Optimal Asset Allocation and Risk Shifting in Money Management

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(1) Motivation and Objective
(2) Model
(3) Empirical Analysis
(4) Costs of Active Management to Investors
1. Motivation and Objective

- Mutual fund managers’ compensation is linked to the value of assets under management
- Implicit incentives due to fund flows to performance relationship
- The flow-performance relationship is
  - positive
  - exhibits convexities
- **Question:** How does a fund manager respond to these incentives?
1. Motivation and Objective

Fund Flow - Performance Relationship (Chevalier and Ellison (1997))

Fig. 1.—Flow-performance relationship \( \hat{f} \) for young funds (age 2) with 90 percent confidence bands.
Summary of Main Testable Implications

- Taking risk \( \neq \) increasing volatility of portfolio

- Gambling entails either an increase or a decrease in portfolio volatility
  - a sufficiently risk averse manager decreases volatility
  - manager manipulates systematic risk rather than idiosyncratic
  - gambling intensifies towards year-end

- Incentives to gamble are state-dependent. For example, the manager’s risk-taking incentives are

\[
\text{year-end flow}
\]

\[
\begin{align*}
&f_H \\
&f_L \\
&0
\end{align*}
\]

\[\text{relative return}\]

- Highest
- Lowest
1. Motivation and Objective

Related Literature

- **Risk-taking of fund managers in response to fund flows:**
  Chevalier and Ellison (1997)

- **Managerial incentives and portfolio choice:**

- **Empirical literature on risk-taking by mutual fund managers:**
2. Model

- Finite horizon, \([0, T]\), Black-Scholes economy

- Assets:
  - Money market account with rate \(r\)
  - Stock follows \(dS_t = \mu S_t dt + \sigma S_t dw_t\)

- Fund manager:
  - evaluated relative to the index \(Y_t\) (fraction \(\beta\) in stock)
  - receives flows at \(T\) at rate \(f_T\)
  - chooses a trading strategy \(\theta\) and terminal portfolio value \(W_T\)

\[
\max_{\theta, W_T} E[u(W_T f_T)] = E \left( \frac{(W_T f_T)^{1-\gamma}}{1-\gamma} \right)
\]

subject to

\[
dW_t = [r + \theta_t(\mu - r)] W_t dt + \theta_t \sigma W_t dw_t
\]
How does one measure risk-taking incentives?

- Conventional view:
  - sensitivity of the payoff’s value to volatility (vega): \( \frac{\partial V(\sigma_t^W; R_t^W - R_t^Y)}{\partial \sigma_t^W} \)

- This paper:
  - optimal volatility \( \hat{\sigma}_t^W = \hat{\theta}_t \sigma \). That is, \( \frac{\partial V(\sigma_t^W; R_t^W - R_t^Y)}{\partial \sigma_t^W} = 0 \Rightarrow \hat{\sigma}_t^W. \)
Manager’s Optimal Risk Exposure

(a) Economies with $\theta^N > \theta^Y$

(b) Economies with $\theta^N < \theta^Y$

$\theta^N$: risk exposure in Merton’s problem, $\theta^Y$: risk exposure of the index
An Alternative Flow-Performance Relationship (Collar-Type)

\[ f_T \]
\[ f_H \]
\[ f_L \]

Can also be reinterpreted as an 80/120 annual bonus plan.
Manager’s Optimal Risk Exposure (Collar-Type)

(a) Economies with $\theta^N > \theta^Y$

(b) Economies with $\theta^N < \theta^Y$

$\theta^N$: risk exposure in Merton’s problem, $\theta^Y$: risk exposure of the index
Further Alternative Flow-Performance Specifications

- Linear-convex (Sirri and Tufano (1998))

\[ f_L \]
\[ 0 \]
\[ \text{relative return} \]

- Linear-linear (asymmetric fee structure)

\[ f_L \]
\[ 0 \]
\[ \text{relative return} \]
Manager’s Optimal Risk Exposure: Dynamics

(a) Economies with $\theta^N > \theta^Y$

(b) Economies with $\theta^N < \theta^Y$

- Manager engages in risk shifting well before the year-end
- Risk shifting more pronounced as the year-end approaches
2. Model

Multiple Stocks

(a) Economies with $\theta_1^N > \theta_1^Y$ and $\theta_2^N > \theta_2^Y$

(b) Economies with $\theta_1^N < \theta_1^Y$ and $\theta_2^N > \theta_2^Y$
2. Model

Idiosyncratic versus Systematic Risk

Economic setup:

\[ dS_{1t} = \mu_1 S_{1t}dt + \sigma_{11} S_{1t}dw_{1t} + \sigma_{12} S_{1t}dw_{2t} \]

\[ dS_{2t} = \mu_2 S_{2t}dt + \sigma_{21} S_{2t}dw_{1t} + \sigma_{22} S_{2t}dw_{2t} \]

\[ \mu = \begin{pmatrix} \mu_1 \\ r \end{pmatrix}, \quad \sigma = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}, \quad Y = S_1 \]
3. Empirical Analysis

Existing Work

- *Brown, Harlow, and Starks (1996)*
  - find that underperforming managers increase volatility towards the year-end

- *Busse (2001)*
  - shows that the above test fails on daily data

- *Chevalier and Ellison (1997)*
  - look at $\sigma(R^W - R^Y)$ towards the year-end; find an increase
  - use monthly data

- *Reed and Wu (2005)*
  - test the results of this paper on daily data
  - distinguish between tournaments- vs. benchmarking-induced incentives
Data

- Daily mutual fund returns from Will Goetzmann and Geert Rouwenhorst (International Center for Finance at Yale SOM).


- Merged with CRSP to find out mutual funds objective codes
  - left only actively managed US equity mutual funds in the aggressive growth, growth and income, and long-term growth categories.

- Used the S&P 500 index as the benchmark.
### Tracking error and standard deviation tests

**Hypothesis 1:** Tracking error variance is higher for underperforming managers.

<table>
<thead>
<tr>
<th>LHS: $\sigma_m(R_{i,t}^W - R_t^Y)$</th>
<th>LHS: $\sigma_m(R_{i,t}^W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point Estimate</strong></td>
<td><strong>t-Statistic</strong></td>
</tr>
<tr>
<td>OVER$_{i,m} \times 10^3$</td>
<td>-0.1819</td>
</tr>
<tr>
<td>Fund-year FE</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.39</td>
</tr>
<tr>
<td>N</td>
<td>40721</td>
</tr>
</tbody>
</table>

$\sigma_m(R_{i,t}^W - R_t^Y)$ – standard deviation of tracking error for month $m$; $Y$ is S&P 500

$\sigma_m(R_{i,t}^W)$ – standard deviation of fund returns for month $m$

OVER$_{i,m}$ – relative performance indicator prior to month $m$
Hypothesis 2: Sufficiently risk-averse managers decrease their portfolio betas when underperforming the market.

\[ R_{i,t}^{W} - R_{t}^{F} = a + (b_{Fund-Year}1_{FY} + b_{Month}1_{M} + b_{UNDER_{i,w}})(R_{t}^{Y} - R_{t}^{F}) + \varepsilon_{i,t} \]

<table>
<thead>
<tr>
<th></th>
<th>Beta(_{T}) below 1</th>
<th>Beta(_{T-1}) below 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>UNDER(<em>{i,w}) \times (R</em>{t}^{Y} - R_{t}^{F})</td>
<td>-0.017 (-6.61)</td>
<td>-0.020 (-7.72)</td>
</tr>
<tr>
<td>Month fixed effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Fund-year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Number of observations</td>
<td>808642</td>
<td></td>
</tr>
</tbody>
</table>
Robustness

- Tried several alternative definitions of the OVER/UNDER indicator
- Included lagged dependent variables to deal with autocorrelation
- Clustered errors by month/day and fund objective code
4. Costs of Active Management to Investors

- Define a measure of gain/loss, $\hat{\lambda}$, in units of investor's initial wealth:

$$V^I((1 + \hat{\lambda})W_0) = \hat{V}(W_0)$$

  - $V^I(\cdot)$ is investor's indirect utility under $\theta^I$
  - $\hat{V}(\cdot)$ is investor's indirect utility under delegation

- Decompose $\hat{\lambda}$ into two components: $1 + \hat{\lambda} = (1 + \lambda^N)(1 + \lambda^Y)$
  - $\lambda^N$: gain/loss due to explicit incentives, solves
    $$V^I((1 + \lambda^N)W_0) = \hat{V}(W_0; f_T=1)$$
  - $\lambda^Y$: gain/loss due to implicit incentives
Costs of Active Management in Economies (a) \((\theta^N > \theta^Y)\)

Fixed parameter values: \(\gamma = 1, \gamma_I = 2, f_L = 0.8, f_H = 1.5, f_L + f_H = 2.3, \beta = 1, \eta_L = -0.08, \eta_H = 0.08, \eta_L + \eta_H = 0, \mu = 0.06, r = 0.02, \sigma = 0.29, W_0 = 1, T = 1.\)

<table>
<thead>
<tr>
<th>Effects of</th>
<th>Cost-benefit measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>(\lambda^Y, \lambda^N)</td>
</tr>
<tr>
<td>Managerial risk aversion</td>
<td>(\lambda) (%)</td>
</tr>
<tr>
<td>0.5</td>
<td>-8.13, -4.19</td>
</tr>
<tr>
<td>1.0</td>
<td>-5.12, -0.47</td>
</tr>
<tr>
<td>2.0</td>
<td>-3.31, 0.00</td>
</tr>
<tr>
<td>3.0</td>
<td>-2.56, -0.05</td>
</tr>
<tr>
<td>4.0</td>
<td>-2.15, -0.11</td>
</tr>
<tr>
<td>Implicit reward for outperformance</td>
<td></td>
</tr>
<tr>
<td>(f_H - f_L)</td>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
<td>-4.29, -0.47</td>
</tr>
<tr>
<td>0.7</td>
<td>-5.12, -0.47</td>
</tr>
<tr>
<td>0.9</td>
<td>-6.01, -0.47</td>
</tr>
<tr>
<td>1.1</td>
<td>-6.88, -0.47</td>
</tr>
<tr>
<td>Risk exposure of the benchmark</td>
<td></td>
</tr>
<tr>
<td>(\theta^Y)</td>
<td>0.50</td>
</tr>
<tr>
<td>0.75</td>
<td>-4.63, -0.47</td>
</tr>
<tr>
<td>1.00</td>
<td>-5.12, -0.47</td>
</tr>
<tr>
<td>1.25</td>
<td>-6.69, -0.47</td>
</tr>
<tr>
<td>1.50</td>
<td>-8.45, -0.47</td>
</tr>
<tr>
<td>Flow threshold differential</td>
<td></td>
</tr>
<tr>
<td>(\eta_H - \eta_L)</td>
<td>0.08</td>
</tr>
<tr>
<td>0.12</td>
<td>-4.70, -0.47</td>
</tr>
<tr>
<td>0.16</td>
<td>-5.12, -0.47</td>
</tr>
<tr>
<td>0.20</td>
<td>-5.67, -0.47</td>
</tr>
<tr>
<td>0.24</td>
<td>-6.21, -0.47</td>
</tr>
</tbody>
</table>

-4.78, -5.15, -5.61, -6.12, -6.65
Fixed parameter values: $\gamma = 1, \gamma_I = 2, f_L = 0.8, f_H = 1.5, f_L + f_H = 2.3, \beta = 1, \eta_L = -0.08, \eta_H = 0.08, \eta_L + \eta_H = 0, \mu = 0.06, r = 0.02, \sigma = 0.29, W_0 = 1, T = 1.$

### Costs of Active Management in Economies (b) ($\theta^N < \theta^Y$)

<table>
<thead>
<tr>
<th>Effects of</th>
<th></th>
<th>Cost-benefit measures $\lambda^Y, \lambda^N, \hat{\lambda}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managerial risk aversion</td>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-8.13, -4.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-11.98</td>
</tr>
<tr>
<td>Implicit reward for outperformance</td>
<td>$f_H - f_L$</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.33, -0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.79</td>
</tr>
<tr>
<td>Risk exposure of the benchmark</td>
<td>$\theta^Y$</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-5.43, -0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-5.88</td>
</tr>
<tr>
<td>Flow threshold differential</td>
<td>$\eta_H - \eta_L$</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.33, -0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.78</td>
</tr>
</tbody>
</table>
5. Conclusion

• Characterized the manager’s optimal behavior in response to incentives induced by the fund flow-performance relationship.

• Identified circumstances in which the manager would like to gamble.

• Gambling may be associated with a decrease in the fund’s volatility.

• Adverse incentives of the manager result in an economically significant cost to the investor.