Spectral Portfolio Theory

Shomesh Chaudhuri and Andrew W. Lo

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An Introduction to Nonparametric Concepts in Economics and Finance

Andrew W. Lo, MIT Sloan School
Q Group Seminar
Fall 1994

Seminar Outline

1. Motivation
2. What Are Nonparametrics?
3. Kernel Regression
4. Neural Networks
5. What Can’t Nonparametrics Do?
6. Practical Considerations
7. Current Research
8. Summary and Conclusions

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The Role of Frequency in Investing
Consider simple mean-reversion strategy of Lehman (1990) and Lo and MacKinlay (1990):

\[
\omega_{it}(q) = -\frac{1}{n} \left( R_{it-q} - \overline{R}_{t-q} \right), \quad \overline{R}_{t-q} \equiv \frac{1}{n} \sum_{i=1}^{n} R_{it-q}
\]

- How correlated are returns from \( q=1 \) and \( q=2 \)?
- 19620716–20151231, correlation is:
- Cheap source of diversification??
125-Day Rolling-Window Correlation of \( q=1 \) and \( q=2 \) Mean Reversion Strategies

20060103 to 20081231
Spectral Analysis

\[ X_t = \alpha \cos(\omega t + \theta) \quad X_t = \cos(\omega t + \theta) \]
Spectral Analysis

\[
\cos(\omega t + \theta) = \cos \theta \cos(\omega t) - \sin \theta \sin(\omega t)
= \alpha \cos(\omega t) + \beta \sin(\omega t)
\]

- Théorie Analytique de la Chaleur

\[
F(x) = \sum_{k=0}^{\infty} \left( \alpha_k \cos(kx) + \beta_k \sin(kx) \right)
\]

- Fourier analysis
- Derived the heat equation
- Discovered the “greenhouse effect”
Spectral Analysis

\[
\frac{4 \sin \theta}{\pi} + \frac{4 \sin 3\theta}{3\pi} + \frac{4 \sin 5\theta}{5\pi} + \frac{4 \sin 7\theta}{7\pi}
\]
Spectral Analysis

- To apply it to time series (stochastic processes):

\[ X_t = \mu + \sum_{k=1}^{m} \left( \alpha_k \cos(\omega_k t) + \beta_k \sin(\omega_k t) \right) \]

\[ 0 = E[\alpha_k] = E[\beta_k] = E[\alpha_j \alpha_k] = E[\beta_j \beta_k], \quad j \neq k \]

\[ X_t = \mu + \int_{0}^{\pi} \alpha(\omega) \cos(\omega t) d\omega + \int_{0}^{\pi} \beta(\omega) \sin(\omega t) d\omega \]

- Statistics of \( X_t \) become functions of frequency \( \omega \)
For example:

\[ X_t = \mu + \alpha \cos(\omega t) + \beta \sin(\omega t) \]

\[ \alpha, \beta \overset{\text{IID}}{\sim} \mathcal{N}(0, \sigma^2) \]

- \[ E[X_t] = \mu \]
- \[ \text{Var}[X_t] = \sigma^2 \]
- \[ \text{Cov}[X_t, X_{t-k}] = \sigma^2 \cos(\omega k) \]
- \[ \text{Corr}[X_t, X_{t-k}] = \cos(\omega k) \]

Autocorrelation

\[ 1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 \]
Main tool of spectral analysis: spectral density

\[ X_t = \mu + \int_0^\pi \alpha(\omega) \cos(\omega t) \, d\omega + \int_0^\pi \beta(\omega) \sin(\omega t) \, d\omega \]

\[ \Rightarrow \text{Cov}[X_t, X_{t-k}] = \int_0^\pi \cos(\omega k) f(\omega) \, d\omega \]

\[ \Rightarrow \text{Var}[X_t] = \int_0^\pi f(\omega) \, d\omega \]

How to estimate \( f(\omega) \)?

\[ f(\omega) = \frac{\sigma^2}{\pi} + \frac{2}{\pi} \sum_{k=1}^{\infty} \text{Cov}[X_t, X_{t-k}] \cos(\omega k) \]
Examples

\[ X_t = \alpha \cos(\omega t + \theta) + \epsilon_t \]

\[ E[\epsilon_t] = E[\epsilon_t \epsilon_{t-k}] = 0 \text{ for } k \neq 0 \]
Multivariate relationships:

\[ X_t = \mu_x + \int_0^\pi \alpha_x(\omega) \cos(\omega t) d\omega + \int_0^\pi \beta_x(\omega) \sin(\omega t) d\omega \]

\[ Y_t = \mu_y + \int_0^\pi \alpha_y(\omega) \cos(\omega t) d\omega + \int_0^\pi \beta_y(\omega) \sin(\omega t) d\omega \]

\[ \Rightarrow \text{Cov}[X_t, Y_{t-k}] = \int_{-\pi}^\pi \exp(-i\omega k) f_{xy}(\omega) d\omega \quad \text{(Cross-Spectrum)} \]

Frequency-dependent versions of:

– covariances, correlations, regression coefficients (Engle, 1974), alpha, beta, VaR, etc.
Financial Applications

- Spectral mean-variance analysis, CAPM, ICAPM, CCAPM, APT, etc.
  
  \[ \mu(\omega), \Sigma(\omega) \]

- Spectral performance attribution
  
  \[ \alpha(\omega), \beta(\omega), \{\rho_{ij}(\omega)\} \]

- Spectral risk management, VaR, stress tests, systemic risk measurement, etc.

What About Buffett vs. Simons?
Expected return decomposition (Lo, 2008):

\[ R_{pt} = \sum_{i=n}^{n} \gamma_{it} R_{it} \quad \gamma_{it} \equiv \gamma_{it}(X_{t-1}, X_{t-2}, \ldots) \]

\[ E[R_{pt}] = \sum_{i=1}^{n} E[\gamma_{it} R_{it}] = \sum_{i=1}^{n} \left( \text{Cov}[\gamma_{it}, R_{it}] + E[\gamma_{it}]E[R_{it}] \right) \]

Profitable strategies are “in phase”

Covariances vary by frequency across investors
- Consider mean-reversion strategy with white noise returns:
  - In phase at high frequencies
  - Out of phase at low frequencies
Mean-reversion strategy with mean reversion (negatively autocorrelated returns):

More power at high frequencies (in phase)
Mean-reversion strategy with momentum (positively autocorrelated returns):

- More power at low frequencies (out of phase)
Spectral Active Ratios

- Application to S&P 500 stocks from 1964 to 2015

\[ \sum_{i=1}^{n} \text{Cov}[\gamma_{it}, R_{it}](\omega) \]

\[ E[R_{pt}] \]
Full-Spectrum Investment Management

Hedge Fund

Active | Active | High | High | High | High | High | High | High | High

Index Fund

Passive | Passive | Low | Low | Low | Low | Low | Low | Low | Low

Untapped Investment Opportunities

alpha, risk control, liquidity, turnover, credit, currency, Sharpe, max DD, skew
Can Financial Engineering Cure Cancer?

Andrew W. Lo, MIT

(based on joint work with Jayna Cummings, David Fagnan, John Frishkopf, Jose-Maria Fernandez, Carole Ho, Austin Gromatzky, Ken Kosik, John McKew, Vahid Montazerhodjat, Roger Stein, Richard Thakor, David Weinstock, Nora Yang)
Consider The Following Investment Opportunity:

- $200MM investment, 10-year horizon
- Probability of positive payoff is 5% (failure rate = 95%)
- If successful, annual profits of $2B for 10-year patent

**Risk and Reward**

\[ E[R] = 11.9\% \]
\[ SD[R] = 423.5\% \]
What If We Invest In 150 Programs Simultaneously?:

- Requires $30B of capital
- Assume programs are IID (can be relaxed)
- Diversification changes the economics of the business:

\[ \mathbb{E}[R] = 11.9\% \]

\[ \text{SD}[R] = \frac{423.5\%}{\sqrt{150}} = 34.6\% \]

- But can we raise $30B??
- It depends on the portfolio’s risk/reward profile (correlations are key)
What If We Invest In 150 Programs Simultaneously?:

- With reduced risk, debt-financing is feasible!

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
<th>Minimum Year-10 NPV</th>
<th>Maximum Year-0 Proceeds at 2.33% (BofAML AA 10-Yr as of 4/6/16)</th>
<th>Maximum Year-0 Proceeds at 2.76% (BofAML A 10-Yr as of 4/6/16)</th>
<th>Maximum Year-0 Proceeds at 4.84% (BofAML Baa 10-Yr as of 4/6/16)</th>
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<tbody>
<tr>
<td>At least 1 hit:</td>
<td>99.95%</td>
<td>$12,289</td>
<td>$9,761</td>
<td>$9,360</td>
<td>$6,084</td>
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<td>At least 2 hits:</td>
<td>99.59%</td>
<td>$24,578</td>
<td>$19,522</td>
<td>$18,720</td>
<td>$12,169</td>
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<tr>
<td>At least 3 hits:</td>
<td>98.18%</td>
<td>$36,867</td>
<td>$29,283</td>
<td>$28,080</td>
<td>$18,253</td>
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<td>At least 4 hits:</td>
<td>94.52%</td>
<td>$49,157</td>
<td>$39,044</td>
<td>$37,440</td>
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<td>At least 5 hits:</td>
<td>87.44%</td>
<td>$61,446</td>
<td>$48,805</td>
<td>$46,800</td>
<td>$30,422</td>
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Can We Afford These Therapies?
There's a difference between price-gouging and genuine breakthrough therapies.

Example: hepatitis C.

12-week treatment cures it!
- Cost of liver transplant: $577,000 in 2011
- Value of statistical life: $9.1 million × 2/3?

But 3 million U.S. patients have hepatitis C!
Can We Afford These Therapies?

Sovaldi Is A Bargain, But The Cost Impact Is Huge!

Suppose we “mortgaged” Sovaldi?

\[
\frac{84,000}{r/12} \left[ 1 - \frac{1}{(1 + r/12)^{12n}} \right] \Rightarrow P = \frac{84,000r/12}{1 - \frac{1}{(1+r/12)^{12n}}}
\]

### Monthly Payment

<table>
<thead>
<tr>
<th>Years</th>
<th>1%</th>
<th>3%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
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<td>1</td>
<td>$7,038</td>
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<tr>
<td>5</td>
<td>$1,436</td>
<td>$1,509</td>
<td>$1,585</td>
<td>$1,785</td>
<td>$1,998</td>
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<tr>
<td>10</td>
<td>$736</td>
<td>$811</td>
<td>$891</td>
<td>$1,110</td>
<td>$1,355</td>
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<tr>
<td>15</td>
<td>$503</td>
<td>$580</td>
<td>$664</td>
<td>$903</td>
<td>$1,176</td>
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<tr>
<td>30</td>
<td>$270</td>
<td>$354</td>
<td>$451</td>
<td>$737</td>
<td>$1,062</td>
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</tbody>
</table>
Can We Afford These Therapies?

Buying cures versus renting health: Financing health care with consumer debt

Vahid Montazerhodjat, David M. Weinstock, Andrew W. Lo

A crisis is building over the prices of new transformative therapies for cancer, HIV, and other serious diseases. The clinical imperative is to offer these therapies early and as widely as possible. We propose a practical way to increase drug affordability—leverage consumer health care loans (HCLs)—the equivalent of mortgages for large health care expenditures. HCLs allow patients in both multipayer and single-payer markets to access a broad range of transformative therapies, including expensive short-duration treatments that are curative and associated with high payment to clinical benefit and should help lower per-patient cost while also accelerating the development of transformative therapies rather than those that offer slow or small advances. Moreover, we propose the use of securitization—a well-known financial engineering method—to finance a large diversified pool of HCLs through both domestic and foreign capital markets. Numerical simulations suggest that securitization is viable for a wide range of mortality rates, survival environments, and cost parameters, allowing a much broader patient population to access transformative therapies while also aligning the interests of patients, payers, and the biopharmaceutical industry.

Senior tranche: 2.1%
Junior tranche: 2.5%
Equity tranche: 12.5%
Conclusion

We Should All Be Harvey Lodish!

Finance Doesn’t Have To Be A Zero-Sum Game
Thank You!
Happy Birthday
Q Group
References for Healthcare Finance

References for Healthcare Finance