Outline

- Introduction
- An Economic Framework
- Econometric Methodology
- Empirical Findings
- Conclusions
Tail Risk in the Big Picture

Value of assets depends on the potential for infrequent, extreme payoff events

1. Peso problems (Krasker, 1980)

2. Potential for rare disasters can explain equity puzzles (Rietz, 1988; Barro, 2006; Weitzman 2007; Gabaix 2009; Wachter 2009)

Plausible mechanism – OR – convenient (though unrealistic) explanation?
Tail risk is difficult to measure, even unconditionally.

Few risks are static: Feasibility of conditional tail measures?

**My solution**: An economically-motivated *conditional tail risk measure* extracted from the *cross section* of asset returns.
Objectives

1. Structural understanding of how tail risk is priced
   - I derive tractable expressions for expected returns as a function of a tail risk state variable
   - I derive the distribution of return tail events implied by the model

2. I econometrically identify the conditional tail distribution of returns
   - Directly estimable from the cross section of asset prices by exploiting restriction implied by economic theory

3. I evaluate theories relating tail risk to risk premia using my estimated series
Tail risk varies substantially over time and is highly persistent.

Tail measure predicts market returns over horizons of one month to five years, outperforms commonly studied predictors.
- A one standard deviation increase in tail risk increases expected returns by 4.4% per year.

Large explanatory power for cross section of returns.
- Stocks that covary highly with tail risk earn annual expected returns 2% to 6% lower than stocks that with low tail risk covariation.
Emergence in varied theoretical settings, for example

1. Long run risks + heavy-tailed shocks (similar to Eraker and Shaliastovich 2008, Drechsler and Yaron 2009)

2. Time-varying rare disasters (similar to Gabaix 2009, Wachter 2009)

3. Long run risks + large swings in confidence (similar to Bansal and Shaliastovich 2009)
A Tail Risk State Variable in the Long Run Risks Framework

- Epstein-Zin preferences:

\[ m_{t+1} = \theta \ln \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1} \]

- Dynamics of the real economy:

\[
\begin{align*}
\Delta c_{t+1} &= \mu + x_t + \sigma_c \sigma_t z_{c,t+1} + \sqrt{\Lambda_t} W_{c,t+1} \\
x_{t+1} &= \rho_x x_t + \sigma_x \sigma_t z_{x,t+1} \\
\sigma_{t+1}^2 &= \bar{\sigma}^2 (1 - \rho_\sigma) + \rho_\sigma \sigma_t^2 + \sigma_\sigma z_\sigma, t+1 \\
\Lambda_{t+1} &= \bar{\Lambda}(1 - \rho_\Lambda) + \rho_\Lambda \Lambda_t + \sigma_\Lambda z_\Lambda, t+1 \\
\Delta d_{i,t+1} &= \mu_i + \phi_i \Delta c_{t+1} + \sigma_i \sigma_t z_{i,t+1} + q_i \sqrt{\Lambda_t} W_{i,t+1}
\end{align*}
\]

\[ f_W(w) = \frac{1}{2} \exp(-|w|), \ w \in \mathbb{R} \]
What Is Tail Risk?

- Gaussian baseline: is variance sufficient to characterize risk of extreme events?
- Illustrative example: Normal-Laplace distribution
- Since at least Mandelbrot (1963) and Fama (1963), economists have argued for power law return tails

\[ P(R > x | R > u) = \left( \frac{x}{u} \right)^{-\zeta}, \quad u \text{ some high threshold} \]

- \(-\zeta \equiv \text{tail risk measure}\)
- Does \(\zeta\) change through time?
- What kind of world?
What Is Tail Risk?

\[ \text{Var}(X_{N-L}) = \sigma^2 + 2\Lambda^2 \]

\[ P(X_{N-L} > x | X_{N-L} > u) = \exp(-[x-u]/\Lambda) \]
What Is Tail Risk?

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\begin{align*}
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x_{t+1} &= \rho x x_t + \sigma x \sigma t Z_{x,t+1} \\
\sigma^2_{t+1} &= \bar{\sigma}^2 (1 - \rho \sigma) + \rho \sigma \sigma_t^2 + \sigma \sigma Z_{\sigma,t+1} \\
\Lambda_{t+1} &= \bar{\Lambda}(1 - \rho \Lambda) + \rho \Lambda \Lambda_t + \sigma \Lambda Z_{\Lambda,t+1} \\
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\end{align*}
\]

\[ f_W(w) = \frac{1}{2} \exp(-|w|), \quad w \in \mathbb{R} \]
Prices and Excess Returns

Proposition

The log wealth-consumption ratio and log price-dividend ratio for asset (i) are linear in state variables,

\[ wc_{t+1} = A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{t+1} + A_\Lambda \Lambda_{t+1} \]

\[ pd_{i,t+1} = A_{i,0} + A_{i,x} x_{t+1} + A_{i,\sigma} \sigma^2_{t+1} + A_{i,\Lambda} \Lambda_{t+1}. \]

Proposition

The expected return on asset (i) in excess of the risk free rate is

\[ E_t[r_{i,t+1} - r_{f,t}] = \beta_{i,c} \lambda_c (\sigma^2_c \sigma^2_t + 2\Lambda_t) + \beta_{i,x} \lambda_x \sigma^2_x \sigma^2_t + \beta_{i,\sigma} \lambda_\sigma \sigma^2_{\sigma} + \beta_{i,\Lambda} \lambda_\Lambda \sigma^2_{\Lambda} - \frac{1}{2} \text{Var}(r_{i,t+1}). \]

Proof
Key Implications

1. Tail risk forecasts excess stock returns
   \textbf{High tail risk} $\Rightarrow$ \textbf{high future returns}

2. Covariance with tail risk impacts cross section of expected returns
   \textbf{High return tail risk beta} $\Rightarrow$ \textbf{low expected returns}
Proposition

The lower and upper tail distributions of arithmetic returns are asymptotically equivalent to a power law,

\[ P_t(R_{i,t+1} < r \mid R_{i,t+1} < u) \sim \left( \frac{r}{u} \right)^{a_i \zeta_t} \]

\[ P_t(R_{i,t+1} > r \mid R_{i,t+1} > u) \sim \left( \frac{r}{u} \right)^{-a_i \zeta_t} \]

where \( a_i = \max(\phi_i, q_i)^{-1} \) and \( \zeta_t = 1 / \sqrt{\Lambda_t} \).
Tail risk state variable drives \textit{risk premia and tail exponent}

1. Tail risk (and thus tail exponent) forecasts excess stock returns
   \textit{High tail risk} \implies \textit{high future returns}

2. Covariance with tail risk (and thus tail exponent) impacts cross section of expected returns
   \textit{High return tail risk beta} \implies \textit{low expected returns}
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Econometric Intuition

\[ P_t(R_{i,t+1} < r \mid R_{i,t+1} < u) \sim \left( \frac{r}{u} \right)^{a_i \zeta_t} \]

- Single process drives tail dynamics of entire panel of returns

Hill estimator applied to cross-section

Cross-Sectional Return Distribution

Time
Definition: Dynamic Power Law Model

Individual returns on asset \((i)\), conditional upon exceeding threshold \(u\) and given \(\mathcal{F}_t\), obey

\[
F_{u,i,t}(r) = P(R_{i,t+1} > r | R_{i,t+1} > u, \mathcal{F}_t) = \left( \frac{r}{u} \right)^{-a_i \zeta_t}
\]

with exponent

\[
\frac{1}{\zeta_{t+1}} = \pi_0 + \pi_1 \frac{1}{\zeta_{upd}} + \pi_2 \frac{1}{\zeta_t}
\]

and observable update

\[
\frac{1}{\zeta_{upd}^t} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u}
\]
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\]

with exponent

\[
\frac{1}{\zeta_{t+1}} = \pi_0 + \pi_1 \frac{1}{\zeta_{upd}} + \pi_2 \frac{1}{\zeta_t} = \frac{\pi_0}{1 - \pi_2} + \pi_1 \sum_{j=0}^{\infty} \pi_j \frac{1}{\zeta_{upd} t-j}
\]

and observable update

\[
\frac{1}{\zeta_{upd}} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u} \quad \text{Hill (1975) Estimator applied to cross section}
\]
Quasi-Likelihood Estimator for Dynamic Power Law Model

Proposal: Assume tail observations in time $t$ cross section are identical and independent

- But theory (and years of empirical work) suggests...
  1. Dependent observations (factor structure)
  2. Heterogeneous volatility
  3. Heterogeneous tail exponent

Result: Despite mis-specification, estimator consistent and asymptotically normal
Assume (provisionally) tail observations are cross-sectionally independent and each obey

\[
\tilde{F}_{u,i,t}(X_{i,t+1}; \pi) = \left( \frac{x}{u} \right)^{-\tilde{\zeta}_t}
\]

\[
\tilde{f}_{u,i,t}(X_{i,t+1}; \pi) = \frac{\tilde{\zeta}_t}{u} \left( \frac{x}{u} \right)^{-1(1+\tilde{\zeta}_t)}
\]

Construct log quasi-likelihood using only \( u \)-exceedences

\[
\mathcal{L}(X; \pi) = \frac{1}{T} \sum_{t=0}^{T} \ln \tilde{f}_{u,t}(X_{t+1}; \pi) = \frac{1}{T} \sum_{t=0}^{T} \sum_{k=1}^{K_{t+1}} \left( \frac{1}{\tilde{\zeta}_t} - \ln \frac{X_{k,t+1}}{u} \right)
\]

Maximize

QML Estimator: \( \hat{\pi}_{QL} \equiv \arg \max_{\pi \in \Pi} \mathcal{L}(X; \pi) \)
Asymptotic Properties of QML Estimator

**Proposition**

Let the true DGP of \( \{ R_t \}_{t=1}^T \) be given by the Dynamic Power Law model with parameter values \( \pi^* \). Under standard GMM regularity conditions,

\[
\hat{\pi}_{QL} \xrightarrow{p} \pi^* \quad \text{and} \quad \sqrt{T}(\hat{\pi}_{QL} - \pi^*) \xrightarrow{d} N(0, \Psi)
\]

where

\[
\Psi = S^{-1}GS^{-1}, \quad S = E[\nabla_\pi s(X_t; \pi^*)], \quad \text{and} \quad G = E[s(X_t; \pi^*)s(X_t; \pi^*)'].
\]
Proof Sketch

- First order condition of quasi-likelihood maximization

\[ s(X_{t+1}; \pi) \equiv \nabla_\pi \ln \tilde{f}_t(X_{t+1}; \pi) = \frac{K_{t+1}}{\tilde{\zeta}_t} - \sum_{k=1}^{K_{t+1}} \ln \frac{X_{k,t+1}}{u} = 0 \]

- MLE identification condition: expected value of FOC equals zero
- Mis-specified MLE is GMM – FOC moment condition holds

Lemma

\[ E[s(X_{t+1}; \pi)] = 0 \text{ when the true model is the Dynamic Power Law.} \]

Proof:

\[
E[s(X_{t+1}; \pi)] = E\left[ E_t\left[ \frac{K_{t+1}}{\tilde{\zeta}_t} - \sum_{k=1}^{K_{t+1}} \ln \frac{X_{k,t+1}}{u} \right] \right]
\]
\[
= E\left[ \frac{K_{t+1}}{\tilde{\zeta}_t} - \frac{1}{n} \sum_i a_i \right]
\]
\[
= 0 \text{ when } \frac{1}{\tilde{\zeta}_t} = \frac{1}{n} \sum_i a_i. \qed
\]
By varying threshold each period, accommodate time-varying volatility

Cross sectional differences in volatility?

Explicitly modeling dependence?
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Data

- Primary sample: Daily NYSE/AMEX/NASDAQ stock returns from CRSP
- Fama-French factors; Ken French’s data library
- Federal Reserve macro data
- Goyal and Welch (2008) data
- OptionMetrics
- Other (VIX, Hao Zhou’s variance risk premium)
Dynamic Power Law Estimates

\[
\frac{1}{\zeta_{t+1}} = \pi_0 + \pi_1 \frac{1}{\zeta_{t}^{upd}} + \pi_2 \frac{1}{\zeta_{t}}
\]

Table: 1963-2008

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<td>(\pi_2)</td>
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<td>(0.015)</td>
<td>(0.011)</td>
<td>(0.092)</td>
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Dynamic Power Law Estimates: Exponent Series

\[ \rho(\text{Exponent}, \log P/D) = -14\% \]
\[ \rho(\text{Lower Exponent}, \log \frac{P}{D}) = -15\%, \ \ \rho(\text{Upper Exponent}, \log \frac{P}{D}) = -14\% \]
Dynamic Power Law Estimates: Threshold

\[ \rho(\text{Volatility}, \text{Threshold}) = 60\% \]

B. Kelly

Risk Premia and Conditional Tails
Testing Model Implications: Predicting Stock Returns

- Theory suggests increases in tail risk forecast increases in excess returns
- Predictive regressions of excess returns on aggregate market over short (one month) and long (up to five year) horizons
- Compare against common alternatives (dividend-price ratio, term spread, etc.)
- Robustness
### Testing Model Implications: Predicting Stock Returns

#### Univariate Prediction

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<th>Five year horizon</th>
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<td>Long term yield</td>
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<td>Net equity expansion</td>
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Testing Model Implications: Predicting Stock Returns

Bivariate Prediction

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<td>7.42</td>
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Out-of-Sample Prediction (Lower Tail)

Monthly Out-of-Sample $R^2 = 1.3\%$
Theory predicts

1. Differential exposure to tail risk state variable implies cross-sectional difference in expected returns
2. Negative price of tail risk: assets with high beta on tail risk have hedge value

Test for cross-sectional relation between individual asset/portfolio return tails and returns

1. Returns on tail risk beta-sorted portfolios
2. Fama-MacBeth tests
3. Robustness to alternative characteristics
## Tail Beta-Sorted Portfolios: NYSE/AMEX/NASDAQ Stocks

### Tail Risk Beta

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<th>Low 1</th>
<th>Low 2</th>
<th>Low 3</th>
<th>Low 4</th>
<th>High 5</th>
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### Panel B: Market Beta / Tail Risk Beta

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### Panel C: Market Equity / Tail Risk Beta

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### Panel D: Book-to-Market / Tail Risk Beta

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Testing Model Implications: The Cross Section of Returns

Stage 2 Fama-MacBeth Results: NYSE/AMEX/NASDAQ Stocks

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</table>

B. Kelly

Risk Premia and Conditional Tails
Hedging Tail Risk

Tail Risk Betas: 25 Size/BM Pts and VIX

B. Kelly
Risk Premia and Conditional Tails
Outline

- Introduction
- An Economic Framework
- Econometric Methodology
- Empirical Findings
- Conclusions
Conclusions

- Derive link between return tails and risk premia in an affine pricing framework with tail risk
- Present new methodology for capturing dynamic extreme risk in the economy
- Identify substantial time variation in tails
- Empirics consistent with predictions of structural model
  1. Large variation in tail risk over time
  2. Tail exponent forecasts excess market returns
  3. Associated with large cross-sectional differences in average returns

⋆

- What next?
  1. Unified pricing with other asset classes (options and credit)