On the Size of the Active Management Industry

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Q-Group Presentation, March 23, 2010
The “Active Management Puzzle”?  

- Track record of the active management (AM) industry is poor  
  - active equity mutual funds in aggregate: $\hat{\alpha} :< 0$ with $t \approx -2$  
- Nonetheless, active management remains popular:  
  - e.g., 87% of mutual funds assets are actively managed
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- Decreasing returns to scale in the active management industry 
  - industry’s \( \alpha \) depends on industry’s size: 
  - \( \alpha \) becomes more elusive as more money chases it 
  - past underperformance \( \Rightarrow \) industry should shrink, but 
  - uncertainty about decreasing returns \( \Rightarrow \) large confidence interval for the size we should expect
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- Decreasing returns to scale in the active management industry
  - industry’s $\alpha$ depends on industry’s size:  
    - $\alpha$ becomes more elusive as more money chases it  
    - past underperformance $\Rightarrow$ industry should shrink, but  
    - uncertainty about decreasing returns $\Rightarrow$ large confidence interval for the size we should expect
- In contrast, industry’s size would seem puzzling if it were known that returns to scale are constant
What We Do

- Develop an equilibrium model of active management (AM) with
  - competing utility-maximizing investors
  - competing fee-maximizing fund managers
- Solve for equilibrium size, alpha, and fees in AM industry
- Relate size of AM industry to past performance
- Analyze learning about returns to scale
What We Find

- AM industry can be large even if the track record is poor
- Investors’ learning about returns to scale is endogenous
  - what they invest depends on what they’ve learned
  - what they learn depends on what they invest
- Investors never learn the degree of returns to scale exactly
- Industry size can be suboptimal for a long time
- Other features of the model
  - industry size crucially depends on the degree of competition
  - $\alpha > 0$
  - “investment externality”
Two types of agents:

- $M$ managers of active funds (have skill but no capital)
- $N$ investors in active funds (have capital but no skill)

Benchmark-adjusted return to investors from fund $i = 1, \ldots, M$:

$$r_i = \alpha_i + u_i$$
$$u_i = x + \epsilon_i$$

$\sigma_x > 0 \Rightarrow$ cannot fully diversify risk by buying many funds
Model: Expected Profits

- Benchmark-adjusted expected total profit to fund $i$’s manager and investors:

$$\pi_i = s_i \left( a - b \frac{S}{W} \right)$$

$s_i$ . . . size of manager $i$’s fund
$S = \sum_{i=1}^{M} s_i$ . . . aggregate size of the AM industry
$W$ . . . total investable wealth of the $N$ investors

- Manager $i$ charges a proportional fee $f_i \Rightarrow$ investors expect benchmark-adjusted rate of return

$$\alpha_i = a - b \frac{S}{W} - f_i$$

- Decreasing returns to aggregate scale: $b > 0$
Figure 1. Decreasing returns to scale in the active management industry.
Model: Optimization

- Each manager $i$ chooses $f_i$ to maximize fee revenue:
  \[
  \max_{f_i} f_i s_i
  \]

- Investors know $f_i$'s before making investment decisions

- Each investor $j$ chooses weights $\delta_j$ on the $M$ funds to maximize
  \[
  \max_{\delta_j} \left\{ \delta_j' E(r|D) - \frac{\gamma}{2} \delta_j' \text{Var}(r|D) \delta_j \right\}
  \]

- All investors have same risk aversion $\gamma > 0$, same wealth, and same information set $D$
- Unrestricted allocations to benchmarks and T-bill
- No short sales of funds ($\delta_j \geq 0$)
Equilibrium with Known $a$ and $b$

- We solve for a symmetric Nash equilibrium
  - First for investors’ allocations, as a function of fees, then for managers’ fees
  - In equilibrium, fees and $\alpha$’s are equal across funds ($f_i = f$, $\alpha_i = \alpha$):

  \[
  f = \frac{a \gamma \sigma^2}{2 \gamma \sigma^2 + (M - 1)p}
  \]

  \[
  \alpha = a \left(1 - \frac{\gamma \sigma^2}{2 \gamma \sigma^2 + (M - 1)p}\right) \left(1 - \frac{Mb}{\gamma \sigma^2 + Mp}\right)
  \]

  \[
  \frac{S}{W} = \frac{Ma}{\gamma \sigma^2 + Mp} \left(1 - \frac{\gamma \sigma^2}{2 \gamma \sigma^2 + (M - 1)p}\right),
  \]

  where

  \[
  p = \frac{N + 1}{N} b + \gamma \sigma^2_x
  \]
Equilibrium Fee

- $M \uparrow \Rightarrow f \downarrow$, due to competition among managers
  - For $M = 1$, we have $f = a/2$
  - For $M \to \infty$, we have $f \to 0$

- Note: $f$ is the discretionary component of the total fee
  - $f$ = fee the manager sets while considering its effect on fund size
    (any competitive proportional fee is part of $a$)

- Also, $f \uparrow$ when $a \uparrow$, $b \downarrow$, $\sigma_\epsilon \uparrow$, $\sigma_x \downarrow$, and $N \uparrow$
In general, the equilibrium alpha is positive, 

$$\alpha > 0$$

because investors

1. demand compensation for risk ($$\sigma_x$$ and possibly also $$\sigma_\epsilon$$)
2. internalize some of the “investment externality”
Investors demand compensation for two kinds of risk:

- $\sigma_\epsilon$: diversifiable if $M \to \infty$
- $\sigma_x$: non-diversifiable even if $M \to \infty$

When $M \to \infty$, diversifiable risk $\sigma_\epsilon$ drops out:

$$\alpha = a \left( \frac{(1/N) b + \gamma \sigma_x^2}{[(N+1)/N] b + \gamma \sigma_x^2} \right)$$

When $N \to \infty$ as well,

$$\alpha = a \left( \frac{\gamma \sigma_x^2}{b + \gamma \sigma_x^2} \right)$$
“Investment externality”:
- new investors impose a negative externality on existing investors by diluting their returns

When $N$ is finite, investors internalize some of the reduction in profits resulting from their own investment

$\Rightarrow \alpha > 0$ even if there is no risk

When $M \to \infty$ and $\sigma_x \to 0$,

$$\alpha = \frac{a}{N + 1}$$

Note: $\alpha \downarrow$ when $N \uparrow$
The effect of $M$ on $\alpha$ is ambiguous; two opposing effects:

- $M \downarrow \Rightarrow \alpha \downarrow$ because fees $\uparrow$
- $M \downarrow \Rightarrow \alpha \uparrow$ because investors demand compensation for $\sigma_\epsilon$
Equilibrium Size of the AM Industry

- When \( N \to \infty \), we obtain a familiar mean-variance result:

\[
\frac{S}{W} = \frac{E(r_A|D)}{\gamma \text{Var}(r_A|D)}
\]

where \( r_A \) is the aggregate benchmark-adjusted return.

- When \( N \to \infty \) and \( M \to \infty \),

\[
\frac{S}{W} = \frac{a}{b + \gamma \sigma_x^2} = \frac{\alpha}{\gamma \sigma_x^2}
\]

- When \( N \to \infty \) and \( M = 1 \),

\[
\frac{S}{W} = \frac{\alpha}{\gamma (\sigma_x^2 + \sigma_\epsilon^2)}
\]
Let $S^* = \text{size maximizing expected total profit}, \ S \left( a - b \frac{S}{W} \right)$

$$\frac{S^*}{W} = \frac{a}{2b}$$

The equilibrium size $S \leq \bar{S} = 2S^*$
\[ a - b \left( \frac{S}{W} \right) \]
Let $S^\ast = \text{size maximizing expected total profit}, \ S \ (a - b \frac{S}{W})$

$$\frac{S^\ast}{W} = \frac{a}{2b}$$

The equilibrium size $S \leq \bar{S} = 2S^\ast$

When $M = 1$, there is underinvestment, $S \leq S^\ast$

- Manager-monopolist charges high fee
- $S = S^\ast$ only if $N \rightarrow \infty$ and $\sigma_x^2 + \sigma_\epsilon^2 = 0$
When $M \to \infty$, 

$$\frac{S}{S^*} = \frac{2b}{\frac{N+1}{N} b + \gamma \sigma_x^2}$$

⇒ *underinvestment* ($S < S^*$) or *overinvestment* ($S > S^*$)

When $M \to \infty$ and $\sigma_x^2 \to 0$, there is *overinvestment*:

$$\frac{S}{S^*} = \frac{2N}{N + 1}$$

$N \to \infty$ ⇒ $S \to \bar{S} = 2S^*$
Unknown $a$ and $b$

- Now suppose $a$ and $b$ are unknown

\[
E\left(\begin{bmatrix} a \\ b \end{bmatrix} \mid D\right) = \begin{bmatrix} \tilde{a} \\ \tilde{b} \end{bmatrix}
\]

\[
\text{Var}\left(\begin{bmatrix} a \\ b \end{bmatrix} \mid D\right) = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix}
\]

- For simplicity, let $M \to \infty$ and $N \to \infty$
  - Then $f \to 0$ and $\alpha = a - b(S/W)$

- Solve for a symmetric Nash equilibrium among investors
Unknown $a$ and $b$ (cont’d)

► In equilibrium, $S/W$ is the (unique) real positive solution to

$$0 = \tilde{a} - \frac{S}{W} \left[ \tilde{b} + \gamma (\sigma_a^2 + \sigma_x^2) \right] + \left( \frac{S}{W} \right)^2 2\gamma \sigma_{ab} - \left( \frac{S}{W} \right)^3 \gamma \sigma_b^2$$

as long as $\tilde{a} > 0$. If $\tilde{a} \leq 0$, then $S/W = 0$.

► In this setting,

$$E(r_A|D) = \tilde{a} - \tilde{b} \frac{S}{W}$$

$$\text{Var}(r_A|D) = \sigma_a^2 + \sigma_x^2 - 2 \left( \frac{S}{W} \right) \sigma_{ab} + \left( \frac{S}{W} \right)^2 \sigma_b^2$$

$$\Rightarrow \frac{S}{W} = \frac{E(r_A|D)}{\gamma \text{Var}(r_A|D)}$$
Prior Beliefs for $a$ and $b$

- One prior for $a$, two priors for $b$
  - **Prior 1**: $b = 0$, known (constant returns to scale)
  - **Prior 2**: $b \geq 0$, unknown (decreasing returns to scale)

- Assume
  - $S/W = 0.9$ is optimal under Prior 2
  - Prior mean of $\alpha = 10\%$ per year at $S/W = 0.9$
  - Risk aversion of $\gamma = 2$
  - Volatility of aggregate active return $\sigma_x = 2\%$ per year
    - Close to empirical estimates for active equity mutual funds
Figure 2. Prior distributions.
Implied Prior Beliefs for $\alpha$

- Implied prior for $\alpha$ depends on $S/W$ when $b \geq 0$
  
  \[ \alpha = a - b(S/W) \]

- **Prior 2 is more pessimistic about $\alpha$** than Prior 1
  
  - $\alpha$ is smaller under Prior 2 for any $S/W > 0$

- Nonetheless, we’ll see that **Prior 2 investors invest more** in AM than Prior 1 investors after a negative track record
Updated Beliefs

- 300,000 samples of simulated AM returns and allocations
  - For each sample, randomly draw $a$ and $b$ from their priors
- In each year $t$, beginning with $t = 1$, we perform three steps:
  1. Solve for equilibrium allocation to AM
     - Restrict $(S/W)_t$ between 0 and 1
  2. Construct AM return
     - $r_{A,t} = a - b(S/W)_t + x_t$, where $x_t \sim N(0, \sigma^2_x)$
  3. Update beliefs about $a$ and $b$
     - Regress $r_A$ on $S/W$ and constant $\Rightarrow$ intercept $a$, slope $-b$

... then back to step 1
Figure 3. Posterior standard deviations.
Figure 4. Deviations from true values.
Learning About Returns to Scale

- “Endogeneity”: learn $\Rightarrow$ invest $\Rightarrow$ learn $\Rightarrow$ invest $\Rightarrow$ ...
- Learning about the intercept and slope from

$$r_t = a - b\left(\frac{S}{W}\right)_t + \epsilon_t$$

- $\left(\frac{S}{W}\right)_{t+1}$ depends on beliefs about $a$ and $b$ at time $t$
- If $\left(\frac{S}{W}\right)_t$ stops changing, learning about $a$ and $b$ stops
  $\Rightarrow$ Never learn $a$ and $b$
- $\left(\frac{S}{W}\right)_t$ converges to optimal level quickly when $b$ is high, but it can stay suboptimal for a long time when $b$ is low
Learning When $b \geq 0$

- Investors learn differently under Priors 1 and 2 because $(S/W)_t$ affects learning when $b \geq 0$ but not when $b = 0$
- Representative examples of learning paths: Figure 5
  - Three values of $b$: “low”, “median”, “high” (5th, 50th, 95th percentiles of the prior distribution)
  - Given $b$, pick $a$ such that the “true” $S/W = 0.5$
  - Use $(a, b)$ to generate random samples of returns
- Results:
  - When $b$ is high, investors find the optimal $S/W$ quickly
  - When $b$ is low, investors can get stuck at “wrong” $S/W$ for a long time
Figure 5. Examples of learning paths.
Role of Historical Performance

- Plot the posterior of the equilibrium $S/W$ conditional on $t(\hat{\alpha})$
  - Note: $t(\hat{\alpha}) = \hat{\alpha}\sqrt{T}/\sigma_x$

- How much is invested in AM when $\hat{\alpha}$ is significantly negative?

- Prior 1 ($b = 0$) investors invest nothing
  - $t(\hat{\alpha})$ is a sufficient statistic for $S/W$

- Prior 2 ($b \geq 0$) investors can invest a lot
  - despite Prior 2 being more pessimistic about $\alpha$ than Prior 1
  - $t(\hat{\alpha})$ is NOT a sufficient statistic for $S/W$
Figure 6. Posterior distribution of the equilibrium allocation to active management conditional on the sample $t$-statistic.
Figure 7. Alternative prior distribution.
Figure 8. Equilibrium allocation to active management under the alternative prior.
Differences from Berk and Green (2004)

- **Focus**
  - BG: Capital flows across funds
  - We: Size of the AM industry

- **Decreasing returns to scale**
  - BG: At individual fund level
  - We: At aggregate industry level

- **Parameter uncertainty**
  - BG: $a$ unknown, $b$ known
  - We: $a$ and $b$ both unknown

- **Fund managers setting fees**
  - BG: Monopoly
  - We: Competition
Differences from Berk and Green (2004) (cont’d)

- Equilibrium alpha
  - BG: $\alpha = 0$; no explicit investor optimization
  - We: $\alpha > 0$; equilibrium outcome for optimizing investors

- Our model comes closest to BG when . . .
  - $M = 1$ (a single fund manager) $\Rightarrow$ same fee setting
  - $N \to \infty$ (many investors) $\Rightarrow$ no investment externality
  - $\sigma_\epsilon = \sigma_x = 0$ (no risk) $\Rightarrow$ no compensation for risk

Equilibrium then features $\alpha = 0$, $S = S^*$, and $f = a/2$, as in BG
Conclusions

- Size of AM industry can be large even if the track record is poor
  - Due to decreasing returns to scale
- Learning about returns to scale is “endogenous” and slow
  - Never learn the degree of returns to scale exactly
  - Industry size can be suboptimal for a long time

- Interesting features of the model
  1. Industry size crucially depends on the degree of competition
  2. $\alpha > 0$
  3. “Investment externality”