are highest for relative-return preferences, followed by preferences for assets under management. The reduced-form performance regression are most competitive, but are still rejected too often in favor of the status model. In conclusion, there is strong statistical support in favor of the status model.

**The fraction of skilled managers** Measuring mutual fund performance has been of great interest to both academics and practitioners. The recent view contends that there are only small number of fund managers who are able to recover their costs. For this group of managers, performance actually persists. Knowing which fraction of the managers possesses skill is key because most investors base their investment in active funds on this premise.\(^{53}\) My approach provides a fresh look at this debate as the controversy

\(^{53}\)Alternatively, investors may choose to invest in mutual funds for time considerations only. Mamaysky and Spiegel (2002) argue that investors can allocate their capital to mutual funds to complete the static investment opportunity set with dynamic strategies even if the dynamic strategies require no private information.
stems from the large uncertainty surrounding the estimates of alpha. Structural models
of delegated management lead to sharper estimates of managerial ability as the cross-
equation restrictions allow me to extract estimates of ability from the volatility of mutual
fund returns.

Figure 1 displays the empirical distribution of mutual fund alphas following from
performance regressions (top row) and the status model (bottom row). The left figures
portray the fund alphas before costs, whereas the right figures subtract the fund’s
expenses. Before fees and expenses, the average alpha is 1.57bp for both performance
regressions and for the status model. These numbers change to an average of 2bp after
costs. This implies that the average alphas are zero, consistent with the prior literature.
However, the distribution of alphas from the structural model is much narrower. The
fraction of alphas that exceed zero is therefore substantially smaller. For the performance
regressions, 46.03% of the after-costs alphas exceed zero, while this number drops to
30.95% in the case of the status model.

The large number of managers that produce fund alphas that exceed fees and expenses
reflects sampling error. I now study the managers who are able to reliably recuperate their
costs. Statistical significance is determined using the asymptotic standard errors.
This is slightly more involved for the status model. To compute the standard errors for the
status model, I first compute the fund’s alpha as \( \alpha(\Theta) = x_{A0}(\hat{\Theta}_A)\hat{\lambda}_A\hat{\sigma}_A \), which I in turn
average over all fund years. I compute the standard errors by applying the delta theorem
as \( \sqrt{T} (\hat{\Theta}_A - \Theta_A) \rightarrow^d N(0, \Sigma_\Theta) \). For each manager, I test whether the after-costs alpha
significantly exceeds zero at the 5% level. For the performance regressions, I can reject
the null in 9.43% of the cases, while this number increases to 13.12% for the status model.
This result is important because it shows that despite the fact that fewer managers recover
their costs based on point estimates (31% instead of 46%), the increased efficiency of the
estimator implies that more fund managers robustly display skill (13% instead of 9%). The
fraction of skilled managers increases by almost 40%. Kosowski, Timmermann, Wermers,
and White (2006) show that even a small number of skilled managers can be economically
important, which underscores the economic relevance of this exercise.

\footnotesize

54 Baks, Metrick, and Wachter (2001) address the question in a Bayesian way, while Kosowski,
Timmermann, Wermers, and White (2006) use a bootstrap analysis to compute the correct, finite-sample
distribution of the estimates. The former paper finds that even skeptical investors may allocate part of
their capital to active management, and the latter paper estimates the fraction of skilled managers to be
about 10%.

55 Note that the risk-adjustment is only via the style benchmark. The results typically look somewhat
worse if one also corrects using a four-factor model. Also, I require three years of return data to estimate
the models, which introduces a survivorship bias. The results in Kosowski, Timmermann, Wermers,
and White (2006) suggest, however, that this effect may be small.

56 Alternatively, I could bootstrap the standard errors as in Kosowski, Timmermann, Wermers,
and White (2006) to construct the finite-sample distribution of the test statistics. This would require frequent
re-sampling and re-estimation of the structural model, which is computationally infeasible.

57 The results are very similar if I sample the fund’s alpha at a monthly frequency and subsequently
average it over all fund years.
Table 10 about here.

To conclude, Table 10 depicts the fraction of managers that reliably recover their costs and expenses by investment style. Skilled managers are concentrated in the small/growth-oriented styles. For most investment styles, the structural estimation results in a more rosy view of ability in the mutual fund industry.

**Cross-sectional stability of ability and risk aversion**  A subset of fund managers in my dataset controls multiple funds belonging to different investment styles. This provides an opportunity to study the stability of ability and risk aversion estimates holding constant the economic environment. Obviously, it may be that a manager is more skilled in the large/value style than in small/growth or vice versa. Likewise, there may be disparity in fund sizes, which induces differences in risk aversion across styles. It would nevertheless be reassuring to detect a positive relationship across styles.

The sample contains 105 style matches for which I have at least three years of data. The resulting scatter plot of risk aversion and ability is displayed in Figure 6. The correlation in risk aversion estimates across styles equals 65.0%; it equals 32.9% for managerial ability. Both are significantly positive at the 1%-level. It implies that risk aversion estimates are stable across styles, which is important. Managerial ability is less stable, which may reflect that risk aversion is more an attribute of the manager, while ability is more asset-class specific.

Figure 6 about here.

**Persistence of managerial ability**  I study whether measures of ability are useful to predict future performance. I use either performance regressions or the status model to sort funds into quartiles based on a three-year selection period. I then form equally-weighted portfolios of funds in a particular quartile and compute the portfolio return and the corresponding equally-weighted benchmark return. I hold the portfolios for one year. This leads to a return series over the full sample period, which I use to compute the annualized information ratio. As an alternative, I include the momentum factor to correct for the returns on passive strategies. Figure 7 displays the main results for only benchmark returns (Panel A) or benchmark returns and the momentum factor (Panel B).

Figure 7 about here.

Regardless of the ranking procedure, the price of risk in the selection period relates positively to the information ratio in the out-of-sample period. The difference between both approaches is small, but performance regressions relate performance monotonically in

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both periods. By comparing Panel A and Panel B, I find that accounting for momentum trading reduces the information ratio, but the main pattern in rankings is unaffected. This implies that, based on this exercise, the structural model does not ameliorate the forecasting power of fund performance.

Note that I would ideally use the status model to measure performance in the out-of-sample period. This would be most consistent with the selection year and enhance efficiency. In addition, funds take heterogeneous and time-varying amounts of active risk and passive risk. It is unclear how this affects standard performance regressions and the predictability results that I report. However, it is computationally too intensive to estimate the model for a large cross-section of fund managers over rolling samples.

**Time series of relative risk aversion and expected returns** The status model is not homogenous in assets under management. This implies that variation in both fund status and the cross-sectional asset distribution lead to variation in the coefficient of relative risk aversion. Both will move around the average coefficient of relative risk aversion across managers. The solid line in Figure 4 displays that average coefficient from 1992 to 2006. In recent equilibrium models featuring habit formation, time variation in risk aversion translates into time variation in risk premia (Campbell and Cochrane (1999)). It is therefore interesting to compare the resulting time series with the time series for expected returns, which is taken from Binsbergen and Koijen (2007). They use a present-value model to estimate the time series of expected returns and expected growth rates, which results in stronger predictors for future returns and dividend growth rates than standard predictive regressions. The dashed line corresponds to the time series of expected returns from 1992 to 2006. The two time series display a strong co-movement; their correlation equals 62.6%. This lends further credibility to the risk aversion estimates and its variation over time. I also compute the average price of active risk over the sample period (not reported). This average is very stable and varies in a range of only .05 over time.

**Correlation risk aversion and managerial ability** One empirical finding that is remarkably robust across all models is that risk aversion and managerial ability are positively correlated. Three potential mechanisms can generate this empirical regularity. First, it may simply be a genetic feature that skilled investors tend to be more conservative. Second, even if ability and risk aversion are uncorrelated in population, selection effects can lead to an increasing and concave relation between ability and managerial risk aversion, consistent with Figure 2. For expositional reasons, I focus on the model of Section 3.2, but the argument applies to all models. Consider an individual who can choose between a job in the mutual fund industry and a less risky job at a savings bank. For argument’s sake, suppose the bank provides a known and constant income $O_T$ at
\( t = T \). I assume that the manager decides which job to take based on one-period utilities, but the argument extends easily to a multi-period framework. As such, the manager compares the value function corresponding to the mutual fund industry \((A_0 = 1)\):

\[
J^{MF} = \frac{1}{1 - \gamma} \exp \left( (1 - \gamma) r + \frac{1 - \gamma}{2\gamma} (\lambda_A^2 + \lambda_B^2) \right),
\]  

with the value function induced by the outside option \(J^{OO} = \frac{1}{1 - \gamma} O_T^{1 - \gamma}\). The indifference locus reads:

\[
\bar{\lambda}_A(\gamma) = \sqrt{(\log O_T - r)2\gamma - \lambda_B^2}.
\]  

Fund managers will opt into the industry only if \(\lambda_A \geq \bar{\lambda}_A(\gamma)\). The right-hand side of (32) is increasing and concave in \(\gamma\). Hence, even when ability and risk aversion are uncorrelated in population, selection effects may lead to the relation between ability and risk aversion. A third explanation would be that the status component in the utility index implicitly proxies for career concerns. Skilled managers may act more cautiously, realizing that they have more at stake than less skilled managers. The status component of the utility function can be interpreted as a value function or continuation utility. Consistent with the prediction that skilled managers are more status concerned, I find that the correlation between \(\lambda_A\) and \(\sigma^2\) is positive and equals 57\%. I show in Section 3.2 that a model with career concerns is unable to affect optimal policies for the relevant range of risk aversion. One reason why this model has so little bite is a peso-problem in measuring career concerns. If all managers avoid particular actions as they know this will induce demotion, then the model of demotion probabilities needs to extrapolate into this region and underestimate true career concerns faced by fund managers.

8 Optimal delegated investment management

In this section, I demonstrate that the joint distribution of risk preferences and managerial ability is a key input for the optimal allocation to active and passive management. I consider three types of investors who allocate capital to cash, a passive style index, and actively-managed mutual funds.\(^{59}\) They differ in how they account for heterogeneity. The first investor optimally accounts for heterogeneity and uses the structural model of Section 6. The second investor ignores heterogeneity and uses sample average values of the parameters that govern ability and risk preferences. The third investor accounts for heterogeneity, but uses performance regressions to characterize heterogeneity. I quantify

\(^{59}\)I focus in the section on the problem of delegated instead of decentralized investment management. The current approach can easily be extended to study the decentralized problem with different investment styles (Binsbergen, Brandt, and Koijen (2007)).
the economic importance of heterogeneity by measuring the utility costs that are induced by these sub-optimal strategies.

**The economic importance of heterogeneity** The investor is assumed to have power-utility preferences with a coefficient of relative risk aversion $\gamma_I$:

$$\max_{\pi} E_0 \left[ \frac{1}{1 - \gamma_I} W_T^{1-\gamma_I} \right],$$

where $W_T$ denotes time-$T$ wealth. For tractability, I assume that the investor implements a constant-proportions strategy, $\pi$, so that wealth evolves as:

$$dW_t = W_t \left( r + \pi' \Sigma_t \Lambda \right) dt + W_t \pi' \Sigma_t dZ_t,$$

with $\Sigma_t \equiv (\sigma_P, \Sigma' x_t(\Theta_A))'$ and $x_t(\Theta_A)$ indicates the manager’s optimal (dynamic) strategy. The returns produced by the active manager depend on her ability and risk preferences, summarized in $\Theta_A$. A key feature of (33) is that the expectation operator not only integrates out financial risks, but also uncertainty about the manager’s type, that is, her ability and risk preferences. Denote the return distribution conditional on the manager’s parameters by $f_{R|\Theta_A}(R_T | \Theta_A)$, and the distribution over the managers’ parameters by $f_{\Theta_A}(\Theta_A)$. Their joint distribution is denoted by $f_{R,\Theta_A}(R_T, \Theta_A) \equiv f_{R|\Theta_A}(R_T | \Theta_A) \cdot f_{\Theta_A}(\Theta_A)$. The problem in (33) can then be reformulated as:

$$\max_{\pi} \int_{\Theta_A} \int_{R} \frac{1}{1 - \gamma_I} W_T^{1-\gamma_I} f_{R|\Theta_A}(R_T | \Theta_A) dR \right] f_{\Theta_A}(\Theta_A) d\Theta_A = \max_{\pi} J(\pi, T, \Theta_A) f_{\Theta_A}(\Theta_A) d\Theta_A = \max_{\pi} \tilde{J}(\pi, T),$$

with $J(\pi, T, \Theta_A)$ denoting the value function corresponding to the strategy $\pi$ when the manager’s parameters are given by $\Theta_A$. To quantify the utility costs induced by ignoring heterogeneity in ability and preferences, I compare the optimal solution to (35), denoted by $\pi_1$, to the optimal strategy of an investor who employs the average parameters instead, denoted by $\pi_2 \equiv \arg \max_{\pi} J(\pi, T, E[\Theta_A])$. The utility costs are computed as the reduction in certainty-equivalent wealth:

$$CEQ(\pi_1, \pi_2) \equiv \left( \frac{\tilde{J}(\pi_2, T)}{\tilde{J}(\pi_1, T)} \right)^{\frac{1}{1-\gamma_I}} - 1.$$
Appendix G provides details on how to compute $CEQ(\pi_1, \pi_2)$. If the investor uses performance regressions, the optimal strategy can be determined along the same lines. The investor allocates her capital to the style benchmark and the managed portfolio with dynamics:

$$dA_t = A_t(r + \beta\sigma_P\lambda_P + \alpha) dt + A_t\beta\sigma_P dZ_t^P + A_t\sigma_\varepsilon dZ_t^A,$$

where the coefficients $\Theta_A = (\alpha, \beta, \sigma_\varepsilon)$ are drawn from the empirical distribution. In this case, heterogeneity is reflected by the trivariate empirical distribution of $\Theta_A$, which the manager integrates out as in (35). Further details are provided in Appendix G.

**Empirical results** Panel A of Figure 8 displays the optimal allocation to the style benchmark (left) and mutual funds (right) for the three investors in the large/value style. The horizontal axis displays the coefficient of relative risk aversion ranging from one to 10, the vertical axis the optimal allocation. First, the investor who ignores heterogeneity in ability and risk aversion (blue solid line) invests more aggressively in mutual funds than the optimal strategy (green dashed line). Heterogeneity acts as a source of background risk or Bayesian parameter uncertainty, which increases the effective risk aversion of the investor (Gollier and Pratt (1996)). Second, if the investor uses performance regressions, she becomes overly conservative. The estimated cross-sectional distribution is a convolution of true heterogeneity and estimation error. For performance regressions (red dotted line), the latter component dominates and the manager overestimates heterogeneity. The utility costs corresponding to both sub-optimal strategies are depicted in Panel B for the large/value style (left) and small/growth style (right). First, the utility costs are economically large, in particular for the small/growth style. For the strategy that ignores heterogeneity (blue solid line), the utility costs can be as high as 380bp per year, while the costs peak above 50bp for the large/value style. Second, the use of performance regressions somewhat mitigates the costs (red dotted line). For the small/growth style, the maximum costs drop to 130bp. In sum, the utility costs are substantial, suggesting that it is crucial for investors to take into account the heterogeneity in ability and risk preferences of their mutual fund managers.

[Figure 8 about here.]

9 Conclusions

I use structural models of delegated portfolio management to recover the cross-sectional distribution of managerial ability and risk aversion. I develop a new likelihood-based estimation procedure to analyze such models empirically. By imposing the cross-equation restrictions that are implied by the structural models, I show that both managerial ability
and risk preference parameters can be estimated from the volatility instead of the mean of fund returns. As such, I obtain sharp estimates of managerial ability, an issue that has plagued the performance literature ever since Jensen (1968). I find that 31% of the managers have positive alphas after costs. Once sampling uncertainty is taken into account, this number drops to 13%.

Two standard models of delegated portfolio management result in economically implausible estimates of either managerial ability or risk aversion. Therefore, I develop a new model that imputes a concern for the relative position in the cross-sectional asset distribution into the preferences of the manager. I find that this model describes fund returns better than the other structural models and reduced-form performance regressions. The resulting estimates of managerial ability and risk aversion are plausible.

The main empirical results can be summarized as follows. First, risk aversion and managerial ability are both right-skewed, and there is more heterogeneity in risk preferences than in ability. Second, risk aversion and managerial ability are positively related. Skilled managers are more cautious. I show that this result can be explained by selection arguments or career concerns. Third, only a small fraction of the cross-sectional variation can be related to observable characteristics, which points to considerable unobserved heterogeneity. Fourth, the model endogenously generates time variation in risk aversion. I find that this time variation strongly co-moves with the equity risk premium; their correlation is 62%.

My results can be extended in several directions. First, the methodology that I develop may be applied to a range of different problems that use martingale techniques. Such applications include dynamic models with strategic interaction (Basak and Makarov (2007)), ability and preferences in the hedge fund industry (Panageas and Westerfield (2007)), and dynamic corporate finance models. Second, learning about managerial ability plays a key role in many theoretical mutual fund models (for instance, Berk and Green (2004) and Dangl, Wu, and Zechner (2007)). The approach in this paper implies that the individual investor’s learning mechanism is much more efficient if the investment problem of the manager is taken into account. It seems therefore interesting to revisit the role of learning when explaining phenomena in mutual fund markets using the approach advocated in this paper. Third, recent models of consumption-based asset pricing use the household’s Euler condition to price the assets. However, most capital invested in financial markets flows through the hands of delegated portfolio managers. Several recent studies show that the manager can become the inframarginal agent that prices the assets (for instance He and Krishnamurthy (2006)). If this is the case, deepening our understanding of the preferences of mutual fund managers is an important component of a better understanding of asset prices. The pronounced co-movement between risk aversion and risk premia I find suggests that there is merit to this conjecture. The results in this
paper provide a first step in modeling the preferences of the managers that decide upon the optimal asset allocation on behalf of most households. Explicitly incorporating the intermediation sector in consumption-based asset pricing models is left for future research. Finally, it is interesting to explicitly model the manager’s private information as in Liu, Peleg, and Subrahmanyam (2007). Information on the manager’s returns and portfolio holdings\(^{61}\) can then be used to extract information about the manager’s quality of private information and risk preferences, which builds upon recent work of Cohen, Coval, and Pastor (2005), Wermers, Yao, and Zhao (2007), and Yuan (2007).

References


\(^{61}\)Dybvig and Rogers (1997) propose a simple estimator of preference parameters based on holdings data.


A Performance regressions in continuous time

This appendix summarizes performance regressions in a continuous-time framework. Nielsen and Vassalou (2004) discuss the link between continuous-time and discrete-time performance measures. I focus on the case with one style benchmark, but extensions to multi-factor benchmark models are trivial. In continuous time, the standard performance regression reads:

$$\frac{dA_t}{A_t} - rd = \alpha dt + \beta \left( \frac{dS^B_t}{S^B_t} - rd \right) + \sigma \varepsilon dZ^A_t. \quad (A.1)$$

This implies that the dynamics of assets under management satisfy:

$$\frac{dA_t}{A_t} = (r + \alpha + \beta \sigma B \lambda_B) dt + \beta \sigma_B dZ^B_t + \sigma \varepsilon dZ^A_t. \quad (A.2)$$

For comparability with the structural models, I perform a two-step procedure in which I compute the likelihood of fund returns, $r^A_{\kappa \times T}$, conditional on benchmark returns, $r^B_{\kappa \times T}$, and the passive parameters, $\Theta_B$, that are estimated in the first step. $\kappa$ denotes the number of fund years available for a particular manager-fund combination. I define $\Theta_C \equiv \{\alpha, \beta, \sigma\}$.

The performance parameters $\Theta_C$ are estimated by maximizing the log-likelihood:

$$\max_{\Theta_C} L \left( r^A_{\kappa \times T} \mid r^B_{\kappa \times T}; \Theta_C, \Theta_B \right) = \max_{\Theta_C} \sum_{t=h}^{\kappa T} \ell \left( r^A_t \mid r^B_t; \Theta_C, \Theta_B \right). \quad (A.3)$$

Given the log-normal structure of the financial market in Section 2, the joint dynamics of the passive return and the mutual fund return are given by:

$$r^B_t = \left( \bar{r} + \sigma_B \lambda_B - \frac{1}{2} \sigma^2_B \right) h + \sigma_B \Delta Z^B_t, \quad (A.4)$$

$$r^A_t = \left( \bar{r} + \alpha + \beta \sigma_B \lambda_B - \frac{1}{2} \beta^2 \sigma^2_B - \frac{1}{2} \sigma^2 \varepsilon \right) h + \beta \sigma_B \Delta Z^B_t + \sigma \varepsilon \Delta Z^A_t, \quad (A.5)$$

with $h = 1/12$ because the parameters are expressed in annual terms, $\bar{r}$ the average 1-month T-bill rate over the relevant year, $\Delta y_t \equiv y_t - y_{t-h}$, and:

$$\begin{pmatrix} \Delta Z^B_t \\ \Delta Z^A_t \end{pmatrix} \sim N \left( 0_{2 \times 1}, hI_{2 \times 2} \right). \quad (A.6)$$

It therefore holds:

$$r^A_t \mid r^B_t \sim N \left( \mu_t, \sigma^2 \right), \quad (A.7)$$

with:

$$\mu_t \equiv \left( \bar{r} + \alpha + \beta \sigma_B \lambda_B - \frac{1}{2} \beta^2 \sigma^2_B - \frac{1}{2} \sigma^2 \varepsilon \right) h + \beta \left( r^B_t - \left( \bar{r} + \sigma_B \lambda_B - \frac{1}{2} \sigma^2_B \right) h \right), \quad (A.8)$$

$$\sigma^2 \equiv \sigma^2 \varepsilon h, \quad (A.9)$$

which results in the log-likelihood in (A.3).

B Career concerns and fund flows

This appendix extends the model in Section 3.2 to allow for career concerns and external fund flows. The model closely follows Chapman, Evans, and Xu (2007). Section B.1 summarizes the model, while Section B.2 provides further details on its calibration. Section B.3 derives the Bellman equation and demonstrates its homogeneity in assets under management. Section B.4 briefly summarizes the numerical procedure. The optimal strategies and results are discussed in Section B.5. Time is expressed in months in this section to simplify notation.

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B.1 The model

The dynamics of assets under management reads \( A_t = \theta_t A_{t-3} \), with \( \theta_t \):

\[
\theta_t \equiv \begin{cases} 
R_t A \exp \left( F_{t-3}(z_{t-3}) + \varepsilon_t^F \right) & \text{w.p.} \ 1 - q_{t|t-3}^P(z_{t-3}, Age_{t-3}) - q_{t|t-3}^D(z_{t-3}, Age_{t-3}) \\
\nu_P & \text{w.p.} \ q_{t|t-3}^P(z_{t-3}, Age_{t-3}) \\
\nu_D & \text{w.p.} \ q_{t|t-3}^D(z_{t-3}, Age_{t-3}) 
\end{cases}
\]

with \( R_t^A \equiv A_t/A_{t-3} \) and \( q_{t|t-3}^P (q_{t|t-3}^D) \) the probability that the manager will be promoted (demoted) at time \( t \) conditional upon the information at time \( t - 3 \). The change in assets under management in case of promotion (demotion) is denoted by \( \nu_P > 1 (\nu_D < 1) \). \( F_{t-3} \) denotes the expected fund flow and \( \varepsilon_t^F \sim N(0, \sigma_F^2) \) is the idiosyncratic risk present in fund flows.\(^{62}\) Fund flows and promotion/demotion probabilities depend on past fund performance via \( z_t \), which evolves as:

\[
z_t = \rho_0 z_{t-3} + \rho_1 (R_t^A - R_t^B), \tag{B.1}
\]

and forms a weighted average of past relative performance. Promotion and demotion probabilities furthermore depend on the number of years that the manager is active in the mutual fund industry, \( Age_t \). Appendix B.2 describes the exact functional forms and calibration in full detail, derives the value function, and provides further details on the numerical method. The decision frequency is quarterly and I assume that the manager follows a constant-proportions strategy at intermediate points in time.\(^{63}\)

B.2 Model specification and calibration details

Fund flows are modeled as a third-order polynomial in past performance:

\[
F_t = \delta_0 + \sum_{i=1}^{3} \delta_i \cdot (z_t)^i,
\]

of which the parameters are given in the Table 11. It also reports the idiosyncratic volatility of fund flows (\( \sigma_F \)) and the increase (decrease) in assets under management, \( \nu_P (\nu_D) \), in case of promotion (demotion).

[Table 11 about here.]

The promotion and demotion probabilities are represented by a multinomial logit model:

\[
q_{t|t-3}^P = \frac{\exp(\varphi_{P} x_t)}{1 + \exp(\varphi_{P} x_t) + \exp(\varphi_{D} x_t)}, \quad q_{t|t-3}^D = \frac{\exp(\varphi_{D} x_t)}{1 + \exp(\varphi_{P} x_t) + \exp(\varphi_{D} x_t)},
\]

with \( x_t \equiv (z_t, Age_t) \). The parameters that describe the promotion and demotion probabilities are depicted in Table 12. The variable \( Age_t \) indicates the period that the manager is active in the industry and is used to compute the dummy variables in Table 12.

[Table 12 about here.]

The performance variable \( z_t \) evolves according to (B.1). The parameters for the value manager equal: \( \rho_0 = 0.51 \) and \( \rho_1 = 0.178 \); for growth managers: \( \rho_0 = 0.59053 \) and \( \rho_1 = 0.15309 \).

B.3 Homogeneity of the value function

The manager’s problem is given by:

\[
\max_{\{x_0,x_3,x_6,x_9\}} E_0 \left[ \frac{1}{1 - \gamma} A_t^{1-\gamma} \right]. \tag{B.2}
\]

\(^{62}\)Uncertainty in fund flows is assumed to be independent of the other financial risks.

\(^{63}\)This assumption follows Campbell and Viceira (1999) and Campbell, Chan, and Viceira (2003). Under this assumption, the investor can hold long and short positions without rendering the strategy inadmissible.
Define the manager's value function as:

$$J(A_t, z_t, t) = \max_{x_t} E_t \left[ \frac{1}{1 - \gamma} A_t^{1-\gamma} \right],$$  \hspace{1cm} (B.3)$$

with $J(A_T, z_T, T) \equiv A_T^{1-\gamma}$. The value function satisfies the Bellman equation:

$$J(A_t, z_t, t) = \max_{x_t} E_t [J(A_{t+3}, z_{t+3}, t + 3)].$$  \hspace{1cm} (B.4)$$

I show that value function has the property:

$$J(A_t, z_t, t) = A_t^{1-\gamma}J(1, z_t, t) = A_t^{1-\gamma}\tilde{J}(z_t, t),$$  \hspace{1cm} (B.5)$$

with $\tilde{J}(z_t, t) \equiv J(1, z_t, t)$. The proof is by induction. At $t = T$, the property trivially holds. Suppose that (B.5) also holds for $s, t < s \leq T$, then it follows:

$$J(A_t, z_t, t) = \max_{x_t} E_t [J(A_{t+3}, z_{t+3}, t + 3)] = \max_{x_t} E_t [A_{t+3} \tilde{J}(z_{t+3}, t + 3)]$$

$$= \max_{x_t} E_t \left[ A_{t+3}^{1-\gamma} \left( 1 - q_{t+3|t}^{P} - q_{t+3|t}^{D} \right) E_t \left[ \tilde{J}(z_{t+3}, t + 3) \right] \right]$$

$$= \max_{x_t} A_t^{1-\gamma} \tilde{J}(z_t, t),$$  \hspace{1cm} (B.6)$$

which establishes the homogeneity of the value function.

### B.4 Numerical procedure

The optimal allocation to the style benchmark and active portfolio are determined by means of numerical dynamic programming. Since the model is specified at a quarterly frequency, the manager needs to make four investment decisions per annum, conditional on prior performance, $x_t^*(z_t)$. The optimal policies are determined using the Bellman equation derived in the previous section. The model features three shocks; two for the financial market and one for idiosyncratic fund flows. However, the latter shock only enters the value function in case the manager is not demoted nor promoted:

$$\left( 1 - q_{t+3|t}^{P} - q_{t+3|t}^{D} \right) E_t \left[ R_{t+3}^{A(1-\gamma)} \exp \left( F_t(1 - \gamma) + \varepsilon_{t+3}^{F}(1 - \gamma) \right) \tilde{J}(z_{t+3}, t + 3) \right],$$

see (B.6). As a result, this shock can be integrated out analytically:

$$\left( 1 - q_{t+3|t}^{P} - q_{t+3|t}^{D} \right) E_t \left[ R_{t+3}^{A(1-\gamma)} \exp \left( F_t(1 - \gamma) + \varepsilon_{t+3}^{F}(1 - \gamma) \right) \tilde{J}(z_{t+3}, t + 3) \right] = \left( 1 - q_{t+3|t}^{P} - q_{t+3|t}^{D} \right) \exp \left( F_t(1 - \gamma) + \frac{1}{2}(1 - \gamma)^2 \sigma_F^2 \right) E_t \left[ R_{t+3}^{A(1-\gamma)} \tilde{J}(z_{t+3}, t + 3) \right],$$

which implies that only two shocks are left. I use bivariate Gaussian quadrature to compute all expectations that arise (Tauchen and Hussey (1991)). For the performance variable $z_t$, I form an equally-spaced grid on $[-0.90, 0.90]$ with steps of size 0.05. I solve for the optimal strategy and the implied value function at each of the grid points. Refining the step sizes does not change the results. The value function in between grid points is interpolated via cubic-spline interpolation.

---

For notational convenience, I omit the arguments of the promotion/demotion probabilities and expected fund flows.
B.5 Optimal strategies

Figure 9 depicts the optimal investment strategy of a large/value manager that works between 3-7 years in the mutual fund industry. The top figure displays the manager’s holdings of the benchmark asset and the bottom figure the optimal allocation to the active portfolio. The axes correspond to the manager’s coefficient of relative risk aversion ($\gamma$) and prior relative performance ($z_t$). The downward sloping plane corresponds to the problem in which there are no incentives ($\theta_t = R^A_t$), while the non-monotone plane describes the optimal solution to the full-fledged model. Three aspects are worth mentioning. First, the optimal allocations are non-monotone in risk aversion. The intuition is as follows. As (B.6) shows, the value function has three components corresponding to promotion, demotion, and no career change. The demotion event results in a drop in assets in which case the value function is pre-multiplied by $\nu^{1-\gamma}$. Since $\nu_D < 1$, this number turns large when $\gamma$ increases, and the manager effectively minimizes the demotion probability, which is virtually linear in performance. As a result, the manager acts as if she is more aggressive. Second, for low levels of risk aversion, incentives do not affect the optimal investment strategy. This region turns out to be key however. Mutual funds have betas with respect to their style benchmarks that are close to one and display relatively little dispersion. In this model, this can be reconciled by either a risk aversion level that is close to $\lambda_B/\sigma_B$ in case incentives have no bite and the manager acts as an asset-only investor. Alternatively, the manager’s risk aversion is such that it exactly balances the demotion probability in the increasing region in the direction of risk aversion. The latter case can however be ruled out for different economic reasons. It is well known that older managers take more risk (Chevalier and Ellison (1997)), which in fact motivates Chapman, Evans, and Xu (2007) to study the impact of career concerns on risk-taking. Older managers are less likely to be demoted, meaning that the increase in risk-taking for more conservative managers becomes less pronounced. Hence, the model predicts that older managers to take on less risk, which clearly is at odds with the data. Therefore, I conclude that incentives, once calibrated to observed career changes and fund flows, hardly affect the optimal policies in the relevant region of risk aversion. As such, I study the reduced-form case with $\theta_t = R^A_t$ from now on for this model. Third, in the direction of past relative performance, there is a slight increase in risk-taking. This is a consequence of the convexities in fund flows. Indeed, if $F_t$ is zero, the slope in this direction is nearly zero. The economic significance is small however, which resonates with the findings of Chapman, Evans, and Xu (2007).

C Relative risk aversion in the status model

The manager’s preferences are given by:

$$E_0 \left[ u \left( A_T, \tilde{A}_T \right) \right] = E_0 \left[ \frac{\eta}{1 - \sigma_1} A_T^{1-\sigma_1} + (1 - \eta) S(1 - \sigma_2) \tilde{A}_T^{1-\sigma_1} \theta \left( \frac{A_T}{\tilde{A}_T} \right)^{1-\sigma_2} \right].$$

(C.1)

The coefficient of relative risk aversion reads:

$$RRA \left( A_T, \tilde{A}_T \right) \equiv \frac{A_T u^{(2,0)} \left( A_T, \tilde{A}_T \right)}{u^{(1,0)} \left( A_T, \tilde{A}_T \right)},$$

(C.2)

---

65 The results for the small/growth managers and different tenure stages are qualitatively similar.

---
with \( u^{(i,j)} \) denoting the \( i \)-th derivative with respect to \( A_T \) and the \( j \)-th derivative to \( \bar{A}_T \). The required derivatives are given by:

\[
\begin{align*}
  u^{(1,0)}(A_T, \bar{A}_T) &= \eta A_T^{-\sigma_1} + (1 - \sigma_2)(1 - \eta) S (1 - \sigma_2) \bar{A}_T^{-\sigma_1} \varrho \left( \frac{A_T}{A_T} \right)^{-\sigma_2} \varrho' \left( \frac{A_T}{A_T} \right) \\
  u^{(2,0)}(A_T, \bar{A}_T) &= -\sigma_1 \eta A_T^{-\sigma_1 - 1} - \sigma_2 (1 - \sigma_2) (1 - \eta) S (1 - \sigma_2) \bar{A}_T^{-\sigma_1 - 1} \varrho \left( \frac{A_T}{A_T} \right)^{-\sigma_2} \varrho' \left( \frac{A_T}{A_T} \right) \\
  &\quad + (1 - \sigma_2)(1 - \eta) S (1 - \sigma_2) \bar{A}_T^{-\sigma_1} \varrho \left( \frac{A_T}{A_T} \right)^{-\sigma_2} \varrho'' \left( \frac{A_T}{A_T} \right) \\
  &= -\sigma_1 \eta A_T^{-\sigma_1 - 1} - \sigma_2 A_T^{-\sigma_2} \varrho \left( \frac{A_T}{A_T} \right)^{-\sigma_2} \varrho'' \left( \frac{A_T}{A_T} \right) \\
  &\quad + \sigma_2 A_T^{-\sigma_2} \varrho' \left( \frac{A_T}{A_T} \right),
\end{align*}
\]

which implies that the Arrow-Pratt measure of relative risk aversion reads:

\[
RRA \left( A_T, \bar{A}_T \right) = \frac{\sigma_1 \eta A_T^{-\sigma_1} + (1 - \sigma_2)(1 - \eta) S (1 - \sigma_2) \bar{A}_T^{-\sigma_1} \varrho \left( \frac{A_T}{A_T} \right)^{-\sigma_2} \varrho' \left( \frac{A_T}{A_T} \right)}{\eta A_T^{-\sigma_1} + (1 - \sigma_2)(1 - \eta) S (1 - \sigma_2) \bar{A}_T^{-\sigma_1} \varrho \left( \frac{A_T}{A_T} \right)^{-\sigma_2} \varrho' \left( \frac{A_T}{A_T} \right)}
\]

\[
= \omega(a_T) \sigma_1 + (1 - \omega(a_T)) \left[ \sigma_2 \frac{\varrho' (a_T) a_T}{\varrho (a_T)} - \frac{\varrho'' (a_T) a_T}{\varrho (a_T)} \right],
\]

with:

\[
\omega(a_T) = \frac{\sigma_1 A_T^{-\sigma_1}}{\eta a_T^{-\sigma_1} + (1 - \sigma_2)(1 - \eta) S (1 - \sigma_2) \varrho (a_T)^{-\sigma_2} \varrho' (a_T)}.
\]

Note that both \( \varrho > 0 \) and \( \varrho' > 0 \), which implies \( \omega(a_T) \in [0, 1] \).

### D The role of \( \sigma_1 \) in passive risk-taking

I show in this appendix that \( \sigma_1 \) controls passive risk taking in the status model of Section 6. I solve for the optimal strategy using the martingale method, which relates to Basak, Pavlova, and Shapiro (2007b). In the martingale approach, I first solve for the optimal terminal asset level. The optimal investment strategy is then given by the strategy that replicates this terminal claim.

Using the homogeneity property of the value function, the manager’s problem can be reformulated as:

\[
\max_{R_T^A \geq 0} E_0 \left[ \frac{\eta}{1 - \sigma_1} \left( a_0 R_T^A \right)^{1 - \sigma_1} + (1 - \eta) S (1 - \sigma_2) R_T^B(1 - \sigma_1) \varrho_T \left( a_0 \frac{R_T^A}{R_T^B} \right)^{1 - \sigma_2} \right],
\]

with \( R_T^A \equiv A_T/A_0 \). The optimization is subject to the static budget constraint (recall that \( \varphi_0 = 1 \)):

\[
E_0 \left[ R_T^A \varphi_T \right] = 1.
\]

The corresponding Lagrangian reads, with \( \xi \) denoting the Lagrange parameter:

\[
Z(R_T^A, R_T^B, \xi) = \frac{\eta}{1 - \sigma_1} \left( a_0 R_T^A \right)^{1 - \sigma_1} + (1 - \eta) S (1 - \sigma_2) R_T^B(1 - \sigma_1) \varrho_T \left( a_0 \frac{R_T^A}{R_T^B} \right)^{1 - \sigma_2} - \xi \varphi_T R_T^A
\]

\[
\propto \frac{\eta}{1 - \sigma_1} \left( a_0 \frac{R_T^A}{R_T^B} \right)^{1 - \sigma_1} + (1 - \eta) S (1 - \sigma_2) \varrho_T \left( a_0 \frac{R_T^A}{R_T^B} \right)^{1 - \sigma_2} - \xi \varphi_T \frac{R_T^A}{R_T^B}
\]

\[
= \frac{\eta}{1 - \sigma_1} \left( a_0 \frac{R_T^A}{R_T^B} \right)^{1 - \sigma_1} + (1 - \eta) S (1 - \sigma_2) \varrho_T \left( a_0 \frac{R_T^A}{R_T^B} \right)^{1 - \sigma_2} - \xi \varphi_T \frac{R_T^A}{R_T^B}
\]

\[
= \tilde{u} \left( R_T^{AB} \right) - \xi \varphi_T R_T^{AB},
\]

(D.3)
where last equality defines \( \tilde{u}(\cdot) \), \( R_{T}^{AB} = R_{T}^{A} / R_{T}^{B} \), and I define:

\[
\tilde{\varphi}_T \equiv \varphi_T \left( R_{T}^{B} \right)^{\sigma_1}.
\] (D.4)

This change of variables shows that I can equivalently optimize over \( R_{T}^{AB} \). One complication is that the objective function may not be globally concave in \( R_{T}^{AB} \) if \( \sigma_2 < 0 \) or if \( g'' > 0 \). Hence, standard first-order conditions are not sufficient. The standard approach is to construct the concavification of \( \tilde{u}(R_{T}^{AB}) \) (Carpenter (2000), Cuoco and Kaniel (2006), and Basak, Pavlova, and Shapiro (2007b)), which is the smallest concave function that dominates \( \tilde{u}(R_{T}^{AB}) \). I call the concavified function \( \hat{u}(R_{T}^{AB}) \). Details on the construction of this function are provided in Appendix E.2. The resulting optimization problem is given by:

\[
\max_{R_{T}^{AB}} \hat{u}(R_{T}^{AB}) - \xi \tilde{\varphi}_T R_{T}^{AB}.
\] (D.5)

Denote the optimal relative terminal asset level by:

\[
R_{T}^{AB*} = f(\xi \tilde{\varphi}_T),
\] (D.6)

which is decreasing in \( \varphi_T \) as can be shown using the techniques in Basak, Pavlova, and Shapiro (2007b). Equipped with the optimal terminal relative return, I can compute the time-\( t \) return on assets:

\[
R_{t}^{A*} = E_{t} \left[ \frac{\varphi_T R_{t}^{B} f(\xi \tilde{\varphi}_T)}{\varphi_t} \right]
= R_{t}^{B} E_{t} \left[ \frac{\varphi_T R_{t}^{B} f(\xi \tilde{\varphi}_T)}{\varphi_t} \right]
= R_{t}^{B} E_{t} \left[ \frac{\varphi_T \left( R_{t}^{B} / R_{t}^{B} \right)^{1-\sigma_1} f(\xi \tilde{\varphi}_T)}{\varphi_t} \right]
= R_{t}^{B} E_{t} \left[ \left( R_{t}^{B} / R_{t}^{B} \right)^{1-\sigma_1} E_{t} \left[ \frac{R_{t}^{B} / R_{t}^{B}}{E_{t} \left( R_{t}^{B} / R_{t}^{B} \right)^{1-\sigma_1}} \frac{\tilde{\varphi}_T f(\xi \tilde{\varphi}_T)}{\varphi_t} \right] \right]
= R_{t}^{B} E_{t} \left[ \left( R_{t}^{B} / R_{t}^{B} \right)^{1-\sigma_1} E_{t}^{G} \left[ \frac{\varphi_T \tilde{\varphi}_T f(\xi \tilde{\varphi}_T)}{\varphi_t} \right] \right],
\] (D.7)

in which I change the measure to an equivalent measure \( G \) via the Radon-Nikodym derivative:

\[
\frac{dG}{d\mathbb{P}} = \left( \frac{R_{t}^{B} / R_{t}^{B}}{E_{t} \left( R_{t}^{B} / R_{t}^{B} \right)^{1-\sigma_1}} \right).
\] (D.8)

All expectations under the equivalent measure \( G \) are denoted by \( E_{t}^{G} [\cdot] \), while \( \mathbb{P} \)-expectations are denoted by \( E_{t} [\cdot] \). Note that the first expectation in (D.7) is a deterministic function of the remaining investment horizon, \( T - t \). Due to the Markovianity of \( (\varphi_t)_{t \geq 0} \), it holds:

\[
R_{t}^{A*} = R_{t}^{B} g(\xi, \tilde{\varphi}_t),
\] (D.9)

with:

\[
g(\xi, \tilde{\varphi}_t) = E_{t} \left[ \left( R_{t}^{B} / R_{t}^{B} \right)^{1-\sigma_1} E_{t}^{G} \left[ \frac{\tilde{\varphi}_T f(\xi \tilde{\varphi}_T)}{\varphi_t} \right] \right].
\] (D.10)

The last step is to compute the optimal investment strategy, \( x_{t}^{*}(\varphi_t) \), which is the replicating portfolio of \( R_{t}^{A*} \). To this end, I match the diffusion term of \( R_{t}^{A} \), which is given by \( R_{t}^{A} x^* \Sigma \Sigma dZ_t \), with the one of \( R_{t}^{A*} \). The latter diffusion term takes the form (Ito’s lemma):

\[
\left( \frac{\partial g(\xi, \tilde{\varphi}_t)}{\partial \tilde{\varphi}_t} \right) R_{t}^{B} \tilde{\varphi}_t \left[ -A' + \sigma_1 \Sigma \right] dZ_t + R_{t}^{A*} \xi \Sigma dZ_t,
\] (D.11)
which leads to:
\[
x^*_t(\tilde{\phi}_t) = \left( x^B_t(\tilde{\phi}_t) \right)_{x^*_t(\tilde{\phi}_t)} = e_1 - \sigma_1 \left( \frac{\partial g(\xi, \tilde{\phi}_t)}{\partial \tilde{\phi}_t} \right) \frac{\tilde{\phi}_t}{\tilde{\phi}_t} R_{t}^{AB*} \left[ \frac{1}{\sigma_1} \Sigma^{-1} \Lambda - e_1 \right],
\]
with \( e_1 \) denoting the first unit vector. Since \( \tilde{u}(R_{t}^{AB}) \) is increasing and concave in \( R_{t}^{AB} \), it holds that \( g(\xi, \tilde{\phi}_t) \) is monotonically decreasing in \( \tilde{\phi}_t \). As a result:
\[
- \sigma_1 \left( \frac{\partial g(\xi, \tilde{\phi}_t)}{\partial \tilde{\phi}_t} \right) \tilde{\phi}_t R_{t}^{AB*},
\]
is positive. If \( \sigma_1 > \lambda_B/\sigma_B \), then \( x^B_t(\tilde{\phi}_t) < 1 \), while, by contrast, \( x^B_t(\tilde{\phi}_t) > 1 \) if \( \sigma_1 < \lambda_B/\sigma_B \). This implies that the manager can increase or decrease the benchmark weight if she wants to deviate from the herd. It depends on \( \sigma_1 \) how the manager deviates and implies that \( \sigma_1 \) controls passive risk-taking. Quantitatively, the deviation also depends on \( \sigma_2 \), which affects \( g(\xi, \tilde{\phi}_t) \). Lower values of \( \sigma_2 \) will make the utility index more convex, thereby enlarging the risk-shifting region and increasing the manager’s tilt away from the benchmark.

The two special cases in which the preferences reduce to the preference specifications in Section 3 can be identified easily. If \( \eta = 0 \), it holds:
\[
- \sigma_1 \left( \frac{\partial g(\xi, \tilde{\phi}_t)}{\partial \tilde{\phi}_t} \right) \tilde{\phi}_t R_{t}^{AB*} = 1,
\]
and the optimal portfolio simplifies to:
\[
x^*_t = \frac{1}{\sigma_1} \Sigma^{-1} \Lambda.
\]
When \( \eta = 1 \), the asset distribution is uniform, and \( \sigma_1 = 1 \), it holds:
\[
- \sigma_1 \left( \frac{\partial g(\xi, \tilde{\phi}_t)}{\partial \tilde{\phi}_t} \right) \tilde{\phi}_t R_{t}^{AB*} = 1 \frac{1}{\sigma_2},
\]
and the optimal portfolio simplifies to:
\[
x^*_t = \frac{1}{\sigma_2} \Sigma^{-1} \Lambda + \left( 1 - \frac{1}{\sigma_2} \right) e_1,
\]
which is the optimal strategy derived in Binsbergen, Brandt, and Koijen (2007).

E Econometric approach

This appendix details the construction of the likelihood of mutual funds returns conditional on passive returns. Section E.1 provides the results for the two benchmark models in Section 3. Section E.2 constructs the likelihood for the model in Section 6 in which the manager care about their relative position in the asset distribution.

E.1 Two benchmark models

In both benchmark models, the optimal strategy is a constant-proportions strategy ((8) and (10)). This implies that the likelihood can be constructed as in Appendix A with:
\[
\alpha = x^A \sigma_A \lambda_A, \quad \beta = x^B, \quad \sigma_\varepsilon = x^A \sigma_A,
\]
and \( \Theta_C \) is replaced by \( \Theta_A = \{ \lambda_A, \gamma \} \).
E.2 Status model

Section 4 and, in more detail, Koijen (2007) uses the mapping from assets under management to the state-price density that is implied by the martingale method to construct the likelihood of fund returns. This mapping is straightforward to construct in case of a globally concave utility index (Koijen (2007)). This appendix provides the procedure for the status model in which the utility index can feature local convexities. I combine martingale techniques in the presence of local convexities (Basak, Pavlova, and Shapiro (2007b)) with the simplifications in Section D.

I outline the main procedure if there is at most one convex region. The method directly extends to multiple convex regions.

1. Check whether the Lagrangian is globally concave:

\[
\hat{u} \left( R_T^{AB} \right) - \xi \hat{\varphi}_T R_T^{AB} = \frac{\eta}{1 - \sigma_1} \left( a_0 R_T^{AB} \right)^{1 - \sigma_1} + (1 - \eta) S \left( 1 - \sigma_2 \right) \vartheta_T \left( a_0 R_T^{AB} \right)^{1 - \sigma_2} - \xi \hat{\varphi}_T R_T^{AB},
\]

(E.4)

for instance, by computing the maximum of \( \hat{u}'' \left( R_T^{AB} \right) \). If the maximum is positive, the function features local convexities; otherwise, the Lagrangian is globally concave.

2. If the objective function is not globally concave, I construct its concavification. The concavified function is the smallest function that dominates \( \hat{u}(\cdot) \), see Carpenter (2000), Cuoco and Kaniel (2006), and Basak, Pavlova, and Shapiro (2007b). This means that the convex region is replaced by a chord between \( R_1 \) and \( R_2 \), where \( R_1 \) and \( R_2 \) solve \( R_1 < R_2 \):

\[
\begin{align*}
\hat{u} (R_1) &= A + BR_1, \\
\hat{u}' (R_1) &= B, \\
\hat{u} (R_2) &= A + BR_2, \\
\hat{u}' (R_2) &= B.
\end{align*}
\]

(E.5-8)

This results in a system of four equations in four unknowns, which simplifies to:

\[
\begin{align*}
\hat{u}' (R_1) &= \hat{u}' (R_2), \\
\hat{u}' (R_1) &= \frac{\hat{u} (R_2) - \hat{u} (R_1)}{R_2 - R_1},
\end{align*}
\]

(E.9-10)

which is a system of two equations in two unknowns \( (R_1 \text{ and } R_2) \) only. The concavified function is then defined as:

\[
\begin{align*}
\hat{u} (R_T^{AB}) &= \hat{u} (R_T^{AB}) \quad \text{, if } R_T^{AB} \notin [R_1, R_2] \\
\hat{u} (R_T^{AB}) &= A + BR_T^{AB} \quad \text{, if } R_T^{AB} \in [R_1, R_2]
\end{align*}
\]

(E.11)

This function is, by construction, concave and continuously differentiable. In summary, I use \( \hat{u}(\cdot) \) if the utility index is globally concave and \( \hat{u}(\cdot) \) in case of local convexities. I will use \( \hat{u}(\cdot) \) in the remainder of the procedure, which can be replaced by \( \hat{u}(\cdot) \) if the utility index is globally concave.

3. Compute the Lagrange parameter, \( \xi \). I need to find \( \xi \) that satisfies the budget constraint: \( E_0 \left[ R_T^{AB} \hat{\varphi}_T \right] = 1 \). Appendix D, Equation (D.7) shows that the budget constraint can be written as:

\[
1 = E_0 \left[ \left( R_T^B \right)^{1 - \sigma_1} \right] E_0 \left[ \hat{\varphi}_T \left( \xi \hat{\varphi}_T \right) \right],
\]

(E.12)

with \( f(\xi \hat{\varphi}_T) \) the optimal terminal asset level relative to the benchmark \( (R_T^{AB}) \) that solves:

\[
\max_{R_T^{AB}} \hat{u} \left( R_T^{AB} \right) - \xi \hat{\varphi}_T R_T^{AB}.
\]

(E.13)

The scaled state-price density \( \hat{\varphi}_t \) is defined in (D.4). The first expectation in (E.12) can be computed analytically given the log-normal structure of the financial market:

\[
E_0 \left[ \left( R_T^B \right)^{1 - \sigma_1} \right] = \exp \left( 1 - \sigma_1 \right) \left( r + \sigma_T \lambda_T - \frac{1}{2} \sigma_1 \sigma_T^2 \right) T.
\]

(E.14)
To compute the second expectation, I use that under $G$ (by Girsanov’s theorem) it holds that (using the Radon-Nikodym derivative in (D.8)):

$$Z_t^{P,G} = Z_t^P - (1 - \sigma_1)\sigma_P t,$$

(E.15)

is a $G$-Brownian motion. $Z_t^A$ is a $P$- and $G$-Brownian motion. This implies that $\hat{\varphi}_T$ given $\hat{\varphi}_t$ can be written as ($\tau \equiv T - t$):

$$\frac{\hat{\varphi}_T}{\hat{\varphi}_t} = \exp \left( \left[ \left( \sigma_1 - 1 \right) r - \frac{1}{2} \left( \Lambda' \Lambda + \sigma_1 \sigma_P^2 \right) + \sigma_1 \sigma_P \lambda_P \right) - \left( \lambda_P - \sigma_1 \sigma_P \right) Z_t^{P,G} \right) \tau,$$

with $\Delta Z_{t:T} \equiv Z_T - Z_t$. It therefore holds:

$$\log \hat{\varphi}_T - \log \hat{\varphi}_t \sim G N \left( \mu_{\hat{\varphi},T}, \sigma^2_{\hat{\varphi},T} \right),$$

(E.16)

with:

$$\mu_{\hat{\varphi},T} \equiv \left[ \left( \sigma_1 - 1 \right) r - \frac{1}{2} \left( \Lambda' \Lambda + \sigma_1 \sigma_P^2 \right) + \sigma_1 \sigma_P \lambda_P \right) - \left( \lambda_P - \sigma_1 \sigma_P \right) \tau,$$

(E.17)

$$\sigma^2_{\hat{\varphi},T} \equiv \left( \lambda_P - \sigma_1 \sigma_P \right)^2 \tau.$$

(E.18)

The expectation in (E.14) is computed using univariate Gaussian quadrature with six points (see Tauchen and Hussey (1991)). This holds true regardless of the number of Brownian motions driving the uncertainty in the financial market.

4. Solve for the transformed state-price density at each point, $\tilde{\varphi}_t$, in time to match the observed mutual fund return:

$$R_{A,t} = R_{B,t} \mathbb{E}_t \left[ \left( \frac{R^B_t}{R^B_{t-1}} \right)^{1-\sigma_1} \right] \mathbb{E}_t^G \left[ \frac{\hat{\varphi}_T}{\hat{\varphi}_t} f (\xi, \hat{\varphi}_T) \right],$$

(E.19)

in which the expectations are computed as in the previous step. This completes the mapping from fund returns, $R^{A,T}$, to a time-series of the (transformed) state-price density, $\hat{\varphi}_T$. Then the log-likelihood contribution follows as a standard application of the change-of-variables theorem:

$$\ell \left( R^A_t \ | \ R^B_t, R^{A,t-h} \right) = \ell \left( R^A_t \ | \ R^B_t, \log \hat{\varphi}_{t-h} \right)$$

$$= \ell \left( \log \hat{\varphi}_t \ | \ R^B_t, \log \hat{\varphi}_{t-h} \right) + \log \left| \frac{\partial R^A_t}{\partial \log \hat{\varphi}_t} \right|^{-1}. \quad (E.20)$$

Note that I do not need to compute the portfolio weights explicitly, as I can compute the likelihood of assets under management directly.

Koijen (2007) provides further details on the exact implementation and explains the method in the model of Section 3.2.

F Hypothesis testing

I formally test competing models of delegated portfolio management to study which model describes the returns produced by fund managers best. I the testing procedure, I distinguish between nested and non-nested models.

Nested models If the models are nested, the likelihood-ratio test can be used to discriminate between models. Denote by $\mathcal{L}^1$ the log-likelihood corresponding the unconstrained model evaluated at the maximum-likelihood estimates, and $\mathcal{L}^0$ the log-likelihood of the constrained model. The likelihood-ratio statistic:

$$LR = 2 (\mathcal{L}^1 - \mathcal{L}^0), \quad (F.1)$$
follows under the null a chi-squared distribution with the degrees of freedom equal to the number of parameter constraints.

Non-nested models If the models are non-nested, the standard likelihood-ratio test cannot be applied. However, Vuong (1989) develops an alternative test that also uses the likelihood ratio as the main input, and which can be used to test non-nested models.\(^{66}\) The different dynamic model of delegated management are not necessarily nested. The log-likelihood of fund returns conditional on the benchmark returns corresponding to Model 1 is denoted by \(L^{(1)}(r_A^T | r_B^T; \Theta_A) = \sum_{t=h}^{T/h} \ell^{(1)}(r_{At} | r_{Bt}, r_{A0}^{t-h}; \Theta_A)\) and for Model 2 by \(L^{(2)}(r_A^T | r_B^T; \Theta_A) = \sum_{t=h}^{T/h} \ell^{(2)}(r_{At} | r_{Bt}, r_{A0}^{t-h}; \Theta_A)\). The null hypothesis reads:

\[
H_0: E_0 \left[ L^{(1)}(r_A^T | r_B^T; \Theta_A^{(1)*}) \right] = E_0 \left[ L^{(2)}(r_A^T | r_B^T; \Theta_A^{(2)*}) \right], \tag{F.2}
\]

in which \(E_0 \left[ \cdot \right]\) denotes the expectation under the true model, and \(\Theta_A^{(j)*}\) the pseudo-true parameters of Model \(j\). The null hypothesis does not require either of the models to be correctly specified. Then under \(H_0:\)

\[
\frac{L^{(1)}(r_A^T | r_B^T; \hat{\Theta}_A^{(1)}) - L^{(2)}(r_A^T | r_B^T; \hat{\Theta}_A^{(2)}) - (p - q)}{\sqrt{\hat{\omega}_T T/h}} \overset{d}{\to} N(0, 1), \tag{F.3}
\]

with \(p\) the number of parameters estimated in Model 1, \(q\) the number of parameters for Model 2, and \(\hat{\Theta}_A^{(k)}\) the maximum-likelihood estimates of Model \(j\). In addition, \(\hat{\omega}_T\) is defined as:

\[
\hat{\omega}_T \equiv \frac{1}{T/h} \sum_{t=h}^{T/h} \left[ \ell^{(1)}(r_{At} | r_{Bt}, r_{A0}^{t-h}; \Theta_A^{(1)}) - \ell^{(2)}(r_{At} | r_{Bt}, r_{A0}^{t-h}; \Theta_A^{(2)}) \right]^2
\]

\[
- \frac{1}{T/h} \sum_{t=h}^{T/h} \left[ \ell^{(1)}(r_{At} | r_{Bt}, r_{A0}^{t-h}; \Theta_A^{(1)}) - \ell^{(2)}(r_{At} | r_{Bt}, r_{A0}^{t-h}; \Theta_A^{(2)}) \right]^2. \tag{F.4}
\]

Implementation All tests will be performed at the manager level. This implies that the test will have relatively little power at a per-manager basis. However, I can take advantage of the large cross-section of managers available. If the significance level is set at 5\%, I expect to reject the null only for 5\% of the managers. If the rejection rate is considerably higher, this provides strong evidence that one of the models provides a better description of mutual fund behavior.

G Utility cost calculation

To compute the loss in certainty-equivalent wealth, I need to determine \(\pi_1, \pi_2\), and \(CEQ(\pi_1, \pi_2)\). First, to compute \(\pi_2\), I only need to integrate out financial uncertainty. This requires me to obtain the joint distribution of the benchmark, \(S_B^T\), and the state-price density \(\varphi_T\) (see Appendix E.2) to compute \(W_T\). Note that in the derivation of the optimal strategy in Appendix E.2, I require the transformed state-price density \(S_t^{\tilde{\gamma}}\varphi_t\). As such, it is sufficient to simulate \(S_B^T\) and \(\varphi_T\) to compute the payoffs of the active and passive asset, and the investor’s wealth in turn. I use Monte Carlo simulations to integrate out the financial uncertainty and I sample the manager’s strategy at a monthly frequency. To compute \(\pi_1\), I also need to integrate out heterogeneity in ability and preferences. I use the empirical distribution function that I estimate in Section 7 and the integration is replaced by summation as a result. The expectation with respect to financial risks is again approximated by Monte Carlo simulations. The value function

\[
\text{follow directly and hence } CEQ(\pi_1, \pi_2). \tag{F.5}
\]

Note that the optimal strategy depends on \(a_0\). For each manager, I take the average over the sample period available for the particular manager and use this value in the simulation exercise.

If the manager uses performance regressions, the computations can be simplified in this case as the

\(^{66}\) This test is also used in St-Amour (2006) to discriminate between structural consumption-based asset pricing models.
value function conditional on the parameters can be computed analytically and is given by \((W_0 = 1)\):

\[
J(\pi, T, \Theta_A) = \frac{1}{1 - \gamma} \exp \left( (1 - \gamma) (x' \Sigma \Lambda + r) - \frac{\gamma (1 - \gamma)}{2} x' \Sigma \Sigma' x \right),
\]

with \(\Sigma\) the volatility matrix of the passive asset and the managed portfolio.
<table>
<thead>
<tr>
<th>Mutual fund style</th>
<th>Selected benchmark</th>
<th>Fraction of observations (%)</th>
<th>Number of managers</th>
<th>Fraction of observations (%)</th>
<th>Number of managers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Years of data ≥ 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large-cap/blend</td>
<td>S&amp;P 500</td>
<td>20.1</td>
<td>714</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large-cap/value</td>
<td>Russell 1000 Value</td>
<td>11.7</td>
<td>427</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large-cap/growth</td>
<td>Russell 1000 Growth</td>
<td>11.6</td>
<td>448</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mid-cap/blend</td>
<td>Russell Mid-cap</td>
<td>10.2</td>
<td>383</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mid-cap/value</td>
<td>Russell Mid-cap Value</td>
<td>6.3</td>
<td>228</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mid-cap/growth</td>
<td>Russell Mid-cap Growth</td>
<td>13.7</td>
<td>526</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small-cap/blend</td>
<td>Russell 2000</td>
<td>7.8</td>
<td>291</td>
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<tr>
<td>Small-cap/value</td>
<td>Russell 2000 Value</td>
<td>6.2</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small-cap/growth</td>
<td>Russell 2000 Growth</td>
<td>12.4</td>
<td>477</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>100.0</td>
<td>3,694</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 1: **Number of manager-fund combinations per investment style**

The table summarizes the number and fraction of manager-fund combinations per investment style. Managers are allocated to a benchmark by performing nine regressions of fund returns in excess of the short rate on excess benchmark returns. I select the benchmark that maximizes the R-squared. The left panel displays the allocation of manager-fund combinations for the full sample, the right panel for manager-fund combinations for which at least 3 years of data is available.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St.dev.</th>
<th>Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10% 25% 50% 75% 90%</td>
</tr>
<tr>
<td>TNA ($mln)</td>
<td>1,042</td>
<td>3,757</td>
<td>22 54 174 630 2,001</td>
</tr>
<tr>
<td>Family TNA ($mln)</td>
<td>14,621</td>
<td>45,837</td>
<td>74 342 1,807 9,050 23,088</td>
</tr>
<tr>
<td>Family size</td>
<td>8.3</td>
<td>9.0</td>
<td>1.0 2.0 5.0 11.0 18.0</td>
</tr>
<tr>
<td>Expense ratio (%)</td>
<td>1.3</td>
<td>0.5</td>
<td>0.8 1.0 1.3 1.6 1.9</td>
</tr>
<tr>
<td>12B-1 fee (bp)</td>
<td>21.5</td>
<td>28.0</td>
<td>0.0 0.0 6.1 34.0 65.0</td>
</tr>
<tr>
<td>Total load (%)</td>
<td>2.1</td>
<td>2.4</td>
<td>0.0 0.0 1.0 4.7 5.5</td>
</tr>
<tr>
<td>Cash holdings (%)</td>
<td>4.2</td>
<td>4.5</td>
<td>0.0 1.0 3.1 6.3 10.2</td>
</tr>
<tr>
<td>Stock holdings (%)</td>
<td>94.9</td>
<td>5.0</td>
<td>87.8 92.4 96.2 98.4 99.7</td>
</tr>
<tr>
<td>Manager tenure (years)</td>
<td>5.2</td>
<td>4.5</td>
<td>1.2 2.1 3.9 6.8 10.6</td>
</tr>
<tr>
<td>Fund age (years)</td>
<td>11.1</td>
<td>12.5</td>
<td>2.2 3.9 6.9 12.3 27.1</td>
</tr>
<tr>
<td>Turnover (%)</td>
<td>89</td>
<td>96</td>
<td>18 36 67 113 175</td>
</tr>
</tbody>
</table>

Table 2: **Summary statistics**

The table provides the summary statistics of manager and fund characteristics. I provide summary statistics for the total net assets under management (TNA), total net assets of the fund family (as defined by Chen, Hong, Huang, and Kubik (2004)), family size (the number of funds that belong to the fund family), expense ratio, 12B-1 fees, the total load (the sum of maximum front-end load fees and maximum deferred and rear-end load fees), cash holdings as reported by the fund, stock holdings as reported by the fund (the sum of common and preferred stock), manager’s tenure, fund age, and turnover.
Panel A: Structural parameters

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>$\lambda_A$</th>
<th>$\gamma$</th>
<th>Mean</th>
<th>Std</th>
<th>10%</th>
<th>50%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_A$</td>
<td>2</td>
<td>0.10</td>
<td>0.08</td>
<td>0.10</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>0.10</td>
<td>0.08</td>
<td>0.10</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_A$</td>
<td>0.1</td>
<td>0.20</td>
<td>0.16</td>
<td>0.19</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1</td>
<td>0.20</td>
<td>0.16</td>
<td>0.19</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_A$</td>
<td>0.2</td>
<td>0.20</td>
<td>0.16</td>
<td>0.19</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>0.20</td>
<td>0.16</td>
<td>0.19</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_A$</td>
<td>0.3</td>
<td>0.30</td>
<td>0.24</td>
<td>0.29</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3</td>
<td>0.30</td>
<td>0.24</td>
<td>0.29</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Structural and reduced-form estimation of fund alphas

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>$\alpha_{ML}$</th>
<th>Mean</th>
<th>Std</th>
<th>10%</th>
<th>50%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_A$</td>
<td>2</td>
<td>0.5%</td>
<td>0.49</td>
<td>0.34</td>
<td>0.47</td>
<td>0.65</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>0.5%</td>
<td>0.49</td>
<td>0.34</td>
<td>0.47</td>
<td>0.65</td>
</tr>
<tr>
<td>$\lambda_A$</td>
<td>0.1</td>
<td>0.2%</td>
<td>0.20</td>
<td>0.14</td>
<td>0.19</td>
<td>0.26</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1</td>
<td>0.2%</td>
<td>0.20</td>
<td>0.14</td>
<td>0.19</td>
<td>0.26</td>
</tr>
<tr>
<td>$\lambda_A$</td>
<td>0.2</td>
<td>0.8%</td>
<td>0.78</td>
<td>0.34</td>
<td>0.47</td>
<td>0.65</td>
</tr>
<tr>
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<td>0.34</td>
<td>0.47</td>
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<td>1.72</td>
<td>2.44</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.8%</td>
<td>1.78</td>
<td>1.21</td>
<td>1.72</td>
<td>2.44</td>
<td>1.89</td>
</tr>
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</table>

Table 3: Simulation experiment to compare model-implied and regression-based alphas
Panel A displays the results of a simulation exercise in which I simulate the model in Section 3.1 for three years on a monthly frequency. Managerial ability takes values in $\lambda_A \in \{1, 2, 3\}$ and the coefficient of relative risk aversion takes values in $\gamma \in \{2, 5, 10\}$. The table provides the mean, standard deviation, and 10%, 50%, and 90% quantiles of the estimates across 2,500 data sets. Both models are estimated by means of likelihood, see Appendix E.1. Panel B displays the fund’s alpha that is implied by the structural model, $\alpha_{ML} = \frac{\lambda_A^2}{\gamma}$, or that follows from a standard performance regression, $\alpha_{OLS}$, see Appendix A.
Table 4: Parameter estimates for the model in Section 3.1
The table summarizes the estimation results for the model in Section 3.1. The model is estimated by means of maximum likelihood for 1,273 managers over the period 1992.1 to 2006.12 for all nine investment styles. Fund managers are included when at least three years of return data is available to estimate the models. The first two columns provides the estimates of the structural model, $\gamma$ and $\lambda_A$. Columns three to five provide the implied estimates for the coefficients of a performance regression, $\alpha$, $\beta$, and $\sigma_\varepsilon$. The last three columns report the results of standard performance regressions (Appendix A). In all cases, I report the cross-sectional mean, median, and standard deviation (St.dev.) of the estimates. The parameters are expressed in annual terms.
### Model-implied Performance regressions

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<tr>
<th></th>
<th>$\gamma$</th>
<th>$\lambda_A$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\sigma_\varepsilon$</th>
<th>Performance regressions</th>
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<tr>
<td>Mean</td>
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<td>4.16%</td>
<td>0.82% 0.96 4.10%</td>
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<td>3.82%</td>
<td>0.67% 0.96 3.75%</td>
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<td>0.11</td>
<td>1.99%</td>
<td></td>
<td>2.98% 0.11 1.97%</td>
</tr>
<tr>
<td><strong>Russell 1000 Value</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.41</td>
<td>0.26</td>
<td>1.21%</td>
<td>0.93</td>
<td>4.13%</td>
<td>0.30% 0.93 4.09%</td>
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<tr>
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<td>St.dev.</td>
<td>0.89 0.10</td>
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<td>0.12</td>
<td>0.86%</td>
<td>0.94</td>
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<td>1.26% 0.94 5.47%</td>
</tr>
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<td>5.03%</td>
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<tr>
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<td>2.53%</td>
<td>0.95</td>
<td>6.54%</td>
<td>1.11% 0.96 6.47%</td>
</tr>
<tr>
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<td>6.03%</td>
<td>0.32% 0.94 6.00%</td>
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<td>1.77%</td>
<td>0.18</td>
<td>1.74%</td>
<td></td>
<td>4.15% 0.18 1.77%</td>
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<td>0.16</td>
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<td>0.95</td>
<td>8.11%</td>
<td>0.40% 0.95 7.98%</td>
</tr>
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<td>7.86%</td>
<td>0.59% 0.95 7.76%</td>
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</tr>
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<td>5.20% 0.18 2.92%</td>
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<td>5.92%</td>
<td>1.46% 0.94 5.93%</td>
</tr>
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<td>1.19%</td>
<td>0.96</td>
<td>9.38%</td>
<td>5.49% 0.96 9.10%</td>
</tr>
<tr>
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<td></td>
<td>6.92% 0.17 4.73%</td>
</tr>
</tbody>
</table>

Table 5: **Parameter estimates for the model in Section 3.2**

The table summarizes the estimation results for the model in Section 3.2. The model is estimated by means of maximum likelihood for 1,273 managers over the period 1992.1 to 2006.12 for all nine investment styles. Fund managers are included when at least three years of return data is available to estimate the models. The first two columns provides the estimates of the structural model, $\gamma$ and $\lambda_A$. Columns three to five provide the implied estimates for the coefficients of a performance regression, $\alpha$, $\beta$, and $\sigma_\varepsilon$. The last three columns report the results of standard performance regressions (Appendix A). In all cases, I report the cross-sectional mean, median, and standard deviation (St.dev.) of the estimates. The parameters are expressed in annual terms.
\[ \sigma_1 = 4.00, a_0 = 1 \quad \sigma_1 = 3.75, \sigma_2 = 0.5 \quad \sigma_1 = 4.25, \sigma_2 = 0.5 \]

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<th>( \sigma_2 )</th>
<th>RRA</th>
<th>( x^B )</th>
<th>( x^A )</th>
<th>( \rho_0(a_0) )</th>
<th>RRA</th>
<th>( x^B )</th>
<th>( x^A )</th>
<th>( \rho_0(a_0) )</th>
<th>RRA</th>
<th>( x^B )</th>
<th>( x^A )</th>
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<td>10%</td>
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<td>92%</td>
<td>23%</td>
</tr>
<tr>
<td>0</td>
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<td>20%</td>
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<td>121%</td>
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<td>30%</td>
<td>1.10</td>
<td>123%</td>
<td>68%</td>
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<td>66%</td>
</tr>
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<td>121%</td>
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<td>40%</td>
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<td>79%</td>
<td>64%</td>
</tr>
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<td>2.10</td>
<td>100%</td>
<td>34%</td>
<td>50%</td>
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<td>120%</td>
<td>60%</td>
<td>50%</td>
<td>1.22</td>
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<td>60%</td>
</tr>
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<td>56%</td>
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<td>1.31</td>
<td>81%</td>
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<td>90%</td>
<td>1.74</td>
<td>86%</td>
<td>42%</td>
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</table>

Table 6: Fund status, risk aversion, and risk-taking
The first four columns display the optimal initial allocation to the benchmark portfolio \((x^B)\) and the active portfolio \((x^A)\) as well as the Arrow-Pratt measure of relative risk aversion for different values of \(\sigma_2\). The next four columns display the optimal initial strategies and coefficient of relative risk aversion for different initial values of assets under management, \(a_0\), expressed in terms of percentile rank \((\rho_0(a_0))\) if \(\sigma_1 = 3.75\). The last four columns provide the results for \(\sigma_1 = 4.25\). I set \(\eta = 0.005\), \(\sigma_2 = 0.5\) and \(\lambda_A = 0.15\) for the results in the last eight columns. The volatility of the active portfolio, \(\sigma_A = 20\%\). The market parameters and the parameters describing the asset distribution are calibrated on the basis of the S&P 500 so that \(\lambda_B/\sigma_B = 4\). The short rate is set to \(r = 5\%\).

<table>
<thead>
<tr>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \lambda_A )</th>
<th>RRA</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \lambda_A )</th>
<th>RRA</th>
</tr>
</thead>
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<tr>
<td>S&amp;P 500</td>
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<td>4.32</td>
<td>11.04</td>
<td>0.23</td>
<td>5.96</td>
<td>R. Mid-cap G.</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>4.09</td>
<td>0.77</td>
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<td>Median</td>
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<tr>
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<td>28.08</td>
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<td>9.27</td>
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<td>St.dev.</td>
</tr>
<tr>
<td>R. 1000 V.</td>
<td>Mean</td>
<td>6.35</td>
<td>11.44</td>
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<td>Mean</td>
</tr>
<tr>
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<tr>
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<td>28.12</td>
<td>0.24</td>
<td>7.04</td>
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<td>St.dev.</td>
</tr>
<tr>
<td>R. 1000 G.</td>
<td>Mean</td>
<td>2.18</td>
<td>9.23</td>
<td>0.22</td>
<td>4.67</td>
<td>R. 2000 V.</td>
<td>Mean</td>
</tr>
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<td>8.15</td>
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<td>5.01</td>
<td>R. 2000 G.</td>
<td>Mean</td>
</tr>
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<td>St.dev.</td>
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Table 7: Parameter estimates for the status model in Section 6
The model is estimated for by means of maximum likelihood for 1,273 managers over the period 1992.1 to 2006.12 for all nine investment styles. I also report the results across all styles (“Overall”). Fund managers are included when at least three years of return data is available to estimate the models. The first three columns provide the estimates of the structural model, \(\sigma_1\), \(\sigma_2\), and \(\lambda_A\). The last column reports the implied coefficient of relative risk aversion. \(\lambda_A\) is expressed in annual terms. In all cases, I report the cross-sectional mean, median, and standard deviation (St.dev.) of the estimates. R. abbreviates Russell, V. Value, and G. Growth.
<table>
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<th>log($\lambda_A$)</th>
<th>log($RRA$)</th>
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<td></td>
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<tr>
<td>Tenure</td>
<td>7.27%**</td>
<td>2.19</td>
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<tr>
<td>Turnover</td>
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<td>12B-1 fees</td>
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</table>

Table 8: **Heterogeneity in risk aversion and ability**
The table displays results of multiple cross-sectional regressions of managerial ability and risk aversion on observable manager and fund characteristics. The characteristics include the fund’s total net assets, the manager’s tenure, turnover, expenses, the investment in common and preferred stocks, loads, 12B-1 fees, the family’s total net assets, and the fund’s age. The cross-sectional regressions include style dummies. The standard errors used to compute t-statistics are robust to heteroscedasticity. ** indicates statistical significance at the 5% level.
Relative-return Preferences for Performance regressions

preferences assets under management

<table>
<thead>
<tr>
<th>Investment style</th>
<th>10%</th>
<th>5%</th>
<th>10%</th>
<th>5%</th>
<th>10%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>77.1%</td>
<td>68.6%</td>
<td>36.0%</td>
<td>29.8%</td>
<td>20.9%</td>
<td>14.0%</td>
</tr>
<tr>
<td>Russell 1000 Value</td>
<td>91.3%</td>
<td>86.6%</td>
<td>43.6%</td>
<td>37.6%</td>
<td>21.5%</td>
<td>12.8%</td>
</tr>
<tr>
<td>Russell 1000 Growth</td>
<td>61.7%</td>
<td>53.2%</td>
<td>33.3%</td>
<td>28.4%</td>
<td>17.7%</td>
<td>10.6%</td>
</tr>
<tr>
<td>Russell Mid-cap</td>
<td>82.5%</td>
<td>72.2%</td>
<td>34.1%</td>
<td>25.4%</td>
<td>7.9%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Russell Mid-cap Value</td>
<td>90.2%</td>
<td>89.0%</td>
<td>35.4%</td>
<td>26.8%</td>
<td>9.8%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Russell Mid-cap Growth</td>
<td>58.1%</td>
<td>47.1%</td>
<td>40.7%</td>
<td>29.1%</td>
<td>18.0%</td>
<td>8.7%</td>
</tr>
<tr>
<td>Russell 2000</td>
<td>68.2%</td>
<td>52.7%</td>
<td>36.4%</td>
<td>26.4%</td>
<td>10.0%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Russell 2000 Value</td>
<td>90.0%</td>
<td>85.0%</td>
<td>31.3%</td>
<td>25.0%</td>
<td>18.8%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Russell 2000 Growth</td>
<td>38.7%</td>
<td>31.6%</td>
<td>45.8%</td>
<td>37.4%</td>
<td>16.1%</td>
<td>12.9%</td>
</tr>
<tr>
<td>Overall</td>
<td>71.2%</td>
<td>62.9%</td>
<td>37.9%</td>
<td>30.2%</td>
<td>16.6%</td>
<td>10.3%</td>
</tr>
</tbody>
</table>

Table 9: Testing competing models
The table displays the results testing competing models to describe fund returns. The models under the null are: (i) relative-return preferences (Section 3.1), (ii) preferences for assets under management (Section 3.2), and (iii) reduced-form performance regressions (Appendix A). The alternative is the status model (Section 6). For the models that are nested, I use the likelihood-ratio test. To compare non-nested models, I use the test developed in Vuong (1989), see Appendix F. I test the models at the manager level and report the average number of rejections at either the 5% or 10% significance level. As such, a model is rejected if the average number of rejections exceeds 5% or 10%.

<table>
<thead>
<tr>
<th>Investment style</th>
<th>Performance regression</th>
<th>Status model</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>5.4%</td>
<td>7.4%</td>
</tr>
<tr>
<td>Russell 1000 Value</td>
<td>4.7%</td>
<td>6.7%</td>
</tr>
<tr>
<td>Russell 1000 Growth</td>
<td>6.4%</td>
<td>6.4%</td>
</tr>
<tr>
<td>Russell Mid-cap</td>
<td>6.3%</td>
<td>15.9%</td>
</tr>
<tr>
<td>Russell Mid-cap Value</td>
<td>2.4%</td>
<td>12.2%</td>
</tr>
<tr>
<td>Russell Mid-cap Growth</td>
<td>5.8%</td>
<td>19.2%</td>
</tr>
<tr>
<td>Russell 2000</td>
<td>23.6%</td>
<td>21.8%</td>
</tr>
<tr>
<td>Russell 2000 Value</td>
<td>2.5%</td>
<td>11.3%</td>
</tr>
<tr>
<td>Russell 2000 Growth</td>
<td>27.1%</td>
<td>21.3%</td>
</tr>
</tbody>
</table>

Table 10: Testing for the fraction of skilled managers
The table reports the fraction of managers that significantly recuperates their fees and expenses at the 5% level. I use either performance regressions or the status model to estimate the fund’s alpha. I subsequently test whether the alpha, after fees and expenses, reliably exceeds zero. For the status model, the standard errors are computed using the delta method. The main text provides further details.
Table 11: **Model parameters**
The table lists the parameters for the model of fund flows in Appendix B. $\delta_i, i = 0, \ldots, 3$, describe how fund flows depend on past performance (see (B.2)) and $\sigma_F$ is the idiosyncratic risk in fund flows. The table also displays the (proportional) reduction in assets under management in case of demotion ($\nu_D$) and the (proportional) increase in case of promotion ($\nu_P$). The estimates are taken from Chapman, Evans, and Xu (2007).

<table>
<thead>
<tr>
<th>(\delta_i)</th>
<th>Value</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.0135</td>
<td>0.0142</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.0928</td>
<td>0.1389</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.0031</td>
<td>0.0411</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>-0.0371</td>
<td>-0.0703</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\nu_D)</th>
<th>(\nu_P)</th>
<th>(\sigma_F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.423</td>
<td>1.72</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 12: **Model parameters**
The table lists the parameters for the model of managerial promotion and demotion in Appendix B. It contains the parameters of the multi-nominal logit model. The estimates are taken from Chapman, Evans, and Xu (2007).

<table>
<thead>
<tr>
<th>Variables contained in (x_t)</th>
<th>Promotion</th>
<th>Demotion</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z_t)</td>
<td>(I_{(\text{Tenure} \leq 3)})</td>
<td>(I_{(\text{Tenure} \in (3,7])})</td>
</tr>
<tr>
<td>Value</td>
<td>0.9327</td>
<td>-4.8866</td>
</tr>
<tr>
<td>Growth</td>
<td>0.9118</td>
<td>-4.8004</td>
</tr>
</tbody>
</table>

| Value | -0.6682 | -3.4784 | -3.5454 | -3.8634 | -0.0884 | -0.2415 |
| Growth | -0.5930 | -3.7300 | -3.7186 | -3.8829 | 0.0331 | -0.1348 |
Figure 1: Estimated distribution of fund alphas
The figure displays the distribution of fund alphas following from performance regressions (top panels) and from the structural status model (bottom panels) in Section 6. The left panels provide the results before fees and expenses, whereas the right panels correspond to the results after fees and expenses. The model is estimated for 1,273 manager-benchmark combinations. The alpha in case of performance regressions is computed as explained in Appendix A.
Figure 2: Managerial ability and risk aversion
The figure displays the cross-sectional distribution of managerial ability and the coefficient of relative risk aversion that follows from the model in Section 6. The model is estimated for 1,273 manager-benchmark combinations. The red line corresponds to a second-order polynomial fitted through the cloud of points to illustrate the relation between managerial ability and the coefficient of relative risk aversion.

Figure 3: Managerial ability and risk aversion across styles
The left panel of this figure displays the distribution of the coefficient of relative risk aversion. The right panel depicts the distribution of managerial ability. The red (blue) lines correspond to the small/growth (large/value) style. The densities are estimated using a standard kernel density estimators based on a normal kernel function.
Figure 4: Time series of risk aversion and expected returns
The blue line depicts the average time-series variation in the coefficient of relative risk aversion that follows from the status model in Section 6. This time series is computed by averaging the coefficients of relative risk aversion in the cross-section of fund managers in each year. The dashed green line corresponds to the time series of the equity risk premium, which is taken from Binsbergen and Koijen (2007).
Figure 5: Fund size and the coefficient of relative risk aversion ($RRA(a_T)$)

The figure displays the coefficient of relative risk aversion (blue dashed-dotted line) on the vertical axis as a function of the fund’s relative size on the horizontal axis for the model in Section 6. The coefficient of relative risk aversion is decomposed into three components, see Equation (28): (i) $\sigma_1$ (green solid line), (ii) $\sigma_2 \rho'(a_T)a_T/\rho(a_T)$ (red dotted line), and (iii) $-\rho''(a_T)a_T/\rho'(a_T)$ (brown dashed line).
Figure 6: Cross-sectional stability of ability and risk aversion
The top panel compares the estimates for the coefficient of relative risk aversion across styles for managers who simultaneously manage funds in different styles. The bottom panel displays the same results for managerial ability. The red line corresponds to the 45-degree line along which the estimates ideally line up. The sample contains 105 managers that are active in multiple styles and that have at least three years of data.
Figure 7: Performance persistence

The figure provides the results of a performance persistence test. I use either performance regressions (red bars) or the status model (blue bars) to sort funds into quartiles based on a three-year selection period. I then form equally-weighted portfolios of funds in a particular quartile and compute the portfolio return and the corresponding (equally-weighted) benchmark return. I hold the portfolios for one year. This leads to return series over the full sample period, which I use to compute the (annualized) information ratio (left panel). The information ratio is computed as the intercept of a performance regression, divided by the standard deviation of the residual. To annualize the information ratio, it is multiplied by the square root of twelve. As an alternative, I include the momentum factor to correct for the returns on passive strategies (right panel).
Risk aversion ($\gamma_I$)

Allocation to mutual funds

Large/value style

Small/growth style

Utility costs (in bp)

Figure 8: Delegated investment management

The top panels display the optimal allocation to the style benchmark (left) and actively-managed mutual funds (right) for an investor in the large/value style. The optimal allocations are determined for three investors and indicated on the vertical axes, the coefficient of relative risk aversion is displayed on the horizontal axis. The green dashed lines in corresponds to the strategy that accounts for heterogeneity using the status model. This case is first-best. The blue solid lines represent the optimal strategy if the investor ignores heterogeneity and uses the average parameter values. The red dotted line corresponds to an investor that (inefficient) performance regressions to account for heterogeneity. The bottom panels compute the loss in certainty-equivalent wealth for both sub-optimal strategies. The solid blue lines corresponds to the investor that ignores heterogeneity, the red dotted line to an investor that uses performance regressions. The bottom left panel corresponds to the large/value investment style; the bottom right panel to small/growth.
Figure 9: Optimal investment strategy in the presence of incentives
The figure displays the optimal allocation to the style benchmark (top panel) and the active portfolio (bottom panel) for different values of the coefficient of relative risk aversion ($\gamma$) and past performance ($z_t$). $z_t$ is defined in (B.1). The monotone planes correspond to the optimal strategy of the model in Section 3.2. The non-monotone planes display the optimal strategies in the model of Appendix B. In this model, assets under management change due to fund returns, performance-sensitive fund flows, and promotion and demotion. The figure depicts the optimal investment strategy of a large/value manager that works between 3-7 years in the mutual fund industry.