A Neoclassical Interpretation of Momentum

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The Q Group
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An economic interpretation of momentum based on the neoclassical $q$-theory of investment

Practical implication: Momentum might be riskier than you thought
Key Result
Price and earnings momentum

- Average realized returns vs. Average predicted returns
- The points (2, 3), (4, 5), (6, 7), (8, 9) are plotted on the graph.
- The linear relationship suggests a strong positive correlation between realized and predicted returns.
Outline

1. The Q Model
2. Average Momentum Profits
3. Momentum Reversal
4. Long-run Risks in Momentum
5. Market States and Momentum
6. The Interaction of Momentum with Firm Characteristics
7. Risk Analysis
Outline

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The Q Model
A microfoundation for the WACC approach to capital budgeting

Marginal benefits of investment at time $t+1$

$$r_{it+1} = \left[ (1 - \tau_{t+1}) \left\{ (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1})a\left( \frac{l_{it+1}}{K_{it+1}} \right) \right] \right\} + \tau_{t+1}\delta_{it+1} + \frac{a}{2} \left( \frac{l_{it+1}}{K_{it+1}} \right)^2 \right]$$

Marginal product plus economy of scale (net of taxes)

Expected continuation value

Marginal costs of investment at time $t$

The weighted average cost of capital

$$w_{it}r_{it+1}^{Ba} + (1 - w_{it})r_{it+1}^{S} = r_{it+1}^{I}$$
The Q Model

Expected stock returns = expected levered investment returns?

\[
E \left[ r_{it+1}^S - \frac{r_{it+1}^l(a, \kappa) - w_{it}r_{it+1}^{Ba}}{1 - w_{it}} \right] = 0,
\]

with the model error, \( \alpha_i^q \), as the sample average of the difference

Construct a \( \chi^2 \) test per Hansen (1982) based on these alphas
The Q Model
Measurement, 1963–2012

- $K_{it}$: Net property, plant, and equipment (PPE)
- $l_{it}$: Capital expenditure minus sales of PPE
- $Y_{it}$: Sales
- $B_{it}$: Long-term debt plus short-term debt
- $P_{it}$: The market value of common equity
- $\delta_{it}$: The amount of depreciation divided by capital
- $r_{it+1}^B$: Imputed bond ratings, assigning corporate bond returns of a given rating to all firms with the same rating
- $\tau_t$: Statutory tax rate of corporate income
The Q Model

Timing alignment, firms with December fiscal yearend

\[ r_{it+1} \]

(from July of year \( t \) to June of \( t + 1 \))

\[ \tau_t, I_{it} \]

(from January of year \( t \) to December of \( t \))

\[ \tau_{t+1}, \delta_{it+1}, Y_{it+1}, I_{it+1} \]

(from January of year \( t + 1 \) to December of \( t + 1 \))

The holding period, February–July of year \( t \), for the first sub-portfolio of a momentum portfolio in July of year \( t \)

The holding period, July–December of year \( t \), for the sixth sub-portfolio of a momentum portfolio in July of year \( t \)
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## Average Momentum Profits

### Point estimates

<table>
<thead>
<tr>
<th></th>
<th>Price momentum</th>
<th>Earnings momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>2.52</td>
<td>5.41</td>
</tr>
<tr>
<td>[se]</td>
<td>[0.94]</td>
<td>[2.51]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>[se]</td>
<td>[0.02]</td>
<td>[0.03]</td>
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## Average Momentum Profits
### Deciles, alphas and overall model performance

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>5</th>
<th>W</th>
<th>W−L</th>
<th>mae</th>
<th>[p-val]</th>
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<td></td>
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</tr>
<tr>
<td>$\bar{r}^S$</td>
<td>4.04</td>
<td>12.36</td>
<td>19.13</td>
<td>15.09</td>
<td></td>
<td></td>
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<tr>
<td>$\alpha^q$</td>
<td>−1.61</td>
<td>1.32</td>
<td>−1.21</td>
<td>0.40</td>
<td>0.83</td>
<td>[0.04]</td>
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<tr>
<td>[t]</td>
<td>−0.39</td>
<td>0.44</td>
<td>−0.29</td>
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<td><strong>Earnings momentum</strong></td>
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<tr>
<td>$\bar{r}^S$</td>
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<td>15.48</td>
<td>18.95</td>
<td>8.47</td>
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<tr>
<td>$\alpha^q$</td>
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<td>1.05</td>
<td>−1.31</td>
<td>−0.92</td>
<td>0.63</td>
<td>[0.09]</td>
</tr>
<tr>
<td>[t]</td>
<td>−0.09</td>
<td>0.25</td>
<td>−0.37</td>
<td>−0.36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Average Momentum Profits
Comparative statics

The investment return, $r_{it+1}^I$

$$
(1 - \tau_{t+1}) \left[ \kappa \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{I_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1})a \left( \frac{I_{it+1}}{K_{it+1}} \right) \right] - w_{it} r_{it+1}^B
$$

The levered investment return, $r_{it+1}^{lw}$

$$
1 + (1 - \tau_{t})a \left( \frac{I_{it}}{K_{it}} \right)
$$

$$
1 - w_{it}
$$

Components of expected stock returns:
$I_{it}/K_{it}$, $Y_{it+1}/K_{it+1}$, $(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$, and $w_{it}$
Average Momentum Profits

Expected return components

<table>
<thead>
<tr>
<th>Loser</th>
<th>5</th>
<th>Winner</th>
<th>W−L</th>
<th>[t]</th>
</tr>
</thead>
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<tr>
<td><strong>Price momentum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{it}/K_{it}$</td>
<td>0.22</td>
<td>0.19</td>
<td>0.25</td>
<td>0.04</td>
</tr>
<tr>
<td>$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$</td>
<td>0.83</td>
<td>0.99</td>
<td>1.15</td>
<td>0.32</td>
</tr>
<tr>
<td>$Y_{it+1}/K_{it+1}$</td>
<td>3.16</td>
<td>3.00</td>
<td>4.10</td>
<td>0.94</td>
</tr>
<tr>
<td>$w_{it}$</td>
<td>0.34</td>
<td>0.25</td>
<td>0.22</td>
<td>−0.12</td>
</tr>
<tr>
<td><strong>Earnings momentum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{it}/K_{it}$</td>
<td>0.19</td>
<td>0.19</td>
<td>0.20</td>
<td>0.01</td>
</tr>
<tr>
<td>$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$</td>
<td>0.95</td>
<td>1.00</td>
<td>1.05</td>
<td>0.10</td>
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<tr>
<td>$Y_{it+1}/K_{it+1}$</td>
<td>3.01</td>
<td>3.06</td>
<td>3.53</td>
<td>0.52</td>
</tr>
<tr>
<td>$w_{it}$</td>
<td>0.29</td>
<td>0.28</td>
<td>0.20</td>
<td>−0.09</td>
</tr>
</tbody>
</table>
### Average Momentum Profits

#### Comparative statics

<table>
<thead>
<tr>
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<th>W−L</th>
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<tr>
<td><strong>Price momentum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{it}/K_{it}$</td>
<td>−2.58</td>
<td>3.77</td>
<td>−7.23</td>
<td>−4.65</td>
</tr>
<tr>
<td>$q_{it+1}/q_{it}$</td>
<td>−7.26</td>
<td>1.00</td>
<td>2.66</td>
<td>9.92</td>
</tr>
<tr>
<td>$Y_{it+1}/K_{it+1}$</td>
<td>−2.59</td>
<td>−0.56</td>
<td>4.13</td>
<td>6.73</td>
</tr>
<tr>
<td>$\overline{w}_{it}$</td>
<td>−1.39</td>
<td>1.22</td>
<td>−1.48</td>
<td>−0.09</td>
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<tr>
<td><strong>Earnings momentum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{it}/K_{it}$</td>
<td>0.62</td>
<td>2.89</td>
<td>−4.54</td>
<td>−5.16</td>
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<td>$q_{it+1}/q_{it}$</td>
<td>−3.20</td>
<td>0.88</td>
<td>0.88</td>
<td>4.07</td>
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<td>$Y_{it+1}/K_{it+1}$</td>
<td>−1.65</td>
<td>0.24</td>
<td>1.71</td>
<td>3.36</td>
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<tr>
<td>$\overline{w}_{it}$</td>
<td>−0.57</td>
<td>1.20</td>
<td>−2.52</td>
<td>−1.95</td>
</tr>
</tbody>
</table>
Outline

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Reversal

Price momentum, $r_{it+1}$ and $r_{it+1}^{lw}$
Reversal

Price momentum, \( \frac{I_{it+1}/K_{it+1}}{I_{it}/K_{it}} \) and \( \frac{Y_{it+1}}{K_{it+1}} \)
Reversal

Earnings momentum, $r_{it+1}^S$ and $r_{it+1}^{lw}$
Reversal

Earnings momentum, $(l_{it+1}/k_{it+1})/(l_{it}/k_{it})$ and $y_{it+1}/k_{it+1}$
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Bansal, Dittmar, and Lundblad (2005) show that aggregate consumption risks in cash flows help explain momentum profits:

$$g_{i,t} = \gamma_i \left( \frac{1}{8} \sum_{k=1}^{8} g_{c,t-k} \right) + u_{i,t}$$

- $g_{i,t}$: demeaned log real dividend growth of momentum decile $i$
- $g_{c,t}$: demeaned log real growth rate of aggregate consumption
- $\gamma_i$: cash flow exposure to long-run consumption growth
Define the cash flow in the investment return as:

\[
D_{it+1}^* = (1 - \tau_{t+1}) \left[ \kappa \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{l_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1}
\]

Let \( g_{i,t}^* \): demeaned log real growth of \( D_{it+1}^* \) for momentum decile \( i \):

\[
g_{i,t}^* = \gamma_{i}^* \left( \frac{1}{8} \sum_{k=1}^{8} g_{c,t-k} \right) + u_{i,t}
\]

- \( \gamma_{i}^* \): cash flow exposure to long-run consumption growth
## Long-run Risks

### Evidence

<table>
<thead>
<tr>
<th></th>
<th>Price momentum</th>
<th>Earnings momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_i$ [se] $\gamma_i^*$ [se]</td>
<td>$\gamma_i$ [se] $\gamma_i^*$ [se]</td>
</tr>
<tr>
<td>5</td>
<td>0.18 [1.27]      5.52 [1.20]</td>
<td>1.21 [2.20]      5.82 [1.49]</td>
</tr>
</tbody>
</table>
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7 Risk Analysis
### Market States and Momentum

Cooper, Gutierrez, and Hameed (2004), UP (DOWN) defined as the market returns nonnegative (negative) over the prior year

<table>
<thead>
<tr>
<th>State</th>
<th>Price momentum</th>
<th>Earnings momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profits $[t]$</td>
<td>Profits $[t]$</td>
</tr>
<tr>
<td>DOWN</td>
<td>2.21 [0.62] $r^S$</td>
<td>1.31 [0.40] $r^S$</td>
</tr>
<tr>
<td>UP</td>
<td>9.89 [5.04] $r^S$</td>
<td>5.04 [6.73] $r^S$</td>
</tr>
<tr>
<td>DOWN</td>
<td>9.19 [4.50] $r^{lw}$</td>
<td>4.50 [2.46] $r^{lw}$</td>
</tr>
<tr>
<td>UP</td>
<td>6.87 [4.64] $r^{lw}$</td>
<td>4.64 [6.51] $r^{lw}$</td>
</tr>
</tbody>
</table>
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## Interaction

GMM tests, price momentum

<table>
<thead>
<tr>
<th></th>
<th>Size</th>
<th>Age</th>
<th>Trading volume</th>
<th>Credit ratings</th>
<th>Stock return volatility</th>
<th>Book-to-market</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>2.33</td>
<td>2.37</td>
<td>2.76</td>
<td>1.97</td>
<td>3.17</td>
<td>3.44</td>
</tr>
<tr>
<td>[se]</td>
<td>0.70</td>
<td>0.95</td>
<td>0.93</td>
<td>0.83</td>
<td>0.82</td>
<td>0.89</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.09</td>
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<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.13</td>
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<td>0.01</td>
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<td>0.01</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>mae</td>
<td>3.66</td>
<td>1.29</td>
<td>1.67</td>
<td>1.68</td>
<td>1.92</td>
<td>3.10</td>
</tr>
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</table>
### Interaction

**GMM tests, earnings momentum**

<table>
<thead>
<tr>
<th></th>
<th>Size</th>
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<th>Trading volume</th>
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<th>Stock return volatility</th>
<th>Book-to-market</th>
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<tbody>
<tr>
<td>$a$</td>
<td>2.74</td>
<td>2.75</td>
<td>2.56</td>
<td>1.14</td>
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<tr>
<td>[se]</td>
<td>0.60</td>
<td>1.55</td>
<td>1.32</td>
<td>0.72</td>
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<td>2.36</td>
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<td>0.12</td>
<td>0.11</td>
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<td>0.16</td>
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<td>0.01</td>
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<td>0.02</td>
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<td><strong>0.00</strong></td>
<td><strong>0.00</strong></td>
<td><strong>0.01</strong></td>
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<td>mae</td>
<td>4.37</td>
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<td>1.35</td>
<td>1.95</td>
<td>2.88</td>
</tr>
</tbody>
</table>
Interaction

Size and price (earnings) momentum

Average predicted returns vs. Average realized returns chart.
Interaction

Book-to-market and price (earnings) momentum
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Risk Analysis
The consumption-investment model

Testing the consumption CAPM and the $Q$ model jointly:

\[ \begin{align*}
E[M_{t+1}(r_{it+1}^S - r_{t+1}^f)] &= 0, \\
E[M_{t+1}(r_{t+1}^f/i_{t+1})] &= 1, \\
E[M_{t+1}(r_{it+1}^{lw} - r_{t+1}^f)] &= 0, \\
E[r_{it+1}^S - r_{it+1}^{lw}] &= 0.
\end{align*} \]

\[ M_{t+1} = \rho \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \] Annual data
Risk Analysis

$R^6$: CCAPM with $r^S$, CCAPM with $r^{lw}$, Q-moment

Average real stock returns vs. Average real predicted stock returns

Average real levered investment returns vs. Average real predicted $r^{lw}$

Average realized stock returns vs. Average levered investment returns
Risk Analysis

SUE: CCAPM with $r^S$, CCAPM with $r^{lw}$, Q-moment
Conclusion
Summary and future work

The Q model broadly consistent with many aspects of momentum:
- Managers align investment properly with costs of capital
- Momentum per se might not imply investor irrationality

Several directions in the future:
- Value and momentum jointly, industry-specific parameters
- Value and momentum in currencies, International Q model
- Recent developments in CCAPM in the joint model