Attention Allocation Over the Business Cycle

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Abstract

The invisibility of information precludes a direct test of attention allocation theories. To surmount this obstacle, we develop a model that uses an observable variable – the state of the business cycle – to predict attention allocation. Attention allocation, in turn, predicts aggregate investment patterns. Because the theory begins and ends with observable variables, it becomes testable. We apply our theory to a large information-based industry, actively managed equity mutual funds, and study its investment choices and returns. Consistent with the theory, which predicts cyclical changes in attention allocation, we find that in recessions, funds' portfolios (1) covary more with aggregate payoff-relevant information, (2) exhibit more cross-sectional dispersion, and (3) generate higher returns. The results suggest that some, but not all, fund managers process information in a value-maximizing way for their clients and that these skilled managers outperform others.

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> "What information consumes is rather obvious: It consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention, and a need to allocate that attention efficiently among the overabundance of information sources that might consume it." Simon (1971)

Most decision makers are faced with an abundance of information and must choose how to allocate their limited attention. Recent work has shown that introducing attention constraints into decision problems can help explain observed price-setting, consumption, and investment patterns.[1] Unfortunately, the invisibility of information precludes direct testing of whether agents actually allocate their attention in a value-maximizing way. To surmount this obstacle, we develop a model of portfolio investment that uses an observable variable – the state of the business cycle – to predict attention allocation. Attention, in turn, predicts aggregate investment patterns. Because the theory begins and ends with observable variables, it becomes testable. To carry out these tests, we use data on actively managed equity mutual funds. A wealth of detailed data on portfolio holdings and returns makes this industry an ideal setting in which to test the rationality of attention allocation.

A better understanding of attention allocation sheds new light on a central question in the financial intermediation literature: Do investment managers add value for their clients? What makes this an important question is that a large and growing fraction of individual investors delegate their portfolio management to professional investment managers.[2] This intermediation occurs despite a significant body of evidence that finds that actively managed portfolios do not outperform passive investment strategies, on average, net of fees, and after controlling for differences in systematic risk exposure.[3] This evidence of zero average "alpha" has led many to conclude that investment managers have no skill. By developing a theory

[1]See, for example, Sims (2003) on consumption, Maćkowiak and Wiederholt (2009a, 2009b) on price setting, and Van Nieuwerburgh and Veldkamp (2009, 2010) or Kondor (2009) on financial investment. Klenow and Willis (2007) test whether price-setting dynamics and experimental outcomes are consistent with inattention theories. A related financial investment literature on information choice includes Grossman and Stiglitz (1980), Verrecchia (1982), Admati (1985), Peress (2004, 2009), Amador and Weill (2008), and Veldkamp (2006). While in rational inattention models agents typically choose the precision of their beliefs, Brunnermeier, Gollier, and Parker (2007) solve a portfolio problem in which investors choose the mean of their beliefs. In models of inattentiveness, e.g. Mankiw and Reis (2002), Gabaix and Laibson (2002) or Abel, Eberly, and Panageas (2007), agents update infrequently, but do not choose to pay more attention to some risks than others.

[2]In 1980, 48% of U.S. equity was directly held by individuals – as opposed to being held through intermediaries; by 2007, that fraction has been down to 21.5% (French (2008), Table 1). At the end of 2008, $9.6 trillion was invested with such intermediaries in the U.S. Of all investment in domestic equity mutual funds, about 85% is actively managed (2009 Investment Company Factbook). A related theoretical literature studies delegated portfolio management; e.g., Cuoco and Kaniel (2010), Vayanos and Woolley (2008), and Chien, Cole, and Lustig (2009).

[3]Among many others, see Jensen (1968), Gruber (1996), Fama and French (2010).

of managers' information and investment choices and finding evidence for its predictions in the mutual fund industry data, we conclude that the data are consistent with a world in which a small fraction of investment managers have skill, meaning that they can acquire and process informative signals about the future values of risky assets.[4] However, the model is also consistent with the empirical literature's finding that skill is hard to detect, on average. The model identifies recessions as times when information choices lead to investment choices that are more revealing of skill.

We argue that recessions and expansions imply different optimal attention allocation strategies for skilled investment managers. Different learning strategies, in turn, prompt different investment strategies, causing the differential performance in recessions and expansions. Specifically, we build a general equilibrium model in which a fraction of investment managers have skill. These skilled managers can observe a fixed number of signals and choose what fraction of those signals will contain aggregate versus stock-specific information. We think of aggregate signals as macroeconomic data that affect future cash flows of all firms, and of stock-specific signals as firm-level data that forecast the part of firms' future cash flows that is independent of the aggregate shocks. Based on their signals, skilled managers form portfolios, choosing larger portfolio weights for assets that are more likely to have high returns.

The model produces three main predictions. The first prediction is that attention should be re-allocated over the business cycle. As in most learning problems, risks that are large in *scale* and high in *volatility* are more valuable to learn about. In our model, aggregate shocks are large in scale, because many asset returns are affected by them, but they have low volatility. Stock-specific shocks are smaller in scale but have higher volatility. As in the data, aggregate shocks are more volatile in recessions, relative to stock-specific shocks.[5] The increased volatility of aggregate shocks makes it optimal to increase the attention paid to aggregate shocks in recessions and to stock-specific shocks in expansions. While the idea that it is more valuable to shift attention to more volatile shocks may not be all that surprising, how to test such a prediction is not obvious.

The second and third predictions do not come from the re-allocation of attention. Rather, they help to distinguish this theory from non-informational alternatives and support the idea

[4]The finding that some managers have skill is consistent with a number of recent papers in the empirical mutual fund literature, e.g., Kacperczyk, Sialm, and Zheng (2005, 2008), Kacperczyk and Seru (2007), Koijen (2010), Baker, Litov, Wachter, and Wurgler (2009), Huang, Sialm, and Zhang (2009).

[5]We show below that the idiosyncratic risk in stock returns, averaged across stocks, does not vary significantly over the business cycle. In contrast, the aggregate risk averaged across stocks is almost twenty-five percent higher in recessions in our sample.

that at least some portfolio managers are engaging in value-maximizing behavior.

The second prediction is counter-cyclical dispersion in portfolios and profits. In recessions, when aggregate shocks to asset payoffs are larger in magnitude, asset payoffs exhibit more comovement. Thus, any portfolio strategies that put (exogenously) fixed weights on assets would have returns that also comove more in recessions. In contrast, when investment managers learn about asset payoffs and manage their portfolios according to what they learn, recessionary fund returns comove less. The reason is that when aggregate shocks become more volatile, managers who learn about aggregate shocks put less weight on their common prior beliefs, which have less predictive power, and more weight on their heterogeneous signals. This generates more heterogeneous beliefs in recessions and therefore more heterogeneous investment strategies and fund returns.

Third, the model predicts time variation in fund performance. The average fund can only outperform the market if there are other, non-fund investors who underperform. Therefore, the model also includes unskilled non-fund investors. Because asset payoffs are more uncertain, recessions are times when information is more valuable. Therefore, the informational advantage of the skilled over the unskilled increases and generates higher returns for informed managers. The average fund's outperformance rises.

We test the model's three main predictions on the universe of actively managed U.S. mutual funds. We employ a rich data set, assembled from a variety of sources, and uniquely suited to test our information-based explanation of mutual fund performance. To test the first prediction, a key insight is that managers can only choose portfolios that covary with shocks they pay attention to. Thus, to detect cyclical changes in attention, we look for changes in covariances. We estimate the covariance of each fund's portfolio holdings with the aggregate payoff shock, proxied by innovations in industrial production growth. This covariance measures a manager's ability to time the market by increasing (decreasing) her portfolio positions in anticipation of good (bad) macroeconomic news. We find that this timing covariance rises in recessions. We also calculate the covariance of a fund's portfolio holdings with asset-specific shocks, proxied by innovations in earnings. This covariance measures managers' ability to pick stocks that subsequently experience unexpectedly high earnings. We find that this stock-picking covariance increases in expansions.

Second, we test for cyclical changes in portfolio dispersion. We find, in recessions, that funds hold portfolios that differ more from one another. As a result, their returns differ more as well. In the model, much of this dispersion comes from taking different bets on market outcomes, which should show up as dispersion in CAPM betas. In the data, the prediction of higher beta dispersion in recessions is also confirmed.

Third, we document fund outperformance in recessions.[6] Risk-adjusted excess fund returns (alphas) are around 1.8 to 2.4% per year higher in recessions, depending on the specification. Gross alphas (before fees) are not statistically different from zero in expansions, but they are positive (2.1%) in recessions. Net alphas (after fees) are negative in expansions (-0.9%) and positive (1.0%) in recessions. These cyclical differences are statistically and economically significant.

The baseline model is as much about volatility as it is about recessions. But there also is an effect of recessions above and beyond that which comes from volatility alone. When we use both a recession indicator and aggregate volatility as explanatory variables, we find that both contribute about equally to our three main results. Our explanation for the additional recession effect is that recessions are times in which not only the quantity of risk, but also the price of risk rises. An extended model shows why a recession that embodies both effects generates more attention reallocation than an increase in volatility alone. Showing that this is truly a business cycle phenomenon is useful because it connects these results with the existing macroeconomics literature on rational inattention, e.g., Maćkowiak and Wiederholt (2009a, 2009b).

Because our theory tells us how skilled managers should invest, it suggests how to construct metrics that could help us identify skilled managers. To show that skilled managers exist, we select the top 25 percent of funds in terms of their stock-picking ability in expansions and show that the same group has significant market-timing ability in recessions; the other funds show no such market-timing ability.[7] Furthermore, these funds have higher *unconditional* returns. They tend to manage smaller, more active funds. By matching fund-level to manager-level data, we find that these skilled managers are more likely to attract new money flows and are more likely to depart later in their careers to hedge funds. Presumably, both are market-based reflections of their ability. Finally, we construct a skill index based on observables and show that it is persistent and that it predicts future performance.

The rest of the paper is organized as follows. Section 1 lays out our model. After describing the setup, we characterize the optimal information and investment choices of

[6] Moskowitz (2000), Kosowski (2006), Lynch and Wachter (2007), and Glode (2010) also document such evidence, but their focus is solely on performance.

[7] This is quite different from a typical approach in the literature, which has studied stock picking and market timing in isolation, and unconditional on the state of the economy. The consensus view from that literature is that there is some evidence of stock-picking ability (on average, over time, and across managers), but no evidence for market timing (e.g., Graham and Harvey (1996), Daniel, Grinblatt, Titman, and Wermers (1997), Wermers (2000) and Kacperczyk and Seru (2007)).

skilled and unskilled investors. We show how equilibrium asset prices are formed. We derive theoretical predictions for funds' attention allocation, portfolio dispersion, and performance. Section 2 tests the model's predictions using the context of actively managed mutual funds. Section 3 discusses the results that use volatility as a conditioning variable instead of or in addition to the recession indicator. Section 4 uses the model's insights to identify a group of skilled mutual funds in the data. Section 5 discusses alternative explanations.

1 Model

We develop a stylized model whose purpose is to understand the optimal attention allocation of investment managers and its implications for asset holdings and equilibrium asset prices.

1.1 Setup

We consider a three-period static model. At time 1, skilled investment managers choose how to allocate their attention across aggregate and asset-specific shocks. At time 2, all investors choose their portfolios of risky and riskless assets. At time 3, asset payoffs and utility are realized. Since this is a static model, the investment world is either in the recession (R) or in the expansion state (E).[8] Our main model holds each manager's total attention fixed and studies its allocation in recessions and expansions. In Section 1.7, we allow a manager to choose how much capacity for attention to acquire.

Assets The model features three assets. Assets 1 and 2 have random payoffs f with respective loadings b_1, b_2 on an aggregate shock a, and face a stock-specific shock s_1, s_2. The third asset, c, is a composite asset. Its payoff has no stock-specific shock and a loading of one on the aggregate shock. We use this composite asset as a stand-in for all other assets to avoid the curse of dimensionality in the optimal attention allocation problem. Formally,

$$
\begin{aligned}
f_i &= \mu_i + b_i a + s_i, \ i \in \{1, 2\} \\
f_c &= \mu_c + a
\end{aligned}
$$

[8] We do not consider transitions between recessions and expansions, although such an extension would be trivial in our setting because assets are short lived and their payoffs are realized and known to all investors at the end of each period. Thus, a dynamic model simply amounts to a succession of static models that are either in the expansion or in the recession state.

where the shocks $a \sim N(0, \sigma_a)$ and $s_i \sim N(0, \sigma_i)$, for $i \in \{1, 2\}$. At time 1, the distribution of payoffs is common knowledge; all investors have common priors about payoffs $f \sim N(\mu, \Sigma)$. Let E_1, V_1 denote expectations and variances conditioned on this information. Specifically, $E_1[f_i] = \mu_i$. The prior covariance matrix of the payoffs, Σ, has the following entries: $\Sigma_{ii} = b_i^2\sigma_a + \sigma_i$ and $\Sigma_{ij} = b_i b_j \sigma_a$. In matrix notation:

$$\Sigma = bb'\sigma_a + \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where the vector b is defined as $b = [b_1\, b_2\, 1]'$. In addition to the three risky assets, there exists a risk-free asset that pays a gross return, r.

We model recessions as periods with high aggregate risk, that is, the prior variance of the aggregate shock in recessions is higher than the one in expansions: $\sigma_a(R) > \sigma_a(E)$. Section 2.2 justifies this assumption by showing that aggregate risk of stocks increases substantially in recessions while idiosyncratic risk does not.

Investors We consider a continuum of atomless investors. In the model, the only ex-ante difference between investors is that a fraction χ of them have *skill*, meaning that they can choose to observe a set of informative signals about the payoff shocks a or s_i. We describe this signal choice problem below. The remaining unskilled investors observe no information other than their prior beliefs.

Some of the unskilled investors are investment managers. As in reality, there are also non-fund investors, all of whom we assume are unskilled.[9] The reason for modeling non-fund investors is that without them, the sum of all funds' holdings would have to equal the market (market clearing) and therefore, the average fund return would have to equal the market return. There could be no excess return in expansions or recessions.

Bayesian Updating At time 2, each skilled investment manager observes signal realizations. Signals are random draws from a distribution that is centered around the true payoff shock, with a variance equal to the inverse of the signal precision that was chosen at time 1. Thus, skilled manager j's signals are $\eta_{aj} = a + e_{aj}$, $\eta_{1j} = s_1 + e_{1j}$, and $\eta_{2j} = s_2 + e_{2j}$, where $e_{aj} \sim N(0, K_{aj}^{-1})$, $e_{1j} \sim N(0, K_{1j}^{-1})$, and $e_{2j} \sim N(0, K_{2j}^{-1})$ are independent of each other

[9]For our results, it is sufficient to assume that the fraction of non-fund investors that are unskilled is higher than that for the investment managers (funds).

and across fund managers. Managers combine signal realizations with priors to update their beliefs, using Bayes' law. Of course, asset prices contain payoff-relevant information as well. We could allow managers to infer this information and subtract the amount of attention required to infer this information from their total attention endowment. However, Lemma S.2 in the Supplementary Appendix[10] establishes that managers always prefer not to use their attention to process the information in prices, when they could instead use the same amount of capacity to process private signals. Therefore, we model managers as if they observed prices, but did not exert the mental effort required to infer the payoff-relevant signals.

Since the resulting posterior beliefs (conditional on time-2 information) are such that payoffs are normally distributed, they can be fully described by posterior means, $(\hat{a}_j, \hat{s}_{ij})$, and variances, $(\hat{\sigma}_{aj}, \hat{\sigma}_{ij})$. More precisely, posterior precisions are the sum of prior and signal precisions: $\hat{\sigma}_{aj}^{-1} = \sigma_a^{-1} + K_{aj}$ and $\hat{\sigma}_{ij}^{-1} = \sigma_i^{-1} + K_{ij}$. The posterior means of the stock-specific shocks, $\hat{s}_{ij}$, are a precision-weighted linear combination of the prior belief that $s_i = 0$ and the signal η_i: $\hat{s}_{ij} = K_{ij}\eta_{ij}/(K_{ij} + \sigma_i^{-1})$. Simplifying yields $\hat{s}_{ij} = (1 - \hat{\sigma}_{ij}\sigma_i^{-1})\eta_{ij}$ and $\hat{a}_j = (1 - \hat{\sigma}_{aj}\sigma_a^{-1})\eta_{aj}$. Next, we convert posterior beliefs about the underlying shocks into posterior beliefs about the asset payoffs. Let $\hat{\Sigma}_j$ be the posterior variance-covariance matrix of payoffs f:

$$\hat{\Sigma}_j = bb'\hat{\sigma}_{aj} + \begin{bmatrix} \hat{\sigma}_{1j} & 0 & 0 \\ 0 & \hat{\sigma}_{2j} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Likewise, let $\hat{\mu}_j$ be the 3×1 vector of posterior expected payoffs:

$$\hat{\mu}_j = [\mu_1 + b_1\hat{a}_j + \hat{s}_{1j},\ \mu_2 + b_2\hat{a}_j + \hat{s}_{2j},\ \mu_c + \hat{a}_j]' \tag{1}$$

For any unskilled manager or investor: $\hat{\mu}_j = \mu$ and $\hat{\Sigma}_j = \Sigma$.

Portfolio Choice Problem We solve this model by backward induction. We first solve for the optimal portfolio at time 2 and substitute in that solution into the time-1 optimal attention allocation problem.

Investors are each endowed with initial wealth, W_0. They have mean-variance preferences over time-3 wealth, with a risk aversion coefficient, ρ. Let E_2 and V_2 denote expectations and variances conditioned on all information known at time 2. Thus, investor j chooses q_j

[10]References denoted by S are in the paper's separate appendix, available from the authors' websites or at http://pages.stern.nyu.edu/~lveldkam/pdfs/mfund_KVNV_appdx.pdf

to maximize time-2 expected utility, U_{2j}:

$$U_{2j} = \rho E_2[W_j] - \frac{\rho^2}{2} V_2[W_j] \tag{2}$$

subject to the budget constraint:

$$W_j = rW_0 + q_j'(f - pr.) \tag{3}$$

After having received the signals and having observed the prices of the risky assets, p, the investment manager chooses risky asset holdings, q_j, where p and q_j are 3-by-1 vectors.

Asset Prices Equilibrium asset prices are determined by market clearing:

$$\int q_j dj = \bar{x} + x, \tag{4}$$

where the left-hand side of the equation is the vector of aggregate demand and the right-hand side is the vector of aggregate supply. As in the standard noisy rational expectations equilibrium model, the asset supply is random to prevent the price from fully revealing the information of informed investors. We denote the 3×1 noisy asset supply vector by $\bar{x} + x$, with a random component $x \sim N(0, \sigma_x I)$.

Attention Allocation Problem At time 1, a skilled investment manager j chooses the precisions of signals about the payoff-relevant shocks a, s_1, or s_2 that she will receive at time 2. We denote these signal precisions by K_{aj}, K_{1j}, and K_{2j}, respectively. These choices maximize time-1 expected utility, U_{1j}, over the fund's terminal wealth:

$$U_{1j} = E_1 \left[\rho E_2[W_j] - \frac{\rho^2}{2} V_2[W_j] \right], \tag{5}$$

subject to two constraints.

The first constraint is the *information capacity constraint.* It states that the sum of the signal precisions must not exceed the information capacity:

$$K_{1j} + K_{2j} + K_{aj} \leq K. \tag{6}$$

Unskilled investors have no information capacity, $K = 0$. In Bayesian updating with normal variables, observing one signal with precision τ^{-1} or two signals, each with precision $\tau^{-1}/2$, is equivalent. Therefore, one interpretation of the capacity constraint is that it allows the manager to observe N signal draws, each with precision K/N, for large N. The investment manager then chooses how many of those N signals will be about each shock.[11]

The second constraint is the *no-forgetting constraint*, which ensures that the chosen precisions are non-negative:

$$K_{1j} \geq 0 \qquad K_{2j} \geq 0 \qquad K_{aj} \geq 0. \tag{7}$$

It prevents the manager from erasing any prior information, to make room to gather new information about another shock.

1.2 Model Solution

Substituting the budget constraint (3) into the objective function (2) and taking the first-order condition with respect to q_j reveals that optimal holdings are increasing in the investor's risk tolerance, precision of beliefs, and expected return on the assets:

$$q_j = \frac{1}{\rho}\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr). \tag{8}$$

Since uninformed managers and non-fund investors have identical beliefs, $\hat{\mu}_j = \mu$ and $\hat{\Sigma}_j = \Sigma$, they hold identical portfolios $\rho^{-1}\Sigma^{-1}(\mu - pr)$.

Appendix S.1 utilizes the market-clearing condition (4) to prove that equilibrium asset prices are linear in payoffs and supply shocks, and to derive expressions for the coefficients A, B, and C in the following lemma:

Lemma 1. $p = \frac{1}{r}\left(A + Bf + Cx\right)$

Substituting optimal risky asset holdings from equation (8) into the first-period objective function (5) yields: $U_{1j} = \frac{1}{2}E_1\left[(\hat{\mu}_j - pr)\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr)\right]$. Because asset prices are linear functions of normally distributed payoffs and asset supplies, expected excess returns, $\hat{\mu}_j - pr$, are normally distributed as well. Therefore, $(\hat{\mu}_j - pr)\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr)$ is a non-central χ^2-

[11] The results are not sensitive to the additive nature of the information capacity constraint. They also hold, for example, for a product constraint on precisions. The entropy constraints often used in information theory take this multiplicative form.

distributed variable, with mean[12]

$$U_{1j} = \frac{1}{2}trace(\hat{\Sigma}_j^{-1}V_1[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr]. \quad (9)$$

1.3 Bridging The Gap Between Model and Data

The following three sections explain the model's three key predictions: attention allocation, dispersion in investors' portfolios, and average performance. For each prediction, we state a hypothesis and explain how we test it. But the payoffs and quantities that have analytical expressions in a model with CARA preferences and normally distributed asset payoffs do not correspond neatly to the returns and portfolio weights that are commonly measured in the data. To bridge this gap, we introduce empirical measures of attention, dispersion, and performance. These standard definitions of returns and portfolio weights have no known moment-generating functions in our model. For example, the asset return is a ratio of normally distributed variables. Therefore, Appendix S.2 uses a numerical example to demonstrate that the empirical and theoretical measures have the same comparative statics.

Specifically, our empirical measures use conventional definitions of asset returns, portfolio returns, and portfolio weights. Risky asset returns are defined as $R^i \equiv \frac{f_i}{p_i} - 1$, for $i \in \{1, 2, c\}$, while the risk-free asset return is $R^0 \equiv \frac{1+r}{1} - 1 = r$. We define the market return as the value-weighted average of the individual asset returns: $R^m \equiv \sum_{i=1}^3 w_i^m R^i$, where $w_i^m \equiv \frac{p_i q_i^j}{\sum_{i=1}^3 p_i q_i^j}$. Likewise, a fund j's return is $R^j \equiv \sum_{i=0}^3 w_i^j R^i$, where $w_i^j \equiv \frac{p_i q_i^j}{\sum_{i=0}^3 p_i q_i^j}$. It follows that end-of-period wealth (assets under management) equals beginning-of-period wealth times the fund return: $W^j = W_0^j(1 + R^j)$.

1.4 Hypothesis 1: Attention Allocation

Each skilled manager ($K > 0$) solves for the choice of signal precisions $K_{aj} \geq 0$ and $K_{1j} \geq 0$ that maximize her time-1 expected utility (9). The choice of signal precision $K_{2j} \geq 0$ is implied by the capacity constraint (6). A robust prediction of our model is that it becomes relatively more valuable to learn about the aggregate shock, a, when the prior aggregate variance increases, that is, in recessions.

[12]If $z \sim N(E[z], Var[z])$, then $E[z'z] = trace(Var[z]) + E[z]'E[z]$, where $trace$ is the matrix trace (the sum of its diagonal elements). Setting $z = \hat{\Sigma}_j^{-1/2}(\hat{\mu}_j - pr)$ delivers the result. Appendix S.1.2 contains the expressions for $E_1[\hat{\mu}_j - pr]$ and $V_1[\hat{\mu}_j - pr]$.

Proposition 1. *If aggregate variance is not too high ($\sigma_a \leq 1$), then the marginal value of a given investor j reallocating an increment of capacity from stock-specific shock $i \epsilon \{1,2\}$ to the aggregate shock is increasing in the aggregate shock variance: If $K_{aj} = \tilde{K}$ and $K_{ij} = K - \tilde{K}$, then $\partial^2 U / \partial \tilde{K} \partial \sigma_a > 0$.*

The proofs of this and all further propositions are in Appendix S.1. Intuitively, in most learning problems, investors prefer to learn about large shocks that are an important component of the overall asset supply, and volatile shocks that have high prior payoff variance. Aggregate shocks are larger in scale, but are less volatile than stock-specific shocks. Recessions are times when aggregate volatility increases, which makes aggregate shocks more valuable to learn about. The converse is true in expansions. The parameter restriction $\sigma_a < 1$, is a sufficient, but not a necessary condition.[13] Note that this is a partial derivative result. It holds information choices fixed. In any interior equilibrium, attention will re-allocate until the marginal utility of learning about aggregate and stock-specific shocks is equalized. But it is the initial increase in marginal utility which drives this re-allocation.

Appendix S.2 presents a detailed numerical example in which parameters are chosen to match the observed volatilities of the aggregate and individual stock returns in expansions and recessions. For our benchmark parameter values, all skilled managers exclusively allocate attention to stock-specific shocks in expansions. In contrast, the bulk of skilled managers learn about the aggregate shock in recessions (87%, with the remaining 13% equally split between shocks 1 and 2). Thus, managers reallocate their attention over the business cycle. Such large swings in attention allocation occur for a wide range of parameters.

As long as the investor's capacity allocation choice is not a corner solution ($K_{aj} \neq 0$ or $K_{aj} \neq K$), a rise in the marginal utility of aggregate shock information increases the optimal K_{aj}. In these environments, skilled investment managers allocate a relatively larger fraction of their attention to learning about the aggregate shock in recessions. But, that effect can break down when assets become very asymmetric because corner solutions arise. For example, if the average supply of the composite asset, $\bar{x}_c$, is too large relative to the supply of the individual asset supplies, $\bar{x}_1$ and $\bar{x}_2$, the aggregate shock will be so valuable to learn about that all skilled managers will want to learn about it exclusively ($K_{aj} = K$) in expansions and recessions. Similarly, if the aggregate volatility, σ_a, is too low, then nobody ever learns about the aggregate shock ($K_{aj} = 0$ always).

[13] Of the seven terms in expected utility, six can be signed without parameter restrictions and one requires this restriction for the derivative to be positive.This constraint does not seem tight in the sense that a value for σ_a of 0.13 in expansions and 0.25 in recessions are the parameter choices that replicate the observed volatility of aggregate stock market returns in our simulation.

Investors' optimal attention allocation decisions are reflected in their portfolio holdings. In recessions, skilled investors predominantly allocate attention to the aggregate payoff shock, a. They use the information they observe to form a portfolio that covaries with a. In times when they learn that a will be high, they hold more risky assets whose returns are increasing in a. This positive covariance can be seen from equation (8) in which q is increasing in $\hat{\mu}_j$ and from equation (1) in which $\hat{\mu}_j$ is increasing in $\hat{a}_j$, which is further increasing in a. The positive covariances between the aggregate shock and funds' portfolio holdings in recessions, on the one hand, and between stock-specific shocks and the portfolio holdings in expansions, on the other hand, directly follow from optimal attention allocation decisions switching over the business cycle. As such, these covariances are the key moments that enable us to test the attention allocation predictions of the model.

We define a fund's *reliance on aggregate information*, RAI, as the covariance between its portfolio weights in deviation from the market portfolio weights, $w_i^j - w_i^m$, and the aggregate payoff shock, a, over a T-period horizon, averaged across assets:

$$RAI_t^j = \frac{1}{TN^j} \sum_{i=1}^{N^j} \sum_{\tau=0}^{T-1} (w_{it+\tau}^j - w_{it+\tau}^m)(a_{t+\tau+1}), \tag{10}$$

where N^j is the number of individual assets held by fund j. The subscript t on the portfolio weights and the subscript $t+1$ on the aggregate shock signify that the aggregate shock is unknown at the time of portfolio formation. Relative to the market, a fund with a high RAI overweights assets that have high (low) sensitivity to the aggregate shock in anticipation of a positive (negative) aggregate shock realization and underweights assets with a low (high) sensitivity.

RAI is closely related to measures of market-timing ability. $Timing$ measures how a fund's holdings of each asset, relative to the market, covary with the systematic component of the stock return, over the next T periods:

$$Timing_t^j = \frac{1}{TN^j} \sum_{i=1}^{N^j} \sum_{\tau=0}^{T-1} (w_{it+\tau}^j - w_{it+\tau}^m)(\beta_{it+\tau+1} R_{t+\tau+1}^m), \tag{11}$$

where β_i measures the covariance of asset i's return, R^i, with the market return, R^m, divided by the variance of the market return. The object $\beta_i R^m$ measures the systematic component of returns of asset i. The time subscripts indicate that the systematic component of the return is unknown at the time of portfolio formation. Before the market return rises, a fund with a high $Timing$ ability overweights assets that have high betas. Likewise, it underweights assets with high betas in anticipation of a market decline.

To confirm that RAI and $Timing$ accurately represent the model's prediction that skilled investors allocate more attention to the aggregate state in recessions, we resort to a numerical simulation. Appendix S.2 details the procedure and the construction of the empirical measures. For brevity, we only discuss the comparative statics in the main text. The simulation results show that RAI and $Timing$ are higher for skilled investors in recessions than they are in expansions. Because of market clearing, not all investors can time the market. Unskilled investors have negative timing ability in recessions. When the aggregate state a is low, most skilled investors sell, pushing down asset prices, p, and making prior expected returns, $(\mu - pr)$, high. Equation (8) shows that uninformed investors' asset holdings increase in $(\mu - pr)$. Thus, their holdings covary negatively with aggregate payoffs, making their RAI and $Timing$ measures negative. Since no investors learn about the aggregate shock in expansions, RAI and $Timing$ are close to zero for both skilled and unskilled. When averaged over all funds (including both skilled and unskilled funds but excluding non-fund investors), we find that RAI and $Timing$ are higher in recessions than in expansions.

When skilled investment managers allocate attention to stock-specific payoff shocks, s_i, information about s_i allows them to choose portfolios that covary with s_i. We define *reliance on stock-specific information*, RSI, which measures the covariance of a fund's portfolio weights of each stock, relative to the market, with the stock-specific shock, s_i:

$$RSI_t^j = \frac{1}{N^j}\sum_{i=1}^{N^j}(w_{it}^j - w_{it}^m)(s_{it+1}) \tag{12}$$

How well the manager can choose portfolio weights in anticipation of future asset-specific payoff shocks is closely linked to her stock-picking ability. $Picking_t^j$ measures how a fund's holdings of each stock, relative to the market, covary with the idiosyncratic component of the stock return:

$$Picking_t^j = \frac{1}{N^j}\sum_{i=1}^{N^j}(w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m) \tag{13}$$

A fund with a high $Picking$ ability overweights assets that have subsequently high idiosyncratic returns and underweights assets with low subsequent idiosyncratic returns. In our simulation, we find that skilled funds have positive RSI and $Picking$ ability in expansions, when they allocate their attention to stock-specific information. Unskilled investors have negative $Picking$ in expansions for the same reason that they have negative $Timing$ in recessions: Price fluctuations induce them to buy when returns are low and sell when returns are high. Across all funds, the model predicts lower RSI and $Picking$ in recessions.

1.5 Hypothesis 2: Dispersion

Perhaps the most controversial implication of the previous finding is that investment managers are processing information at all. Our second and third predictions speak directly to that question.

In recessions, as aggregate shocks become more volatile, the firm-specific shocks to assets' payoffs account for less of the variation, and the comovement in stock payoffs rises. Since asset payoffs comove more, the payoffs to all investment strategies that put fixed weights on assets should also comove more. But when investment managers are processing information, this prediction is reversed. To see why, consider the Bayesian updating formula for the posterior mean of asset payoffs. It is a weighted average of the prior mean μ and the fund j's signal $\eta_j|f \sim N(f, \Sigma_\eta)$, where each is weighted by their relative precision:

$$E[f|\eta_j] = \left(\Sigma^{-1} + \Sigma_\eta^{-1}\right)^{-1} \left(\Sigma^{-1}\mu + \Sigma_\eta^{-1}\eta_j\right) \tag{14}$$

In recessions, when the variance of the aggregate shock, σ_a, rises, the prior beliefs about asset payoffs are more uncertain: Σ rises and Σ^{-1} falls. This makes the weight on prior beliefs μ decrease and the weight on the signal η_j increase. The prior μ is common across agents, while the signal η_j is heterogeneous. When informed managers weigh their heterogeneous signals more, their resulting posterior beliefs become more different from each other and more different from the beliefs of uninformed managers. More disagreement about asset payoffs results in more heterogeneous portfolios and portfolio returns.

Thus, the model's second prediction is that in recessions, the cross-sectional dispersion in funds' investment strategies and returns rises. The following Proposition shows that funds' portfolio returns, $q_j'(f - pr)$, display higher cross-sectional dispersion when aggregate risk is higher, in recessions.

Proposition 2. *If the average manager has sufficiently low capacity, $\chi K < \sigma_a^{-1}$, then for given K_{aj} and K_{ij}, an increase in aggregate risk, σ_a, increases the dispersion of funds' portfolios $E[\sum_{i\epsilon\{1,2,c\}}(q_{ij} - \bar{q}_i)^2]$, and their portfolio returns $E[((q_j - \bar{q})'(f - pr))^2]$, where $\bar{q} \equiv \int q_j dj$.*

As before, the parameter restriction is sufficient, but not necessary and is not very tight when calibrated to the data.

To connect this proposition to the data, we use several measures of portfolio dispersion, commonly used in the empirical literature. The first one, proposed by Kacperczyk, Sialm, and Zheng (2005), is the sum of squared deviations of fund j's portfolio weight in asset i at time t, w_{it}^j, from the average fund's portfolio weight in asset i at time t, w_{it}^m, summed over all assets held by fund j, N^j:

$$Portfolio\ Dispersion_t^j = \sum_{i=1}^{N^j} \left(w_{it}^j - w_{it}^m\right)^2 \tag{15}$$

This portfolio dispersion measure is the same as the one in Proposition 2, except that the number of shares q are replaced with portfolio weights w. Our numerical example shows that the model's fund $Portfolio\ Dispersion$, defined over portfolio weights w, is higher in recessions as well. In recessions, the portfolios of the informed managers differ more from each other and more from those of the uninformed investors. Part of this difference comes from a change in the composition of the risky asset portfolio and part comes from differences in the fraction of assets held in riskless securities. Fund j's portfolio weight w_{it}^j is a fraction of the fund's assets, including both risky and riskless, held in asset i. Thus, when one informed fund gets a bearish signal about the market, its w_{it}^j for all risky assets i falls. Dispersion can rise when funds take different positions in risky assets, even if the fractional allocation among the risky assets remains identical.

The higher dispersion across funds' portfolio strategies translates into a higher cross-sectional dispersion in fund abnormal returns $(R^j - R^m)$. To facilitate comparison with the data, we define the dispersion of variable X as $|X^j - \bar{X}|$. The notation $\bar{X}$ denotes the equally weighted cross-sectional average across all investment managers (excluding non-fund investors).

When funds get signals about the aggregate state a that are heterogenous, they take different directional bets on the market. Some funds tilt their portfolios to high-beta assets and other funds to low-beta assets, thus creating dispersion in fund betas. To look for evidence of this mechanism, we form a CAPM regression for fund j:

$$R_t^j = \alpha^j + \beta^j R_t^m + \sigma_\varepsilon^j \varepsilon_t^j. \tag{16}$$

Our numerical results confirm that there is higher dispersion in the funds' betas, β^j and their abnormal returns, in recessions.

1.6 Hypothesis 3: Performance

The third prediction of the model is that the average performance of investment managers is higher in recessions than it is in expansions. To measure performance, we want to measure the portfolio return, adjusted for risk. One risk adjustment that is both analytically tractable in our model and often used in empirical work is the certainty equivalent return, which is also an investor's objective (5). The following proposition shows that the average certainty equivalent of skilled funds' returns exceeds that of unskilled funds by more when aggregate risk is higher, that is, in recessions.

Proposition 3. *If investor j knows more about the aggregate shock than the average investor does ($\hat{\sigma}_{aj} < \bar{\sigma}_a$), then an increase in aggregate shock variance increases the difference between j's expected certainty equivalent return and the expected certainty equivalent return of an uninformed investor: $\partial(U_j - U^U)/\partial\sigma_a > 0$.*

Corollary 1 in Appendix S.1.7 shows that a similar result holds for (risk unadjusted) abnormal portfolio returns, defined as the fund's portfolio return, $q_j'(f - pr)$, minus the market return, $\bar{q}'(f - pr)$.

Because asset payoffs are more uncertain, recessions are times when information is more valuable. Therefore, the advantage of the skilled over the unskilled increases in recessions. This informational advantage generates higher returns for informed managers. In equilibrium, market clearing dictates that alphas average to zero across all investors. However, because our data only include mutual funds, our model calculations similarly exclude non-fund investors. Since investment managers are skilled or unskilled, while other investors are only unskilled, an increase in the skill premium implies that an average manager's risk-adjusted return rises in recessions.

Our numerical simulations confirm that abnormal returns and alphas, defined as in the empirical literature, and averaged over all funds, are higher in recessions than in expansions. Skilled investment managers have positive excess returns, while the uninformed ones have negative excess returns. Aggregating returns across skilled and unskilled funds results in higher average alphas in recessions, the third main prediction of the model.

1.7 Endogenous Capacity Choice

So far, we have assumed that skilled investment managers choose how to allocate a fixed information-processing capacity, K. We now extend the model to allow for skilled managers to add capacity at a cost $\mathcal{C}(K)$.[14] We draw three main conclusions. First, the proofs of Propositions 1-3 hold for any chosen level of capacity K, below an upper bound, no matter the functional form of $\mathcal{C}$. Endogenous capacity only has quantitative, not qualitative implications. Second, because the marginal utility of learning about the aggregate shock is increasing in its prior variance (Proposition 1), skilled managers choose to acquire higher capacity in recessions. This extensive-margin effect amplifies our benchmark, intensive-margin result. Third, the degree of amplification depends on the convexity of the cost function, $\mathcal{C}(K)$. The convexity determines how elastic equilibrium capacity choice is to the cyclical changes in the marginal benefit of learning. Appendix S.2.4 discusses numerical simulation results from the endogenous-K model; they are similar to our benchmark results.

2 Evidence from Equity Mutual Funds

Our model studies attention allocation over the business cycle, and its consequences for investors' strategies. We now turn to a specific set of investment managers, active mutual fund managers, to test the predictions of the model. The richness of the data makes the mutual fund industry a great laboratory for these tests. In principle, similar tests could be conducted for hedge funds, other professional investment managers, or even individual investors.

2.1 Data

Our sample builds upon several data sets. We begin with the Center for Research on Security Prices (CRSP) survivorship bias-free mutual fund database. The CRSP database provides comprehensive information about fund returns and a host of other fund characteristics, such as size (total net assets), age, expense ratio, turnover, and load. Given the nature of our tests and data availability, we focus on actively managed open-end U.S. equity mutual funds. We further merge the CRSP data with fund holdings data from Thomson Financial. The total number of funds in our merged sample is 3,477. In addition, for some of our exercises, we map funds to the names of their managers using information from CRSP, Morningstar, Nelson's Directory of Investment Managers, Zoominfo, and Zabasearch. This mapping results in a sample with 4,267 managers. We also use the CRSP/Compustat stock-level database,

[14]We model this cost as a utility penalty, akin to the disutility from labor in business cycle models. Since there are no wealth effects in our setting, it would be equivalent to modeling a cost of capacity through the budget constraint. For a richer treatment of information production modeling, see Veldkamp (2006).

which is a source of information on individual stocks' returns, market capitalizations, book-to-market ratios, momentum, liquidity, and standardized unexpected earnings (SUE). The aggregate stock market return is the value-weighted average return of all stocks in the CRSP universe.

We use changes in monthly industrial production, obtained from the Federal Reserve Statistical Release, as a proxy for aggregate shocks. Industrial production is seasonally adjusted. We measure recessions using the definition of the National Bureau of Economic Research (NBER) business cycle dating committee. The start of the recession is the peak of economic activity and its end is the trough. Our aggregate sample spans 312 months of data from January 1980 until December 2005, among which 38 are NBER recession months (12%). We consider several alternative recession indicators and find our results to be robust.[15]

2.2 Recessions Are Periods of Higher Aggregate Risk

At the outset, we present empirical evidence for the main assumption in our model: Recessions are periods in which individual stocks contain more aggregate risk. Table 1 shows that an average stock's aggregate risk increases substantially in recessions whereas the change in idiosyncratic risk is not statistically different from zero. The table uses monthly returns for all stocks in the CRSP universe. For each stock and each month, we estimate a CAPM equation based on a twelve-month rolling-window regression, delivering the stock's beta, β_t^i, and its residual standard deviation, $\sigma_{\varepsilon t}^i$. We define the aggregate risk of stock i in month t as $|\beta_t^i \sigma_t^m|$ and its idiosyncratic risk as $\sigma_{\varepsilon t}^i$, where σ_t^m is formed monthly as the realized volatility from daily return observations. Panel A reports the results from a time-series regression of the aggregate (Columns 1 and 2) and the idiosyncratic risk (Columns 3 and 4), both averaged across stocks, on the NBER recession indicator variable.[16] The aggregate risk is twenty percent higher in recessions than it is in expansions (6.69% versus 8.04% per month), an economically and statistically significant difference. In contrast, the stock's idiosyncratic risk is essentially identical in expansions and in recessions. The results are similar whether one con-

[15] We have confirmed our results using an indicator variable for negative real consumption growth, the Chicago Fed National Activity Index (CFNAI), and an indicator variable for the 25% lowest stock market returns as alternative recession indicators. While its salience makes the NBER indicator a natural benchmark, the other measures may be available in a more timely manner. Also, the CFNAI has the advantage that it is a continuous variable, measuring the strength of economic activity. As an example, Table S.12 shows that the results on performance are, if anything, stronger using the CFNAI measure than they are with the NBER indicator. Other results are omitted for brevity but are available from the authors upon request.

[16] The reported results are for equally weighted averages. Unreported results confirm that value-weighted averaging across stocks delivers the same conclusion.

trols for other aggregate risk factors (Columns 2 and 4) or not (Columns 1 and 3). Panel B reports estimates from pooled (panel) regressions of a stock's aggregate risk (Columns 1 and 2) or idiosyncratic risk (Columns 3 and 4) on the recession indicator variable, *Recession*, and additional stock-specific control variables including size, book-to-market ratio, and leverage. The panel results confirm the time-series findings.

A large literature in economics and finance presents evidence supporting the results in Table 1. First, Ang and Chen (2002), Ribeiro and Veronesi (2002), and Forbes and Rigobon (2002) document that stocks exhibit more comovement in recessions, consistent with stocks carrying higher systematic risk in recessions. Second, Schwert (1989), Hamilton and Lin (1996), Campbell, Lettau, Malkiel, and Xu (2001), and Engle and Rangel (2008) show that aggregate stock market return volatility is much higher during periods of low economic activity. Diebold and Yilmaz (2008) find a robust cross-country link between volatile stock markets and volatile fundamentals. Third, Bloom, Floetotto, and Jaimovich (2009) find that the volatilities of GDP and industrial production growth, obtained from GARCH estimation, and the volatility implied by stock options are much higher during recessions. The same result holds for the uncertainty in several establishment-, firm- and industry-level payoff measures they consider.[17]

2.3 Testing Hypothesis 1: Attention Allocation

We begin by testing the first and most direct prediction of our model, that skilled investment managers reallocate their attention over the business cycle. Learning about the aggregate payoff shock in recessions makes managers choose portfolio holdings that covary more with the aggregate shock. Conversely, in expansions, their holdings covary more with stock-specific information. To this end, we estimate the following regression model:

$$Attention_t^j = a_0 + a_1 Recession_t + \mathbf{a_2 X_t^j} + \epsilon_t^j, \tag{17}$$

where $Attention_t^j$ denotes a generic attention variable, observed at month t for fund j. $Recession_t$ is an indicator variable equal to one if the economy in month t is in recession, as defined by the NBER, and zero otherwise. X is a vector of fund-specific control variables, including the fund age (natural logarithm of age in years since inception, $\log(Age)$), the

[17]Several other pieces of evidence also corroborate the link between volatility and recessions. First, labor earnings volatility is substantially counter-cyclical (Storesletten, Telmer, and Yaron (2004)). Second, small firms face more risk in recessions (Perez-Quiros and Timmermann (2000)). Finally, the notion of Shumpeterian creative destruction is also consistent with such link.

Table 1: **Individual Stocks Have More Aggregate Risk in Recessions**

For each stock i and month t, we estimate a CAPM equation based on twelve months of data (a twelve-month rolling-window regression). This estimation delivers the stock's beta, β_t^i, and its residual standard deviation, $\sigma_{\varepsilon t}^i$. We define stock i's aggregate risk in month t as $\left|\beta_t^i \sigma_t^m\right|$ and its idiosyncratic risk as $\sigma_{\varepsilon t}^i$, where σ_t^m is the realized volatility from daily market return observations. Panel A reports results from a time-series regression of the average stock's aggregate risk, $\frac{1}{N}\sum_{i=1}^{N}\left|\beta_t^i \sigma_t^m\right|$, in Columns 1 and 2, and of the average idiosyncratic risk, $\frac{1}{N}\sum_{i=1}^{N}\sigma_{\varepsilon t}^i$, in Columns 3 and 4 on *Recession*. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. In Columns 2 and 4 we include several aggregate control variables: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD). The data are monthly from 1980-2005 (309 months). Standard errors (in parentheses) are corrected for autocorrelation and heteroscedasticity. Panel B reports results of panel regressions of each stock's aggregate risk, $\left|\beta_t^i \sigma_t^m\right|$, in Columns 1 and 2 and of its idiosyncratic risk, $\sigma_{\varepsilon t}^i$, in Columns 3 and 4 on *Recession*. In Columns 2 and 4 we include several firm-specific control variables: the log market capitalization of the stock, $log(Size)$, the ratio of book equity to market equity, $B-M$, the average return over the past year, $Momentum$, the stock's ratio of book debt to book debt plus book equity, $Leverage$, and an indicator variable, $NASDAQ$, equal to one if the stock is traded on NASDAQ. All control variables are lagged one month. The data are monthly and cover all stocks in the CRSP universe for 1980-2005. Standard errors (in parentheses) are clustered at the stock and time dimensions.

	(1)	(2)	(3)	(4)
	Aggregate Risk		Idiosyncratic Risk	
Panel A: Time-Series Regression				
Recession	1.348	1.308	0.058	0.016
	(0.693)	(0.678)	(1.018)	(1.016)
MKTPREM		-4.034		-1.865
		(3.055)		(3.043)
SMB		8.110		12.045
		(3.780)		(4.293)
HML		0.292		9.664
		(5.458)		(8.150)
UMD		-4.279		-1.112
		(2.349)		(3.888)
Constant	6.694	6.748	13.229	13.196
	(0.204)	(0.212)	(0.286)	(0.276)
Observations	309	309	309	309
Panel B: Pooled Regression				
Recession	1.203	1.419	0.064	0.510
	(0.242)	(0.238)	(0.493)	(0.580)
Log(Size)		-0.145		-1.544
		(0.021)		(0.037)
B-M Ratio		-0.934		-2.691
		(0.056)		(0.086)
Momentum		0.097		2.059
		(0.101)		(0.177)
Leverage		-0.600		-1.006
		(0.074)		(0.119)
NASDAQ		0.600		1.937
		(0.075)		(0.105)
Constant	4.924	4.902	12.641	12.592
	(0.092)	(0.095)	(0.122)	(0.144)
Observations	1,312,216	1,312,216	1,312,216	1,312,216

fund size (natural logarithm of total net assets under management in millions of dollars, $\log(TNA)$), the average fund expense ratio (in percent per year, *Expenses*), the turnover rate (in percent per year, $Turnover$), the percentage flow of new funds (defined as the ratio of $TNA_t^j - TNA_{t-1}^j(1 + R_t^j)$ to TNA_{t-1}^j, $Flow$), and the fund load (the sum of front-end and back-end loads, additional fees charged to the customers to cover marketing and other expenses, *Load*). Also included are the fund style characteristics along the size, value, and momentum dimensions.[18] To mitigate the impact of outliers on our estimates, we winsorize $Flow$ and $Turnover$ at the 1% level.

We estimate this and most of our subsequent specifications using pooled (panel) regression model and calculating standard errors by clustering at the fund and time dimensions. This approach addresses the concern that the errors, conditional on independent variables, might be correlated within fund and time dimensions (e.g., Moulton (1986) and Thompson (2009)). Addressing this concern is especially important in our context since our variable of interest, $Recession$, is constant across all fund observations in a given time period. Also, we demean all control variables so that the constant a_0 can be interpreted as the level of the attention variable in expansions, and a_1 indicates how much the variable increases in recessions.

The first attention variable we examine is reliance on aggregate information, RAI, as in equation (10). We proxy for the aggregate payoff shock with the innovation in log industrial production growth.[19] A time series for RAI_t^j is obtained by computing the covariance of the innovations and each fund j's portfolio weights using twelve-month rolling windows. Our hypothesis is that RAI should be higher in recessions, which means that the coefficient on $Recession$, a_1, should be positive.

Our estimates of the parameters appear in Table 2. Column 1 shows the results for a univariate regression. In expansions, RAI is not different from zero, implying that funds' portfolios do not comove with future macroeconomic information in those periods. In recessions, RAI increases. Both findings are consistent with the model. The increase amounts to

[18]The size style of a fund is the value-weighted score of its stock holdings' percentile scores calculated with respect to their market capitalizations (1 denotes the smallest size percentile; 100 denotes the largest size percentile). The value style is the value-weighted score of its stock holdings' percentile scores calculated with respect to their book-to-market ratios (1 denotes the smallest B/M percentile; 100 denotes the largest B/M percentile). The momentum style is the value-weighted score of a fund's stock holdings' percentile scores calculated with respect to their past twelve-month returns (1 denotes the smallest return percentile; 100 denotes the largest return percentile). These style measures are similar in spirit to those defined in Kacperczyk, Sialm, and Zheng (2005).

[19]We regress log industrial production growth at $t+1$ on log industrial production growth in month t, and use the residual from this regression. Because industrial production growth is nearly i.i.d, the same results obtain if we simply use the log change in industrial production between t and $t+1$.

ten percent of a standard deviation of RAI. It is measured precisely, with a t-statistic of 3. To remedy the possibility of a bias in the coefficient due to omitted fund characteristics correlated with recession times, we turn to a multivariate regression. Our findings, in Column 2, remain largely unaffected by the inclusion of the control variables.

Table 2: **Attention Allocation is Cyclical**

Dependent variables: Fund j's RAI_t^j is defined in equation (10), where the rolling window T is 12 months and the aggregate shock a_{t+1} is the change in industrial production growth between t and $t+1$. A fund j's RSI_t^j is defined as in equation (12), where s_{it+1} is the change in asset i's earnings growth between t and $t+1$. $Timing_t^j$ and $Picking_t^j$ are defined in equations (11) and (13), where each stock's β_{it} is measured over a twelve-month rolling window. All are multiplied by 10,000 for readability.
Independent variables: *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. *Log(Age)* is the natural logarithm of fund age in years. *Log(TNA)* is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Turnover* is the fund turnover ratio. *Flow* is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered by fund and time.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	RAI		RSI		Timing		Picking	
Recession	0.011	0.011	-0.682	-0.696	0.140	0.139	-0.144	-0.146
	(0.004)	(0.004)	(0.159)	(0.150)	(0.070)	(0.068)	(0.047)	(0.047)
Log(Age)		-0.002		0.423		0.006		0.004
		(0.001)		(0.060)		(0.006)		(0.004)
Log(TNA)		-0.001		-0.173		0.000		-0.003
		(0.000)		(0.029)		(0.004)		(0.003)
Expenses		-0.330		88.756		1.021		-0.815
		(0.244)		(11.459)		(1.280)		(0.839)
Turnover		-0.004		-0.204		0.007		0.017
		(0.001)		(0.053)		(0.013)		(0.010)
Flow		-0.008		1.692		-0.001		0.058
		(0.010)		(0.639)		(0.078)		(0.088)
Load		0.017		-9.644		0.033		0.156
		(0.023)		(1.972)		(0.180)		(0.131)
Constant	-0.001	-0.001	3.084	3.086	0.007	0.007	-0.010	-0.010
	(0.001)	(0.001)	(0.069)	(0.070)	(0.024)	(0.024)	(0.018)	(0.018)
Observations	224,257	224,257	166,328	166,328	221,306	221,306	221,306	221,306

Next, we repeat our analysis using funds' reliance on stock-specific information (RSI) as a dependent variable. Following equation (12), RSI is computed in each month t as a cross-sectional covariance across the assets between the fund's portfolio weights and firm-specific earnings shocks.[20,21] In the model, the fund's portfolio holdings and its returns covary more

[20]We regress earnings per share in a given quarter on earnings per share in the previous quarter (earnings are reported quarterly), and use the residual from this regression. Suppose month t and $t+3$ are end-of-quarter months. Then RSI in months t, $t+1$, and $t+2$ are computed using portfolio weights from month t and earnings surprises from month $t+3$.

[21]We have verified that the firm-specific earnings shocks are uncorrelated with the aggregate earnings

with subsequent firm-specific shocks in expansions. Therefore, our hypothesis is that RSI should fall in recessions, or that a_1 should be negative.

Columns 3 and 4 of Table 2 show that the average RSI across funds is positive in expansions and substantially lower in recessions. The effect is statistically significant at the 1% level. It is also economically significant: RSI decreases by approximately ten percent of one standard deviation. Overall, the data support the model's prediction that portfolio holdings are more sensitive to aggregate shocks in recessions and more sensitive to firm-specific shocks in expansions.

Next, we examine the variation in market-timing, $Timing_t^j$, and stock-picking ability, $Picking_t^j$, defined in equations (11) and (13). The benefit of using these measures is that they have an exact analog in the model. In contrast, for RAI and RSI, we need to take a stance on the empirical proxy for the aggregate and idiosyncratic shocks. The stock betas, β_i, utilized in $Timing$ and $Picking$, are computed using the twelve-month rolling-window regressions of stock excess returns on market excess returns.

Columns 5 and 6 of Table 2 show that the average market-timing ability across funds increases significantly in recessions. In turn, we find no evidence of market timing in expansions. Since expansion months constitute the bulk of our sample, this result is consistent with the literature which fails to find evidence for market timing, on average. However, we find that market timing is positive and statistically different from zero in recessions. The increase is 25 percent of a standard deviation of the $Timing$ measure, which is economically meaningful. Likewise, Columns 7 and 8 show that stock-picking ability deteriorates substantially in recessions, again consistent with our theory. The reduction in recessions is about 20 percent of a standard deviation of the $Picking$ measure.

Table S.5 reports several robustness checks. First, we compute an alternative RAI measure, in which the aggregate shock is proxied by surprises in non-farm employment growth, another salient macroeconomic variable. Second, we use an alternative stock-specific shock s_{it+1} in our RSI measure: the residual from a regression of earnings per share in a given year on earnings per share in that same quarter one year (instead of one quarter) earlier, as in Bernard and Thomas (1989). Third, to check the market-timing results, we use the R^2 from a CAPM regression at the fund level (equation 16) to measure how funds' excess returns (as opposed to their portfolio weights) covary with the market's excess return. All the results are similar to our benchmark results, and in the case of employment growth, are estimated even more precisely.

shocks. The median correlation across stocks is below 0.01, with a cross-sectional standard deviation of 0.28.

To further explain how funds improve their market timing in recessions, we show they increase their cash holdings, reduce their holdings of high-beta stocks, and tilt their portfolios towards more defensive sectors. Tables S.6, S.7, and S.8 present the results; a more detailed discussion is in Appendix S.3.1.

2.4 Testing Hypothesis 2: Dispersion

The second prediction of the model states that heterogeneity in fund investment strategies and portfolio returns rises in recessions. To test this hypothesis, we estimate the following regression specification, using various return and investment heterogeneity measures, generically denoted as $Dispersion_t^j$, the dispersion of fund j at month t.

$$Dispersion_t^j = b_0 + b_1 Recession_t + \mathbf{b_2 X_t^j} + \epsilon_t^j, \tag{18}$$

The definitions of *Recession* and other controls mirror those in regression (17). Our coefficient of interest is b_1.

We begin by examining *Portfolio Dispersion*, which is defined in equation (15). It measures a fund's deviation from the investment strategies from a passive market strategy, and hence in equilibrium from the strategies of other investors. The results, in Columns 1 and 2 of Table 3, indicate an increase in average *Portfolio Dispersion* across funds in recessions. The increase is statistically significant at the 1% level. It is also economically significant: The value of portfolio dispersion in recessions goes up by about 15% of a standard deviation.

Since dispersion in fund strategies should generate dispersion in fund returns, we next look for evidence of higher return dispersion in recessions. To measure dispersion, we use the absolute deviation between fund j's return and the equally weighted cross-sectional average, $|return_t^j - \overline{return}_t|$, as the dependent variable in (18). Columns 3 and 4 of Table 3 show that return dispersion increases by 80% in recessions. Finally, portfolio and return dispersion in recessions should come from different directional bets on the market. This should show up as an increase in the dispersion of portfolio betas. Columns 5 and 6 show that the CAPM-beta dispersion also increases by about 30% in recessions, all consistent with the predictions of our model.

Table S.9 shows that counter-cyclical dispersion in fund performance is a finding that is robust to a variety of ways of measuring performance. Recessions are also times of more active portfolio management, as measured by changes in fund style.

Table 3: **Portfolio and Return Dispersion Rises in Recessions**

Dependent variables: Portfolio dispersion is the Herfindahl index of portfolio weights in stocks $i \in \{1, \cdots, N\}$ in deviation from the market portfolio weights $\sum_{i=1}^{N}(w_{it}^{j} - w_{it}^{m})^2 \times 100$. Return dispersion is $|return_t^j - \bar{return}_t|$, where $\bar{return}$ denotes the (equally weighted) cross-sectional average. The CAPM beta comes from twelve-month rolling-window regressions of fund-level excess returns on excess market returns (and returns on SMB, HML, and MOM). Beta dispersion is constructed analogously to return dispersion. The right-hand side variables, the sample period, and the standard error calculation are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)
	Portfolio Dispersion		Return Dispersion		Beta Dispersion	
Recession	0.205	0.147	0.530	0.561	0.082	0.083
	(0.027)	(0.026)	(0.108)	(0.101)	(0.015)	(0.014)
Log(Age)		0.203		-0.064		-0.009
		(0.028)		(0.007)		(0.002)
Log(TNA)		-0.179		0.029		0.003
		(0.014)		(0.004)		(0.001)
Expenses		28.835		13.816		5.460
		(4.860)		(1.152)		(0.235)
Turnover		-0.092		0.074		0.020
		(0.025)		(0.006)		(0.001)
Flow		0.122		0.479		0.022
		(0.104)		(0.088)		(0.017)
Load		-1.631		-1.738		-0.444
		(0.907)		(0.182)		(0.042)
Constant	1.525	1.524	0.586	0.659	0.229	0.229
	(0.024)	(0.022)	(0.018)	(0.027)	(0.006)	(0.006)
Observations	230,185	230,185	226,745	226,745	227,159	227,159

2.5 Testing Hypothesis 3: Performance

The third prediction of our model is that recessions are times when information allows funds to earn higher average risk-adjusted returns. We evaluate this hypothesis using the following regression specification:

$$Performance_t^j = c_0 + c_1 Recession_t + \mathbf{c_2 X_t^j} + \epsilon_t^j \tag{19}$$

where $Performance_t^j$ denotes fund j's performance in month t, measured as fund abnormal returns, or CAPM, three-factor, or four-factor alphas. *Recession* and the control variables, X, are defined as before. All returns are net of management fees. Our coefficient of interest is c_1.

Table 4, Column 1, shows that the average fund's net return is 3bp per month lower than the market return in expansions, but it is 34bp per month higher in recessions. This difference is highly statistically significant and becomes even larger (42bp), after we control for fund characteristics (Column 2). Similar results (Columns 3 and 4) obtain when we use

Table 4: **Fund Performance Improves in Recessions**

Dependent variables: *Abnormal Return* is the fund return minus the market return. The alphas come from twelve-month rolling-window regressions of fund-level excess returns on excess market returns for the CAPM alpha, additionally on the SMB and the HML factors for the three-factor alpha, and additionally on the UMD factor for the four-factor alpha. The independent variables, the sample period, and the standard error calculations are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Abnormal Return		CAPM Alpha		3-Factor Alpha		4-Factor Alpha	
Recession	0.342	0.425	0.337	0.404	0.043	0.073	0.107	0.139
	(0.056)	(0.058)	(0.048)	(0.047)	(0.034)	(0.028)	(0.041)	(0.032)
Log(Age)		-0.031		-0.036		-0.028		-0.039
		(0.009)		(0.008)		(0.006)		(0.006)
Log(TNA)		0.046		0.033		0.009		0.012
		(0.005)		(0.004)		(0.003)		(0.003)
Expenses		-1.811		-2.372		-7.729		-7.547
		(1.046)		(0.945)		(0.782)		(0.745)
Turnover		-0.023		-0.044		-0.074		-0.065
		(0.016)		(0.010)		(0.010)		(0.008)
Flow		2.978		2.429		1.691		1.536
		(0.244)		(0.172)		(0.097)		(0.096)
Load		-0.809		-0.757		-0.099		-0.335
		(0.226)		(0.178)		(0.131)		(0.141)
Constant	-0.027	-0.033	-0.059	-0.063	-0.059	-0.060	-0.050	-0.052
	(0.027)	(0.026)	(0.025)	(0.024)	(0.020)	(0.018)	(0.023)	(0.021)
Observations	226,745	226,745	226,745	226,745	226,745	226,745	226,745	226,745

the CAPM alpha as a measure of fund performance, except that the alpha in expansions becomes negative. When we use alphas based on the three- and four-factor models, the recession return premiums diminish (Columns 5-8). But in recessions, the four-factor alpha still represents a non-trivial 1% per year risk-adjusted excess return, 1.6% higher than the -0.6% recorded in expansions (significant at the 1% level). The advantage of this cross-sectional regression model is that it allows us to include a host of fund-specific control variables. The disadvantage is that performance is measured using past twelve-month rolling-window regressions. Thus, a given observation can be classified as a recession when some or even all of the remaining eleven months of the window are expansions.

To verify the robustness of our cross-sectional results, we also employ a time-series approach. In each month, we form the equally weighted portfolio of funds and calculate its net return, in excess of the risk-free rate. We then regress this time series of fund portfolio returns on *Recession* and common risk factors. We adjust standard errors for heteroscedasticity and autocorrelation (Newey and West (1987)). Table S.10 shows that our previous results remain largely unchanged.

Our results are robust to alternative performance measures. Table S.11 uses *gross* fund returns and alphas. In unreported results, we also use the information ratio (the ratio of the CAPM alpha to the CAPM residual volatility) as a performance measure. It increases sharply in recessions. Finally, we find similar results when we lead alpha on the left-hand side by one month instead of using a contemporaneous alpha. All results point in the same direction: Outperformance clusters in recessions.

While our model is silent about the distinction between funds and fund managers, in practice, skill could be embodied in the manager or be produced by the organizational setup the fund provides that manager. We estimate our main results using the cross section of data at the manager level. Table S.14 shows that the recession effects on RAI/RSI, dispersion, and performance are essentially unchanged, suggesting that the distinction is not important for our results.

3 Recession versus Volatility

In our model, recessions are times of higher aggregate payoff volatility, and hence higher stock return volatility. In Section 2.2, we show that the same is true in the data. This motivates our use of recessions in the empirical work. The link between recessions and aggregate volatility, however, suggests an additional way of testing the model: We could replace the recession indicator with an indicator for high aggregate payoff volatility. The high-volatility indicator variable equals one in months with the highest volatility of aggregate earnings growth, where aggregate volatility is estimated from Shiller's S&P 500 earnings growth data.[22] As predicted by our theory, we find that RAI, dispersion, and performance all rise in high-volatility months, while RSI falls; see Table S.13.

Given that both recessions and aggregate volatility qualitatively deliver the same predictions, it is natural to ask which of the two effects is stronger. To that end, we include both NBER recession and high aggregate payoff volatility indicators as explanatory variables in an empirical horse race. For brevity, Table 5 combines the headline results for RAI/RSI, portfolio dispersion, and performance. It shows that both recession and volatility contribute to a lower RSI in expansions and a higher RAI in recessions, to a higher portfolio dispersion in recessions, and to a higher performance in recessions (four-factor alpha).[23] For some of the results the recession effect is slightly stronger, while for others the volatility effect is slightly stronger. Clearly, there is an effect of recessions beyond the one coming through volatility.

[22]We choose the volatility cutoff such that 12% of months are selected, the same fraction as NBER recession months.

[23]The results without controls are similar, as are the results for other dispersion and performance measures.

Table 5: **Recession vs. Volatility**

The dependent variables are funds' reliance on aggregate information (RAI), funds' reliance on stock-specific information (RSI), funds' (*Portfolio Dispersion*), the cross-sectional dispersion in fund CAPM betas, and the funds' four-factor alpha. Their definitions are in the captions of Tables 2, 3, and 4. *Recession* is defined in Table 2. *Volatility* is an indicator variable for periods of volatile earnings. We calculate the twelve-month rolling-window standard deviation of the year-to-year log change in the earnings of S&P 500 index constituents; the earnings data are from Robert Shiller for 1926-2008. Volatility equals one if this standard deviation is in the highest 10% of months in the 1926-2008 sample. During 1985-2005, 12% of months are such high volatility months. The remaining right-hand side variables, the sample period, and the standard error calculation are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)
	RAI	RSI	Portfolio Dispersion	CAPM Beta Disp	4-Factor Alpha
Recession	0.011	-0.503	0.185	0.062	0.100
	(0.004)	(0.166)	(0.027)	(0.013)	(0.035)
Volatility	0.000	-0.477	0.206	0.077	0.125
	(0.003)	(0.106)	(0.030)	(0.018)	(0.064)
Log(Age)	-0.002	0.416	0.198	-0.004	-0.036
	(0.001)	(0.060)	(0.027)	(0.002)	(0.006)
Log(TNA)	-0.001	-0.167	-0.176	0.002	0.010
	(0.000)	(0.029)	(0.014)	(0.001)	(0.003)
Expenses	-0.331	90.911	29.859	3.695	-8.187
	(0.247)	(11.599)	(4.868)	(0.211)	(0.787)
Turnover	-0.004	-0.199	-0.089	0.012	-0.067
	(0.001)	(0.063)	(0.025)	(0.001)	(0.008)
Flow	-0.008	1.668	0.108	0.008	1.536
	(0.010)	(0.632)	(0.104)	(0.016)	(0.096)
Load	0.017	-10.009	-1.789	-0.252	-0.223
	(0.023)	(1.984)	(0.909)	(0.040)	(0.141)
Constant	-0.001	3.124	1.546	0.221	-0.065
	(0.001)	(0.074)	(0.023)	(0.005)	(0.021)
Observations	224,257	166,328	226,745	226,745	226,745

To understand why recessions have an incremental effect over volatility in explaining attention allocation, we consider an augmented model in which both the quantity and the price of risk rise in recessions. The idea that the price of risk rises in recessions is supported by a large asset pricing literature (e.g., the external habit model of Campbell and Cochrane (1999) or the variable rare disasters model of Gabaix (2009)). The parameter that governs the price of risk in our model is risk aversion. The following result shows that an increase in the price of risk (risk aversion) in recessions is an independent force driving the reallocation of attention from stock-specific to aggregate shocks. Because of this additional channel, recessions should generate more attention reallocation than a rise in aggregate volatility alone, just as we see in the data. The proof is in Appendix S.1.8.

Proposition 4. *If the size of the composite asset $\bar{x}_c$ is sufficiently large, then an increase in risk aversion increases the marginal utility of reallocating a unit of capacity from the firm-specific shock to the aggregate shock:* $\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) > 0$.

The intuition for this result is that the aggregate shock affects a large fraction of the value of one's portfolio. Therefore, a marginal reduction in the uncertainty about an aggregate shock reduces total portfolio risk by more than the same-sized reduction in the uncertainty about a stock-specific shock. In other words, learning about the aggregate shock is the most efficient way to reduce portfolio risk. The more risk averse an agent is, the more attractive aggregate attention allocation becomes.

4 Using Theory and Data to Identify Skilled Managers

The fact that the data are consistent with our main predictions suggests that this model could successfully identify skilled investment managers in the data. In particular, we exploit the prediction that skilled managers display market-timing ability in recessions and stock-picking ability in expansions. Market-timing and stock-picking ability are defined by equations (11) and (13).

4.1 The Same Managers Do Switch Strategies

We first test the prediction that the *same* investment managers with stock-picking ability in expansions display market-timing ability in recessions. To this end, we first identify funds with superior stock-picking ability in expansions: For all expansion months, we select all fund-month observations that are in the highest 25% of the $Picking_t^j$ distribution. We form an indicator variable *Skill Picking* ($SP_j \in \{0,1\}$) that is equal to 1 for the 25% of funds (884 funds) with the highest fraction of observations in the top, relative to the total number of observations (in expansions) for that fund. Then, we estimate the following pooled regression model, separately for expansions and recessions:

$$Ability_t^j = d_0 + d_1 SP_t^j + \mathbf{d_2 X_t^j} + \epsilon_t^j, \qquad (20)$$

where *Ability* denotes either $Timing$ or $Picking$. X is a vector of previously defined control variables. Our coefficient of interest is d_1.

In Table 6, Column 3, we confirm that SP funds are significantly better at picking stocks in expansions, after controlling for fund characteristics. This is true by construction. The main point is that these same SP funds are on average better at market timing in recessions. This result is evident from positive coefficient on SP in Column 2, that is statistically significant at the 5% level. Finally, the funds in SP do not exhibit superior market-timing ability in expansions (Column 1) nor superior stock-picking ability in recessions (Column 4),

Table 6: **Successful Funds Switch Strategies**

In columns (1)-(4), we divide all fund-month observations into Recession and Expansion subsamples. *Recession* is defined in Table 1; *Expansion* $\equiv 1 - Recession$. The dependent variables CAPM alpha, three-factor alpha, and four-factor alpha are obtained from a twelve-month rolling-window regression of a fund's excess returns, before expenses, on a set of common risk factors. *Skill Picking* is an indicator variable equal to one for all funds whose *Picking* measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. The dependent variables, $Timing_t^j$ and $Picking_t^j$, the control variables, the sample period, and the standard error calculation are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Market Timing		Stock Picking		CAPM Alpha	3-Factor Alpha	4-Factor Alpha
	Expansion	Recession	Expansion	Recession			
Skill Picking	0.000	0.017	0.056	-0.096	0.076	0.056	0.064
	(0.004)	(0.009)	(0.004)	(0.017)	(0.040)	(0.021)	(0.018)
Log(Age)	0.009	-0.025	-0.001	0.029	-0.039	-0.028	-0.038
	(0.002)	(0.006)	(0.002)	(0.007)	(0.008)	(0.006)	(0.006)
Log(TNA)	-0.001	0.005	0.000	-0.023	0.032	0.013	0.014
	(0.001)	(0.003)	(0.001)	(0.003)	(0.005)	(0.004)	(0.004)
Expenses	0.868	1.374	-1.291	-4.434	4.956	0.627	0.241
	(0.321)	(1.032)	(0.376)	(1.378)	(1.066)	(0.793)	(0.739)
Turnover	0.009	-0.011	0.017	-0.006	-0.009	-0.047	-0.041
	(0.003)	(0.007)	(0.004)	(0.012)	(0.014)	(0.012)	(0.009)
Flow	0.056	-0.876	0.138	-0.043	2.579	1.754	1.602
	(0.024)	(0.112)	(0.037)	(0.093)	(0.173)	(0.102)	(0.101)
Load	0.094	-0.076	0.131	0.615	-0.744	-0.090	-0.289
	(0.049)	(0.151)	(0.055)	(0.195)	(0.214)	(0.136)	(0.145)
Constant	0.016	0.059	-0.021	-0.148	0.057	0.038	0.049
	(0.001)	(0.004)	(0.001)	(0.005)	(0.017)	(0.015)	(0.018)
Observations	204,330	18,354	204,330	18,354	227,183	227,183	227,183

which validates the point that SP funds switch strategies.

Having identified a subset of skilled ($SP_j = 1$) funds, the model predicts that this group should outperform the unskilled funds both in recessions and in expansions. Columns 5-7 compares the *unconditional* performance of the SP funds to that of all other funds. After controlling for various fund characteristics, the CAPM, three-factor, and four-factor alphas are 70-90 basis points per year higher for the SP portfolio, a difference that is statistically and economically significant.[24]

[24]The existence of skilled mutual funds with cyclical investment strategies is not a fragile result. First, the results survive if we change the cutoff levels for the inclusion in the SP portfolio. Second, we show that the top 25% RSI funds in expansions have higher RAIs in recessions and higher unconditional alphas (Tables S.16 and S.17). Third, we verify our results using Daniel, Grinblatt, Titman, and Wermers (1997)'s definitions of market timing (CT) and stock picking (CS). Finally, we reverse the sort, to show that funds in the top 25% of market-timing ability in recessions, have statistically higher stock-picking ability in expansions and higher unconditional alphas (Tables S.18 and S.19).

Table 7: **Comparing "Skill-Picking" Funds to Other Funds**

We divide all fund-month observations into Recession and Expansion subsamples. *Expansion* equals one every month the economy is not in recession according to the NBER, and zero otherwise. *Skill Picking* is one for any fund with a *Picking* measure (defined in Table 2) in the highest 25th percentile in expansions, and zero otherwise. Panel A reports fund-level characteristics. *Age*, *TNA*, *Expenses*, *Turnover*, *Flow* and *RAI* are defined in Table 2. *Portfoliodispersion* is the concentration of the fund's portfolio, measured as the Herfindahl index of portfolio weights in deviation from the market portfolio's weights. *Stock Number* is the number of stocks in the fund's portfolio. *Industry* is the industry concentration of the fund's portfolio, measured as the Herfindahl index of portfolio weights in a given industry in deviation from the market portfolio's weights. *Beta Deviation* is the absolute difference between the fund's beta and the average beta in its style category. Panel B reports manager-level characteristics. *MBA* or *Ivy* equals one if the manager obtained an MBA degree or graduated from an Ivy League institution, and equals zero otherwise. *Age* and *Experience* are the fund manager's age and experience in years. *Gender* equals one if the manager is a male and zero if female. *Hedge Fund* equals one if the manager ever departed to a hedge fund, and zero otherwise. $SP1 - SP0$ is the difference between the mean values of the groups for which *Skill Picking* equals one and zero, respectively. $p - values$ measure statistical significance of the difference. The data are monthly from 1980-2005.

	Skill Picking = 1			Skill Picking = 0			Difference	
	Mean	Stdev.	Median	Mean	Stdev.	Median	SP1-SP0	p-value
				Panel A: Fund Characteristics				
Age	10.01	8.91	7	15.20	15.34	9	-5.19	0.000
TNA	621.13	2027.04	129.60	1019.45	4024.29	162.90	-398.32	0.002
Expenses	1.48	0.47	1.42	1.22	0.47	1.17	0.26	0.000
Turnover	130.41	166.44	101.00	79.89	116.02	58.00	50.52	0.000
Flow	0.22	7.39	-0.76	-0.07	6.47	-0.73	0.300	0.008
Portfolio dispersion	1.68	1.60	1.29	1.33	1.50	0.99	0.35	0.000
Stock Number	90.83	110.20	68	111.86	187.13	69	-21.03	0.000
Industry	8.49	7.90	6.39	5.37	7.54	3.54	3.12	0.000
Beta Deviation	0.18	0.38	0.13	0.13	0.23	0.10	0.05	0.000
RAI	4.13	5.93	1.82	2.77	3.97	1.26	1.37	0.000
				Panel B: Fund Manager Characteristics				
MBA	42.09	49.37	0	39.49	48.88	0	2.60	0.128
Ivy	25.36	43.51	0	27.94	44.87	0	-2.57	0.205
Age	53.02	10.42	50	54.11	10.06	52	-1.08	0.081
Experience	26.45	10.01	24	28.14	10.00	26	-1.69	0.003
Gender	90.89	28.77	100	90.50	29.31	100	0.39	0.681
Hedge Fund	10.43	30.57	0	6.12	23.96	0	4.31	0.000

In Panel A of Table 7, we further compare the characteristics of the funds in the *Skill-Picking* portfolio to those of funds not included in the portfolio. We note several differences. First, funds in *SP* portfolio are younger (by five years on average). Second, they have less wealth under management (by $400 million), suggestive of decreasing returns to scale at the fund level. Third, they tend to charge higher expenses (by 0.26% per year), suggesting rent extraction from customers for the skill they provide. Fourth, they exhibit higher turnover rates (130% per year, versus 80% for other funds), consistent with a more active management style. Fifth, they receive higher inflows of new assets to manage, presumably a market-based reflection of their skill. Sixth, the SP funds tend to hold portfolios with fewer stocks and higher stock-level and industry-level portfolio dispersion. Seventh, their betas deviate more from their peers, suggesting a strategy with different systematic risk exposure. Finally, they rely significantly more on aggregate information. Taken together, fund characteristics, such as age, TNA, expenses, and turnover explain 14% of the variation in SP, the skill indicator (Table S.15). Including attributes that our theory links to skilled funds, such as stock and industry portfolio dispersion, beta deviation, and RAI, increases the R^2 to 19%. Thus, these findings paint a rough picture of what a typical skilled fund looks like.

Table 7, Panel B, examines *manager* characteristics. SP fund managers are 2.6% more likely to have an MBA, are one year younger, and have 1.7 fewer years of experience. Interestingly, they are much more likely to depart for hedge funds later in their careers, suggesting that the market judges them to have superior skills.

4.2 Creating a Skill Index

If one is going to use the model to identify skilled investment managers, it is important that she can identify these managers in real time, without looking at the full sample of the data. To this end, we construct a *Skill Index* that is informed by the main predictions of our model that attention allocation and investment strategies change over the business cycle. We define the Skill Index as a weighted average of $Timing$ and $Picking$ measures, in which the weights we place on each measure depend on the state of the business cycle:

$$Skill\,Index_t^j(z) = w(z_t)Timing_t^j + (1 - w(z_t))Picking_t^j, \text{ with } z_t \in \{E, R\}.$$

We demean $Timing$ and $Picking$, divide each by its standard deviation, and set $w(R) = 0.8 > w(E) = 0.2$ (the exact number is not crucial).

Subsequently, we examine whether the time-t $Index$ can predict future fund performance, measured by the CAPM, three-factor, and four-factor alphas one month (and one year) later. Table 8 shows that funds with a higher $SkillIndex$ have higher average alphas. For example, when $Skill\ Index$ is zero (its mean), the alpha is -4bp per month. However, when the $Skill\ Index$ is one standard deviation (0.83%) above its mean, the alpha is 1.1% (four-factor) or 2.4% (CAPM) higher per year. The three most right columns show similar predictive power of the $Skill\ Index$ for one-year ahead alphas. As a robustness check, we construct a

Table 8: **Skill Index Predicts Performance**

The dependent variable is the fund's cumulative CAPM, three-factor, or four-factor alpha, calculated from a twelve-month rolling regression of observations in month $t+2$ in the three left columns and in month $t+13$ in the three most right columns. For each fund, we form the following skill index in month t. $Skill\ Index_t^j = w(z_t)Timing_t^j + (1 - w(z_t))Picking_t^j, z_t \in \{Expansion, Recession\}$, w(Recession)=0.8 > w(Expansion) = 0.2. *Picking* and *Timing* are defined in Table 2, except that now they are normalized so that they are mean zero and have a standard deviation of one over the full sample. The other right-hand side variables, the sample period, and the standard error calculation are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)
	One Month Ahead			One Year Ahead		
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Index	0.239	0.118	0.107	0.224	0.104	0.106
	(0.044)	(0.022)	(0.019)	(0.031)	(0.025)	(0.014)
Log(Age)	-0.034	-0.024	-0.036	-0.019	-0.009	-0.024
	(0.009)	(0.006)	(0.007)	(0.008)	(0.005)	(0.006)
Log(TNA)	0.026	0.010	0.011	-0.016	-0.018	-0.011
	(0.005)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)
Expenses	-2.977	-7.063	-7.340	-5.793	-9.093	-9.308
	(1.620)	(1.004)	(0.957)	(1.578)	(0.917)	(0.887)
Turnover	-0.010	-0.047	-0.039	-0.001	-0.041	-0.036
	(0.016)	(0.014)	(0.010)	(0.016)	(0.014)	(0.010)
Flow	2.409	1.664	1.519	0.237	0.210	0.227
	(0.151)	(0.097)	(0.095)	(0.119)	(0.086)	(0.071)
Load	-0.762	-0.093	-0.313	-0.683	0.213	-0.044
	(0.233)	(0.144)	(0.157)	(0.225)	(0.129)	(0.149)
Constant	-0.030	-0.055	-0.041	-0.043	-0.070	-0.056
	(0.024)	(0.018)	(0.021)	(0.024)	(0.019)	(0.022)
Observations	219,338	219,338	219,338	187,668	187,668	187,668

second skill index based on RAI and RSI instead of $Timing$ and $Picking$. A one-standard-deviation increase in this skill index increases one-month-ahead alphas by 0.3-0.5% per year, a statistically significant effect (Table S.20).

5 Alternative Explanations

Stock price patterns generate effects We briefly explore other candidate explanations. The first alternative is that our effects arise mechanically from the properties of asset returns. To rule this out, we calculate means, volatilities, alphas, betas, and idiosyncratic volatilities of individual stock returns, in the same way as we do it for mutual fund returns. None of these moments differ between expansions and recessions (except for higher volatility of asset returns in recessions, our driving force). Using a simulation, we verify that a mechanical mutual fund investment policy that randomly selects 50, 75, or 100 stocks cannot produce the observed counter-cyclical fund returns.

Sample selection Suppose that managers have heterogeneous skill, but they do not display the cyclical variation in attention allocation we envision. Furthermore, suppose that the best managers leave the sample in good times, maybe because they go to a hedge fund. Then the composition effect would deliver lower alphas and less dispersion in expansions. If for some reason skill is associated with high RAI and low RSI, it could also explain the attention allocation results. There are at least three ways to refute this story. First, we redo our results with managers (instead of funds) as the unit of observation and include manager fixed effects. Table S.21 shows that our results go through unchanged. Including fixed effects in a regression model is a standard response to sample selection concerns. We find that fund fixed effects do not change our fund-level results. Second, in Section 4 we show that the same managers who have high RSI in expansions also have high RAI in recessions, a finding supporting active behavior of fund managers and working against the composition effect explanation. Third, even though in Table S.22 we show a higher chance of being promoted or picked off by a hedge fund in expansions, we also show a higher likelihood of being fired or demoted in recessions. The latter effect works in the opposite direction of the first, especially with respect to the dispersion result. Fourth, we find no systematic differences in age, educational background, or experience of managers in recessions versus expansions (Table S.23).

Career concerns Chevalier and Ellison (1999) show that young managers with career concerns may have an incentive to herd. Now imagine that in expansions the incentive to herd is strong, while in recessions young managers have to deviate from the pack to safeguard their jobs. We would then expect to see higher dispersion in strategies and performance in recessions. In order to investigate this hypothesis, we first estimate our dispersion regressions at the manager level adding the manager's log age and log age interacted with the recession indicator as independent variables. The career-concerns hypothesis predicts a negative sign on the interaction term: Younger managers should deviate more from the pack in recessions. Instead, we find a significantly positive interaction effect for our Portfolio Dispersion measure. The effect on beta dispersion is not statistically different from zero. It is worth noting that the sign on manager age itself is positive and significant, in line with the findings of Chevalier and Ellison (1999).[25] In sum, the results do not provide much evidence for the career-concerns

[25]Detailed results are omitted for brevity, but available upon request.

hypothesis. Moreover, it is not clear how the hypothesis would account for the RAI/RSI and performance results. While labor market considerations may be important to understand many aspects of the behavior of mutual fund managers, the above argument suggests that they cannot account for the patterns we document.

Time-varying marginal utility Glode (2010) argues that funds outperform in recessions because their investors' marginal utility is highest in such periods. While complementary to our explanation, his work remains silent on what strategies investment managers pursue to achieve this differential performance, and hence on our first and second hypothesis. In sum, while various explanations can account for some of the facts, we conclude that they are unlikely to account for all facts jointly.

6 Conclusion

Do investment managers add value for their clients? The answer to this question matters for problems ranging from the discussion of market efficiency to a practical portfolio advice for households. The large amount of randomness in financial asset returns makes it a difficult question to answer. The multi-billion investment management business is first and foremost an information-processing business. We model investment managers not only as agents making optimal portfolio decisions, but also as ones who optimally allocate a limited amount of attention or information-processing capacity. Since the optimal attention allocation varies with the state of the economy, so do investment strategies and fund returns. As long as a subset of investment managers can process information about future asset payoffs, the model predicts a higher covariance of portfolio holdings with aggregate information, more dispersion in returns across funds, and a higher average outperformance, in recessions. We observe these patterns in investments and returns of actively managed U.S. mutual funds. Hence, the data are consistent with a world in which some investment managers have skill, but that skill is often hard to detect. Recessions are times when differences in performance are magnified and skill is easier to detect.

Beyond the mutual fund industry, a sizeable fraction of GDP currently comes from industries that produce and process information. Ever increasing access to information has made the problem of how to best allocate a limited amount of information-processing capacity even more relevant. While information choices have consequences for real outcomes, they are often poorly understood because they are difficult to measure. By predicting how information choices are linked to observable variables (such as the state of the economy) and by tying information choices to real outcomes (such as portfolio investment), we show how models of information choices can be brought to the data. This information-choice-based approach could be useful in examining other information-processing sectors of the economy.

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Supplementary Appendix for "Attention Allocation over the Business Cycle"

S.1 Proofs of Propositions

S.1.1 Mathematical Preliminaries

Expressing matrices in terms of fundamental variances To determine the effect of changes in aggregate shock variance on dispersion and profits, we need to express some of the matrices in terms of σ_a^{-1}. If we can decompose the matrices into components that depend on σ_a and those that do not, we can differentiate the expressions more easily.

First, we decompose the payoff precision matrices. To do this decomposition, we need to invert Σ. Doing it by hand yields

$$\Sigma^{-1} = \begin{bmatrix} \sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\ 0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\ -b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & \sigma_a^{-1} + b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1} \end{bmatrix} \tag{S.1}$$

Similarly, the posterior precision matrix for investor j is

$$\hat{\Sigma}_j^{-1} = \begin{bmatrix} \hat{\sigma}_1^{-1} & 0 & -b_1\hat{\sigma}_1^{-1} \\ 0 & \hat{\sigma}_2^{-1} & -b_2\hat{\sigma}_2^{-1} \\ -b_1\hat{\sigma}_1^{-1} & -b_2\hat{\sigma}_2^{-1} & \hat{\sigma_a}^{-1} + b_1^2\hat{\sigma}_1^{-1} + b_2^2\hat{\sigma}_2^{-1} \end{bmatrix} \tag{S.2}$$

It is useful to separate out the terms that depend on σ_a from those that do not. Define

$$S \equiv \begin{bmatrix} \sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\ 0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\ -b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1} \end{bmatrix} \tag{S.3}$$

and let $\hat{S}$ be the posterior S, meaning that each σ_1 is replaced with the posterior variance $\hat{\sigma}_1$ and each σ_2 is replaced with the posterior variance $\hat{\sigma}_2$.

$$\Upsilon_a \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \Upsilon_1 \equiv \begin{bmatrix} 1 & 0 & -b_1 \\ 0 & 0 & 0 \\ -b_1 & 0 & b_1^2 \end{bmatrix} \tag{S.4}$$

so that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_a}^{-1} = \Upsilon_a$ and $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_1}^{-1} = \Upsilon_1$.

Then,

$$\Sigma^{-1} = S + \sigma_a^{-1}\Upsilon_a \tag{S.5}$$

$$\hat{\Sigma}_j^{-1} = \hat{S} + \hat{\sigma_a}_j^{-1}\Upsilon_a \tag{S.6}$$

Define the average posterior precision matrix when a fraction χ of investment managers have capacity to be

$(\bar{\Sigma})^{-1} \equiv \int_j \hat{\Sigma}_j^{-1} dj$. Similarly, let $S^a \equiv \int_j \hat{S}_j dj$ and let $\bar{K}_a$ be the average amount of capacity that an agent devotes to processing aggregate information. For example, if a fraction χ of investors are skilled, and all skilled investors devote all their capacity K to processing aggregate information, $\bar{K}_a = \chi K$. Then,

$$(\bar{\Sigma})^{-1} = S^a + (\sigma_a^{-1} + \chi K^a)\Upsilon_a \tag{S.7}$$

An expression that recurs frequently below is the difference between the precision of an informed manager's posterior beliefs and the precision of the average manager's posterior beliefs. This difference becomes

$$\Sigma^{-1} - (\bar{\Sigma})^{-1} = S - S^a + (1-\chi)K\Upsilon_a \tag{S.8}$$

Second, we decompose the variance matrices. In particular, we need to know average variance, which requires inverting $(\bar{\Sigma})^{-1}$. Replacing σ_a with $(\sigma_a^{-1}+\chi K)^{-1}$, and following the same inversion steps backwards, we get

$$\bar{\Sigma} = (\sigma_a^{-1} + \bar{K}_a)^{-1} bb' + \Phi, \tag{S.9}$$

where b is the 3×1 vector of loadings of each asset on aggregate risk, and if $\bar{K}_1$ and $\bar{K}_2$ represent the average amount of capacity devoted to processing information about assets 1 and 2,

$$\Phi \equiv \begin{bmatrix} (\sigma_1^{-1} + \bar{K}_1)^{-1} & 0 & 0 \\ 0 & (\sigma_2^{-1} + \bar{K}_2)^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Finally, let $\tilde{\sigma}_a \equiv (\sigma_a^{-1} + \bar{K}_a)^{-1}$.

Portfolio holdings The optimal portfolio for investor j is

$$q_j = \frac{1}{\rho}\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr) \tag{S.10}$$

This comes from the first order condition and is a standard expression in any portfolio problem with CARA or mean-variance utility.

Next, compute the portfolio of the average investor. Let the average of all investors' posterior precision be $(\bar{\Sigma})^{-1} \equiv \int \hat{\Sigma}_j^{-1} dj$. Use the fact that $\hat{\mu}_j = \hat{\Sigma}_j \Sigma^{-1}\mu + (I - \hat{\Sigma}_j\Sigma^{-1})\eta_j$ and the fact that the signal noise is mean-zero to get that $\int \hat{\Sigma}_j^{-1}\hat{\mu}_j dj = \Sigma^{-1}\mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1})f$. This is true because the mean of all investors' signals are the true payoffs f and because the signal errors are uncorrelated with (but of course, not independent of) signal precision.

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho}\left(\Sigma^{-1}\mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1})f - (\bar{\Sigma})^{-1} pr\right) \tag{S.11}$$

Using Bayes' rule for the posterior variance of normal variables, we can rewrite this as

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho}\left(\Sigma^{-1}\mu + \bar{\Sigma_\eta}^{-1} f - (\bar{\Sigma})^{-1} pr\right) \tag{S.12}$$

where $\bar{\Sigma_\eta}^{-1} \equiv \bar{\Sigma}^{-1} - \Sigma^{-1}$ is the average investor's signal precision.

S.1.2 Proof of Lemma 1

Proof. Following Admati (1985), we conjecture that the price vector p is linear in the payoff vector f and the supply vector x: $pr = A + Bf + Cx$. We now verify that conjecture by imposing market clearing

$$\int q_j dj = \bar{x} + x \tag{S.13}$$

Using (S.11) to substitute out the left hand side, and rearranging,

$$pr = -\rho\bar{\Sigma}(\bar{x} + x) + f + \bar{\Sigma}\Sigma^{-1}(\mu - f)$$

Thus, the coefficients A, B, and C are given by

$$A = -\rho\bar{\Sigma}\bar{x} + \bar{\Sigma}\Sigma^{-1}\mu \tag{S.14}$$

$$B = I - \bar{\Sigma}\Sigma^{-1} \tag{S.15}$$

$$C = -\rho\bar{\Sigma} \tag{S.16}$$

which verifies our conjecture. □

S.1.3 Lemma 2: Investors Prefer Not To Learn Price Information

The idea behind this result is that an investor who learns from price information, will infer that the asset is valuable when its price is high and infer that the asset is less valuable when its price is low. Buying high and selling low is generally not a way to earn high profits. This effect shows up as a positive correlation between $\hat{\mu}$ and pr, which reduces the variance $V_1[\hat{\mu}_j - pr]$.

Mathematical Preliminaries: Note that $B^{-1}(pr - A) = f + B^{-1}Cx$. Since x is a mean-zero shock, this is an unbiased signal about the true asset payoff f. The precision of this signal is $\Sigma_p^{-1} \equiv \sigma_x^{-1}B'(CC')^{-1}B$.

Lemma 2. *A manager who could choose either learning from prices and observing a signal $\tilde{\eta}|f \sim N(f, \tilde{\Sigma}_\eta)$ or not learning from prices and instead getting a higher-precision signal $\eta|f \sim N(f, \Sigma_\eta)$, where the signals are conditionally independent across agents, and where $\Sigma_\eta^{-1} = \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1}$, would prefer not to learn from prices.*

Proof. From (9) in the main text, we know that expected utility is

$$U_{1j} = \frac{1}{2}trace(\hat{\Sigma}_j^{-1}V_1[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr]$$

Since the two options yield equally informative signals, by Bayes' rule, they yield equally informative posterior beliefs: $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_\eta^{-1}$, which is also equal to $\Sigma^{-1} + \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1}$. Likewise, sice both possibilities give the manager unbiased signals, beliefs are a martingale, meaning that $E_1[\hat{\mu}_j - pr]$, is identical under the two options.

Thus, the only term in expected utility that is affected by the decision to learn information from prices is $V_1[\hat{\mu}_j - pr]$. Let $\hat{\mu}_j = E[f|\eta]$ be the posterior expected value of payoffs for the manager who learns from the conditionally independent signal. By Bayes' Law,

$$\hat{\mu}_j = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_\eta^{-1}\eta).$$

The signal η can be broken down into the true payoff, plus noise: $\eta = f + \epsilon$, where $\epsilon \sim N(0, \Sigma_\eta)$. Using the expression for $\hat{\mu}_j$ and the pricing equation $pr = A + Bf + Cx$, we write

$$\hat{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1}\mu - A + \hat{\Sigma}_j\Sigma_\eta^{-1}\epsilon + (\hat{\Sigma}_j\Sigma_\eta^{-1} - B)f - Cx.$$

Since μ and A are constants, and ϵ, f, and x are mutually independent, the variance of this expression is

$$V_1[\hat{\mu}_j - pr] = \hat{\Sigma}_j\Sigma_\eta^{-1}\hat{\Sigma}_j + (\hat{\Sigma}_j\Sigma_\eta^{-1} - B)\Sigma(\hat{\Sigma}_j\Sigma_\eta^{-1} - B)' - \sigma_x CC'.$$

Next, consider the manager who chooses to learn information in prices. This person will have different posterior belief about f. Let $E[f|p, \tilde{\eta}] = \tilde{\mu}$. Using Bayes' law, he will combine information from his prior, prices and the signal $\tilde{\eta}$ his posterior belief:

$$\tilde{\mu} = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_p^{-1}B^{-1}(pr - A) + \tilde{\Sigma}_\eta^{-1}\tilde{\eta}).$$

Again, breaking up the signal into truth and noise ($\tilde{\eta} = f + \tilde{\epsilon}$), and using the price equation, we can write

$$\hat{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1}\mu + \hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1}(f + \epsilon) + (\hat{\Sigma}_j\Sigma_p^{-1}B^{-1} - I)(A + Bf + Cx) - \hat{\Sigma}_j\Sigma_p^{-1}B^{-1}A.$$

$$= \hat{\Sigma}_j\Sigma^{-1}\mu + (\hat{\Sigma}_j\Sigma_p^{-1}(I - B^{-1}) - I)A + (\hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1} - B)f + \hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1}\tilde{\epsilon} + (\hat{\Sigma}_j\Sigma_p^{-1} - I)Cx$$

Since μ and A are constants, and ϵ, f, and x are mutually independent, the variance of this expression is

$$V_1[\tilde{\mu} - pr] = (\hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1} - B)\Sigma(\hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1}B - B)' + \hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1}\hat{\Sigma}_j + \sigma_x(\hat{\Sigma}_j\Sigma_p^{-1} - I)CC'(\hat{\Sigma}_j\Sigma_p^{-1} - I)'$$

We have assumed that $\tilde{\Sigma}_\eta^{-1} + \Sigma_p^{-1} = \Sigma_\eta^{-1}$. Therefore, the first term $\hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1} - B = \hat{\Sigma}_j\Sigma_\eta^{-1} - B$, which is the same quantity as in the first term of $V_1[\hat{\mu}_j - pr]$.

Thus, when we subtract one expression from the other,

$$V_1[\hat{\mu}_j - pr] - V_1[\tilde{\mu} - pr] = \hat{\Sigma}_j(\Sigma_\eta^{-1} - \tilde{\Sigma}_\eta^{-1})\hat{\Sigma}_j - \sigma_x(\hat{\Sigma}_j\Sigma_p^{-1}CC'\Sigma_p^{-1}\hat{\Sigma}_j - 2\hat{\Sigma}_j\Sigma_p^{-1}CC').$$

Since $\Sigma_\eta^{-1} = \tilde{\Sigma}_\eta^{-1} + \Sigma_p^{-1}$ and Σ_p^{-1} is positive semi-definite (an inverse variance matrix always is), $\hat{\Sigma}_j(\Sigma_\eta^{-1} - \tilde{\Sigma}_\eta^{-1})\hat{\Sigma}_j$ is positive semi-definite. Thus, the difference is positive semi-definite if $2I - \Sigma_p^{-1}\hat{\Sigma}_j$ is. Since for the investor that learns about prices, Bayes' rule tells us that $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1}$. This means that $I - \Sigma_p^{-1}\hat{\Sigma}_j = (\Sigma^{-1} + \tilde{\Sigma}_\eta^{-1})\hat{\Sigma}_j$, which is positive semi-definite. Therefore, $2I - \Sigma_p^{-1}\hat{\Sigma}_j$ is also positive semi-definite.

Thus, the difference in utility from learning conditionally independent information and learning price information is, $1/2trace(\hat{\Sigma}_j^{-1}(V_1[\hat{\mu}_j - pr] - [V_1[\tilde{\mu} - pr]))$. Since the expression inside the trace is a product of positive semi-definite matrices, the trace and therefore the difference in expected utilities is positive. □

S.1.4 Proof of Proposition 1

If aggregate variance is not too high ($\sigma_a \leq 1$), then the marginal value of a given investor j reallocating an increment of capacity from stock-specific shock $i \,\epsilon\, \{1,2\}$ to the aggregate shock is increasing in the aggregate shock variance: If $K_{aj} = \tilde{K}$ and $K_{ij} = K - \tilde{K}$, then $\partial^2 U / \partial \tilde{K} \partial \sigma_a > 0$.

Proof. From (9) in the main text, we know that expected utility is

$$U_{1j} = \frac{1}{2} trace(\hat{\Sigma}_j^{-1} V_1[\hat{\mu}_j - pr]) + \frac{1}{2} E_1[\hat{\mu}_j - pr]' \hat{\Sigma}_j^{-1} E_1[\hat{\mu}_j - pr]$$

The first step is to work out the variance $V_1[\hat{\mu}_j - pr]$.

$$\hat{\mu}_j - pr = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_{\eta j}^{-1}\eta_j) - A - Bf - Cx$$

The signal η can be expressed as the true asset payoff f, plus orthogonal signal noise ϵ_j.

$$\hat{\mu}_j - pr = \hat{\Sigma}_j \Sigma^{-1}\mu - A + (\hat{\Sigma}_j \Sigma_{\eta j}^{-1} - B)f + \hat{\Sigma}_j \Sigma_{\eta j}^{-1}\epsilon_j - Cx$$

Since μ and A are known constants and f, ϵ_j, and x are independent, with variances Σ, $\Sigma_{\eta j}$, and σ_x respectively,

$$V_1[\hat{\mu}_j - pr] = (\hat{\Sigma}_j \Sigma_{\eta j}^{-1} - B)\Sigma(\hat{\Sigma}_j \Sigma_{\eta j}^{-1} - B)' + \hat{\Sigma}_j \Sigma_{\eta j}^{-1} \hat{\Sigma}_j + CC'\sigma_x$$

Substituting in for the price coefficients using (S.14), (S.15), and (S.16) yields

$$V_1[\hat{\mu}_j - pr] = (\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \hat{\Sigma}_j \Sigma_{\eta}^{-1} \hat{\Sigma}_j + \rho^2 \bar{\Sigma}\bar{\Sigma}\sigma_x$$

Next, work out the second term by using the expression above for $\hat{\mu}_j - pr$ and taking the expectation: $E[\hat{\mu}_j - pr] = \hat{\Sigma}_j \Sigma^{-1}\mu - A + (\hat{\Sigma}_j \Sigma_{\eta j}^{-1} - B)\mu$. Substituting in the coefficients A and B, and simplifying reveals that $E[\hat{\mu}_j - pr] = \rho \bar{\Sigma}\bar{x}$. Thus,

$$E_1[\hat{\mu}_j - pr]' \hat{\Sigma}_j^{-1} E_1[\hat{\mu}_j - pr] = \rho^2 \bar{x}' \bar{\Sigma} \hat{\Sigma}_j^{-1} \bar{\Sigma} \bar{x}$$

Note that cyclic permutations of symmetric matrices are allowed inside a trace operator. Thus, expected utility is

$$U_{1j} = \frac{1}{2} trace\left(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \Sigma_{\eta j}^{-1}\hat{\Sigma}_j + \rho^2 \hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

In the next step, we want to take a cross-partial derivative of utility with respect to σ_a and $\tilde{K}$. To do this, we will substitute out the $\hat{\Sigma}_j^{-1}$ terms using (S.5), (S.6), and (S.7). Then, we will use the fact that, by the chain rule, $\partial U / \partial \tilde{K} = \partial U / \partial K_{aj} - \partial U / \partial K_{ij}$. Therefore, $\partial^2 U / \partial \tilde{K} \partial \sigma_a = \partial^2 U / \partial K_{aj} \partial \sigma_a - \partial^2 U / \partial K_{ij} \partial \sigma_a$ We consider each of these two cross-partial derivatives separately.

Part a: *The marginal value of a given investor j having additional capacity K_{aj} devoted to learning about the aggregate shock a is increasing in the aggregate shock variance: $\partial^2 U / \partial K_{aj} \partial \sigma_a > 0$.*

Sign last term: $\frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$.

Note that K_{aj} appears only in $\hat{\Sigma}_j^{-1}$. Recall that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_a}^{-1} = \Upsilon_a$. Since $\hat{\sigma_a}^{-1} = \sigma_a{}^{-1} + K_{aj}$, the chain rule implies that $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus, the last term has K_{aj} derivative $(\rho^2/2)\bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x}$. The only term in this expression that varies in σ_a is $\bar{\Sigma}$. Since $\bar{\Sigma}$ has every entry increasing in σ_a (equation S.9), and $\bar{\Sigma}$ and Υ_a are positive semi-definite matrices, this term has a positive cross-partial derivative $\partial^2/\partial K_j\partial\sigma_a > 0$.

Thus, a sufficient condition for $\partial^2 U/\partial K_j\partial\sigma_a > 0$ is for the trace term to have a positive cross partial derivative. Since the trace of a sum is the sum of the traces, we can break this term up into 3 major parts.

Sign term 3: $Tr(\rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x)$.

This takes the same form as the term outside the trace. K_{aj} appears only in $\hat{\Sigma}_j^{-1}$. Its derivative is $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus, the term has derivative $\rho^2\Upsilon_a\bar{\Sigma}\bar{\Sigma}\sigma_x$. The only term in this expression that varies in σ_a is $\bar{\Sigma}$. Since $\bar{\Sigma}$ and Υ_a are positive semi-definite matrices, and ρ^2 and σ_x are positive constants, the trace must be positive. Since $\bar{\Sigma}$ has every entry increasing in σ_a (equation S.9), the $\partial Tr(\cdot)/\partial K_{aj}$ is increasing in σ_a. In other words, $\partial^2/\partial K_j\partial\sigma_a > 0$.

Sign term 2: $Tr(\Sigma_{\eta j}^{-1}\hat{\Sigma}_j)$

By Bayes' Law, the signal precision is the difference between the posterior and prior precisions, $\Sigma_{\eta j}^{-1} = \hat{\Sigma}_j^{-1} - \Sigma^{-1}$. Substituting this into the trace term yields $Tr(I - \Sigma^{-1}\hat{\Sigma}_j) = 3 - Tr(\Sigma^{-1}\hat{\Sigma}_j)$, where the 3 comes from the fact that the variance matrices are all (3×3). The $-Tr(\Sigma^{-1}\hat{\Sigma}_j)$ will cancel out with the first term.

Sign term 1: $Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)')$

We can further break this term up into a sum of three parts, which I will call 1a, 1b and 1c.

Term 1a is $Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\hat{\Sigma}_j')$, which is equal to $Tr(\Sigma^{-1}\hat{\Sigma}_j')$. This term cancels out the $-Tr(\Sigma^{-1}\hat{\Sigma}_j)$ from term 2.

Term 1b is $-2Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\bar{\Sigma}')$, which is equal to $-2Tr(\Sigma^{-1}\bar{\Sigma}')$. This term only depends on prior variance and average posterior variance, not on investor j's information choice. Since it has no K_j in it, its derivative with respect to K_{aj} is 0.

Term 1c is $Tr(\hat{\Sigma}_j^{-1}\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}')$. Investor j's information choice K_{aj} shows up only in $\hat{\Sigma}_j^{-1}$, where $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus $\partial Tr(\cdot)/\partial K_{aj} = Tr(\Upsilon_a\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}')$. Next, replace $\bar{\Sigma}$ with (S.9) and Σ^{-1} with (S.5) and then take the derivative with respect to σ_a^{-1}. That delivers

$$\frac{\partial^2 Tr(\cdot)}{\partial K_{aj}\partial\sigma_a^{-1}} = Tr\left[-2\Upsilon_a(\sigma_a^{-1} + \bar{K}_a)^{-2}bb'((\sigma_a^{-1} + \bar{K}_a)^{-1}bb' + \bar{\Phi})(S + \sigma_a^{-1}\Upsilon_a)\right.$$
$$\left. + \Upsilon_a((\sigma_a^{-1} + \bar{K}_a)^{-1}bb' + \bar{\Phi})((\sigma_a^{-1} + \bar{K}_a)^{-1}bb' + \bar{\Phi})'\Upsilon_a\right]$$

If we want the cross-partial derivative with σ_a to be positive, then we are looking for a negative sign here.

Since $\bar{\Phi}$ only has non-zero entries in the (1,1) and (2,2) spots and Υ_a has only a 1 in the (3,3) entry, $\Upsilon_a\bar{\Phi} = 0$. Thus,

$$\frac{\partial^2 Tr(\cdot)}{\partial K_{aj}\partial\sigma_a^{-1}} = (\sigma_a^{-1} + \bar{K}_a)^{-2} Tr\left[-2(\sigma_a^{-1} + \bar{K}_a)^{-1}\Upsilon_a bb'bb'(S + \sigma_a^{-1}\Upsilon_a) + \Upsilon_a bb'bb'\Upsilon_a\right]$$

Since Υ_a, bb' and S are all positive semi-definite matrices, their product must have a positive trace and $-2Tr((\sigma_a^{-1} + \bar{K}_a)^{-1}\Upsilon_a bb'bb'S) < 0$. That leaves $Tr(\Upsilon_a bb'bb'\Upsilon_a)$, which is positive since Υ_a and bb' are positive semi-definite, times a constant $1 - 2\sigma_a^{-1}/(\sigma_a^{-1} + \bar{K}_a)$. This is negative or zero as long as $\bar{K}_a \le \sigma_a^{-1}$.

We can get an alternative sufficient condition for this term to be positive by using the fact that $\Sigma^{-1} = (S + \sigma_a^{-1}\Upsilon_a)$ and therefore $\Upsilon_a = (\Sigma^{-1} - S)\sigma_a$. Thus, we can rewrite

$$\frac{\partial^2 Tr(\cdot)}{\partial K_{aj}\partial\sigma_a^{-1}} = (\sigma_a^{-1} + \bar{K}_a)^{-2} Tr\left[\Upsilon_a bb'bb'((\Sigma^{-1} - S)\sigma_a - 2\frac{1}{\sigma_a^{-1} + \bar{K}_a}\Sigma^{-1}\right]$$

The term $Tr[-\Upsilon_a bb'bb'S\sigma_a]$ is negative, as before. The remaining term is $Tr[\Upsilon_a bb'bb'\Sigma^{-1}]$, which is positive, times $(\sigma_a - 2/(\sigma_a^{-1} + \bar{K}_a)$. This is negative or zero if $\bar{K}_a \geq \sigma_a$.

If $\sigma_a \leq 1$, then one of the sufficient conditions for the last term to have $\partial^2/\partial K_{aj}\partial\sigma_a > 0$ is always satisfied. In sum, when $\sigma_a \leq 1$, all the terms have non-negative cross-partial derivatives and therefore $\partial^2 U/\partial K_j \partial\sigma_a > 0$.

Part b: *The marginal value of a given investor j having additional capacity K_{ij} devoted to learning about stock-specific shock i is constant in the aggregate shock variance: $\partial^2 U/\partial K_{ij}\partial\sigma_a = 0$.*

Without loss of generality, we consider reallocating capacity from the asset 1 shock to the aggregate shock ($i = 1$). The same proof follows if it were asset 2 instead.

Sign last term: $\frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$.

Note that K_{1j} appears only in $\hat{\Sigma}_j^{-1}$. Recall that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_1}^{-1} = \Upsilon_1$. Since $\hat{\sigma_1}^{-1} = \sigma_1{}^{-1} + K_{1j}$, using the chain rule, we get $\partial\hat{\Sigma}_j^{-1}/\partial K_{1j} = \Upsilon_1$. Therefore, $\partial/\partial K_{1j}(\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}) = \bar{x}'\bar{\Sigma}\Upsilon_1\bar{\Sigma}\bar{x}$.

Because of the structure of the Υ_1 matrix, it turns out that using (S.4) and (S.9) to multiply out the three matrices $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ delivers

$$\bar{\Sigma}\Upsilon_1\bar{\Sigma} = \begin{bmatrix} (\sigma_1^{-1} + \bar{K}_1)^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{S.17}$$

Since this has no σ_a term in it and both $\bar{x}$ and ρ are exogenous, the cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a$ of the last terms is zero.

Sign term 3: $Tr(\rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x)$.

This takes the same form as the term outside the trace. The term inside the trace has derivative $\partial/\partial K_{1j} = \rho^2\bar{\Sigma}\Upsilon_1\bar{\Sigma}\sigma_x$. Since ρ and σ_x are exogenous and $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ does not depend on σ_a, the derivative of the trace is invariant in σ_a. Its cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a = 0$.

Sign term 2: $Tr(\Sigma_{\eta j}^{-1}\hat{\Sigma}_j)$

As in part a, this term cancels out Term 1a.

Sign term 1: $Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)')$

We can further break this term up into a sum of three parts, which I will call 1a, 1b and 1c.

Term 1a, $Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\hat{\Sigma}_j')$, cancels out term 2.

Term 1b is $-2Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\bar{\Sigma}')$, which is equal to $-2Tr(\Sigma^{-1}\bar{\Sigma}')$. This term only depends on prior variance and average posterior variance, not on investor j's information choice. Since it has no K_j in it, its derivative with respect to K_{1j} is 0.

Term 1c is $Tr(\hat{\Sigma}_j^{-1}\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}')$. Investor j's information choice K_{1j} shows up only in $\hat{\Sigma}_j^{-1}$, where $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_1$. Therefore, $\partial/\partial K_{1j}(\hat{\Sigma}_j^{-1}\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}') = \Upsilon_1\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}'$. As before, the sparse form of Υ_1 causes the matrix multiplication to turn out neatly. Using (S.1), (S.4) and (S.9) to multiply out the four matrices delivers

$$\Upsilon_1\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}' = \begin{bmatrix} \sigma_1^{-1}(\sigma_1^{-1}+\bar{K}_1)^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ -b_1\sigma_1^{-1}(\sigma_1^{-1}+\bar{K}_1)^{-2} & 0 & 0 \end{bmatrix}. \tag{S.18}$$

The trace of this matrix is $\sigma_1^{-1}(\sigma_1^{-1}+\bar{K}_1)^{-2}$. Since this has no σ_a term in it, the cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a$ is zero.

Since $\partial^2 U/\partial K_{aj}\partial\sigma_a > 0$ and $\partial^2 U/\partial K_{ij}\partial\sigma_a = 0$, the difference of the two terms is positive. Thus, the marginal value of a given investor j reallocating an increment of capacity from shock 1 to the aggregate shock is increasing in the aggregate shock variance: $\partial^2 U/\partial\tilde{K}\partial\sigma_a = \partial^2 U/\partial K_{aj}\partial\sigma_a - \partial^2 U/\partial K_{ij}\partial\sigma_a > 0$. □

S.1.5 Proof of Proposition 2

Part a: *If the average manager has sufficiently low capacity $\chi K < \sigma_a^{-1}$, then for a given K_{aj}, K_{ij}, an increase in aggregate risk σ_a increases the dispersion of fund portfolios $E[(q_j-\bar{q})'(q_j-\bar{q})]$, where $\bar{q} \equiv \int q_j dj$.*

Proof. Using the optimal portfolio expressions, (S.10) and (S.12), and Bayes' rule ($\hat{\mu}_j = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_\eta^{-1}\eta_j)$), the difference in portfolios $(q_j-\bar{q})$ is

$$(q_j-\bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}\eta_j - (\Sigma_\eta^a)^{-1}f + ((\bar{\Sigma})^{-1} - \hat{\Sigma}_j^{-1})pr\right]$$

where $(\Sigma_\eta^a)^{-1} \equiv \int \Sigma_{\eta j}^{-1} dj$ is the average manager's signal precision.

Next, we need to take into account that signals and payoffs are correlated. To do this, replace the signal η_j with the true payoff, plus signal noise: $\eta_j = f + e_j$,

$$(q_j-\bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + (\Sigma_\eta^{-1} - (\Sigma_\eta^a)^{-1})f + ((\bar{\Sigma})^{-1} - \hat{\Sigma}_j^{-1})pr\right]$$

Bayes' rule for variances of normal variables is $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_\eta^{-1}$. Integrating the left and right sides of this expression over managers j yields $(\bar{\Sigma})^{-1} = \Sigma^{-1} + (\Sigma_\eta^a)^{-1}$. Subtracting one expression from the other yields $\hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1} = \Sigma_\eta^{-1} - (\Sigma_\eta^a)^{-1}$. Define $\Delta \equiv \hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1}$. Substituting this in and combining terms yields

$$(q_j-\bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + \Delta(f - pr)\right]. \tag{S.19}$$

Now replace pr with $A + Bf + Cx$, where A, B, and C are given by Appendix S.1.2,

$$(q_j-\bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + \Delta((I-B)f - A - Cx)\right] \tag{S.20}$$

Substituting in the coefficients in the pricing equation reveals that $(I-B)\mu - A = \rho\bar{\Sigma}\bar{x}$, that $I - B = \bar{\Sigma}\Sigma^{-1}$, and that $C = -\rho\bar{\Sigma}$.

To work out the expectation of this quantity squared, recognize that this is the square of a sum of one constant and three, independent, mean-zero, normal variables. Since e_j, $f-\mu$ and x are independent, all the cross terms drop out, leaving

$$E[(q_j-\bar q)'(q_j-\bar q)] = Tr(\Sigma_{\eta j}^{-1}) + Tr(\bar\Sigma\Sigma^{-1}\Delta\Delta\bar\Sigma) + \rho^2\bar x'\bar\Sigma\Delta\Delta\bar\Sigma\bar x + \rho^2\sigma_x Tr(\bar\Sigma\Delta\Delta\bar\Sigma)$$

The first term depends only on information choice variables. So, holding choices fixed, the partial derivative with respect to σ_a is zero.

For the second term, it is easier to take the partial derivative with respect to σ_a^{-1} and show that it is negative. This is equivalent to showing that the derivative with respect to σ_a is positive. First, use (S.9) to show that $\partial\bar\Sigma/\partial\sigma_a^{-1}$ is $-(\sigma_a^{-1}+\bar K_a)^{-2}bb'$. Next, use (S.5) to show that $\partial\Sigma^{-1}/\partial\sigma_a^{-1}=\Upsilon_a$. Recall that Δ depends only on information choices, which we hold fixed.

Then, using the product rule, the derivative is the sum of two terms:

$$\frac{\partial}{\partial\sigma_a^{-1}}Tr(\bar\Sigma\Sigma^{-1}\Delta\Delta\bar\Sigma) = \frac{-2}{(\sigma_a^{-1}+\bar K_a)^2}Tr(\bar\Sigma\Delta\Delta bb') + Tr(\bar\Sigma\Upsilon_a\Delta\Delta\bar\Sigma)$$

Using (S.9), we can write $\bar\Sigma\Upsilon_a = (\sigma_a^{-1}+\bar K_a)^{-1}bb'\Upsilon_a + \Phi\Upsilon_a$. Recall that Φ has only non-zero $(1,1)$ and $(2,2)$ entries and the Υ_a has only a non-zero $(3,3)$ entry. Therefore, $\Phi\Upsilon_a=0$. Letting $\bar\sigma_a \equiv (\sigma_a^{-1}+\bar K_a)^{-1}$, we can rewrite

$$\frac{\partial}{\partial\sigma_a^{-1}}Tr(\bar\Sigma\Sigma^{-1}\Delta\Delta\bar\Sigma) = \bar\sigma_a Tr(\bar\Sigma(\Upsilon_a - 2\bar\sigma_a\Sigma^{-1})\Delta\Delta bb')$$

This is negative if $(\Upsilon_a-2\bar\sigma_a\Sigma^{-1})$ is negative semi-definite. Using (S.5) to rewrite Σ^{-1} as $S+\sigma_a^{-1}\Upsilon_a$ and substituting in reveals that $((1-2\bar\sigma_a)\Upsilon_a - 2\bar\sigma_a S)$ must be negative semi-definite. Since S is a positive semi-definite matrix, a sufficient condition is $1-2\bar\sigma_a\le 0$. Substituting back in the definition of $\bar\sigma_a$ and rearranging yields $\bar K_a \le \sigma_a^{-1}$. Thus, if $\bar K_a\le\sigma_a^{-1}$, the second term is decreasing in σ_a^{-1} and therefore increasing in σ_a.

<u>Term 3</u>: The product $\partial/\partial\sigma_a^{-1}(\bar x'\bar\Sigma\Delta\Delta\bar\Sigma\bar x)$ is non-positive iff $\partial/\partial\sigma_a^{-1}(\bar\Sigma\Delta\Delta\bar\Sigma)$ is negative semi-definite. As before Δ is a choice variable, which we hold fixed. $\partial\bar\Sigma/\partial\sigma_a^{-1} = -(\sigma_a^{-1}+\bar K_a)^{-2}bb'$. Therefore,

$$\frac{\partial}{\partial\sigma_a^{-1}}\bar\Sigma\Delta\Delta\bar\Sigma = \frac{-2}{(\sigma_a^{-1}+\bar K_a)^2}\bar\Sigma\Delta\Delta bb'$$

Since $\bar\Sigma$, Δ and bb' are positive semi-definite, this is negative semi-definite.

<u>Term 4</u>: As show in the previous step, $\partial/\partial\sigma_a^{-1}(\bar\Sigma\Delta\Delta\bar\Sigma)$ is negative semi-definite. Therefore, the derivative of the trace, which is the trace of the derivative, is negative: $\partial/\partial\sigma_a^{-1}Tr(\bar\Sigma\Delta\Delta\bar\Sigma)<0$.

Since all four terms in the expression for dispersion are decreasing in σ_a^{-1}, dispersion is increasing in σ_a. □

<u>Part b</u>: *If the average manager has sufficiently low capacity, $\chi K < \sigma_a^{-1}$, then for a given K_{aj}, K_{ij}, $\forall j$, an increase in aggregate risk, σ_a, increases the dispersion of funds' portfolio returns $E[((q_j-\bar q)'(f-pr))^2]$.*

Proof. From the previous part (equation S.20), we know that we can use the optimal portfolio expressions, (S.10) and (S.12), and Bayes' rule to express the portfolio difference as a function of three underlying random variables, e_j, $f-\mu$ and x. The $f-pr$ can likewise be expressed as a function of $f-\mu$ and x:

$$(q_j-\bar q)'(f-pr) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + \Delta((I-B)f-A-Cx)\right]'((I-B)f-A-Cx) \quad \text{(S.21)}$$

Substituting in the price coefficients, this is

$$= \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + \Delta\bar{\Sigma}(\rho\bar{x} + \Sigma^{-1}(f-\mu) + \rho x)\right]' \bar{\Sigma}(\rho\bar{x} + \Sigma^{-1}(f-\mu) + \rho x) \tag{S.22}$$

Since a linear combination of two normal variables is also a normal variable, we can write $\bar{\Sigma}(\Sigma^{-1}(f - \mu) + \rho x) = V^{1/2}z$ where $z \sim N(0, I)$ and $V \equiv \bar{\Sigma}(\Sigma^{-1} + \rho^2\sigma_x I)\bar{\Sigma}$. Likewise, we can use a shorthand for the constant term $w \equiv \rho\bar{\Sigma}\bar{x}$. Then, dispersion in fund profits becomes

$$E[((q_j - \bar{q})'(f - pr))^2] = E\left[\left(\frac{1}{\rho}[\Sigma_\eta^{-1}e_j + \Delta V^{1/2}z + \Delta w]'(w + V^{1/2}z)\right)^2\right] \tag{S.23}$$

Only terms with even powers of the standard normal variables are non-zero. Recall that symmetric matrices can be permuted within a trace operator. This leaves

$$= \frac{1}{\rho^2}\left[\Sigma_\eta^{-1} + (w'\Delta w)^2 + w'w(Tr(V\Delta\Delta) + 4Tr(V\Delta)w'\Delta w + Tr(V)w'\Delta\Delta w + Tr(\Delta VV\Delta) + Tr(V\Delta)^2)\right] \tag{S.24}$$

where the last two terms come from the expectation of a multivariate normal variable (z), raised to the fourth.

To sign the partial derivative of dispersion, with respect to σ_a, we proceed term-by-term. The first term Σ_η^{-1} depends only on choice variables, which we hold fixed. Similarly, Δ is also choice variables.

The constant w is $\rho\bar{\Sigma}\bar{x}$, where ρ and $\bar{x}$ are positive constants and every entry of $\bar{\Sigma}$ is increasing in σ_a. Therefore, all the $w'w$ and are increasing in σ_a. Furthermore, since wherever Δ appears, it shows up twice, whether it is positive or negative makes no difference. If it is negative, the two negative signs cancel. Thus, it remains to be shown that $\partial V/\partial\sigma_a$ is a positive semi-definite matrix. If so, its trace will be positive.

Recall that $V \equiv \bar{\Sigma}(\Sigma^{-1} + \rho^2\sigma_x I)\bar{\Sigma}$. Thus,

$$\partial V/\partial\sigma_a = 2\bar{\Sigma}(\Sigma^{-1} + \rho^2\sigma_x I)(\sigma_a^{-2}\bar{\sigma}_a^2)bb' - \bar{\Sigma}\sigma_a^{-2}\Upsilon_a\bar{\Sigma}$$

where $\bar{\sigma}_a^{-1} \equiv \sigma_a^{-1} + \int K_{aj}dj$ is the average posterior precision of beliefs about the aggregate shock. Note that $\bar{\Sigma}$ can be written as $\bar{\sigma}_a bb'$, plus a matrix that is all zeros, except for the (1,1) and (2,2) entries. Note that pre-multiplying this latter matrix by Υ_a, which has only a non-zero (3,3) entry, yields zero. Therefore, $\Upsilon_a\bar{\Sigma} = \Upsilon_a\bar{\sigma}_a bb'$. Thus, we can rewrite

$$\partial V/\partial\sigma_a = \sigma_a^{-2}\bar{\sigma}_a\bar{\Sigma}\left[2(\Sigma^{-1} + \rho^2\sigma_x I)\bar{\sigma}_a - \Upsilon_a\right]bb'.$$

Since the matrices outside the square brackets are positive semidefinite, and positive constants times the identity matrix ($\rho^2\sigma_x I\bar{\sigma}_a$) are positive semidefinite, it remains to be shown that $2\bar{\sigma}_a\Sigma^{-1} - \Upsilon_a$ is positive semidefinite as well. Recall from the mathematical preliminaries that $\Sigma^{-1} = \sigma_a^{-1}\Upsilon_a + \sigma_1^{-1}\Upsilon_1 + \sigma_2^{-1}\Upsilon_2$. Since Υ_1 and Υ_2 are positive semidefinite, a sufficient condition is that $2\bar{\sigma}_a\sigma_a^{-1}\Upsilon_a - \Upsilon_a$ is positive semi-definite. This is true whenever $2\bar{\sigma}_a\sigma_a^{-1} > 1$, which implies that $\bar{\sigma}_a^{-1} < 2\sigma_a^{-1}$. Using the definition $\bar{\sigma}_a^{-1} \equiv \sigma_a^{-1} + \bar{K}_a$, we get $\bar{K}_a < \sigma_a^{-1}$. Since the maximum possible average capacity devoted to the aggregate shock is the total capacity of informed investors K times the fraction of informed investors, a sufficient condition for $\partial V/\partial\sigma_a$ to be positive semidefinite is that $\chi K < \sigma_a^{-1}$. Thus, if $\chi K < \sigma_a^{-1}$, the proposition holds. □

S.1.6 Proof of Proposition 3

If investor j knows more about the aggregate shock than the average investor does ($\hat{\sigma}_{aj} < \bar{\sigma}_a$), then an increase in aggregate shock variance increases the difference between j's expected certainty equivalent return and the expected certainty equivalent return of an uninformed investor: $\partial(U_j - U^U)/\partial\sigma_a > 0$.

Proof. From (5) in the main text, we know that time-1 expected utility is

$$U_j = \frac{1}{2}trace(\hat{\Sigma}_j^{-1}V_1[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr].$$

The proof of proposition 1 works out the expectation and variance terms and shows that this expected utility is

$$U_j = \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \Sigma_{\eta j}^{-1}\hat{\Sigma}_j + \rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x},$$

for an agent who has posterior belief precision $\hat{\Sigma}_j^{-1}$. Since the result is about the difference in utility between an informed and an uninformed agent, who has $\hat{\Sigma}_j - \Sigma$, we take the difference of these two expected utilities:

$$U_j - U^U = \frac{1}{2}Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)') - \frac{1}{2}Tr\left(\Sigma^{-1}(\bar{\Sigma} - \Sigma)\Sigma^{-1}(\bar{\Sigma} - \Sigma)'\right)$$

$$+\frac{1}{2}Tr(\Sigma_{\eta j}^{-1}\hat{\Sigma}_j) + \frac{\rho^2}{2}Tr((\hat{\Sigma}_j^{-1} - \Sigma^{-1})\bar{\Sigma}\bar{\Sigma}\sigma_x) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}(\hat{\Sigma}_j^{-1} - \Sigma^{-1})\bar{\Sigma}\bar{x}$$

Note that $\hat{\Sigma}_j^{-1} - \Sigma^{-1} = \Sigma_\eta^{-1}$. This precision matrix of signals is a choice variable, and thus does not change in σ_a. But the following terms will vary in σ_a:

$$\frac{\partial\hat{\Sigma}_j^{-1}}{\partial\sigma_a} = \frac{-1}{\sigma_a^2}\Upsilon_a$$

$$\frac{\partial\hat{\Sigma}_j}{\partial\sigma_a} = \frac{\hat{\sigma}_{aj}^2}{\sigma_a^2}bb'$$

$$\frac{\partial\bar{\Sigma}}{\partial\sigma_a} = \frac{\bar{\sigma}_a^2}{\sigma_a^2}bb'$$

$$\frac{\partial\Sigma^{-1}(\bar{\Sigma} - \Sigma)}{\partial\sigma_a} = \frac{-1}{\sigma_a^2}\Upsilon_a\bar{\Sigma} + \frac{\bar{\sigma}_a^2}{\sigma_a^2}\Sigma^{-1}bb'$$

When the pairs of matrices $\Upsilon_a\bar{\Sigma}$ and $\Sigma^{-1}bb'$ are multiplied out, many terms cancel out, leaving $\Upsilon_a\bar{\Sigma} = \bar{\sigma}_a\begin{bmatrix} 0 & 0 & b \end{bmatrix}'$ and $\Sigma^{-1}bb' = \sigma_a^{-1}\begin{bmatrix} 0 & 0 & b \end{bmatrix}'$, where the matrix $\begin{bmatrix} 0 & 0 & b \end{bmatrix}'$ is a 3×3 with all zeros, except for the third row, which contains the vector $[b_1\, b_2\, 1]$. Thus,

$$\frac{\partial\Sigma^{-1}(\bar{\Sigma} - \Sigma)}{\partial\sigma_a} = \frac{\bar{\sigma}_a(\bar{\sigma}_a - \sigma_a)}{\sigma_a^3}\begin{bmatrix} 0 & 0 & b \end{bmatrix}'$$

Likewise, we can simplify $\partial\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)/\partial\sigma_a$ using the same steps to get

$$\frac{\partial\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)}{\partial\sigma_a} = \frac{\bar{\sigma}_a(\bar{\sigma}_a\hat{\sigma}_{aj}^{-1} - 1)}{\sigma_a^2}\begin{bmatrix} 0 & 0 & b \end{bmatrix}'$$

Finally, we can use the same steps to simplify

$$\frac{\partial \Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)}{\partial \sigma_a} = \frac{\bar{\sigma}_a(\bar{\sigma}_a - \sigma_a) - (\hat{\sigma}_{aj}^2 - \sigma_a \bar{\sigma}_a)}{\sigma_a^3} \begin{bmatrix} 0 & 0 & b \end{bmatrix}'$$

$$= \frac{\bar{\sigma}_a^2 - \hat{\sigma}_{aj}^2}{\sigma_a^3} \begin{bmatrix} 0 & 0 & b \end{bmatrix}'$$

Applying these results to the whole partial derivative, and using the property that the trace is a linear operator and therefore we can interchange the trace and the derivative, we get

$$\frac{\partial (U_j - U^U)}{\partial \sigma_a} = \frac{1}{2} \frac{\bar{\sigma}_a(\bar{\sigma}_a \hat{\sigma}_{aj}^{-1} - 1)}{\sigma_a^2} Tr(\begin{bmatrix} 0 & 0 & b \end{bmatrix}' \Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)')$$

$$+ \frac{1}{2} \frac{\bar{\sigma}_a^2 - \hat{\sigma}_{aj}^2}{\sigma_a^3} Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j) \begin{bmatrix} 0 & 0 & b \end{bmatrix}') - \frac{\bar{\sigma}_a(\bar{\sigma}_a - \sigma_a)}{\sigma_a^3} Tr\left(\begin{bmatrix} 0 & 0 & b \end{bmatrix}' \Sigma^{-1}(\bar{\Sigma} - \Sigma)' \right)$$

$$+ \frac{1}{2} \frac{\hat{\sigma}_{aj}^2}{\sigma_a^2} Tr(\Sigma_{\eta j}^{-1} bb') + \rho^2 \sigma_x \frac{\bar{\sigma}_a^2}{\sigma_a^2} Tr(\Sigma_{\eta j}^{-1} bb' \bar{\Sigma}) + \rho^2 \frac{\bar{\sigma}_a^2}{\sigma_a^2} \bar{x}' \bar{\Sigma} \Sigma_{\eta j}^{-1} bb' \bar{x}$$

Next, we can evaluate the trace terms, one-by-one. Multiplying matrices out term-by-term reveals that

$$Tr(\begin{bmatrix} 0 & 0 & b \end{bmatrix}' \Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)') = \frac{\bar{\sigma}_a - \hat{\sigma}_{aj}}{\sigma_a}$$

$$Tr\left(\begin{bmatrix} 0 & 0 & b \end{bmatrix}' \Sigma^{-1}(\bar{\Sigma} - \Sigma)' \right) = \frac{\bar{\sigma}_a - \sigma_a}{\sigma_a}$$

Since the trace operator allows us to cycle terms without changing the trace,

$$Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j) \begin{bmatrix} 0 & 0 & b \end{bmatrix}') = Tr(\begin{bmatrix} 0 & 0 & b \end{bmatrix}' \hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)) = \frac{\bar{\sigma}_a - \hat{\sigma}_{aj}}{\hat{\sigma}_{aj}}$$

The product $\Sigma_{\eta j}^{-1} bb'$ appears many times over. Multiplying these two matrices out yields $K_{aj} \begin{bmatrix} 0 & 0 & b \end{bmatrix}'$, where K_{aj} is the precision of j's signal about shock a. The trace of this product is K_{aj}.

$$Tr(\Sigma_{\eta j}^{-1} bb' \bar{\Sigma}) = K_{aj}(b_1^2 + b_2^2 + 1)\bar{\sigma}_a$$

and finally, multiplying out the inner product yields

$$\bar{x}' \bar{\Sigma} \Sigma_{\eta j}^{-1} bb' \bar{x} = K_{aj} \bar{x}' \bar{\Sigma} \begin{bmatrix} 0 & 0 & b \end{bmatrix}' \bar{x} = K_{aj} \bar{x}' \bar{\sigma}_a bb' \bar{x}$$

$$= K_{aj} \bar{\sigma}_a (\bar{x}' b)^2 = K_{aj} \bar{\sigma}_a (b_1 \bar{x}_1 + b_2 \bar{x}_2 + \bar{x}_3)^2.$$

Adding all these terms together delivers

$$\frac{\partial (U_j - U^U)}{\partial \sigma_a} = \frac{1}{2} \frac{\bar{\sigma}_a(\bar{\sigma}_a \hat{\sigma}_{aj}^{-1} - 1)}{\sigma_a^2} \frac{\bar{\sigma}_a - \hat{\sigma}_{aj}}{\sigma_a} + \frac{1}{2} \frac{\bar{\sigma}_a^2 - \hat{\sigma}_{aj}^2}{\sigma_a^3} \frac{\bar{\sigma}_a - \hat{\sigma}_{aj}}{\hat{\sigma}_{aj}}$$

$$-\frac{\bar{\sigma}_a(\bar{\sigma}_a-\sigma_a)}{\sigma_a^3}\frac{\bar{\sigma}_a-\hat{\sigma}_{aj}}{\hat{\sigma}_{aj}}+\frac{1}{2}\frac{\hat{\sigma}_{aj}^2}{\sigma_a^2}K_{aj}$$

$$+\rho^2\sigma_x\frac{\bar{\sigma}_a^2}{\sigma_a^2}K_{aj}(b_1^2+b_2^2+1)\bar{\sigma}_a+\rho^2\frac{\bar{\sigma}_a^2}{\sigma_a^2}K_{aj}\bar{\sigma}_a(\bar{x}'b)^2$$

Each one of these terms is positive. The first term is equivalent to $\bar{\sigma}_a(\bar{\sigma}_a-\hat{\sigma}_{aj})^2/(2\sigma_a^3\hat{\sigma}_{aj})$. This product of squares and variances is positive. The second term is positive as long as $\bar{\sigma}_a > \hat{\sigma}_{aj}$. In other words, the average investor is more uncertain about the aggregate shock than the informed investor is. This same condition ensures that the third term, including the minus sign, is also positive. Since the average investor's posterior variance cannot be higher than his prior variance (no forgetting of information), $\bar{\sigma}_a - \sigma_a < 0$. If $\bar{\sigma}_a - \hat{\sigma}_{aj} > 0$, then the product is negative, and minus the product is positive. The last three terms are sums and products of variances, squares and numbers or parameters that must be positive ($\rho > 0$). Therefore, these last three terms and thus the whole expression is positive. □

S.1.7 Proof of Corollary 1

If some managers are uninformed $\chi < 1$, but all informed managers learn about aggregate risk, and the average manager has sufficiently low capacity $\chi K < \sigma_a^{-1}$, then for a given K_{aj}, K_{ij}, an increase in aggregate risk σ_a increases the expected profit of an informed fund, $E[(q_j-\bar{q})'(f-pr)]$, where $\bar{q} \equiv \int q_j dj$.

Proof. Assume that all informed investors use their capacity K to learn about the aggregate risk. We show that when σ_a^{-1} falls (in recessions), that expected excess returns of the informed traders rise.

Begin by taking the expectation of (S.21) to get expected profits. Since the supply shocks and the signal noise are mean-zero and independent of all other shocks, we can take their expectations separately. Using the formula for the expectation of a chi-square variable,

$$E[(q_j-\bar{q})'(f-pr)] = \frac{-\sigma_x}{\rho}Tr[C'(1-\chi)K\Upsilon_a C] + \frac{(1-\chi)K}{\rho}E\left\{((I-B)f-A)'\Upsilon_a'((I-B)f-A)\right\} \quad \text{(S.25)}$$

Since $((I-B)f-A)$ is normally distributed, the remaining expectation is also the mean of a chi square

$$\begin{aligned} E[(q_j-x-\bar{x})'(f-pr)] &= \frac{\sigma_x(1-\chi)K}{\rho}Tr[C'\Upsilon_a C] + \frac{(1-\chi)K}{\rho}((I-B)\mu-A)'\Upsilon_a((I-B)\mu-A) \\ &+ \frac{(1-\chi)K}{\rho}Tr[(I-B)'\Upsilon_a\Sigma(I-B)] \end{aligned}$$

Finally, substitute in for A, B, and C from (S.14), (S.15), and (S.16).

$$E[(q_j-x-\bar{x})'(f-pr)] = (1-\chi)K\left\{\sigma_x\rho Tr[\bar{\Sigma}\Upsilon_a\bar{\Sigma}] + \rho\bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x} + \frac{1}{\rho}Tr[\bar{\Sigma}\Sigma^{-1}\Upsilon_a\bar{\Sigma}]\right\}$$

$\bar{\Sigma}$ is equal to $1/(\sigma_a^{-1}+\chi K)bb'$ in its 3rd column and 3rd row entries, which are the only entries that the Υ_a matrix does not zero out. Therefore, $\bar{\Sigma}\Upsilon_a\bar{\Sigma} = (1/(\sigma_a^{-1}+\chi K))^2 bb'\Upsilon_a bb'$, and

$$E[(q_j-x-\bar{x})'(f-pr)] = (1-\chi)K\left\{\frac{\sigma_x\rho}{(\sigma_a^{-1}+\chi K)^2}Tr[bb'\Upsilon_a bb'] + \frac{\rho}{(\sigma_a^{-1}+\chi K)^2}\bar{x}'bb'\Upsilon_a bb'\bar{x} + \frac{1}{\rho}Tr[\bar{\Sigma}\Sigma^{-1}\Upsilon_a\bar{\Sigma}]\right\}$$

Since $bb'\Upsilon_a bb'$ does not depend on σ_a, but is positive semi-definite, and $(1/(\sigma_a^{-1} + \chi K))^2$ is increasing in σ_a, the first two terms are increasing in σ_a.

The last term can be rewritten using the relationship that $\Sigma^{-1} = S + \sigma_a^{-1}\Upsilon_a$.

$$Tr[\bar{\Sigma}\Sigma^{-1}\Upsilon_a\bar{\Sigma}] = Tr[\bar{\Sigma}S\Upsilon_a\bar{\Sigma}] + \sigma_a^{-1}Tr[\bar{\Sigma}\Upsilon_a\Upsilon_a\bar{\Sigma}]$$

Note that $\bar{\Sigma}$ and S are both positive-definite matrices and therefore have positive eigenvalues. Even Υ_a has non-negative eigenvalues. Since the trace is the sum of the eigenvalues and sums and products of non-negative eigenvalues are non-negative, the first term is positive. Furthermore, $\bar{\Sigma}$ has every entry increasing in σ_a. Therefore, the first trace term is increasing in σ_a.

In the second trace term, the matrix $\Upsilon_a\Upsilon_a = \Upsilon_a$. Using the value derived for $Tr[\bar{\Sigma}\Upsilon_a\bar{\Sigma}]$ above, we can rewrite the remaining term as

$$\sigma_a^{-1}Tr[\bar{\Sigma}\Upsilon_a\Upsilon_a\bar{\Sigma}] = \frac{\sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^2}Tr[bb'\Upsilon_a bb']$$

This is increasing in σ_a if $\partial/\partial\sigma_a^{-1}(\sigma_a^{-1}/(\sigma_a^{-1} + \chi K)^2) < 0$, which is true if

$$\frac{\partial}{\partial\sigma_a^{-1}}\frac{\sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^2} = \frac{\chi K - \sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^3} < 0$$

Thus,

$$\chi K < \sigma_a^{-1}$$

is a sufficient, but not a necessary, condition for profits to be increasing in σ_a. □

S.1.8 Proof of Proposition 4

If the size of the composite asset $\bar{x}_3$ is sufficiently large, then an increase in risk aversion increases the marginal utility of reallocating a unit of capacity from the idiosyncratic shock to the aggregate shock: $\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) > 0$.

Proof. We can rewrite $\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1}))$ as $\partial^2 U/\partial\rho\partial\hat{\sigma}_{aj}^{-1} - \partial^2 U/\partial\rho\partial\hat{\sigma}_{1j}^{-1})) > 0$.

We will work out each of these two terms separately. But first, both depend on the partial derivative of utility with respect to risk aversion. Risk aversion shows up in utility only through the asset prices. Recall from (S.14)-(S.16) that price can be expressed as $pr = -\rho\bar{\Sigma}(\bar{x} + x) + f + \bar{\Sigma}\Sigma(\mu - f)$.

Since $E[x] = 0$, the expected price has partial derivative $\partial E[pr]/\partial\rho = \bar{\Sigma}\bar{x}$. Note that price variance is $V[pr] = \rho^2 vx\bar{\Sigma}\bar{\Sigma} + (I - \bar{\Sigma}\Sigma)\Sigma(I - \bar{\Sigma}\Sigma)$. The partial derivative of this is $\partial V[pr]/\partial\rho = 2\rho vx\bar{\Sigma}\bar{\Sigma}$. Taking the partial derivative of utility and substituting these two expression in yields

$$\frac{\partial U}{\partial\rho} = \rho\sigma_x Tr[\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}] + \frac{1}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}. \tag{S.26}$$

The next step is to differentiate (S.26) with respect to $\hat{\sigma}_{aj}^{-1}$. Since $\hat{\sigma}_{aj}^{-1}$ is the precision of agent j's information, it does not affect aggregate variables such as $\bar{\Sigma}$. Recalling that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma}_{aj}^{-1} = \Upsilon_a$,

$$\frac{\partial^2 U}{\partial\rho\partial\hat{\sigma}_{aj}^{-1}} = \rho\sigma_x Tr[\Upsilon_a \bar{\Sigma}\bar{\Sigma}] + \frac{1}{2}\bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x}. \tag{S.27}$$

Next, we follow the same steps to differentiate (S.26) with respect to $\hat{\sigma}_{1j}^{-1}$, which also affects only $\hat{\Sigma}_j$. Recalling that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma}_{1j}^{-1} = \Upsilon_1$, and using the fact that the trace is invariant to matrix ordering,

$$\frac{\partial^2 U}{\partial\rho\partial\hat{\sigma}_{1j}^{-1}} = \rho\sigma_x Tr[\bar{\Sigma}\Upsilon_1\bar{\Sigma}] + \frac{1}{2}\bar{x}'\bar{\Sigma}\Upsilon_1\bar{\Sigma}\bar{x}. \tag{S.28}$$

The extent to which risk aversion affects the utility of reallocating precision from risk 1 to risk a is the difference of (S.27) and (S.28):

$$\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) = \rho\sigma_x Tr[\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}] + \frac{1}{2}\bar{x}'\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}\bar{x}. \tag{S.29}$$

Multiplying out term-by-term $\bar{\Sigma}\Upsilon_a\bar{\Sigma}$ reveals that it equals $\bar{\sigma}_a^2 bb'$. Multiplying out $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ reveals that it equals

$$\bar{\Sigma}\Upsilon_1\bar{\Sigma} = \begin{bmatrix} \bar{\sigma}_1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus,

$$Tr[\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}] = \bar{\sigma}_a^2(b_1^2 + b_2^2 + 1) - \bar{\sigma}_1^2$$

and

$$\bar{x}'\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}\bar{x} = \bar{\sigma}_a^2(b_1\bar{x}_1 + b_2\bar{x}_2 + \bar{x}_3)^2 - \bar{x}_1^2\bar{\sigma}_1^2$$

If these two terms are positive, then $\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) > 0$.

Note that the whole second partial derivative is increasing in x_3, the supply of the composite asset:

$$\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) = \rho\sigma_x(\bar{\sigma}_a^2(b_1^2 + b_2^2 + 1) - \bar{\sigma}_1^2) + \frac{1}{2}\bar{\sigma}_a^2(b_1\bar{x}_1 + b_2\bar{x}_2 + \bar{x}_3)^2 - \frac{1}{2}\bar{x}_1^2\bar{\sigma}_1^2. \tag{S.30}$$

Thus, as long as the composite asset, meant to represent the entire market capitalization, aside from the two assets 1 and 2, is large enough relative to assets 1 and 2, the cross-partial derivative will be positive. □

S.2 Model Simulation

In this Section, we use a numerical example to illustrate the model's predictions for the same measures of attention, portfolio dispersion, and performance as the ones we measure in the data. The goal of this exercise is to confirm that the model makes the same qualitative predictions for these observables as for the slightly different measures of attention allocation, portfolio dispersion, and fund performance for which we formally proved our propositions. Notably, we do not attempt to quantitatively account for all time-series and cross-sectional moments of actively managed fund portfolio holdings and returns. Such a task would be beyond the scope of this paper and indeed beyond the current state of the literature. Our model is too stylized along many dimensions to deliver on such a task. For example, it has only three assets and no heterogeneity in risk aversion, prior beliefs, or initial wealth among funds, and no heterogeneity in information capacity among skilled managers. Adding such features could improve the predictions, but only at the cost of obscuring the main mechanisms operating in the model.

S.2.1 Parameter Choices

The following explains how we choose the parameters of our model. The simplicity of the model prevents a full calibration. Instead, we pursue a numerical example that matches some salient properties of stock return data. Our benchmark parameter choices are listed in Table S.1. Section S.2.3 below shows that the qualitative results are robust to a wide range of parameter choices.

Our procedure is to simulate 3000 draws of the shocks $(x_1, x_2, x_c, s_1, s_2, a)$ in recessions and 3000 draws of the shocks in expansions. Since our model is static, each simulation is best interpreted as different draws of a random variable, and not as a period (months). The model's recessions differ from expansions in *two respects.*

First, the variance of the aggregate payoff shock σ_a is higher. It is set to replicate the fact the market return volatility is about 25% higher in recessions than in expansions. In the numerical example, the volatility of the market return is 4.0% in expansions and 5.0% in recessions, straddling the observed market return volatility of 4.5%. Setting the variance of the asset supply vector $\sigma_x = .05^2\bar{x}$ allows us to match this level of market return volatility.

Second, recessions are also characterized by lower *realized* stock market returns (despite high expected returns). In order to generate lower realized market returns and higher expected returns in a static model, we have to assume that agents are surprised by unexpectedly low returns in recessions. We accomplish this in the numerical example by replicating the bottom $m = 2.5\%$ of market return realizations among the 3000 simulations of the model in recessions, in effect simulating the economy in recessions for $3000 * (1 + .025) > 3000$ draws. This choice for m is conservative because the 0.03% difference between market returns in expansions (0.87% per month) and recessions (0.84% per month) it generates is lower than the 0.20% per month difference in the data. In the robustness section below, we consider a case that generates a 0.20% return difference. The results are qualitatively and quantitatively similar.

To get the average market return right, we choose the mean of asset payoffs μ (equal for all assets) and the coefficient of absolute risk aversion, ρ, to achieve an average equilibrium market return of about 0.85% per month.

We think of assets 1 and 2 as two large industries and the composite asset as summarizing all other industries. Therefore, we normalize the mean asset supply of assets 1 and 2 to 1, and set the supply of the composite asset, $\bar{x}_c$, to 7. The variance of the firm-specific shocks is chosen to match the fact that individual industry returns are about 30% more volatile than the market return over our sample from 1980 to 2005. We use data from the 30 industry portfolios of Fama and French (1997). In the example, the average volatility of assets 1 and 2 is 6.5% in recessions and 5.8% in expansions, 29% and 45% higher than that of the market return. This choice matches the proportion of the average industry's return variance that is idiosyncratic. We choose the asset loadings on the aggregate payoff shock, b_1 and b_2, to be different from each other so as to generate some spread in asset betas. The chosen values generate average market betas of 0.9 and a

Table S.1: **Numerical Example**

The first column lists the parameter in question, the second column is its symbol, the third column lists its numerical value, and the last column briefly summarizes how we chose that value.

Parameter	*Symbol*	*Value*	*How Chosen?*
CARA	ρ	0.525	*Asset return mean*
mean of payoffs 1,2,c	μ_1, μ_2, μ_c	10, 10, 10	*Asset return mean*
variance aggr. payoff comp. a	σ_a	0.1225 (E), 0.2625 (R)	*Market return vol in expansions vs. recessions*
variance idio. payoff comp. s_i	σ_i	0.25	*Asset return vol vs. market return vol*
a-sensitivity of payoffs	b_1, b_2	0.25,0.50	*Asset beta level + dispersion*
mean asset supply 1,2	$\bar{x}_1 = \bar{x}_2$	1,1	*Normalization*
mean asset supply 1,2	$\bar{x}_c$	7	*Asset return volatility*
variance asset supply	σ_x	$(.05 * \bar{x})^2$	*Asset return idio vol*
risk-free rate	r	0.0022	*Average T-bill return*
initial wealth	W_0	90	*Average cash position*
difficulty learning aggr. info	ψ	1	*Simplicity*
information capacity	K	1	
skilled fraction	χ	0.20	

dispersion in betas of 33%. This is reasonably close to the average beta of 0.95 and the dispersion of 23% for the 30-industry portfolios.

We set the average risk-free rate equal to 0.22% per month, the average of the 1-month yield minus inflation in our sample. We set initial wealth, W_0, to generate average holdings in the risk-free asset around 0%.

For simplicity, we set capacity K for skilled investment managers equal to 1. This implies that learning can increase the precision of one of the idiosyncratic shocks (or the aggregate shock) by 25% (by 18%). We will vary K in our robustness exercise below. Likewise, we have no strong prior on the fraction of skilled funds, χ. In our benchmark, we set it equal to 20%, and we will vary it for robustness. The model is simulated for 800 investors, of which 175 are skilled (20%). We assume that 20% of all investors are non-investment managers ("other investors"). The unskilled managers (60% of the populations) and other investors differ in name only. We note that the parameter conditions in Propositions 2 through 4 are satisfied by these parameter choices.

As in our empirical work on mutual funds in Section 2, we compute all statistics of interest as equally-weighted averages across all investment managers (i.e., without the 20% other investors). We also report results separately for skilled and unskilled investment managers.

S.2.2 Main Simulation Results

Every skilled manager ($K > 0$) solves for the choice of signal precisions $K_{aj} \geq 0$ and $K_{1j} \geq 0$ that maximize time-1 expected utility (9). We assume that these choice variables lie on a 25×25 grid in $\mathbb{R}^2_+$. The signal precision choice $K_{2j} \geq 0$ is implied by the capacity constraint (6).

We simulate a sequence of $T = 3000$ draws (months) for the random variables in each of the recession and expansion states, as explained above. We form a $T \times 1$ time series for the three individual asset returns, for the market return, for each fund's return, and for each fund's (and the market's) portfolio weights in each asset. For each asset i, we then estimate a CAPM regression of the asset's excess return on the market excess return. This delivers the asset's CAPM beta, β_i; one value in expansions and one in recessions. We define the systematic component of returns as $\beta_i R_t^m$, for $t = 1, \cdots, T$ and $i = 1, 2, 3$. Stacking the different is and ts results in a $3T \times 1$ vector of systematic returns. Similarly, we define the idiosyncratic return as $R_t^i - \beta_i R_t^m$.

To compute RAI in equation (10) for fund j, we stack its portfolio weights in deviation from the market's weights for the three assets and the T draws into a $3T \times 1$ vector. We also create a $3T \times 1$ vector of aggregate shocks by stacking three identical repetitions of each aggregate shock realization a. We calculate RAI as the covariance between these two variables. Likewise, we form $Timing$ in equation (11) as the covariance between the time series of portfolio weights, in deviation from the market's weights, and the systematic component of returns. The procedure delivers one RAI and one $Timing$ measure per fund in recessions and one set of measures in expansions. We multiply RAI by 1000 and $Timing$ by 10,000 because the aggregate shocks are an order of magnitude larger than the systematic returns.

Table S.2 summarizes the predictions of the model for the main statistics of interest. The left panel shows the results for recessions, while the right panel shows the results for expansions. In each panel, we present three columns. Column *skilled* reports the equally weighted average of the statistic in question for the group of skilled investors (20% of investors have $K > 0$ in our benchmark parametrization). Column *unskilled* is the equally weighted average across the unskilled funds (60% of investors are unskilled investment managers). Column *all* is the equally weighted average across all funds (80% of investors). The 20% unskilled other investors are excluded from the table because we do not observe them in the data. However, the model's predictions for this group are identical to those for the unskilled funds. These two groups differ in name only.

Rows 1 and 2 of Table S.2 show that RAI and $Timing$ are higher for skilled investors in recessions (left panel) than in expansions (right panel). Because of market clearing, unskilled investors are the flip side of the skilled ones, their RAI and $Timing$ measures are negative. Since no investors learn about the aggregate shock in expansions, RAI and $Timing$ are essentially zero for both skilled and unskilled. The net effect of the skilled and unskilled is listed in Column *all*. This combination of all investment managers, skilled and unskilled, is what we have data on. Hence, the first testable implication of the model is that RAI and $Timing$ should be higher for all funds in recessions than in expansions.

In a similar fashion, we construct RSI measure, defined in equation (12), and the stock-picking measure $Picking$, defined in equation (13). That is, we stack the stock-specific shocks, s_i, the idiosyncratic returns, $R_t^i - \beta_i R_t^m$, and the fund's portfolio weights into $3T \times 1$ vectors and compute the respective covariances. Rows 3 and 4 summarize the predictions of the model for RSI (multiplied by 1000) and $Picking$ (multiplied by 10,000). Across all funds (skilled and unskilled), the model predicts lower RSI and $Picking$ in recessions. Skilled funds have a high RSI and $Picking$ ability in expansions, when they allocate their attention to stock-specific information. Unskilled investors exhibit a negative $Picking$ in expansions for the same reason that they have a negative $Timing$ in recessions: Price fluctuations induce them to buy when returns are low and sell when returns are high. The RSI and $Picking$ measures are close to zero for all investors in recessions. Hence, the second testable implication of the model is that RSI and $Picking$ should be lower for all funds in recessions than in expansions.

Next, we turn to the measures of portfolio and return dispersion. Row 5 of Table S.2 shows the results for the $Concentration$ measure, defined in equation (15), in our numerical example. We calculate

Table S.2: **Benchmark Simulation Results from the Model**

This table provides the main statistics for a simulation of the model under the benchmark parameter values summarized in Table S.1. Panel A reports moments related to attention allocation, Panel B reports the moments related to portfolio dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text, the next three columns report the predictions for the model simulated in a recession, the last three columns report the results for the model simulated in an expansion. All moments are generated from a simulation of 3,000 draws and 800 investors. For both recessions and expansions, we list the equally-weighted average across *all* investment managers (the 20% skilled and the 60% investment managers), and separately for the *skilled* and the *unskilled* investment managers.

	Recessions			**Expansions**		
	All managers	Skilled	Unskilled	All managers	Skilled	Unskilled
			Panel A: Attention Allocation			
1. RAI	10.55	162.54	-40.11	0.07	0.94	-0.22
2. Timing	9.91	155.78	-38.72	0.06	3.31	-1.02
3. RSI	2.15	33.46	-8.28	15.61	249.72	-62.42
4. Picking	1.66	25.84	-6.40	10.02	160.11	-40.01
			Panel B: Dispersion			
5. Concentration	3.75	13.15	0.00	3.12	11.48	0.00
6. Idiosyncratic volatility	5.09	15.21	1.72	4.33	13.50	1.28
7. Dispersion in abnormal return	3.54	10.05	1.36	3.37	9.82	1.22
8. Dispersion in CAPM alpha	2.52	5.05	1.68	2.28	4.55	1.52
9. Dispersion in CAPM beta	6.37	13.20	4.09	1.46	4.30	0.51
			Panel C: Performance			
10. Abnormal return	0.346	5.471	-1.363	0.302	4.867	-1.220
11. CAPM Alpha	0.353	5.401	-1.330	0.307	4.861	-1.211

Concentration^j for fund j by stacking all squared deviations of fund j's portfolio from the market portfolio into a $3T \times 1$ vector, and by summing over its entries, and dividing by T. We obtain one number for recessions and one for expansions. We find that *Concentration* is higher for all funds in recessions than in expansions. This increase is driven entirely by the informed; the uninformed are all holding the exact same portfolio because of common prior beliefs.

More concentrated portfolios are also less diversified. For each fund j, we estimate CAPM regression (16) by regressing the fund's excess return on the market's excess return. This delivers the fund's α^j, β^j, and σ^j_ε. We use the idiosyncratic risk σ^j_ε as our second measure of portfolio dispersion. If all funds held the market portfolio, their idiosyncratic risk would be zero, and there would be zero cross-sectional dispersion. In simulation, the skilled funds take on more idiosyncratic risk than the unskilled ones, and more in recessions than in expansions. As a result, idiosyncratic risk is higher in recessions than in expansions for all funds.

Rows 7 through 9 report the results for the dispersion across funds' abnormal returns, CAPM alphas, and CAPM betas. All three metrics show increasing dispersion in recessions, driven largely by the heterogeneity in the choices of the skilled investors.

Finally, we study performance measures. Rows 10 and 11 of Table S.2 show that skilled investment managers have large excess returns, as measured by abnormal fund returns or fund alphas ($R^j - R^m$ and α^j), at the expense of the uninformed. The average investment manager has a slightly higher alpha in recessions than in expansions. While quantitatively modest (4.6bp per month or 55bp per year), the positive difference in average alphas between recessions and expansions is a robust finding of the model.

The numerical results also reveal that the regression residual variance $(\sigma_{\varepsilon}^{j})^2$ is higher in recessions. This effect arises because a fund that gets different signal draws (information) in each period holds a portfolio with a beta that varies over time. The CAPM equation (16) estimates an unconditional beta instead. The difference between the true, conditional beta and the estimated, constant beta shows up in the regression residual. Since recessions are times when funds learn more new information each period about the aggregate shock, these are times when true fund betas fluctuate more and the regression residuals are more volatile.

S.2.3 Robustness of Simulation Results

This section discusses the robustness of the model to alternative parameter choices. We conduct several experiments in which we vary one key parameter at a time, while holding all other parameters fixed at their benchmark levels. Table S.3 summarizes these robustness checks. For brevity, we only report the results averaged over all investment managers and omit the results broken out for skilled and unskilled managers, separately. We find that none of the comparative statics are sensitive to variation in the key parameters of the model.

In our benchmark model, we assume that 20% ($\chi = .20$) of investors are skilled mutual funds (60% are unskilled mutual funds and 20% unskilled other investors). We first study two different values for the fraction of skilled investment managers: $\chi = 10\%$ and $\chi = 30\%$. When there are fewer skilled funds, they have a comparatively larger advantage over the unskilled. This results in investment choices that exploit their informational advantage more aggressively. $Timing$ for the skilled increases from 156 to 210 in recessions while their $Picking$ reading in expansions increases from 160 in the baseline to 181. At the same time, there are fewer skilled investors exploiting more unskilled investors than in the baseline, so that the unskilled investors have less negative average $Timing$ values in recessions and less negative $Picking$ values in expansions. As a result, the $Timing$ value in recessions and $Picking$ value in expansions, averaged across across *all* investment managers (80% of the investor population), fall relative to the benchmark (from 9.9 to 5.8). Similarly, RAI increases in recessions for all funds and RSI decreases, but the changes are smaller than in the benchmark case. Likewise, our measures of portfolio dispersion continue to be higher in recessions than in expansions, but all dispersion levels are somewhat lower than before. The reason is that there is no dispersion among the unskilled, and there are more of them than in the benchmark. Finally, the performance results remain intact as well. The skilled investors make higher abnormal returns and alphas than in the benchmark, which means the unskilled loose more in total. However, they loose less per unskilled investor. As a result, alphas averaged across all funds are lower than in the benchmark: 18.6bp per month in recessions (versus 35.3bp) and 15.6bp in expansions (versus 30.7bp).

The opposite effects occur when we increase the fraction of skilled investors to 30 percent. The increase in RAI and $Timing$ and the decrease in RSI and $Picking$ in recessions are larger than those in the benchmark model. The same is true for portfolio dispersion and performance. For example, the average alpha is now 50.6bp per month in recessions and 45.4bp in expansions; the difference is slightly higher than in the benchmark. In expansions, all skilled investors continue to learn about the stock-specific information. In recessions, about 70% of attention is allocated to the aggregate shock in recessions and 15% to each of the stock-specific shocks. This 70% is lower than the 87% of skilled managers who learn about the aggregate shock in recessions in our benchmark parametrization. This is a general equilibrium effect, which we label *strategic substitutability.* When many informed investors learn about the aggregate shock, and buy assets that load heavily on that shock, they push up the price of these assets, making it less desirable to learn about for other informed investors ceteris paribus. This leads some to learn about the stock-specific shocks instead. Hence, the higher average RSI of the informed in recessions compared to the benchmark. Why is the reverse not happening in expansions? Because the volatility of the aggregate shock is low enough in expansions that it turns out not to be optimal for any of the 30% informed investors to deviate from the full attention allocation to the idiosyncratic shocks.

The second variational experiment is to decrease and increase the amount of attention allocation capacity K that skilled investors have. In our benchmark, $K = 1$, which amounts to the ability to increase the precision on any one signal by 25% of the prior precision of the stock-specific information through learning. We now consider $K = .5$ and $K = 2$. When the 20% of skilled have twice as much capacity, their RAI and $Timing$ increase substantially in recessions ($Timing$ goes up from 157 in the benchmark to 220), and their RSI and $Picking$ increase in expansions ($Picking$ goes up from 160 in the benchmark to 311). In contrast to the previous exercise, the $Timing$ measure for the unskilled becomes more negative in recessions and their $Picking$ more negative in expansions than in the benchmark. The reason is that there are as many unskilled as in the benchmark, but they are now at a larger informational disadvantage. The net effect of the skilled and the unskilled is an increase in $Timing$ in recessions from 9.9 in the benchmark to 14.0. Likewise, $Picking$ in expansions increases from 10.0 to 19.5. Giving 30% of investors $K = 1$ has similar effects as giving 20% of investors $K = 2$. Portfolio dispersion increases substantially with higher K. The result is driven by the more concentrated portfolios of the skilled, which creates both more dispersion among the skilled and a bigger difference with the unskilled. The skilled investors make abnormal returns and alphas that are about twice as high as those in the benchmark, and the unskilled loose about twice as much. The net effect are average fund alphas that are substantially higher than in the benchmark: 67.2bp per month in recessions (versus 35.3bp) and 59.3bp in expansions (versus 30.7bp). The opposite happens when we lower K to 0.5.

We recall that recessions in the model are periods with not only a higher variance of the aggregate shock, but also with lower realized market returns. We implement the latter by first simulating the model in recessions for 3000 periods, then taking the bottom m% of return realizations, and adding them to the 3000 draws when calculating the moments of interest. In our third robustness check we verify how robust our results are to different values for m. We explore $m = 0$ and $m = 0.08$, while our benchmark is $m = 0.025$. When $m = .08$, realized market returns are 22 basis points per month lower in recessions than in expansions (0.54 versus 0.76% per month). This corresponds to the return difference in the data. The results for $Timing$, $Picking$, RAI, and RSI are slightly stronger, but the magnitudes are quite close to the benchmark. The same is true for all dispersion measures, except for the beta dispersion. The latter is quite a bit lower in recessions than in the benchmark (3.91 instead of 6.37), driven by a reduction in the beta dispersion of the skilled. Because of the lower returns in recessions, skilled managers have both lower betas and less differences in their betas compared to the unskilled in recessions. Finally, the performance results are similar to the benchmark. Alphas are slightly higher than in the benchmark: 38.5bp per month in recessions (versus 35.3bp) and 31.6bp in expansions (versus 30.7bp). The difference between recessions and expansions grows to 7bp per month.

The case of $m = 0$ corresponds to a world in which assets have realized payoffs that are symmetrically distributed around the same mean in expansions and in recessions. However, because recessions are times in which returns are more volatile, *expected* (and unconditional average) returns must be higher to compensate the investors for bearing higher risk. In particular, the average market return is 1.30% in recessions and 0.95% in expansions. The results on the fund moments are opposite from the case with higher m, but still quantitatively similar to our benchmark case. For example, the difference in average alphas between recessions and expansions is 4.1bp per month compared to 4.6bp in the benchmark.

Table S.3: **Robustness of Predictions of the Model**

Panel A reports moments related to attention allocation, Panel B reports the moments related to dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text. The other pairs of columns report results for the benchmark parameters and for six robustness exercises. In each pair of columns, the first column reports the predictions for the model simulated in a recession and the second column for the model simulated in an expansion. All moments are generated from a simulation of 3000 draws and 750 investors. For both recessions and expansions, we list the equally weighted average across all investment managers (the 20% skilled and the 60% investment managers). The parameters are the same as in the benchmark model, except for the parameter listed in the first row.

	baseline		$\chi = .10$		$\chi = .30$		$K = 0.5$		$K = 2$		$m = 0$		$m = .08$	
	R	E	R	E	R	E	R	E	R	E	R	E	R	E
Panel A: Attention Allocation														
1. RAI	10.55	0.07	6.11	0.04	12.96	-0.08	6.23	0.01	14.53	-0.08	9.30	-0.04	11.01	-0.02
2. Timing	9.91	0.06	5.77	-0.09	11.56	0.12	5.93	-0.05	13.96	0.13	9.34	0.09	10.93	0.06
3. RSI	2.15	15.61	0.01	7.76	7.23	23.21	-0.03	7.89	12.66	30.14	2.08	16.16	2.14	15.76
4. Picking	1.66	10.02	0.13	5.04	5.10	14.83	0.18	5.04	8.19	19.45	1.68	9.31	1.65	10.28
Panel B: Dispersion														
5. Concentration	3.75	3.12	1.99	1.59	5.27	4.62	1.78	1.50	7.71	6.74	3.74	3.12	3.77	3.14
6. Idiosyncratic volatility	5.09	4.33	2.90	2.27	6.66	6.07	3.34	2.80	7.38	6.71	4.83	4.09	5.29	4.33
7. Dispersion in abnormal return	3.54	3.37	1.93	1.82	4.94	4.71	2.21	2.13	5.97	5.51	3.42	3.21	3.74	3.44
8. Dispersion in CAPM alpha	2.52	2.28	1.52	1.35	3.06	2.84	1.32	1.15	4.85	4.46	2.22	2.15	2.76	2.35
9. Dispersion in CAPM beta	6.37	1.46	4.70	1.03	5.76	2.45	3.45	0.98	10.84	2.67	21.03	2.08	3.91	1.38
Panel C: Performance														
10. Abnormal return	0.346	0.302	0.178	0.150	0.500	0.450	0.184	0.151	0.664	0.589	0.330	0.283	0.379	0.312
11. CAPM Alpha	0.353	0.307	0.186	0.156	0.506	0.454	0.192	0.156	0.672	0.593	0.329	0.288	0.385	0.316

S.2.4 Endogenous Capacity Model

Finally, we consider an extended model in which skilled managers can freely choose not only how to allocate their information processing capacity, but also how much capacity to acquire. We let the cost of acquiring K units of capacity be $\mathcal{C}(K)$. Each skilled fund solves for the choice of signal precisions $K_{aj} \geq 0$ and $K_{1j} \geq 0$, and capacity K that maximize time-1 expected utility, as in (9) but adjusted for a penalty term $-\mathcal{C}(K)$. In our numerical work, we assume that these choice variables lie on a 25×25 grid in $\mathbb{R}^3_+$. The choice of signal precision $K_{2j} \geq 0$ is implied by the capacity constraint (6).

In our numerical exercise, we consider two different functional forms for $\mathcal{C}(K)$. The first one is $\mathcal{C}_1(K) = c_1 \exp(K)$ and the second one is $\mathcal{C}_2(K) = c_2 K^{\psi}$. For ease of comparison with our exogenous K results, we choose the scalars c_1 and c_2 such that the optimal capacity choice is $K = 1$ on average across expansions and recessions. This is the same capacity choice we assume in our benchmark parametrization. Clearly, increasing (lowering) the scalars c_1 and c_2 will lead to lower (higher) optimal capacity choice. These scalars can be interpreted as (shadow) prices of capacity. All other parameters are the same as in our benchmark model.

More interesting than the level of K that is chosen is how that choice differs between recessions and expansions. We find that for both cost functions, investors acquire more capacity in recessions than in expansions. Nothing in the cost function makes it cheaper to acquire capacity in either expansions or recessions. This result is solely driven by the fact that the higher (aggregate) uncertainty in recessions makes it optimal to acquire more capacity and to allocate it to the aggregate shock. This extensive-margin effect acts as an amplification to our intensive-margin effect. How elastic capacity choice is to changes in prior aggregate uncertainty, and hence how large the amplification effect is, *does* depend on the functional form of the cost function. For cost function 1, we find that capacity choice is 1.02 in recessions and 0.97 in expansions. For cost function 2, the elasticity is much higher, with a capacity choice of 1.15 in recessions and 0.92 in expansions. The reason for the higher elasticity is that the marginal cost function 2 is less steep in capacity. As a result, a given change in the marginal benefit of acquiring information leads to larger equilibrium changes in capacity. Since we have no strong prior over the functional form, we conduct our numerical simulation for both cost functions.

Table S.4 summarizes the main moments of interest for the endogenous K model, alongside the benchmark, exogenous K results. For brevity, we only report the results averaged over all investment managers and omit the results broken out for skilled and unskilled managers, separately. Overall, we find that the results are very similar to those in our exogenous K model, not only qualitatively, but also quantitatively. The moments for cost function 2 (two most right columns) tend to be higher in recessions than do the benchmark numbers, and lower in expansions. Hence, there is amplification of the difference between recessions and expansions. For example, average fund alphas are somewhat higher than in the benchmark in recessions (40.7bp per month versus 35.3bp) and somewhat lower in expansions (27.7bp versus 30.7bp). The resulting difference between recessions and expansions grows substantially from 4.6bp to 13bp per month. For cost function 1 (two middle columns), the moments are slightly higher in recessions since the skilled investment managers choose to acquire slightly more capacity than what they are endowed with in the benchmark ($K = 1.02$ versus 1). The moments are slightly lower in expansions, since they have slightly lower capacity ($K = 0.97$ versus 1). Overall, the difference in our key variables between recessions and expansions is usually very similar to that in our benchmark model.

Table S.4: **Endogenous Capacity Model**

This table provides the results from an extension of the model where skilled funds endogenously choose how much capacity to acquire. It reports on the main predictions of the model. Panel A reports moments related to attention allocation, Panel B reports the moments related to dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text. The other pairs of columns report results for the benchmark parameters and for two versions of the endogenous K model with different cost functions. The cost function in the first one is $\mathcal{C}_1(K) = c_1 \exp(K)$, while the cost function in the second one is $\mathcal{C}_2(K) = c_2 K^{\psi}$. We set $c_1 = 1.057$, $c_2 = 2.4$, and $\psi = 1.2$. All other parameters are the same as in the benchmark model. In each pair of columns, the first column reports the predictions for the model simulated in a recession (R) and the second column for the model simulated in an expansion (E). All moments are generated from a simulation of 2000 draws and 100 investors. For both recessions and expansions, we list the equally weighted average across all investment managers (the 20% skilled and the 60% investment managers).

	Baseline		$\mathcal{C}_1(K) = c_1 \exp(K)$		$\mathcal{C}_2(K) = c_2 K^{\psi}$	
	R	E	R	E	R	E
Panel A: Attention Allocation						
1. RAI	10.55	0.07	10.53	-0.12	11.58	-0.01
2. Timing	9.91	0.06	9.83	0.10	10.65	0.13
3. RSI	2.15	15.61	2.35	15.43	3.69	14.37
4. Picking	1.66	10.02	1.76	9.91	2.65	8.87
Panel B: Dispersion						
5. Concentration	3.75	3.12	3.82	3.13	4.37	2.85
6. Idiosyncratic volatility	5.09	4.33	5.20	4.53	5.34	4.15
7. Dispersion in abnormal return	3.54	3.37	3.59	3.34	3.97	3.16
8. Dispersion in CAPM alpha	2.52	2.28	2.57	2.29	2.92	2.06
9. Dispersion in CAPM beta	6.37	1.46	5.18	3.01	5.28	1.94
Panel C: Performance						
10. Abnormal return	0.346	0.302	0.348	0.302	0.399	0.271
11. CAPM Alpha	0.353	0.307	0.355	0.307	0.407	0.277

S.3 Robustness Checks for Empirical Results

S.3.1 Attention Allocation Results

Table S.5 reports several robustness checks. First, we compute an alternative RAI measure in which the aggregate shock is proxied by surprises in non-farm employment growth, another salient macroeconomic variable, instead of industrial production growth. Second, we compute an alternative RSI measure in which earnings surprises are defined as the residual from a regression of earnings per share in a given year on earnings per share in that same quarter one year earlier (instead of one quarter earlier), as in Bernard and Thomas (1989). Third, to check the market-timing results, we also study the R^2 from a CAPM regression at the fund level (as in equation 16). It measures how the funds' excess returns (as opposed to their portfolio weights) covary with the aggregate state, as measured by the market's excess return. The average R^2 across all funds rises from 77% in expansions to 80% in recessions. All three findings are consistent with the hypothesis that recessions are times when funds learn about the aggregate shock, making their portfolio choices and therefore their fund returns more sensitive to changes in market returns.

To get further insight into why the RAI and the market-timing measure increase in recessions, we conduct several other exercises. First, we ask whether managers actively change their cash holdings in recessions. Cash is measured either as Reported Cash, from CRSP, or Implied Cash, backed out from fund size and its equity holdings. Table S.6 shows a slightly higher cash position in recessions than in expansions. In expansions, funds hold about 5% of their portfolio in cash. In recessions, the fraction of their holdings in cash rises by about 0.3% for the Reported Cash measure and by about 3% for the Implied Cash measure. Both increases are statistically significant, and each represents about a 0.1 standard-deviation change. The last two columns report the month-over-month change in the Implied Cash position. In recessions, cash holdings increase by 0.5%. The effect is modest, but measured precisely. In sum, one way in which funds lower their portfolio beta in recessions is to increase their cash positions.

The second question we ask is whether fund managers also invest in lower-beta stocks in recessions. For each individual stock, we compute the beta (from twelve-month rolling-window regressions). Based, on the individual stock holdings of each mutual fund, we construct the funds' (value-weighted) *equity betas*. Table S.7 shows that this beta is 1.11 in expansions and 0.99 in recessions; the 0.118 difference has a t-statistic of 4.5. This means that funds not only keep more cash in recessions, but also hold different types of stocks, namely lower-beta stocks.

Finally, we investigate whether funds change their portfolio allocations towards *defensive sectors* sectors over the business cycle. Table S.8 shows that, in recessions, funds increase their portfolio weights (relative to those in the market portfolio) in low-beta sectors such as Healthcare, Non-Durables (which includes Food and Tobacco), Wholesale, and Utilities. They reduce their portfolio weight (relative to those in the market portfolio) in high-beta sectors such as Telecom, Business Equipment and Services, Manufacturing, Energy, and Durables. Hence, funds do engage in sector rotation over the course of the business cycle.

Table S.5: **Robustness: Alternative RAI and RSI Measures**

The dependent variables are funds' reliance on aggregate information $RAI2$, funds' reliance on stock-specific information $RSI2$, and the CAPM $R - squared$. A fund j's $RAI2_t^j$ is defined as the (twelve-month rolling window time series) covariance between the funds' portfolio holdings in deviation from the market $(w_{it}^j - w_{it}^m)$ in month t and changes in non-farm employment growth between t and $t+1$. A fund j's $RSI2_t^j$ is defined as the (across stock) covariance between the funds' holdings in deviation from the market $(w_{it}^j - w_{it}^m)$ in month t and changes in earnings growth between $t-11$ and $t+1$. $R - squared$ is obtained from the twelve-month rolling-window regression model of a fund's excess returns on excess market returns. $RAI2$, and $RSI2$ are multiplied by 10,000 and $R - squared$ is multiplied by 100 for ease of readability. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Turnover* is the fund turnover ratio. *Flow* is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	RAI2		RSI2		R-squared	
Recession	0.004	0.004	-0.886	-0.897	3.040	2.891
	(0.001)	(0.001)	(0.201)	(0.191)	(1.451)	(1.315)
Log(Age)		-0.001		0.452		2.126
		(0.000)		(0.076)		(0.190)
Log(TNA)		0.000		-0.229		0.258
		(0.000)		(0.034)		(0.074)
Expenses		-0.158		111.982		-582.087
		(0.058)		(12.954)		(26.684)
Turnover		0.000		-0.329		-1.242
		(0.000)		(0.074)		(0.110)
Flow		-0.001		2.570		-6.614
		(0.003)		(0.723)		(2.885)
Load		0.021		-12.614		68.883
		(0.007)		(2.317)		(5.434)
Constant	-0.001	-0.001	3.962	3.962	77.361	77.331
	(0.000)	(0.000)	(0.089)	(0.089)	(0.854)	(0.846)
Observations	224,257	224,257	166,328	166,328	227,159	227,159

Table S.6: **Robustness: Cash Holdings**

The dependent variables are three measures of funds' cash holdings. *ReportedCash* is the cash position reported by mutual funds to CRSP in their quarterly statements, relative to the size of the fund (expressed as a percent). *ImpliedCash* is based on the portfolio holdings of the fund. In particular, it is the difference between the total size of the fund (monthly) as reported in the data and the implied size of the equity portio based on the observed holdings and their prices. It is also expressed as a percent of total holdings. *%ChangeCash* is defined as the percentage change in the implied cash measure. *Recession* equals one for every month the economy is in the recession according to the NBER, and zero otherwise. We use the following control variables. *Log(Age)* is the natural logarithm of fund age. *Log(TNA)* is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the fund new money growth. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	Implied Cash		Reported Cash		% Change Cash	
Recession	2.491	3.278	0.230	0.362	0.643	0.545
	(0.537)	(0.535)	(0.089)	(0.087)	(0.051)	(0.050)
Log(Age)		-0.453		0.309		-0.075
		(0.517)		(0.081)		(0.037)
Log(TNA)		1.676		-0.047		-0.092
		(0.277)		(0.040)		(0.018)
Expenses		163.772		-46.153		24.280
		(119.092)		(18.459)		(6.859)
Turnover		-0.059		-0.168		0.119
		(0.413)		(0.064)		(0.031)
Flow		13.794		3.893		0.189
		(2.840)		(0.315)		(0.301)
Load		-5.033		15.169		-1.144
		(14.366)		(2.837)		(1.196)
Constant	5.316	5.252	4.672	4.656	-1.505	-1.495
	(0.481)	(0.479)	(0.065)	(0.062)	(0.033)	(0.031)
Observations	230,185	230,185	209,516	209,516	225,374	225,374

Table S.7: **Robustness: Equity Betas**

The dependent variable is the fund's *Equity Beta*. For each individual stock, we compute the market beta from a twelve-month rolling-window regression. We then construct the funds' equity beta as the value-weighted average of the individual stock betas, where the weights are the fund's dollar holdings in that stock divided by the dollar holdings in all stocks. *Recession* equals one for every month the economy is in the recession according to the NBER, and zero otherwise. We use the following control variables. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the fund new money growth. $Turnover$ is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)
	Equity Beta	
Recession	-0.118	-0.106
	(0.026)	(0.027)
Log(Age)		0.013
		(0.002)
Log(TNA)		0.008
		(0.001)
Expenses		3.113
		(0.385)
Turnover		0.035
		(0.002)
Flow		0.056
		(0.037)
Load		0.561
		(0.062)
Constant	1.112	1.111
	(0.006)	(0.008)
Observations	226,094	226,094

Table S.8: **Robustness: Sector Rotation**

The dependent variable is the portfolio weight of fund j in sector l in deviation from the market portfolio's weight in sector l: $w^j_{lt} - w^m_{lt}$. Each column represent a different sector. The sectors are the ten Fama-French industry sectors: (1) Consumer non-durables, (2) Consumer durables, (3) Healthcare, (4) Manufacturing, (5) Energy, (6) Utilities, (7) Telecom, (8) Business Equipment and Services, (9) Wholesale and Retail, (10) Finance. *Recession* equals one for every month the economy is in the recession according to the NBER, and zero otherwise. We use the following control variables. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the fund new money growth. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	NDRBL	DRBL	HLTH	MFCT	ENER	UTIL	TEL	BUSEQ	WHLS	FIN
Recession	0.817	0.123	0.541	-0.278	-0.311	0.246	-0.493	-1.565	0.384	0.741
	(0.085)	(0.111)	(0.215)	(0.121)	(0.397)	(0.269)	(0.111)	(0.563)	(0.081)	(0.154)
Log(Age)	0.301	-0.022	0.728	-0.086	-0.557	-0.702	0.037	0.278	0.085	0.036
	(0.028)	(0.016)	(0.032)	(0.031)	(0.044)	(0.035)	(0.022)	(0.064)	(0.027)	(0.054)
Log(TNA)	-0.246	-0.076	-0.387	-0.114	0.270	0.247	0.257	0.001	-0.060	-0.003
	(0.014)	(0.013)	(0.013)	(0.013)	(0.025)	(0.016)	(0.010)	(0.025)	(0.015)	(0.016)
Expenses	42.310	2.132	-54.607	-72.195	67.829	28.997	22.623	-63.774	11.964	29.247
	(3.641)	(2.849)	(4.630)	(5.043)	(7.958)	(4.180)	(3.902)	(10.138)	(4.611)	(8.483)
Turnover	-0.181	-0.134	0.330	-0.586	0.328	0.040	0.307	1.276	0.021	-1.325
	(0.030)	(0.025)	(0.040)	(0.039)	(0.025)	(0.017)	(0.031)	(0.075)	(0.044)	(0.041)
Flow	-0.256	-0.423	-0.320	-1.241	-1.554	-1.298	1.308	3.289	0.057	0.450
	(0.314)	(0.296)	(0.443)	(0.431)	(0.602)	(0.503)	(0.423)	(1.219)	(0.377)	(0.580)
Load	-4.833	0.764	6.123	8.589	-12.164	-9.617	3.758	23.523	-1.886	-15.841
	(0.656)	(0.563)	(0.658)	(0.894)	(1.219)	(0.705)	(0.473)	(1.729)	(0.871)	(0.978)
Constant	-0.471	-0.777	0.173	2.499	-1.171	-1.769	-1.489	3.600	2.404	-2.660
	(0.046)	(0.065)	(0.050)	(0.073)	(0.080)	(0.084)	(0.053)	(0.201)	(0.059)	(0.101)
Observations	207,382	207,382	207,382	207,382	207,382	207,382	207,382	207,382	207,382	207,382

S.3.2 Dispersion Results

Table S.9 considers additional measures of portfolio and return dispersion. First, managers shift their investment styles more in recessions, consistent with them pursuing a more active portfolio management strategy. The results are again highly significant, both economically and statistically. Investment managers also display a somewhat greater industry concentration in recessions. Column 5 and 6 show that the dispersion of abnormal returns (fund returns minus the market return) nearly doubles in recessions. The last two columns show that the dispersion in 3-factor alphas increases from 0.5 in expansions to 0.7 in recessions, an economically and statistically large increase. Similar results hold for the dispersion in CAPM alpha.

In unreported results, we obtain similar results for the dispersion in CAPM alpha and beta when funds' alphas and betas are calculated not by using twelve-month rolling-window regressions, but by estimating their dependence on several state variables (the dividend-price ratio of the aggregate stock market, the term spread, the short-term interest rate, and the default spread) in one full-sample regression.

Finally, we study the dispersion of the information ratio, defined as the ratio of the CAPM alpha to the CAPM residual volatility. Its dispersion increases from 0.31 in expansion to 0.39 in recessions. The increase is measured precisely (t-statistic of 4.7). These results bolster our evidence for increased dispersion in recessions.

Table S.9: **Robustness: Dispersion in Funds' Portfolio Strategies and Returns**

The dependent variables are *Style Shifting*, *Sector Deviation*, and $|X_t^j - \bar{X}_t|$, where X_t^j is the *Abnormal Return* or $3-$*Factor Alpha* and $\bar{X}$ denotes the (equally weighted) cross-sectional average. *Style Shifting* for fund j at time t is the absolute value of the change between time t and time $t-1$ in the the fund's investment style index, defined in footnote 18. *Sector Deviation* for fund j at time t is calculated as the mean square root of the sum of squared differences between the share of fund j's assets in each of 10 industry sectors and the mean share in each sector in quarter t among all funds in fund j's objective class (aggressive growth, growth, or value). To identify the investment objectives, we use the Thomson Financial's style categories 2, 3, and 4. Industry sectors are defined using a modified 10-industry classification of Fama and French, as in Kacperczyk, Sialm, and Zheng (2005). The three-factor alphas are obtained from twelve-month rolling window regressions of fund-level excess returns on excess market returns, SMB, and HML. The abnormal return is the fund return minus the market return, also from a twelve-month rolling-window regression. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. *Log*(*Age*) is the natural logarithm of fund age. *Log*(*TNA*) is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Style Shifting		Sector Deviation		Abnormal Return		3-Factor Alpha	
Recession	5.246	5.082	0.003	0.003	0.530	0.561	0.201	0.212
	(0.826)	(0.798)	(0.002)	(0.001)	(0.108)	(0.102)	(0.042)	(0.039)
Log(Age)		0.014		0.000		-0.064		-0.013
		(0.080)		(0.002)		(0.007)		(0.003)
Log(TNA)		-0.214		-0.006		0.029		-0.001
		(0.047)		(0.001)		(0.004)		(0.001)
Expenses		101.35		2.084		13.816		9.178
		(13.908)		(0.301)		(1.152)		(0.465)
Turnover		1.111		-0.001		0.074		0.064
		(0.109)		(0.002)		(0.006)		(0.004)
Flow		-1.749		0.008		0.479		0.227
		(1.101)		(0.007)		(0.088)		(0.034)
Load		-4.517		-0.175		-1.738		-0.622
		(2.284)		(0.052)		(0.182)		(0.076)
Constant	14.725	14.807	0.185	0.186	0.661	0.659	0.497	0.497
	(0.283)	(0.284)	(0.001)	(0.001)	(0.029)	(0.027)	(0.010)	(0.010)
Observations	191,109	191,109	72,708	72,708	226,745	226,745	226,745	226,745

S.3.3 Performance Results

The cross-sectional regression model allows us to include a host of fund-specific control variables, thereby making use of rich panel data. But because it measures performance using past twelve-month rolling-window regressions, a given month observation for the dependent variable can be classified as a recession when some or even all of the remaining eleven months of the window are expansions. To avoid this problem and verify our results, we also employ a time-series approach which is free of this issue and potentially provides a cleaner estimate of the economic magnitudes of interest. In each month, we form the equally weighted portfolio of funds and calculate its (net) return in excess of the risk-free rate. This procedure results in a series of portfolio returns which we then use to estimate a time-series regression model of the portfolio returns on *Recession* and common risk factors. We adjust standard errors for any heteroscedasticity and autocorrelation using the procedure in Newey and West (1987). Table S.10 presents the results.

In Column 1, we control for the excess market return. The intercept of the regression, equal to -6bp per month, measures the CAPM alpha in expansions. The coefficient on *Recession* measures the incremental CAPM alpha in recessions, and is equal to 27bp per month. This result is equivalent to an economically large and statistically positive alpha in recessions of around 2.5% per year. In Columns 2 and 3, we include additional two and three factors. The resulting three- and four-factor alphas are both estimated to be 16bp per month higher in recessions than in expansions; they are also fairly negative in expansions. The time-series approach allows us to include an additional illiquidity factor, defined as in Pástor and Stambaugh (2003). The results, in Column 4, show that fund returns do not load significantly on this factor; consequently, our previous results remain largely unchanged.

To allow for the possibility that factor loadings may vary over the business cycle, we re-estimate the regression specifications in Columns 1 through 4, allowing for interactions of the factors with *Recession*. The results, in Columns 5 through 8, show that the coefficients on the interaction terms are typically statistically insignificant, with the exception of the interaction term with the value factor. Our results are largely unaffected by the time-varying factor exposures of fund returns. If anything, the magnitudes of the incremental returns in recessions become slightly stronger. As an example, in Column 8, the risk-adjusted excess return is 1.7% per year in recessions, 2.7% higher than the -1% return in expansions. This difference is statistically and economically significant. The magnitudes of the estimates in Table S.10 are reassuring as they are consistent with our panel-regression estimates in Table 4.

We also study the robustness to additional ways of measuring performance. Table S.11 uses *gross* fund returns and alphas instead of returns and alphas net of fees. They are constructed by adding back the management fees. Gross returns are about zero in expansions and substantially higher in recessions. In unreported results, we also examine the information ratio, defined as the ratio of the CAPM alpha to the CAPM residual volatility. The results are, if anything, stronger. The information ratio is -0.07 in expansions and increases by 0.14 (t-statistic of 7.8) to 0.072 in recessions. The 0.14 increase in recessions can be interpreted as a Sharpe ratio gain. Finally, we study a specification in which we lead the alpha on the left-hand side of equation (19) by one month, instead of using its contemporaneous value. The increase in alpha in recessions falls slightly, compared to the benchmark specification, but the effect remains statistically and economically significant.

Table S.10: **Robustness: Time-Series Approach to Fund Performance**

Each month, we form an equally weighted portfolio of all funds. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. We regress the portfolio (net) return in excess of the risk-free rate on *Recession* and a set of four risk factors: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD), and the return on the illiquid-minus-liquid stock portfolio (LIQ) of Pástor and Stambaugh (2003). We also consider specifications in which the risk factors are interacted with Recession. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are corrected for autocorrelation and heteroscedasticity.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Recession	0.274	0.151	0.156	0.157	0.265	0.215	0.198	0.222
	(0.105)	(0.079)	(0.076)	(0.074)	(0.104)	(0.094)	(0.097)	(0.081)
MKTPREM	1.000	0.976	0.978	0.981	0.998	0.979	0.980	0.984
	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)
SMB		0.169	0.168	0.168		0.170	0.169	0.169
		(0.024)	(0.025)	(0.026)		(0.026)	(0.027)	(0.027)
HML		-0.007	-0.003	-0.001		0.001	0.005	0.007
		(0.030)	(0.030)	(0.030)		(0.032)	(0.033)	(0.033)
UMD			0.014	0.014			0.015	0.015
			(0.019)	(0.020)			(0.021)	(0.022)
LIQ				-0.008				-0.007
				(0.007)				(0.008)
Recession*MKT					0.011	-0.029	-0.018	-0.025
					(0.024)	(0.023)	(0.025)	(0.024)
Recession*SMB						-0.027	-0.024	-0.013
						(0.047)	(0.049)	(0.041)
Recession*HML						-0.088	-0.098	-0.124
						(0.049)	(0.050)	(0.044)
Recession*UMD							0.017	0.027
							(0.033)	(0.035)
Recession*LIQ								-0.046
								(0.026)
Constant	-0.064	-0.059	-0.074	-0.075	-0.062	-0.065	-0.079	-0.081
	(0.057)	(0.038)	(0.047)	(0.047)	(0.058)	(0.038)	(0.048)	(0.048)
Observations	309	309	309	309	309	309	309	309

Table S.11: **Robustness: Using Gross Returns**

Each month, we calculate value-weighted or equally weighted gross portfolio returns (returns + expenses) of all mutual funds. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. We regress the returns on *Recession* and a set of four risk factors: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD), and the return on the illiquid-minus-liquid stock portfolio (LIQ) of Pástor and Stambaugh (2003). We also consider specifications in which the risk factors are interacted with Recession. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are corrected for autocorrelation and heteroskedasticity.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Recession	0.268	0.146	0.150	0.151	0.259	0.210	0.195	0.218
	(0.104)	(0.078)	(0.076)	(0.073)	(0.103)	(0.092)	(0.096)	(0.081)
MKTPREM	1.000	0.976	0.978	0.981	0.998	0.979	0.980	0.984
	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)
SMB		0.169	0.168	0.168		0.170	0.169	0.169
		(0.024)	(0.025)	(0.026)		(0.026)	(0.027)	(0.027)
HML		-0.007	-0.003	-0.002		0.001	0.005	0.007
		(0.030)	(0.030)	(0.030)		(0.032)	(0.033)	(0.033)
UMD			0.014	0.014			0.015	0.015
			(0.019)	(0.020)			(0.021)	(0.022)
LIQ				-0.008				-0.007
				(0.007)				(0.008)
Recession*MKTPREM					0.011	-0.029	-0.019	-0.026
					(0.024)	(0.023)	(0.025)	(0.024)
Recession*SMB						-0.028	-0.025	-0.014
						(0.047)	(0.048)	(0.040)
Recession*HML						-0.090	-0.099	-0.125
						(0.049)	(0.049)	(0.044)
Recession*UMD							0.016	0.026
							(0.033)	(0.035)
Recession*LIQ								-0.045
								(0.026)
Constant	0.037	0.042	0.027	0.026	0.039	0.036	0.022	0.020
	(0.058)	(0.038)	(0.047)	(0.047)	(0.058)	(0.038)	(0.048)	(0.048)
Observations	309	309	309	309	309	309	309	309

S.3.4 Alternative Recession Indicators

Table S.12 reports the results of the performance analysis based on the pooled-regression specification. It shows that the average fund outperformance continues to be higher in recessions when we replace the NBER recession indicator with the Chicago Fed National Activity Index (CFNAI). The CFNAI is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity trends, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend. The CFNAI has a correlation of -70% with the NBER recession indicator. The CFNAI has the advantage that it is a continuous variable, which measures the *strength* of economic activity, i.e., how deep the recession or how strong the expansion is. Table S.12 shows that, on average (when CFNAI is zero), the average fund alpha is negative. But, in recessions, when economic activity is low (CFNAI is negative), an average fund's performance increases significantly. The CFNAI is constructed to have a standard deviation of one in the full sample (1967-2008). In our 1980-2005 sample, it has a standard deviation of 0.644. Hence, a one standard-deviation decrease in CFNAI increases the CAPM alpha by 12.8bp per month and the four-factor alpha by 5.7bp points per month. These effects are measured precisely and quantitatively similar to our benchmark results. The RAI/RSI and dispersion results also continue to go through; the tables are omitted for brevity.

Next, we find performance results that are similar, and somewhat stronger, when we replace the NBER recession indicator with an indicator which is one when real consumption growth is negative. We also find similar results when we use a dummy that captures the 25% lowest stock market returns as a recession indicator. The latter two tables are omitted for brevity.

As a second alternative recession indicator we use a measure of fundamental volatility. More specifically, we calculate the twelve-month rolling-window standard deviation of aggregate earnings growth. Aggregate earnings growth is the year-to-year (e.g., March to March) log change in aggregate earnings. Aggregate earnings are based on the earnings of all firms in the S&P 500 index; the aggregate earnings data are from Robert Shiller for the period from 1926 until 2008. The variable $Volatility$ equals one if the standard deviation of aggregate earnings growth is in the highest 10% of months in the 1926-2008 sample. Twelve percent of months in our 1980-2005 sample are such high-volatility months. Table S.13 shows that alphas are negative in low-volatility months and substantially higher in high-volatility months. The incremental CAPM alpha in recessions is 54bp per month and the incremental four-factor alpha is 14.8bp per month. Both are statistically significant. All results point in the same direction: Outperformance clusters in periods of recessions. The RAI/RSI and dispersion results also continue to go through with the high-volatility indicator; the tables are omitted for brevity.

Table S.12: **Robustness: Performance Using the CFNAI as Recession Indicator**

The dependent variables are funds' $CAPM\ Alpha$, $3-Factor\ Alpha$, and $4-Factor\ Alpha$. All alphas are obtained from twelve-month rolling-window regressions of fund-level excess returns on excess market returns for the CAPM alpha, additionally on the SMB and the HML factors for the three-factor alpha, and additionally on the UMD factor for the four-factor alpha. The abnormal return is the fund return minus the market return. $CFNAI$, or Chicago Fed National Activity Index, is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward trend growth rate over time, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. $Expenses$ is the fund expense ratio. $Flow$ is the percentage growth in a fund's new money. $Turnover$ is the fund turnover ratio. $Load$ is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimension, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. $Flow$ and $Turnover$ are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	CAPM Alpha		3-Factor Alpha		4-Factor Alpha	
CFNAI	-0.163	-0.198	-0.046	-0.061	-0.071	-0.088
	(0.034)	(0.033)	(0.024)	(0.021)	(0.028)	(0.025)
Log(Age)		-0.035		-0.028		-0.038
		(0.007)		(0.006)		(0.006)
Log(TNA)		0.032		0.009		0.012
		(0.004)		(0.003)		(0.003)
Expenses		-3.407		-7.996		-7.967
		(0.922)		(0.786)		(0.747)
Turnover		-0.045		-0.075		-0.066
		(0.010)		(0.010)		(0.008)
Flow		2.439		1.695		1.541
		(0.169)		(0.096)		(0.095)
Load		-0.611		-0.064		-0.278
		(0.169)		(0.128)		(0.136)
Constant	-0.031	-0.031	-0.055	-0.055	-0.042	-0.042
	(0.022)	(0.021)	(0.018)	(0.016)	(0.021)	(0.019)
Observations	226,745	226,745	226,745	226,745	226,745	226,745

Table S.13: **Robustness: Performance Using Volatility as Recession Indicator**

The dependent variables are funds' reliance on aggregate information (RAI), funds' reliance on stock-specific information (RSI), funds' portfolio concentration (*Concentration*), the cross-sectional dispersion in fund CAPM alphas (CAPM Alpha Disp) and betas (CAPM Beta Disp), and the funds' four-factor alpha (4-Factor Alpha). The definitions of the dependent variables are listed in the captions of Tables 2, 3, and 4. *Volatility* is a dummy variable indicating periods of high volatility in fundamentals. We calculate the twelve-month rolling-window standard deviation of aggregate earnings growth. Aggregate earnings growth is the year-to-year log change in the earnings of S&P 500 index constituents; the aggregate earnings data are from Robert Shiller for the period from 1926 until 2008. Volatility equals one if the standard deviation of aggregate earnings growth is in the highest 10% of months in the 1926-2008 sample. Twelve percent of months in our 1985-2005 sample are such high volatility months. *Log*(*Age*) is the natural logarithm of fund age. *Log*(*TNA*) is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Turnover* is the fund turnover ratio. *Flow* is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	RAI	RSI	Concentration	CAPM Alpha Disp	CAPM Beta Disp	4-Factor Alpha
Volatility	0.003	-0.642	0.184	0.304	0.092	0.148
	(0.003)	(0.104)	(0.028)	(0.042)	(0.019)	(0.061)
Log(Age)	-0.001	0.411	0.201	-0.037	-0.004	-0.035
	(0.001)	(0.061)	(0.028)	(0.004)	(0.002)	(0.006)
Log(TNA)	-0.001	-0.163	-0.179	0.011	0.001	0.009
	(0.000)	(0.029)	(0.014)	(0.003)	(0.001)	(0.003)
Expenses	-0.353	92.072	29.520	7.709	3.564	-8.396
	(0.247)	(11.620)	(4.871)	(0.581)	(0.218)	(0.803)
Turnover	-0.004	-0.201	-0.088	0.045	0.013	-0.067
	(0.001)	(0.063)	(0.025)	(0.004)	(0.001)	(0.008)
Flow	-0.008	1.680	0.100	0.311	0.007	1.534
	(0.010)	(0.628)	(0.104)	(0.044)	(0.015)	(0.097)
Load	0.021	-10.207	-1.714	-0.812	-0.228	-0.185
	(0.023)	(1.985)	(0.908)	(0.099)	(0.041)	(0.145)
Constant	-0.001	3.097	1.556	0.572	0.224	-0.060
	(0.001)	(0.073)	(0.023)	(0.017)	(0.005)	(0.021)
Observations	224,257	166,328	226,745	226,745	226,745	226,745

S.3.5 Managers Instead of Funds as Unit of Observation

We redo the main exercises using our managers as the unit of observation rather than the funds themselves. We follow a manager over time, even as (s)he switches funds. This allows us to investigate to what extent our results hold up at the manager level, and to what extent they reflect skill at the level of the fund versus at the level of the manager. For brevity, we combine our three main results: Columns 1 and 2 of Table S.14 show RAI and RSI results, Columns 3 through 5 show dispersion results (Concentration, alpha dispersion and beta dispersion), and column 6 shows the performance results (four-factor alpha). The results for the other metrics of performance and dispersion are similar. The results without the control variables are similar to the results with controls, which we present. The table shows higher RAI and lower RSI, higher dispersion, and higher performance in recessions. All effects remain statistically significant and the magnitudes of the recession effects are similar at the manager level than they were at the fund level. They are the same for RSI, slightly higher for dispersion and performance, and slightly lower for RAI. This suggests that our results hold both at the fund level and at the manager level.

Table S.14: **Robustness: Managers as the Unit of Observation**

The dependent variables are reliance on aggregate information (RAI), reliance on stock-specific information (RSI), portfolio concentration (*Concentration*), the cross-sectional dispersion in fund CAPM alphas (CAPM Alpha Disp) and betas (CAPM Beta Disp), and the four-factor alpha (4-Factor Alpha), all of which are tracked at the manager level. The definitions of the dependent variables are listed in the captions of Tables 2, 3, and 4. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. $Turnover$ is the fund turnover ratio. $Flow$ is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of the manager along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	RAI	RSI	Concentration	CAPM Alpha Disp	CAPM Beta Disp	4-Factor Alpha
Recession	0.008	-0.701	0.105	0.363	0.113	0.167
	(0.003)	(0.130)	(0.031)	(0.047)	(0.014)	(0.035)
Log(Age)	-0.002	0.460	0.154	-0.041	-0.003	-0.032
	(0.001)	(0.066)	(0.032)	(0.004)	(0.002)	(0.006)
Log(TNA)	-0.000	-0.126	-0.131	0.022	0.004	0.007
	(0.000)	(0.032)	(0.018)	(0.002)	(0.001)	(0.004)
Expenses	0.130	127.222	43.920	10.247	4.590	-8.225
	(0.105)	(13.972)	(6.012)	(0.738)	(0.253)	(0.794)
Turnover	-0.005	-0.287	-0.127	0.053	0.012	-0.081
	(0.002)	(0.077)	(0.030)	(0.005)	(0.001)	(0.010)
Flow	-0.009	1.037	0.154	0.331	0.015	1.832
	(0.009)	(0.613)	(0.121)	(0.055)	(0.017)	(0.097)
Load	-0.009	-16.064	-4.674	-1.415	-0.400	-0.426
	(0.017)	(2.393)	(1.284)	(0.098)	(0.044)	(0.151)
Constant	-0.002	2.966	1.438	0.605	0.230	-0.045
	(0.001)	(0.072)	(0.026)	(0.019)	(0.006)	(0.024)
Observations	332,676	249,942	332,776	332,776	332,776	332,776

S.3.6 Identifying Skilled Fund Managers

Table S.15 reports the estimation results from the linear probability regression model of the *SP* indicator variable on fund age, TNA, expenses, and turnover. The R^2 of the baseline regression is 14%.

The existence of a group of skilled mutual funds who switch in terms of their learning and investment strategies between recessions and expansions is not very sensitive to the exact specification. First, we investigate a different cutoff level for the inclusion in the *SP* portfolio. Including more than 25% of funds in the *SP* portfolio considerably weakens the funds' average market-timing ability in recessions as well as their unconditional alpha, because skill dissipates with the size of the group. Conversely, including fewer than 25% of funds strengthens both results.

Second, we examine funds that are among the top 25% in terms of their RSI metric in expansions. These funds have higher RAI in recessions and higher unconditional alphas (Tables S.16 and S.17).

Third, we verify our results using alternative definitions of market timing (CT) and stock picking (CS), originally proposed in Daniel, Grinblatt, Titman, and Wermers (1997). The funds that are among the 25% best in terms of their CS metric in expansions have significantly higher CT in recessions. They also have higher unconditional alphas. We also find the same results for fund and manager characteristics as those in Table 7, but using the *SP* portfolio based on the CT and CS variables instead. Fund age, size, expenses, and turnover explain 8% of the selection into the *SP* portfolio. Adding stock concentration, beta deviation, and RAI improves that R^2 to 18%. For brevity, we omit these tables. They are available upon request.

Fourth, we perform the reverse sort. We verify that funds in the top 25% of market-timing ability in recessions, have statistically higher stock-picking ability, *Picking*, in expansions (Table S.18) and higher unconditional alphas (Table S.19). Finally, Table S.20 shows that a one-standard-deviation increase in a skill index, based on *RAI* and *RSI*, instead of *Timing* and *Picking*, increases one-month ahead alphas by 0.3-0.5% per year, a statistically significant effect.

Table S.15: **Robustness: Characteristics of the Picking-Skill Funds**

Expansion is every month the economy is not in recession. We define the stock picking ability of a fund as $Picking_t^j = \sum_{i=1}^{N}(w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m)$. *Switching Portfolio* is an indicator variable equal to one for all funds whose *Picking* in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. *Age* is the fund's age. *TNA* is the fund 's total net assets. *Expenses* is the fund's expense ratio. *Turnover* is the fund's turnover ratio. *Industry Concentration* is the industry concentration of the fund's portfolio. *Stock Concentration* is the stock concentration of the fund's portfolio. *Beta Deviation* is the absolute difference between the fund beta and the average beta in its style category. *Number of Stocks* is the number of stocks in the fund portfolio. *RAI* is the fund manager's reliance on aggregate information (RAI), defined as the R-squared from the regression of fund portfolio returns on contemporaneous changes in industrial production. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)
	Picking Skill	Picking Skill
Log(Age)	-0.073	-0.069
	(0.010)	(0.010)
Log(TNA)	0.010	0.017
	(0.006)	(0.005)
Expenses	21.677	18.715
	(2.151)	(2.095)
Turnover	0.126	0.117
	(0.011)	(0.011)
Industry Concentration		1.063
		(0.170)
Stock Concentration		1.338
		(0.497)
Beta Deviation		0.146
		(0.054)
Number of Stocks		-0.002
		(0.002)
RAI		0.844
		(0.075)
Constant	0.255	0.255
	(0.010)	(0.010)
Observations	180,997	180,997
R-squared	0.14	0.19

Table S.16: **Robustness: Same Managers with High RSI in Expansions Have High RAI in Recessions**

We divide all mutual fund-month observations into Recession and Expansion subsamples. *Recession* is an indicator variable equal to one every month the economy is in recession according to the NBER, and zero otherwise; *Expansion* is one every month the economy is not in recession. The dependent variables are fund managers' reliance on aggregate information (RAI) and fund managers' reliance on stock-specific information (RSI). RAI is defined as the R-squared from the regression of fund portfolio returns on contemporaneous changes in industrial production. RSI is defined as the R-squared from the regression of changes in a mutual fund's stock holdings on contemporaneous changes in equity analysts' stock recommendations. *Skill Picking* 2 is an indicator variable equal to one for all funds whose RSI measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)
	RAI		RSI	
	Expansion	Recession	Expansion	Recession
Skill Picking 2	0.138	0.811	25.899	22.030
	(0.124)	(0.411)	(0.602)	(1.164)
Log(Age)	-0.178	-1.372	0.195	2.020
	(0.091)	(0.257)	(0.374)	(0.734)
Log(TNA)	0.122	0.530	-0.728	-1.459
	(0.038)	(0.110)	(0.185)	(0.374)
Expenses	111.586	140.244	217.636	505.439
	(16.334)	(41.952)	(75.370)	(164.300)
Turnover	0.054	2.304	-0.268	0.068
	(0.085)	(0.246)	(0.312)	(0.663)
Flow	1.189	-13.400	4.402	18.798
	(0.553)	(1.327)	(2.047)	(9.011)
Load	-11.771	4.776	-68.071	-91.943
	(2.366)	(6.244)	(13.560)	(30.653)
Constant	8.318	13.943	31.903	33.704
	(0.083)	(0.225)	(0.263)	(0.623)
Observations	206,204	18,264	111,198	8,463

Table S.17: **Robustness: Unconditional Performance of the Skill Picking Funds using RAI/RSI**

We divide all mutual fund-month observations into Recession and Expansion subsamples. *Expansion* is one every month the economy is not in recession according to the NBER, and zero otherwise. $Skill\,Picking\,2$ is an indicator variable equal to one for all funds whose RSI measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. The dependent variable is the CAPM alpha, three-factor alpha, or four-factor alpha of the mutual fund, obtained from a twelve-month rolling-window regression. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Picking 2	0.144	0.009	0.037
	(0.039)	(0.013)	(0.015)
Log(Age)	-0.024	-0.030	-0.036
	(0.007)	(0.006)	(0.006)
Log(TNA)	0.027	0.013	0.013
	(0.005)	(0.004)	(0.003)
Expenses	-4.480	-6.791	-7.530
	(1.187)	(0.877)	(0.797)
Turnover	-0.005	-0.041	-0.036
	(0.016)	(0.013)	(0.010)
Flow	2.571	1.753	1.600
	(0.173)	(0.102)	(0.101)
Load	-0.412	-0.125	-0.251
	(0.150)	(0.130)	(0.129)
Constant	-0.091	-0.058	-0.057
	(0.013)	(0.015)	(0.017)
Observations	227,183	227,183	227,183

Table S.18: **Robustness: Same Funds with Market-Timing Ability in Recessions Have Stock-Picking Ability in Expansions**

We divide all fund-month observations into Recession and Expansion subsamples. *Recession* is one every month the economy is in recession according to the NBER; *Expansion* is one every month the economy is not in recession. The dependent variables are $Timing_t^j$ and $Picking_t^j$. They are defined as follows: $Timing_t^j = \sum_{i=1}^{N}(w_{it}^j - w_{it}^m)(\beta_i R_{t+1}^m)$ and $Picking_t^j = \sum_{i=1}^{N}(w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m)$. *Skill Picking* 3 is an indicator variable equal to one for all funds whose *Timing* measure in recessions is in the highest 25th percentile of the distribution, and zero otherwise. *Log*(*Age*) is the natural logarithm of fund age. *Log*(*TNA*) is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)
	Market Timing		Stock Picking	
	Expansion	Recession	Expansion	Recession
Skill Picking 3	0.000	0.017	0.056	-0.096
	(0.004)	(0.009)	(0.004)	(0.017)
Log(Age)	0.009	-0.025	-0.001	0.029
	(0.002)	(0.006)	(0.002)	(0.007)
Log(TNA)	-0.001	0.005	0.000	-0.023
	(0.001)	(0.003)	(0.001)	(0.003)
Expenses	0.868	1.374	-1.291	-4.434
	(0.321)	(1.032)	(0.376)	(1.378)
Turnover	0.009	-0.011	0.017	-0.006
	(0.003)	(0.007)	(0.004)	(0.012)
Flow	0.056	-0.876	0.138	-0.043
	(0.024)	(0.112)	(0.037)	(0.093)
Load	0.094	-0.076	0.131	0.615
	(0.049)	(0.151)	(0.055)	(0.195)
Constant	0.016	0.059	-0.021	-0.148
	(0.001)	(0.004)	(0.001)	(0.005)
Observations	204,330	18,354	204,330	18,354

Table S.19: **Robustness: Unconditional Performance of the Reverse-Sort Funds**

We divide all fund-month observations into Recession and Expansion subsamples. *Expansion* equals one every month the economy is not in recession according to the NBER. $Picking_t^j = \sum_{i=1}^{N}(w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m)$. *Skill Picking* 3 is an indicator variable equal to one for all funds whose *Timing* measure in recessions is in the highest 25th percentile of the distribution, and zero otherwise. The dependent variable is the CAPM alpha, three-factor alpha, or four-factor alpha of the mutual fund, obtained from a twelve-month rolling-window regression of excess gross fund returns on a set of various risk factors. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Picking 3	0.076	0.056	0.064
	(0.040)	(0.021)	(0.018)
Log(Age)	-0.039	-0.028	-0.038
	(0.008)	(0.006)	(0.006)
Log(TNA)	0.032	0.013	0.014
	(0.005)	(0.004)	(0.004)
Expenses	4.956	0.627	0.241
	(1.066)	(0.793)	(0.739)
Turnover	-0.009	-0.047	-0.041
	(0.014)	(0.012)	(0.009)
Flow	2.579	1.754	1.602
	(0.173)	(0.102)	(0.101)
Load	-0.744	-0.090	-0.289
	(0.214)	(0.136)	(0.145)
Constant	0.057	0.038	0.049
	(0.017)	(0.015)	(0.018)
Observations	227,183	227,183	227,183

Table S.20: **Robustness: Skill Index using RAI/RSI Predicts Performance**

The dependent variable is the fund's cumulative CAPM, three-factor, or four-factor alpha, calculated from a twelve-month rolling regression of observations in month $t+2$ in the three left columns and in month $t+13$ in the three most right columns. For each fund, we form the following skill index in month t. $Skill\,Index\,2_t^j = w(z_t)RAI_t^j + (1-w(z_t))RSI_t^j, z_t \in \{Expansion, Recession\}$, w(Recession)=0.8 > w(Expansion)= 0.2, where RAI is the fund manager's reliance on aggregate information and RSI is the fund manger's reliance on stock-specific information. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. $Expenses$ is the fund expense ratio. $Flow$ is the percentage growth in a fund's new money. $Turnover$ is the fund turnover ratio. $Load$ is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. $Flow$ and $Turnover$ are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	One Month Ahead			One Year Ahead		
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Index 2	0.021	0.035	0.020	0.005	0.011	0.011
	(0.009)	(0.009)	(0.007)	(0.008)	(0.006)	(0.007)
Log(Age)	-0.037	-0.025	-0.036	-0.024	-0.012	-0.027
	(0.009)	(0.006)	(0.007)	(0.009)	(0.006)	(0.007)
Log(TNA)	0.028	0.011	0.012	-0.015	-0.017	-0.010
	(0.005)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
Expenses	-2.489	-7.040	-7.199	-4.985	-8.916	-8.979
	(1.600)	(0.988)	(0.945)	(1.586)	(0.908)	(0.863)
Turnover	0.000	-0.043	-0.035	0.011	-0.035	-0.029
	(0.017)	(0.014)	(0.010)	(0.018)	(0.014)	(0.010)
Flow	2.483	1.691	1.546	0.329	0.252	0.270
	(0.173)	(0.106)	(0.104)	(0.115)	(0.083)	(0.067)
Load	-0.818	-0.074	-0.280	-0.698	0.251	0.001
	(0.238)	(0.144)	(0.157)	(0.223)	(0.129)	(0.148)
Constant	-0.034	-0.058	-0.044	-0.042	-0.070	-0.055
	(0.024)	(0.019)	(0.022)	(0.025)	(0.019)	(0.022)
Observations	218,104	218,104	218,104	183,845	183,845	183,845

S.3.7 Alternative Explanations

To rule out a sample selection (composition) effects explanation, where the best managers leave our data set in good times, we return to our manager-level results and include manager-level fixed effects as explanatory variables. If a selection/composition effect drives the increase in RAI, dispersion, or performance in recessions, we should not find any results from recession on our dependent variables once we control for fixed effects. Table S.21 shows that all our manager-level results survive the inclusion of manager fixed effects. Likewise, inserting fund fixed effects does not change our fund-level results either. These results are omitted for brevity, but available from the authors upon request.

Table S.21: **Alternatives: Manager Fixed Effects Regressions**

The dependent variables are reliance on aggregate information (RAI), reliance on stock-specific information (RSI), portfolio concentration (*Concentration*), the cross-sectional dispersion in fund CAPM alphas (CAPM Alpha Disp) and betas (CAPM Beta Disp), and the four-factor alpha (4-Factor Alpha), all of which are tracked at the manager level. The definitions of the dependent variables are listed in the captions of Tables 2, 3, and 4. We include manager-level fixed effects as independent variables. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Turnover* is the fund turnover ratio. *Flow* is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of the manager along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	RAI	RSI	Concentration	CAPM Alpha Disp	CAPM Beta Disp	4-Factor Alpha
Recession	0.007	-0.824	0.142	0.324	0.103	0.141
	(0.002)	(0.132)	(0.024)	(0.041)	(0.013)	(0.035)
Log(Age)	-0.004	0.268	0.017	-0.056	-0.014	-0.069
	(0.001)	(0.065)	(0.021)	(0.005)	(0.003)	(0.008)
Log(TNA)	-0.000	-0.139	-0.093	0.032	0.010	0.003
	(0.000)	(0.033)	(0.011)	(0.003)	(0.001)	(0.005)
Expenses	0.060	45.894	13.241	3.311	1.547	-10.590
	(0.146)	(13.867)	(4.504)	(0.789)	(0.279)	(1.138)
Turnover	-0.004	0.210	-0.014	0.059	0.011	-0.035
	(0.002)	(0.090)	(0.020)	(0.005)	(0.001)	(0.009)
Flow	-0.016	1.186	-0.374	0.329	-0.010	1.483
	(0.009)	(0.554)	(0.101)	(0.045)	(0.014)	(0.089)
Load	-0.069	-5.832	-0.231	-0.787	-0.095	0.097
	(0.023)	(2.307)	(0.809)	(0.131)	(0.042)	(0.172)
Constant	-0.002	2.977	1.447	0.608	0.230	-0.043
	(0.001)	(0.068)	(0.008)	(0.015)	(0.006)	(0.021)
Observations	332,676	249,942	332,776	332,776	332,776	332,776

The data show that outside labor market options of investment fund managers deteriorate in recessions. Not only do their assets under management and therefore their wages shrink, they are also more likely to get fired or demoted. Table S.22 shows less turnover in the labor market for investment managers in recessions (Columns 1 and 2), and a smaller incidence of promotion to a larger mutual fund in a different fund family (Columns 3 and 4), a higher incidence of demotion to a smaller mutual fund in a different fund family (Columns 5 and 6), and a lower incidence of departure to a hedge fund (Columns 7 and 8). Finally, Table S.23 shows that an average manager's age, experience, and education are not higher in recessions.

Table S.22: **Alternatives: Managerial Turnover**

The dependent variables are: The manager's departure (*Departure*) for any reason, the manager's promotion to a larger mutual fund in a different fund family (*Promotion*), the manager's departure to a smaller mutual fund in a different fund family (*Demotion*), and the manager's departure to a hedge fund (*Hedge Fund*). *Departure* includes those managers in the variables *Promotion*, *Demotion*, *Hedge Fund*, as well as the managers that disappear from the investment management industry. *Recession* is an indicator variable that equals one for every month the economy is in the recession according to the NBER, and zero otherwise. *Log*(*Age*) is the natural logarithm of a fund's age. *Log*(*TNA*) is the natural logarithm of a fund's total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. *Log*(*Manage*) is the natural logarithm of the fund manager's age in years. *Log*(*Experience*) is the natural logarithm of a manager experience in years. *Ivy* is an indicator variable equal to one if the manager graduated from an Ivy League academic institution, and zero otherwise. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Departure		Promotion		Demotion		Hedge Fund	
Recession	-1.118	-1.470	-0.247	-0.263	0.333	0.326	-0.037	-0.041
	(0.616)	(0.602)	(0.095)	(0.099)	(0.130)	(0.131)	(0.013)	(0.014)
Log(Age)		-1.079		0.097		0.021		0.001
		(0.125)		(0.043)		(0.044)		(0.009)
Log(TNA)		0.600		-0.065		0.018		-0.001
		(0.094)		(0.021)		(0.022)		(0.005)
Expenses		239.641		-9.060		12.793		0.771
		(41.513)		(7.985)		(8.238)		(1.730)
Turnover		2.823		0.061		0.052		0.021
		(0.197)		(0.037)		(0.035)		(0.009)
Flow		-5.175		0.170		-0.605		-0.073
		(1.871)		(0.537)		(0.403)		(0.180)
Load		56.096		4.587		-1.945		0.322
		(5.244)		(1.601)		(1.203)		(0.308)
Log(Manage)		-5.210		-0.497		-0.699		-0.139
		(1.854)		(0.403)		(0.289)		(0.086)
Log(Experience)		4.157		-0.477		0.473		0.020
		(0.787)		(0.236)		(0.165)		(0.034)
Ivy		1.103		0.137		0.085		0.036
		(0.433)		(0.071)		(0.072)		(0.020)
Constant	7.598	7.624	0.548	0.549	0.419	0.420	0.037	0.038
	(0.332)	(0.320)	(0.056)	(0.058)	(0.056)	(0.056)	(0.013)	(0.013)
Observations	86,823	86,823	86,823	86,823	86,823	86,823	86,823	86,823

Table S.23: **Alternatives: Managers' Age, Experience, and Education**

The dependent variables are: The natural logarithm of the fund manager's age in years ($Log(Manage)$); the natural logarithm of a manager experience in years ($Log(Experience)$); an indicator variable (Ivy) that is equal to one if the manager graduated from an Ivy League University, and zero otherwise. *Recession* is an indicator variable equal to one for every month the economy is in the recession according to the NBER, and zero otherwise. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at both fund and time dimensions.

	(1)	(2)	(3)
	Log(Manage)	Log(Experience)	Ivy
Recession	-0.011	-0.023	0.003
	(0.009)	(0.017)	(0.004)
Constant	3.974	3.287	0.253
	(0.003)	(0.006)	(0.001)
Observations	91,879	91,879	91,879