Fund Tradeoffs

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Abstract

We derive equilibrium relations among active mutual funds' key characteristics: fund size, expense ratio, turnover, and portfolio liquidity. As our model predicts, funds with smaller size, higher expense ratios, and lower turnover hold less liquid portfolios. A portfolio's liquidity, a concept introduced here, depends not only on the liquidity of the portfolio's holdings but also on the portfolio's diversification. We derive simple, theoretically motivated measures of portfolio liquidity and diversification. Both measures have trended up over time. We also find larger funds are cheaper, funds trading less are larger and cheaper, and excessively large funds underperform, as our model predicts.

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1. Introduction

Active mutual funds manage tens of trillions of dollars in an effort to beat their benchmarks. These funds differ in numerous respects, including their size, expense ratio, turnover, diversification, and the liquidity of portfolio holdings. Are there any tradeoffs among these characteristics? For example, are funds with higher expense ratios more diversified, or less? Do larger funds trade more heavily, or less? Are the holdings of better-diversified funds more liquid, or less? We attempt to answer such questions, both theoretically and empirically.

Our focus on the tradeoffs among active funds' characteristics can be motivated by the arguments of Berk and Green (2004). If each fund's expected benchmark-adjusted return is zero in equilibrium then a fund's performance—the object of much empirical scrutiny—is rather uninformative. Our study therefore turns to fund characteristics as a richer source of insights into the economics of mutual funds.

We derive equilibrium relations among four key fund characteristics: fund size, expense ratio, turnover, and portfolio liquidity. This last characteristic is novel. While the literature presents a variety of liquidity measures for individual securities, it offers little guidance for assessing liquidity at the portfolio level. We introduce the concept of portfolio liquidity and show that funds trade off this characteristic against others in important ways.

A portfolio is more liquid if it has lower trading costs. More precisely, if two equally sized funds trade the same fraction of their portfolios, the fund with lower trading costs has greater portfolio liquidity. When assessing portfolio liquidity, it seems natural to begin with the average liquidity of the portfolio's constituents. For example, portfolios of small-cap stocks tend to be less liquid than portfolios of large-cap stocks. While this assessment is a useful starting point, it is incomplete. We show that a portfolio's liquidity depends not only on the liquidity of the stocks held in the portfolio, but also on the degree to which the portfolio is diversified:

$$\text{Portfolio Liquidity} = \text{Stock Liquidity} \times \text{Diversification}\ . \tag{1}$$

The more diversified a portfolio, the less costly is trading a given fraction of it. For example, a fund trading just 1 stock will incur higher costs than a fund spreading the same dollar amount of trading over 100 stocks, even if all of the stocks are equally liquid. Throughout, we focus on equity portfolios, but our ideas are more general.

Starting from a simple trading cost function, we derive a measure of portfolio liquidity that is easy to calculate from the portfolio's composition. Following equation (1), our measure has two components. The first, stock liquidity, reflects the average market capitalization of the portfolio's holdings. The second component, diversification, has its own intuitive decomposition:

$$\text{Diversification} = \text{Coverage} \times \text{Balance}\ . \tag{2}$$

Coverage reflects the number of stocks in the portfolio. Portfolios holding more stocks have greater coverage. Balance reflects how the portfolio weights the stocks it holds. Portfolios with weights closer to market-cap weights have greater balance.

Diversification's role in portfolio liquidity is important empirically. We compute our measures of portfolio liquidity and diversification for the portfolios of 2,789 active U.S. equity mutual funds from 1979 through 2014. We find that fund portfolios have become more liquid over time. Average portfolio liquidity almost doubled over the sample period, driven by diversification. Diversification quadrupled, as both of its components in equation (2) rose steadily. Coverage rose because the number of stocks held by the average fund grew from 54 to 126. Balance rose because funds' portfolio weights increasingly resembled market-cap weights.[1]

Diversification's role in portfolio liquidity goes beyond its strong time trend. We show that diversification is an important cross-sectional determinant of mutual fund portfolio liquidity. Moreover, diversification explains why the typical active fund's portfolio is far less liquid than passive benchmark portfolios. The typical fund actually tilts toward stocks of above-average size, but that positive effect on portfolio liquidity is more than offset by the relatively low diversification inherent to active management. The 126 stocks held by the average active fund in 2014 cover only a small fraction of available stocks.

We develop an equilibrium model relating portfolio liquidity to three other fund characteristics: fund size, expense ratio, and turnover. In the model, funds face decreasing returns to scale. When choosing their portfolio liquidity and turnover, funds recognize that lower liquidity and higher turnover raise expected gross profits but also raise transaction costs. Investors allocate money to funds up to the point at which each fund's net alpha is driven to 0. This equilibrium determination of fund size follows Berk and Green (2004).

The model implies that funds whose portfolios are less liquid should have smaller size, higher expense ratios, and lower turnover. This equilibrium relation provides a novel theoretical link between the four key mutual fund characteristics. Intuitively, if a fund trades a lot or holds an illiquid portfolio, diseconomies of scale force the fund to be small. The role of the expense ratio involves the fund's skill. A more skilled fund can afford to charge a higher fee and to trade a less liquid portfolio.

[1] The increased resemblance of active funds' portfolios to the market benchmark is also apparent from measures such as active share and tracking error (e.g., Cremers and Petajisto, 2009, and Stambaugh, 2014).

The equilibrium relation among fund characteristics delivers a regression of portfolio liquidity on fund size, expense ratio, and turnover. We estimate this cross-sectional regression in our panel dataset and find strong support for the model. All three slopes have their predicted signs and are highly significant, both economically and statistically, with t-statistics ranging from 4.9 to 13.8. Funds that are smaller, more expensive, and trading less indeed tend to hold less liquid portfolios, as the model predicts.

The model also makes strong predictions about diversification. In equilibrium, funds with more diversified portfolios should be larger and cheaper, they should trade more, and their stock holdings should be less liquid. We find strong empirical support for all four predictions. The negative relation between diversification and stock liquidity implies that these components of portfolio liquidity are substitutes: funds holding less liquid stocks make up for it by diversifying more, and vice versa. The components of diversification, coverage and balance, are also substitutes: portfolios with lower coverage tend to be better balanced, and vice versa. Both substitution effects are predicted by our model.

The model also makes predictions for correlations among fund characteristics. First, larger funds should be cheaper. In the data, the correlation between fund size and expense ratio is indeed strongly negative, both in the cross section (-32%) and in the time series (-25%). In our model, a fund's skill uniquely determines its fee revenue, so fund size and expense ratio trade off negatively as long as skill does not vary too much. The model also predicts that fund turnover should be negatively related to fund size and positively related to expense ratio. These relations hold strongly in the data as well: funds that trade less are larger and cheaper, both across funds and over time.

Finally, we extend our model by allowing fund size to deviate from its equilibrium value. Guided by our model, we estimate excess fund size as the residual from the regression of portfolio liquidity on fund size, expense ratio, and turnover. We find that excess fund size gets corrected over time, yet it is highly persistent.

The model also predicts that excess fund size should be negatively related to future fund performance, due to diseconomies of scale. We find empirical support for this prediction in two ways. First, in panel regressions of benchmark-adjusted fund returns on lagged excess fund size, we find negative and significant slopes at all return horizons up to four years. Second, in portfolio sorts on lagged excess fund size, average benchmark-adjusted returns decrease monotonically across the portfolios. Moreover, the negative relation between excess fund size and future performance is stronger for more expensive funds, as the model predicts. Among high-expense-ratio funds, high-excess-size funds underperform low-excess-size funds by 1.3% per year ($t = -2.47$) on a benchmark-adjusted basis. This out-of-sample analysis provides additional support for the model.

Our study relates to the literature on decreasing returns to scale in active management. This literature explores the hypothesis that as a fund's size increases, its ability to outperform its benchmark declines (Berk and Green, 2004).[2] This hypothesis is motivated by liquidity constraints: Being larger erodes performance because a larger fund trades larger dollar amounts, and trading larger dollar amounts incurs higher proportional trading costs. The hypothesis has received a fair amount of empirical support. Fund size negatively predicts fund performance, especially among funds holding small-cap stocks (Chen et al., 2004) and less liquid stocks (Yan, 2008), suggesting that the adverse effects of scale are related to liquidity.[3] We establish the same link from a different angle. We find that larger funds tend to have lower turnover and higher portfolio liquidity. This evidence is in line with our model, in which diseconomies of scale lead larger funds to trade less and hold more-liquid portfolios, either by holding more-liquid stocks or by diversifying more. Our results represent strong evidence of decreasing returns to scale. It is not clear what other mechanism could explain why larger funds trade less and hold more-liquid portfolios.

Two other studies provide related evidence on returns to scale. Pollet and Wilson (2008) find that mutual funds respond to asset growth mostly by scaling up existing holdings rather than by increasing the number of stocks held. But the authors also find that larger funds and small-cap funds are less reluctant to diversify in response to growth, exactly as our theory predicts. In their comprehensive analysis of mutual fund trading costs, Busse et al. (2017) report that larger funds trade less and hold more-liquid stocks. This evidence, which overlaps with our findings, also supports our model. In the language of equation (1), Busse et al. show that larger funds have higher stock liquidity; we show they also have higher diversification. The evidence of Busse et al. is based on a sample much smaller than ours (583 funds in 1999 through 2011), dictated by their focus on trading costs. Neither Busse et al. nor Pollet and Wilson do any theoretical analysis.

Our study is also related to the literature on portfolio diversification. We propose a new measure of diversification that has strong theoretical motivation. Our measure exhibits features of two common ad-hoc measures: the number of stocks held and the Herfindahl index of portfolio weights. By using our measure, we show that mutual funds have become substantially more diversified over time. Nevertheless, their diversification remains relatively

[2] This is the hypothesis of fund-level decreasing returns to scale. A complementary hypothesis of industry-level decreasing returns to scale is that as the size of the active mutual fund industry increases, the ability of any given fund to outperform declines (see Pástor and Stambaugh, 2012, and Pástor, Stambaugh, and Taylor, 2015). In this paper, we focus on the fund-level hypothesis.

[3] For additional evidence on returns to scale in mutual funds, see Bris et al. (2007), Pollet and Wilson (2008), Reuter and Zitzewitz (2015), Pástor, Stambaugh, and Taylor (2015), Harvey and Liu (2017), etc.

low.[4] We also derive theoretical predictions for the determinants of diversification. Funds with more-diversified portfolios should be larger and cheaper, they should trade more, and their holdings should be less liquid, on average. We find strong empirical support for all of these predictions.

The rest of the paper is organized as follows. Section 2 introduces our measures of portfolio liquidity and diversification. Section 3 examines the tradeoffs among fund characteristics. Section 4 analyzes the predictability of fund performance by excess fund size. Section 5 concludes. Formal proofs of all assertions are in Appendix A. Additional empirical results are in the Internet Appendix, which is available on the authors' websites.

2. Portfolio Liquidity and Diversification

We begin this section by deriving our measure of portfolio liquidity. We then examine the key properties of this measure, including its decomposition into stock liquidity and diversification. We go on to discuss our measure of diversification. Finally, we examine the time series and cross section of portfolio liquidity and its components in our mutual fund sample.

2.1. Introducing Portfolio Liquidity

The definition of portfolio liquidity is based on trading costs: If two equally sized funds trade the same fraction of their portfolios, the fund incurring lower costs has greater portfolio liquidity. We show that this fundamental concept is captured by the following measure:

$$L = \left(\sum_{i=1}^{N} \frac{w_i^2}{m_i} \right)^{-1} , \tag{3}$$

where N is the number of stocks in the portfolio, w_i is the portfolio's weight on stock i, and m_i denotes the weight on stock i in a market-cap-weighted benchmark portfolio containing N_M stocks. The latter portfolio can be the overall market, the most familiar benchmark, or it can be the portfolio of all stocks in the sector in which the fund trades, such as large-cap growth. We apply both choices in our empirical analysis.

To derive this measure, we begin with the fund's total dollar trading costs, given by

$$C = \sum_{i=1}^{N} D_i \, C_i \ , \tag{4}$$

[4] Low diversification by institutional investors is also reported by Kacperczyk, Sialm, and Zheng (2005), Pollet and Wilson (2008), and others. Household portfolios also exhibit low diversification, as shown by Blume and Friend (1975), Polkovnichenko (2005), Goetzmann and Kumar (2008), and others.

where D_i is the dollar amount traded of stock i and C_i is the cost per dollar traded of the same stock. We assume that the cost per dollar traded is larger when trading a larger fraction of the stock's market capitalization:

$$C_i = \tilde{c}\,\frac{D_i}{M_i}\ , \tag{5}$$

where M_i is the market capitalization of stock i and $\tilde{c} > 0$. Equation (5) reflects the basic idea that larger trades have higher proportional trading costs, such as price impact. Empirical support for this idea is extensive (e.g., Keim and Madhavan, 1997). The linearity of equation (5) implies that trading, say, 1% of a stock's market capitalization costs twice as much per dollar traded compared to trading 0.5% of the stock's capitalization.[5] The total dollar amount traded by the fund is the product of the fund's size (i.e., assets under management), denoted by A, and the fund's turnover, T. We assume that the fund expects to turn over its portfolio proportionately, trading larger dollar amounts of stocks that occupy bigger shares of the portfolio. Specifically, the amount traded in stock i obeys

$$D_i = ATw_i(1+\varepsilon_i)\ , \tag{6}$$

where ε_i has a mean of 0 and variance of σ_ε^2. That is, the expected dollar amount traded in stock i is $\mathrm{E}(D_i) = ATw_i$, but the actual amount traded can differ from this expectation. This assumption, while nontrivial, seems plausible, and it allows us to derive the simple measure of portfolio liquidity in equation (3).[6] Denoting the total market capitalization of all stocks in the benchmark portfolio by M, we have $m_i = M_i/M$. Combining equations (4) through (6), we can write the fund's expected trading cost as

$$\mathrm{E}(C) = (c/M)\ (AT)^2\ \underbrace{\left(\sum_{i=1}^{N}\frac{w_i^2}{m_i}\right)}_{L^{-1}}\ , \tag{7}$$

where $c = (1+\sigma_\varepsilon^2)\tilde{c}$ and c/M is a constant with respect to the fund's choices. Equation (7) links portfolio liquidity from equation (3) to the fund's trading cost function: less liquid portfolios have a higher expected cost of trading a given dollar amount, AT. Trading that dollar amount is also costlier the smaller is the aggregate stock value, M. In addition, it is useful to rewrite equation (7) as

$$\mathrm{E}(C) = (c/M)\ V^2\ , \tag{8}$$

[5] A linear function for the proportional trading cost in a given stock is entertained, for example, by Kyle and Obizhaeva (2016). That study examines portfolio transition trades and concludes that a linear function fits the data only slightly less well than a nonlinear square-root specification. We generalize the simplifying assumption of linearity in Appendix B.

[6] The assumption has some empirical support in the evidence of Pollet and Wilson (2008) who find that mutual funds tend to respond to asset growth by scaling up their existing investments. In Appendix B, we modify equation (6) by allowing D_i to depend also on stock-level turnover.

where V is the fund's liquidity-adjusted dollar volume of trading:

$$V = ATL^{-\frac{1}{2}} . \tag{9}$$

Equation (8) shows that the expected dollar trading cost is increasing and convex in V. As we explain later in Section 3.1, this convexity results in the fund facing decreasing returns to scale, with V being the implied measure of fund scale.

2.2. Properties of Portfolio Liquidity

Our measure of portfolio liquidity (L) from equation (3) exhibits several desirable properties. First, the measure is derived theoretically under plausible assumptions about the trading cost function. Second, the measure always takes values between 0 and 1.

The least liquid portfolio is fully invested in a single stock: the one with the smallest market capitalization among stocks in the benchmark. The liquidity of this portfolio is equal to the benchmark's market-cap weight on that smallest stock, so L can be nearly 0.

A portfolio can be no more liquid than its benchmark, for which $L = 1$. This statement is proven in Appendix A, but its simple intuition follows from the trading-cost assumption in equation (5). When trading a given dollar amount of the benchmark portfolio, which has market-cap weights, the proportional cost of trading each stock is equal across stocks. With this cost denoted by κ, the proportional cost of the overall trade is also κ. If the benchmark portfolio is perturbed by buying one stock and selling another, then more weight is put on a stock whose proportional cost is now greater than κ, and less weight is put on a stock whose proportional cost is now smaller than κ. Therefore, the proportional cost of trading the same dollar amount of this alternative portfolio exceeds κ.

Portfolio liquidity from equation (3) can be decomposed as

$$L = \underbrace{\frac{1}{N}\sum_{i=1}^{N}\mathcal{L}_i}_{\text{Stock Liquidity}} \times \underbrace{\left(\frac{N}{N_M}\right)\left[1 + \text{Var}^*\left(\frac{w_i}{m_i^*}\right)\right]^{-1}}_{\text{Diversification}}, \tag{10}$$

We discuss the second component, "diversification," in the following subsection. The first component, "stock liquidity," is the equal-weighted average of $\mathcal{L}_i = M_i/\overline{M}$, with $\overline{M}$ denoting the average market capitalization of stocks in the benchmark: $\overline{M} = \frac{1}{N_M}\sum_{j=1}^{N_M} M_j$. Variable $\mathcal{L}_i$ captures the liquidity of stock i relative to all stocks in the benchmark. Stock liquidity is larger (smaller) than 1 if the portfolio's holdings have a larger (smaller) average market capitalization than the average stock in the benchmark.

Implicit in our measure L is the use of a stock's market capitalization to measure liquidity at the stock level. This result follows from our assumption (5), which implies that trading \$1 of stock i incurs a cost proportional to $1/m_i$. This is intuitive—trading a fixed dollar amount of a small-cap stock (whose m_i is small) incurs a larger price impact than trading the same amount of a large-cap stock (whose m_i is large). Moreover, market capitalization is closely related to other measures of stock liquidity in the data. For example, we calculate the correlations between the log of market capitalization and the logs of two popular measures, the Amihud (2002) measure of illiquidity and dollar volume, across all common stocks. The two correlations average -0.91 and 0.85, respectively, across all months in our sample period (1979 to 2014, as described later). Also, it makes little difference whether market capitalization is float-adjusted or not: the correlation between the logs of float-adjusted and unadjusted market capitalization is 0.98.[7] We use unadjusted market capitalization in our empirical analysis to maximize data coverage.

2.3. Portfolio Diversification

A portfolio's liquidity depends not only on the liquidity of the portfolio's constituents but also on the extent to which the portfolio is diversified. Better-diversified portfolios are more liquid because they incur lower trading costs compared to more concentrated portfolios with the same turnover. Diversification is an essential part of portfolio liquidity (equation (10)), which is a key determinant of trading costs (equation (7)).

Diversification is a foundational concept in finance, yet there is no accepted standard for measuring it. In an important early contribution, Blume and Friend (1975) use two measures. The first one is the number of stocks in the portfolio. This measure is also used by Goetzmann and Kumar (2008), Ivkovich, Sialm, and Weisbenner (2008), Pollet and Wilson (2008), and others. The idea is that portfolios holding more stocks are better diversified. While this idea is sound, the measure is far from perfect. Consider two portfolios holding the same set of 500 stocks. The first portfolio weights the stocks in proportion to their market capitalization. The second portfolio is 99.9% invested in a single stock while the remaining 0.1% is spread across the remaining 499 stocks. Even though both portfolios hold the same number of stocks, the first portfolio is clearly better diversified.

The second measure of diversification used by Blume and Friend is the sum of squared deviations of portfolio weights from market weights, essentially a market-adjusted Herfindahl index. The Herfindahl index measures portfolio concentration, the inverse of diversification.

[7]We compute this correlation using data on the Russell 3000 stocks from 2011 to 2014. Data on stocks' shares outstanding are from CRSP. Data on float-adjusted shares outstanding are from Russell.

Studies that use various versions of this measure include Kacperczyk, Sialm, and Zheng (2005), Goetzmann and Kumar (2008), and Cremers and Petajisto (2009), among others.

Our measure of portfolio diversification, which we derive formally from the trading cost function, blends the ideas from both of the above measures. As one can see from equation (10), our measure can be further decomposed as

$$\text{Diversification} = \underbrace{\left(\frac{N}{N_M}\right)}_{\text{Coverage}} \times \underbrace{\left[1 + \text{Var}^*\left(\frac{w_i}{m_i^*}\right)\right]^{-1}}_{\text{Balance}}. \tag{11}$$

The first component of diversification, "coverage," is the number of stocks in the portfolio (N) divided by the total number of stocks in the benchmark (N_M) . Dividing by the latter number makes sense. If all firms in the benchmark were to merge into one big conglomerate, a portfolio holding only the conglomerate's stock would be perfectly diversified despite holding only a single stock. Given N_M, portfolios holding more stocks have larger coverage. The value of coverage is always between 0 and 1, with the maximum value reached if the portfolio holds every stock in the benchmark.

The second component, "balance," measures how diversified the portfolio is across its holdings, regardless of their number. A portfolio is highly balanced if its weights are close to market-cap weights. The degree to which a portfolio's weights are close to market-cap weights is captured by the term $\text{Var}^*(w_i/m_i^*)$, which is the variance of w_i/m_i^* with respect to the probability measure defined by scaled market-cap weights $m_i^* = m_i/\sum_{i=1}^{N} m_i$, so that $\sum_{i=1}^{N} m_i^* = 1$.[8] If portfolio weights equal market-cap weights, so $w_i/m_i^* = 1$, then $\text{Var}^*(w_i/m_i^*) = 0$ and balance equals 1. Like coverage, balance is always between 0 and 1.

Equation (11) shows that a portfolio is well diversified if it holds a large fraction of the benchmark's stocks and if its weights are close to market-cap weights. Given the ranges of coverage and balance, diversification is always between 0 and 1. The benchmark portfolio has coverage and balance both equal to 1.

Our measure of portfolio diversification is easy to calculate from equation (11). A simple two-step approach is available to those wishing to circumvent the calculation of variance with respect to the m^* probability measure. One can simply compute L from equation (3) and then divide it by stock liquidity, following equation (10).

[8]Note that $\sum_{i=1}^{N_M} m_i = 1$, but $\sum_{i=1}^{N} m_i \leq 1$, because $N \leq N_M$. $\text{Var}^*(.)$ can be easily computed using the expression $\text{Var}^*(w_i/m_i^*) = \sum_{i=1}^{N} w_i^2/m_i^* - 1$. Details are in Appendix A.

2.4. Empirical Evidence

We compute our measures of portfolio liquidity and diversification for a sample of 2,789 actively managed U.S. domestic equity mutual funds covering the 1979–2014 period. To construct this sample, we begin with the dataset constructed by Pástor, Stambaugh, and Taylor (2015, 2017), which combines data from the Center for Research in Securities Prices (CRSP) and Morningstar. We add three years of data and merge this dataset with the Thomson Reuters dataset of fund holdings. We restrict the sample to fund-month observations whose Morningstar category falls within the traditional 3×3 style box (small-cap/mid-cap/large-cap interacted with growth/blend/value). This restriction excludes non-equity funds, international funds, and industry-sector funds. We also exclude index funds, funds of funds, and funds smaller than $15 million. A more detailed description of our sample, including the variable definitions and their summary statistics, is in Appendix C.

For each fund and quarter-end, we compute portfolio liquidity from the fund's quarterly holdings data. Initially, we compute portfolio liquidity relative to the market portfolio. Our definition of the market portfolio is guided by the end-of-sample holdings of the world's largest mutual fund, Vanguard's Total Stock Market Index fund. This fund tracks the CRSP US Total Market Index, which is designed to track the entire U.S. equity market. We find that 98.9% of the fund's holdings are either ordinary common shares (CRSP share code, *shrcd*, with first digit equal to 1) or REIT shares of beneficial interest ($shrcd = 48$). We therefore define the market as all CRSP securities with these share codes. This definition includes foreign-incorporated firms ($shrcd = 12$), many of which are deemed domestic by CRSP (they make up 1.4% of the Vanguard fund's holdings), but it excludes securities such as ADRs ($shrcd = 31$) and units or limited partnerships (*shrcd* first digit equal to 7). When computing mutual funds' portfolio liquidity, we exclude all fund holdings that are not included in the above definition of the market portfolio. For the median fund/quarter in our sample, 2.3% of holding names and 1.9% of holding dollars are outside the market.

2.4.1. Time Series

Panel A of Figure 1 plots the time series of the cross-sectional means of portfolio liquidity, L, across all funds. The figure offers two main observations. First, fund portfolios became substantially more liquid in the last two decades of the 20th century, with average L doubling between 1980 and 2000. Most of this increase took place in the late 1990s. Second, since 2000, average L has been relatively stable around 0.05.

To understand these patterns, Panel B of Figure 1 plots the time series of the two components of L: stock liquidity and diversification. Stock liquidity rose sharply in the late 1990s, single-handedly explaining the contemporaneous increase in L observed in Panel A. The post-2000 patterns are more interesting. Stock liquidity declined steadily in the 21st century, falling from 17.8 in 2000 to 7.4 in 2014.[9] Judging by this large decline in the liquidity of fund holdings, one might expect fund portfolios to have become less liquid in the 21st century, but that is not the case, as shown in Panel A. The reason is that fund portfolios have become much more diversified: diversification almost tripled between 2000 and 2014. The two opposing effects—the decrease in stock liquidity and the increase in diversification—roughly cancel out, resulting in a flat pattern in L since 2000.

The sharp increase in diversification after 2000 is remarkable. To shed more light on this increase, Panel C of Figure 1 plots the components of diversification: balance and coverage. Both components rise steadily, especially after 2000.[10] Between 2000 and 2014, balance rose from 0.31 to 0.43. Coverage rose even faster: it doubled. The portfolios of active mutual funds have thus become more index-like: they hold an increasingly large fraction of all stocks in the market, and their weights increasingly resemble market weights.

Finally, we dissect the sharp increase in coverage, which is equal to N/N_M, by plotting the time series of the cross-sectional averages of N and N_M. Panel D of Figure 1 shows that funds hold an increasingly large number of stocks. The average N rises essentially linearly from 54 in 1980 to 126 in 2014. In addition, the number of stocks in the market plummets from about 8,600 in the late 1990s to fewer than 5,000 in 2014. The observed increase in coverage is thus driven by a combination of a rising N and falling N_M.

Koijen and Yogo (2016) show that the price impact of mutual funds' trades declines between 1980 and 2014. Our Figure 1 suggests that this decline is driven by the rising diversification of mutual fund portfolios. Both coverage and balance of fund portfolios increase substantially over that period, making the portfolios more liquid. The rising diversification is also consistent with the growth of closet indexing (e.g., Cremers and Petajisto, 2009).

More broadly, the rise in L reduces the stock market's vulnerability to large capital redemptions from mutual funds. Such redemptions are often triggered by poor fund performance, especially for funds holding illiquid assets (e.g., Chen, Goldstein, and Jiang, 2010). Funds facing large redemptions must liquidate some of their holdings, and the resulting price pressure can move stock prices (e.g., Coval and Stafford, 2007). This price pressure is alleviated by the rising liquidity of active mutual funds' portfolios. The growth of index funds,

[9]This decrease in stock liquidity is consistent with the evidence of Blume and Keim (2017) that institutional investors steadily increased their holdings of smaller stocks in recent decades.

[10]The upward trends in both components of diversification, as well as the resulting upward trend in portfolio liquidity, are statistically significant, as we show in the Internet Appendix.

which are particularly liquid, also reduces the stock market's fragility.

2.4.2. Cross Section

Figure 2 plots the cross-sectional distribution of L and its components at the end of our sample, in 2014Q4. The left-hand set of panels uses the market portfolio as a benchmark (as in Figure 1); the right-hand set uses the appropriate sector benchmark. We consider nine sectors corresponding to the traditional 3×3 style box used by Morningstar.[11] To calculate L with respect to a fund's sector, we divide the fund's market-based L by the fraction of the total market capitalization accounted for by that sector. We calculate those sector-specific fractions from the holdings of the Vanguard index fund tracking the sector-specific benchmark.[12] To calculate a fund's sector-based stock liquidity, we multiply the fund's market-based stock liquidity by the ratio of the average market cap of all stocks in the market to the average market cap of all stocks held by the sector-specific Vanguard index fund. To calculate sector-based diversification and coverage, we multiply their market-based values by the ratio of the number of stocks in the market to the number of stocks held by the corresponding Vanguard index fund. Balance is unaffected by benchmark choice.

Figure 2 shows that active mutual funds hold relatively illiquid portfolios. Market-based L, plotted in the top left panel, is mostly below 0.15, far below its potential maximum of 1. Sector-based L, plotted in the top right panel, is larger than market-based L, by construction. But even sector-based L is far below 1, mostly below 0.5.

Are the low portfolio liquidities caused by funds' preference for illiquid stocks? The answer is no. For the vast majority of funds, stock liquidity, plotted in the second row of Figure 2, exceeds 1. In fact, market-based stock liquidity often exceeds 10, suggesting that the average stock held by the fund is more than ten times bigger than the average stock in the market. Sector-based stock liquidity also exceeds 1 for most funds, though it rarely exceeds 4. In short, mutual funds tend to hold more-liquid stocks than their benchmarks. This evidence is consistent with that of Falkenstein (1996), Gompers and Metrick (2001), and others. The high stock liquidity makes fund portfolios more liquid, not less. Instead, the story behind funds' low portfolio liquidity is diversification. Market-based diversification

[11] Morningstar assigns funds to style categories based on the funds' reported portfolio holdings, and it updates these assignments over time. Since the assignments are made by Morningstar rather than the funds themselves, there is no room for benchmark manipulation of the kind documented by Sensoy (2009). The benchmark assigned by Morningstar can differ from that reported in the fund's prospectus.

[12] These sector-specific fractions are 0.403, 0.748, and 0.362 for large-cap value, blend, and growth funds (Vanguard tickers VIVAX, VLACX, VIGRX), 0.069, 0.134, and 0.070 for mid-cap value, blend, and growth funds (Vanguard tickers VMVIX, VIMSX, VMGIX), and 0.067, 0.123, 0.061 for small-cap value, blend, and growth funds (Vanguard tickers VISVX, NAESX, VISGX).

is mostly below 0.02, and sector-based diversification is largely below 0.4. To gain more insight, we examine the components of diversification. While balance spreads across most of the range between 0 and 1, coverage tends to be lower. Even sector-based coverage takes values mostly below 0.5. This result is not surprising, since the average fund holds only 126 stocks (recall Panel D of Figure 1). We thus conclude that the relatively low liquidity of active mutual funds is largely due to their low diversification, and that the low diversification is driven mostly by the low coverage of the funds' portfolios.

2.4.3. Correlations

How much of the variance in portfolio liquidity is contributed by each of its components? To answer this question, Table 1 reports the correlations between market-based L and stock liquidity, diversification, coverage, and balance. We compute these correlations in four ways: across all panel observations (Panel A), across funds (Panel B), across funds within the same sector (Panel C), and over time within funds (Panel D). In all four panels, L is positively correlated with both stock liquidity and diversification, which is not surprising. But the correlation with stock liquidity is higher in Panels A and B, whereas the correlation with diversification is higher in Panels C and D. This difference is driven by dispersion in stock liquidity across sectors (e.g., large-cap stocks are more liquid than small-cap stocks). Therefore, when we do not control for sector differences, the primary driver of L is stock liquidity (Panels A and B), but when we do, the primary driver is diversification (Panels C and D).

The two components of L, stock liquidity and diversification, are negatively correlated in all four panels. Funds holding less liquid stocks tend to be more diversified, in terms of both coverage and balance. Stock liquidity and diversification seem to act as substitutes: funds tend to make up for the low liquidity of their holdings by diversifying more.

Portfolio diversification is highly correlated with both of its components, coverage and balance. The correlations are of similar magnitudes, indicating that coverage and balance are roughly equally important in explaining diversification. Coverage and balance are mildly positively correlated, but their correlation turns negative after controlling for other fund characteristics, as we show later in Table 2.

3. Tradeoffs Among Fund Characteristics

In this section, we examine the relations among key fund characteristics: portfolio liquidity, fund size, expense ratio, and turnover. We first derive such relations theoretically, from optimizing behavior of fund managers and investors, and then verify them empirically.

3.1. Expected Fund Profits

A fund's profits depend both on its skill and how actively it applies that skill. To capture this interaction, we model the fund's expected benchmark-adjusted return, before costs and fees, as

$$a = \underbrace{\mu}_{\text{Skill}} \times \underbrace{\left(TL^{-\frac{1}{2}}\right)}_{\text{Activeness}} , \tag{12}$$

where μ is a fund-specific positive constant reflecting skill in identifying profitable trading opportunities. The more actively such skill is applied, the greater is the profit before costs. Our "activeness" measure, $TL^{-1/2}$, is increasing in turnover, T, and decreasing in portfolio liquidity, L. A fund is more active if it trades more and if it holds a less liquid portfolio.

The role of T in activeness is consistent with the theory and empirical evidence of Pástor, Stambaugh, and Taylor (2017), who establish a positive link between a funds's turnover and its performance. Intuitively, higher turnover means the fund is more frequently applying its skill in identifying profit opportunities.

Recall from equation (10) that L is the product of stock liquidity and diversification, so a fund's activeness is decreasing in both of those quantities. This role of stock liquidity in activeness reflects evidence that mispricing is greater among less liquid and smaller stocks (e.g., Sadka and Scherbina, 2007, and Stambaugh, Yu, and Yuan, 2015), consistent with arguments that arbitrage is deterred by higher trading costs and greater volatility (e.g., Shleifer and Vishny, 1997, Pontiff, 2006). A fund tilting toward such stocks is more actively pursuing mispricing where it is most prevalent.

Both components of diversification, coverage and balance, explain diversification's role in activeness. By holding fewer stocks (i.e., lower coverage), a fund can focus on its best trading ideas, leading to higher expected gross profits. By deviating more from market-cap weights (i.e., lower balance), a fund can place larger bets on its better ideas, again boosting performance. Theoretical settings in which portfolio concentration (lower diversification) arises optimally include Merton (1987), van Nieuwerburgh and Veldkamp (2010), and Kacperczyk, van Nieuwerburgh, and Veldkamp (2016). Empirical evidence linking portfolio concentration to performance includes results in Kacperczyk, Sialm, and Zheng (2005), Ivkovich, Sialm, and Weisbenner (2008), and Choi et al. (2017).

In untabulated results, we find that L has a -79% correlation with the active share measure of Cremers and Petajisto (2009), in logs. This negative correlation makes sense because both measures capture deviations of portfolio weights from benchmark weights. Cremers and Petajisto interpret active share as the extent to which a fund engages in active management. Our measure of activeness incorporates L, micro-founded in the trading cost function, as well as T. Activeness has a 55% correlation with active share, again in logs.

The functional form in which T and L enter equation (12) delivers a simple form of decreasing returns to scale, with scale captured by the fund's liquidity-adjusted dollar volume, $V = ATL^{-\frac{1}{2}}$ (equation (9)). To understand this implied concept of scale, observe from equation (12) that aA, the fund's expected dollar profit (before cost, benchmark-adjusted), equals μV. This gross profit thus increases in proportion to V, whereas the fund's expected dollar trading cost increases in proportion to V^2, as noted earlier in equation (8). As a result, expected dollar profit net of trading cost,

$$\Pi = \mu V - (c/M)V^2 \ , \tag{13}$$

is hump-shaped with respect to V, consistent with the fund facing decreasing returns to scale in its liquidity-adjusted dollar volume. Specifying equation (12) to imply expected profit proportional to V, given that expected trading cost is proportional to V^2, allows a transparent solution to the fund's optimization problem presented below.

3.2. Fund Characteristics in Equilibrium

The fund chooses its turnover, T, portfolio liquidity, L, and expense ratio ("fee" for short), denoted as f. The choice of f determines the fund's equilibrium size, A, but does not affect its equilibrium fee revenue or expected net profit, as explained below. The fund chooses T and L to maximize its expected net profit, given A. Those choices of T and L determine V in equation (9). The value of V that maximizes the net profit in equation (13) is given by $V = (\mu M)/(2c)$. Therefore, using equation (9), the product of the fund's size and its activeness, $TL^{-\frac{1}{2}}$, is uniquely determined as

$$ATL^{-\frac{1}{2}} = \frac{\mu M}{2c} \ . \tag{14}$$

This equation governs the tradeoffs faced by the fund. Its right-hand side is pinned down by the fund's skill; its left-hand side is determined by choices under the fund's control. Since T and L affect net profit only through the product $TL^{-\frac{1}{2}}$ and its square, the fund chooses that product—activeness—while being indifferent between the various ways of achieving it. That is, if the fund chooses a less liquid portfolio, it also chooses to trade less, and vice versa. In addition, equation (14) establishes a tradeoff between the fund's size and its activeness.

Combining equations (7) and (12), the fund's net alpha is

$$\begin{aligned}\alpha &= a - E(C)/A - f \\ &= \mu T L^{-\frac{1}{2}} - (c/M)\, A T^2 L^{-1} - f \;. \end{aligned} \tag{15}$$

Following Berk and Green (2004), we assume that competing investors allocate the amount of assets A to the fund such that

$$\alpha = 0 \;. \tag{16}$$

Combining equations (14), (15), and (16) implies the fund's equilibrium size is given by

$$A = \frac{\mu^2 M}{4cf} \;. \tag{17}$$

Note from equation (17) that the fund can set the fee rate f arbitrarily, in that fee revenue, fA, is invariant to f. If the fund charges a higher f, investors allocate less to the fund, leaving fee revenue unchanged. Fee revenue is determined by the fund's skill, μ. Combining equations (14) and (17), we obtain

$$T L^{-\frac{1}{2}} = \frac{2f}{\mu} \;. \tag{18}$$

The fund's choice of f therefore determines not only the fund's size but also its activeness: more expensive funds end up being smaller and more active.[13]

From equations (9), (13), (15), and (16), the fund's equilibrium fee revenue satisfies

$$\begin{aligned} fA &= \mu A T L^{-\frac{1}{2}} - (c/M)\, A^2 T^2 L^{-1} \\ &= \mu V - (c/M) V^2 = \Pi \;. \end{aligned} \tag{19}$$

The fund reaps all the profit from trading because investors earn zero alpha. The profit-maximizing choice of V in equation (14) thus also maximizes fee revenue in equilibrium.

Substituting for μ from equation (18) into equation (14) and taking logs, we obtain

$$\log(L) = \log(A) - \log(f) + 2\log(T) + \text{constant} \;. \tag{20}$$

The "constant" term is the log of c/M, which is fixed and exogenous with respect to funds' tradeoffs. Equation (20) shows how the four fund characteristics—portfolio liquidity L, fund

[13] The relations we derive bear some algebraic resemblance to equations that appear in Berk and Green (2004). The contexts are different, however. In Berk and Green, the equations arise when a fund allocates between an active strategy and a zero-alpha passive strategy. In contrast, our setting has funds choosing portfolio liquidity and turnover among alternative positive-alpha strategies. Moreover, the quadratic cost function delivering decreasing returns to scale in Berk and Green's example is exogenously specified. In contrast, decreasing returns to scale in our setting are micro-founded in the trading cost function, with scale defined in a richer way, as the fund's liquidity-adjusted dollar volume of trading (see equation (9)).

size A, expense ratio f, and turnover T—are jointly determined in equilibrium. Funds with more-liquid portfolios should be larger and cheaper, and they should trade more. To see the intuition, imagine changing one variable on the right-hand side of equation (20) while holding the other two constant. Holding f and T constant, when fund size increases, diseconomies of scale lead the fund to hold a more liquid portfolio. Holding f and A constant, fee revenue, and thus skill, are also constant. For a given level of skill, if a fund trades more, it chooses a more liquid portfolio to reduce transaction costs. Holding A and T constant, if a fund has a higher f, it has higher fee revenue and hence higher skill. A more skilled fund can more effectively offset the higher trading costs associated with a less liquid portfolio. For example, it can afford to concentrate its portfolio on its best ideas or to trade in less liquid stocks, which are more susceptible to mispricing.

3.3. Evidence

To test the predictions from equation (20), we interpret the equation as a regression of $\log(L)$ on the other fund characteristics, and we estimate the regression using our mutual fund dataset. The unit of observation is the fund/quarter. We include sector-quarter fixed effects in the regression, which offers three important benefits. First, the fixed effects isolate variation across funds, which is appropriate because our model applies period by period. Second, by including sector-quarter fixed effects, we effectively use L defined with respect to a sector-specific benchmark rather than the market. As noted earlier, sector-based L is equal to market-based L divided by the fraction of the total stock market capitalization accounted for by the sector. Since that fraction is sector-specific within a given quarter, sector-based $\log(L)$ is equal to market-based $\log(L)$ minus a sector-quarter-specific constant that is absorbed by our fixed effects. Third, our model assumes c is constant, and this assumption is more likely to hold across funds within a given sector and quarter. The sector-quarter fixed effects absorb variation in $\log(c/M)$, the constant in equation (20), both across sectors and over time. Our specification therefore allows liquidity conditions to vary over time and across sectors. Results using only quarter fixed effects, equivalent to using market-based L, are very similar (see the Internet Appendix).

Column 1 of Table 2 provides strong support for the model's predictions in equation (20). The slope coefficients on all three regressors have their predicted signs. Moreover, all three slopes are highly significant, both statistically and economically. The slope on fund size ($t = 13.76$) shows that larger funds tend to have more-liquid portfolios. A one-standard-deviation increase in the logarithm of fund size is associated with a sizeable 0.21 increase in $\log(L)$, which corresponds, for example, to an increase in L from 0.20 to 0.25 (cf. top right panel of Figure 2). The slope on expense ratio ($t = -11.26$) shows that cheaper funds tend to have more-liquid portfolios. The economic significance of expense ratio is comparable to that of fund size: a one-standard-deviation increase in $\log(f)$ is associated with a 0.22 decrease in $\log(L)$. Finally, the slope on turnover ($t = 4.93$) shows that funds that trade more tend to have more-liquid portfolios. A one-standard-deviation increase in $\log(T)$ is associated with a 0.09 increase in $\log(L)$, which corresponds to an increase in L from 0.20 to 0.22. We conclude that funds with less liquid portfolios trade less and are smaller and more expensive, fully in line with our theory.

Having tested the central predictions of equation (20), we turn to the additional predictions following from equations (1) and (2). Equation (1) implies that

$$\log(L) = \log(\text{Stock Liquidity}) + \log(\text{Diversification}) \ . \tag{21}$$

Combined with equation (20), this equation implies

$$\log(\text{Diversification}) = \log(A) - \log(f) + 2\log(T) - \log(\text{Stock Liquidity}) + \text{constant} \ . \tag{22}$$

This equation makes strong predictions about the determinants of portfolio diversification. In equilibrium, funds with more diversified portfolios should be larger and cheaper, they should trade more, and their stock holdings should be less liquid, on average.

Column 2 of Table 2 provides strong support for all of these predictions. Fund size, expense ratio, and turnover help explain diversification with their predicted signs, and the slopes have magnitudes similar to those in column 1. The new regressor, stock liquidity, also enters with the right sign and is highly significant, both statistically ($t = -21.61$) and economically. A one-standard-deviation increase in log(Stock Liquidity) is associated with a 0.95 decrease in log(Diversification), for example, a decrease in diversification from 0.26 to 0.10 (cf. middle right panel of Figure 2). Stock liquidity and diversification are thus substitutes, as noted earlier. This evidence fits our model, which predicts the L a fund should choose, but not how to achieve that L by combining its components.

Next, we drill deeper by decomposing diversification following equation (2):

$$\log(\text{Diversification}) = \log(\text{Coverage}) + \log(\text{Balance}) \ . \tag{23}$$

Combined with equation (22), this equation implies

$$\log(\text{Coverage}) = \log(A) - \log(f) + 2\log(T) - \log(\text{Stock Liq.}) - \log(\text{Balance}) + \text{constant}$$

and

$$\log(\text{Balance}) = \log(A) - \log(f) + 2\log(T) - \log(\text{Stock Liq.}) - \log(\text{Coverage}) + \text{constant}.$$

These equations make predictions about the determinants of portfolio coverage and balance.

Columns 3 and 4 of Table 2 support those predictions. In both regressions, all the variables enter with their predicted signs. Most of the variables are highly significant; only turnover in column 4 is marginally significant. Notably, the slopes on balance in column 3 and coverage in column 4 are both negative. Therefore, controlling for other fund characteristics, coverage and balance are substitutes: funds that are less diversified in terms of coverage tend to be better diversified in terms of balance, and vice versa.

Finally, Column 5 of Table 2 tests the prediction analogous to that in equation (22), except that diversification and stock liquidity switch sides: the former appears on the right-hand side and the latter on the left-hand side of the regression. The evidence again supports the model, though a bit less strongly than the first four columns. Three of the four slopes have the right sign and are all significant (the t-statistic on stock liquidity is -24.49!). The slope on turnover is negative but not significantly different from 0.

In a robustness exercise, we split the sample into two subsamples, 1979 through 2004 and 2005 through 2014, which contain roughly the same number of fund-quarter observations. The counterparts of Table 2 for both subsamples look very similar to the original, leading to the same conclusions. We show both tables in the Internet Appendix.

3.4. Correlations Among Fund Characteristics

The tradeoffs among fund characteristics are also apparent in simple correlations. With additional assumptions, our model helps us understand these correlations.

3.4.1. Larger Funds Are Cheaper

The sharpest additional prediction is that larger funds should have lower expense ratios. This prediction follows from equation (17): fund size and expense ratio are perfectly negatively correlated across funds if the skill parameter μ is constant. Since a fund's net alpha is 0 in equilibrium, the fund's fee revenue ($A \times f$) equals the fund's profits, which depend on skill. Therefore, f goes down if A goes up, holding μ constant. If μ varies across funds, the correlation between A and f is no longer perfect, but it remains negative as long as μ is not too highly correlated with f across funds. Specifically, let $\beta_{\mu,f}$ denote the slope from the cross-sectional regression of $\log(\mu)$ on $\log(f)$. Our model implies a negative cross-sectional correlation between fund size and expense ratio as long as $\beta_{\mu,f} < 1/2$. It makes sense for $\beta_{\mu,f}$ to be positive, in that more skilled funds should be able to charge higher fees.

Nonetheless, it seems plausible that $\beta_{\mu,f} < 1/2$ because in practice, expense ratios have a variety of institutional determinants beyond skill (marketing, distribution, etc.).

Empirical evidence strongly supports this prediction. Table 3 reports correlations between fund characteristics, again measured in logs. In our mutual fund dataset, the cross-sectional within-sector correlation between fund size and expense ratio is -31.5% ($t = -15.30$). Larger funds clearly charge lower expense ratios. This evidence is consistent with our model. Other studies have already reported a negative correlation between fund size and expense ratio (e.g., Ferris and Chance, 1987). But we appear to be the first to provide a theoretical justification for this strong stylized fact.[14]

The correlation between fund size and expense ratio is also strongly negative in the time series for the typical fund, -25.1% ($t = -17.61$). We mainly view our model's predictions as being cross-sectional, applying within a single period. In principle, one could also view our model as describing a given fund solving a series of single-period problems. However, applying the model to a fund's time series requires the trading-cost parameter c to be constant over time, in tension with ample evidence that liquidity conditions fluctuate.

In computing the time-series correlations in Panel B of Table 3, we need to account for the substantial growth in the dollar values of stocks that renders dollar assets under management (AUM) unappealing as a time-series measure of fund size: AUM values in the 1980s are not comparable to those today. To address this fact, we divide each fund's AUM by the contemporaneous total stock market capitalization. This scaling is motivated by our theory, in which fund size A enters the key relations as a multiple of M. For example, equation (20) can be rewritten so that the first term on the right-hand side is $\log(A/M)$ while the constant term is $\log(c)$. Equations (14) and (17) can also be rewritten to feature the ratio A/M. Even beyond our model, it seems sensible to deflate fund size by stock-market value when analyzing a time series of fund size. For example, Pástor, Stambaugh, and Taylor (2015) also follow this approach.

3.4.2. Funds That Trade Less Are Larger and Cheaper

The next set of predictions involves turnover (T) and portfolio liquidity (L). Recall from Section 3.2 that the fund effectively chooses $TL^{-\frac{1}{2}}$, or activeness, to maximize its profit. Our model predicts a negative correlation between $TL^{-\frac{1}{2}}$ and A (equation (14)) and a positive correlation between $TL^{-\frac{1}{2}}$ and f (equation (18)). Both correlations should be perfect if skill

[14]Although not noted by Berk and Green (2004), a negative correlation between A and f could also be derived from their model, which also has skill determining fee revenue.

(μ) is constant. The intuition is that larger funds, facing diseconomies of scale, optimally reduce their trading costs by either trading less or increasing their portfolio liquidity. Low-fee funds also trade less and hold more-liquid portfolios, because such funds are larger, holding μ constant. If μ varies, both correlations should retain their signs as long as μ is not too highly correlated with f or A. Specifically, let $\beta_{\mu,A}$ denote the slope from the regression of $\log(\mu)$ on $\log(A)$. The model implies a negative correlation between $TL^{-\frac{1}{2}}$ and A as long as $\beta_{\mu,A} < 1$ and a positive correlation between $TL^{-\frac{1}{2}}$ and f as long as $\beta_{\mu,f} < 1$.

Empirical evidence strongly supports both predictions. Table 3 shows that the cross-sectional within-sector correlation of $TL^{-\frac{1}{2}}$ with fund size is strongly negative, -23.1% ($t = -13.27$), while the correlation with expense ratio is strongly positive, 25.5% ($t = 13.58$). The correlations are similar when computed from the time series: -26.8% ($t = -17.60$) for fund size and 13.6% ($t = 9.20$) for expense ratio.

While the product $TL^{-\frac{1}{2}}$ is interesting from the theoretical perspective, it also makes sense to look at T and L individually. Our predictions for $TL^{-\frac{1}{2}}$ imply that, controlling for L, T should be negatively related to fund size and positively related to expense ratio. This is indeed true in the data, and the relations are so strong that they hold even without controlling for L. In Table 3, T is negatively correlated with fund size, both in the cross section and in the time series: the correlations are -10.5% ($t = -6.01$) and -14.7% ($t = -12.17$), respectively. In addition, T is positively correlated with expense ratio: the correlation is 13.0% ($t = 6.35$) in the cross section and 10.5% ($t = 7.57$) in the time series. In short, funds that trade less are larger and cheaper, as predicted by our model.

3.4.3. Funds with More-Liquid Portfolios Are Larger and Cheaper

Our predictions for $TL^{-\frac{1}{2}}$ also imply that, controlling for T, L should be positively related to fund size and negatively related to expense ratio. Again, both relations hold strongly in the data, even in simple correlations, as shown in Table 3. The correlations between L and fund size are 28.5% ($t = 17.90$) and 30.8% ($t = 18.23$) in the cross section and time series, respectively. The correlations between L and expense ratio are -29.1% ($t = -13.39$) and -11.8% ($t = -6.87$). In short, funds with more-liquid portfolios are larger and cheaper, as predicted by our model. This evidence is also consistent with our multiple-regression results reported in Column 1 of Table 2.

The cross-sectional correlations that involve L are extremely robust. The correlations in Panel A of Table 3 are computed from panel regressions with quarter-sector fixed effects, which isolate cross-sectional correlations within sectors.[15] Those correlations are therefore weighted averages of cross-sectional correlations, where the averaging is across all quarters in our sample. It turns out that the cross-sectional relations involving L hold not only on average, but also in every single quarter in our sample. This stunning fact is plotted in Figure 3. Both correlations involving L retain the same sign in every quarter between 1980 and 2014. In fact, in each quarter, their magnitudes exceed 20% in absolute value.

Two other cross-sectional correlations discussed earlier are similarly strong, which is why we plot their time series in Figure 3. The correlation between fund size and expense ratio, analyzed in Section 3.4.1, is negative in every single quarter, varying between -0.74 and -0.23 across quarters. The correlation between turnover and expense ratio, analyzed in Section 3.4.2, is positive in every quarter, varying between 0.10 and 0.36. It is rare to see a model's theoretical predictions hold so strongly in the data.

While Figure 3 plots cross-sectional correlations, the time-series correlations reported in Table 3 are of similar magnitudes. The time-series correlation between L and fund size, 30.8%, is particularly strong. It shows that when a fund gets larger, its portfolio becomes more liquid. This fact is easily interpreted in the context of our theory. Consider a fund that receives a large inflow. Cognizant of decreasing returns to scale, the fund's manager makes the fund's portfolio more liquid. And vice versa—after a large outflow, a fund can afford to make its portfolio less liquid.

To illustrate these effects, we pick the example of Fidelity Magellan, the largest mutual fund at the turn of the millenium. Figure 4 plots the time series of Magellan's AUM and its portfolio liquidity. The similarity between the two series is striking. Between 1980 and 2000, Magellan's assets grew rapidly, in large part due to the fund's stellar performance under Peter Lynch in 1977 through 1990. Over the same period, and especially after 1993, the liquidity of Magellan's portfolio also grew rapidly. From 1993 to 2001, Magellan's L grew from 0.1 to 0.4, a remarkable increase equal to nearly five standard deviations of the sample distribution of L. After 2000, though, Magellan's assets shrank steadily, and by 2014, they were down by almost 90%. Over the same period, Magellan's L was down also, back to about 0.1. A natural interpretation is that Magellan's large size around 2000 forced the fund's managers to increase the liquidity of Magellan's portfolio to shelter the fund from the pernicious effects of decreasing returns to scale.

[15] We also compute plain cross-sectional correlations (i.e., including quarter fixed effects instead of sector-quarter fixed effects). The results are very similar to those in Panel A of Table 3 so we report them only in the Internet Appendix. In that Appendix, we also show the results from another robustness exercise, in which we recompute Table 3 for two subperiods containing roughly the same number of fund-month observations. The results in both subsamples look very similar to the full-sample ones.

4. Predicting Fund Returns

So far, we have assumed that investors allocate capital perfectly to funds. Next, we consider the possibility of capital misallocation, whereby fund sizes differ from their equilibrium values. We relate excess fund size to portfolio liquidity and other fund characteristics. This theoretical relation allows us to estimate excess fund size as the residual from the regression implied by equation (20). We then show that our model implies a negative relation between excess fund size and future fund performance. Finally, we verify such a relation in the data.

4.1. Excess Fund Size

Building on Section 3.2, let $\left\{\bar{A}, \bar{f}, \bar{L}, \bar{T}\right\}$ denote the equilibrium values of a given fund's characteristics. Also let $\{A, f, L, T\}$ denote the actual values of the same characteristics. Suppose that investors misallocate capital to the fund, so that

$$A = \bar{A}\left(1 + \delta\right) \ . \tag{24}$$

We refer to δ as excess fund size. Funds with $\delta > 0$ are too big, whereas those with $\delta < 0$ are too small relative to their equilibrium size.

How does the fund respond to this misallocation? We assume that changing the previously announced fee is not an option, so that $f = \bar{f}$. This assumption is motivated by the high persistence of fund fees over time. Instead, the fund responds by adjusting either its turnover or its portfolio liquidity, or both. Specifically, the fund still chooses the profit-maximizing value of activeness according to equation (14), so that the fund's actual $TL^{-\frac{1}{2}}$ is equal to $\bar{T}\bar{L}^{-\frac{1}{2}}$ divided by $(1+\delta)$. That is, funds that are too big ($\delta > 0$) choose either to trade less or to hold more-liquid portfolios, or both. Funds that are too small ($\delta < 0$) do the opposite. Intuitively, if a fund receives more capital than its skill level can support, it optimally reduces trading costs, by either trading less or holding a more liquid portfolio.

Aided by the approximation $\delta \approx \log(1+\delta)$, we show in Appendix A that

$$\log(L) \approx \log(A) - \log(f) + 2\log(T) + \log(c/M) + \delta \ . \tag{25}$$

This equation is identical to equation (20) except for the extra term δ. Assuming that δ is drawn from a random distribution with mean 0, we can view δ as the residual from the regression of $\log(L)$ on the logs of the other three fund characteristics (A, f, and T). This is how we estimate excess fund size in our subsequent empirical analysis. As before, we include sector-quarter fixed effects to absorb variation in $\log(c/M)$.

One complication in running regression (25) is that the regression residual, δ, is correlated with one of the regressors, fund size. This correlation is apparent from equation (24). As a result, OLS estimation of regression (25) is inconsistent. To get around this problem, we estimate the regression by using an instrumental variables (IV) approach.

Our instrument for fund size is the size of the mutual fund family to which the fund belongs. To calculate a given fund's family size, we first add up the AUM across all funds in the fund's family, excluding the fund itself to avoid any mechanical correlation between family size and fund size. We then augment each family size observation by \$15 million, our minimum threshold for fund AUM. The addition of \$15 million ensures a positive value for family size for single-fund families. We need family size to be positive because we use its logarithm to instrument for $\log(A)$. The correlation between the logs of family size and fund size is positive and significant, suggesting that the relevance condition is satisfied. The exclusion restriction is likely to be satisfied as well because it is not clear why family size should be correlated with misallocation to the fund (δ). The F-statistic for the first-stage regression is large, 583, indicating that our IV specification does not suffer from the weak instrument problem.

In our first application of the IV approach, we use it to reestimate the regressions from Table 2. The results, reported in Table 4, are very similar to the OLS results from Table 2. All of the slope estimates in Table 4 have the same signs as their counterparts in Table 2. Moreover, all the slopes that are statistically significant in Table 2 are also significant in Table 4. The results from Table 2 are thus robust to a different estimation approach. Our main results are the same whether or not we allow for capital misallocation.

Next, we analyze the persistence of excess fund size, δ. We compute δ as the residual from regression (25) estimated by the IV approach. Full-sample estimates from this regression are reported in column 1 of Table 4, though we reestimate this regression every month. For each month t, we run the panel IV regression by using data from 1979 through month $t-12$. We denote this regression's residual for fund j in month $t-12$ by $\delta_{j,t-12}$. We use a 12-month lag of δ to ensure that this variable is observable at the end of month t. One of the regressors in regression (25), turnover, is computed over the fund's current fiscal year, so we lag δ by 12 months to accommodate turnover's annual reporting. Finally, we use a ten-year burn-in period for estimating the IV regression. Since holdings data are very scarce in 1979, we run the initial IV estimation over the ten-year period 1980 through 1989. Given the 12-month lag of $\delta_{j,t-12}$, the first month t for which we report results is January 1991. All empirical analysis in this section therefore covers the period January 1991 through December 2014.

Figure 5 examines the persistence of the δ estimates. At the end of each month t, we sort funds into five quintile portfolios within sectors based on funds' values of $\delta_{j,t-12}$. For each horizon τ between 1 and 48 months, and for each quintile, we compute the average value of $\delta_{j,t+\tau} - \delta_{j,t}$ across all funds j in the quintile and all months t, weighting each fund/month observation equally. We plot these average changes in δ against horizon τ for quintiles 1, 3, and 5. We refer to quintile 1 as low-δ funds and to quintile 5 as high-δ funds. Since the δ's average to 0, by construction, low-δ (high-δ) funds have negative (positive) values of $\delta_{j,t-12}$.

Figure 5 shows that fund δ's trend toward 0 in a highly persistent fashion. For low-δ funds, the value of δ rises, essentially linearly, at all horizons out to at least four years. For high-δ funds, the value of δ falls at a similar rate at the same horizons. This evidence suggests that fund sizes trend toward their equilibrium values, but this convergence is relatively slow. Capital misallocation gradually gets corrected, but it persists for years.

Our simple model makes no predictions as to where capital misallocation comes from or how long it persists. The model does, however, make predictions about how this misallocation affects fund performance.

4.2. Predicting Fund Performance

Given our assumptions about misallocation from Section 4.1, equation (15) turns into

$$\alpha = -\left(\frac{\delta}{1+\delta}\right) f \ . \tag{26}$$

Due to decreasing returns to scale, funds that are too big ($\delta > 0$) have $\alpha < 0$; funds that are too small ($\delta < 0$) have $\alpha > 0$. When δ is small, as the approximation leading to equation (25) assumes, then $\frac{\delta}{1+\delta} \approx \delta$, so that $\alpha \approx -\delta f$. Even when δ is large, equation (26) implies that α decreases in δ, especially when f is high. Intuitively, high-fee funds face steeper decreasing returns to scale because they choose to be more active in equilibrium (see equation (18)). In short, our model implies that high excess fund size predicts low benchmark-adjusted fund returns, and that this relation is stronger for higher-fee funds.

We examine this prediction empirically in two ways, first using regressions and then using portfolio sorts. We run panel regressions of benchmark-adjusted fund returns in month $t+\tau$ on funds' $\delta_{j,t-12}$, including sector-month fixed effects to isolate cross-sectional variation within sectors. That is, we compare future returns across funds within the same sector. We benchmark-adjust fund returns by subtracting the return on Morningstar's designated benchmark. The sample period is 1991 through 2014, as explained earlier.

The regression results, reported in Table 5, show that δ is indeed able to predict benchmark-adjusted fund returns. All estimated slope coefficients are negative, indicating that higher-δ funds have lower future returns. This predictive relation is statistically significant at all forecasting horizons up to three years. The relation is also economically significant. A one-standard-deviation increase in $\delta_{j,t-12}$ is associated with a 3.2 basis points decline in fund j's benchmark-adjusted return in month $t+1$, which translates into a nontrivial 38 basis points per year. While the predictive power of δ tends to decline with the forecasting horizon, the decline is modest. Even at the four-year horizon, a one-standard-deviation increase in $\delta_{j,t-12}$ is associated with a return decline of 13 basis points per year. In short, excess fund size negatively predicts benchmark-adjusted fund returns, consistent with the model.[16]

Next, we analyze the predictive power of δ in portfolio sorts. At the end of each month t, we sort funds within sectors into five portfolios based on their values of $\delta_{j,t-12}$. We then compute the returns of each of the five portfolios in month $t+1$ by equal-weighting the Morningstar-benchmark-adjusted returns of all funds on the portfolio. We repeat this procedure each month, generating monthly time series of benchmark-adjusted returns on the five portfolios between January 1991 and December 2014.

Table 6 shows that average benchmark-adjusted returns decrease monotonically across the five portfolios, from -0.028% per month for the lowest-δ quintile to -0.082% per month for the highest-δ quintile. The high-minus-low difference of -0.053% per month, or -0.64% per year, is substantial but not statistically significant ($t = -1.32$). Why is the statistical significance in Table 6 weaker than in Table 5? The predictive regressions in Table 5 are estimated in sample, whereas the portfolio sorts in Table 6 are out of sample, with portfolios formed each month based on publicly available information. Out-of-sample predictability is often weaker than in-sample predictability (e.g., Goyal and Welch, 2003).

Nonetheless, even the out-of-sample results in Table 6 become statistically significant when we focus on funds with high expense ratios, following the model's guidance. Recall from equation (26) that the negative relation between α and δ should be stronger for funds with higher f. To examine the role of f, we split funds into two groups. In each month t, we classify funds as either high-f or low-f depending on their expense ratio's position relative to the median across all funds in the given sector in month $t-1$.

Table 6 shows that the negative relation between α and δ is much stronger for high-f funds. Among high-f funds, high-δ funds underperform low-δ funds by 0.109% per month, or 1.31% per year. This difference is statistically significant ($t = -2.47$) and twice as large

[16] In additional tests, we run the same predictive panel regressions with a different regressor: instead of $\delta_{j,t-12}$, we use the product $\delta_{j,t-12} f_{j,t-12}$, where $f_{j,t-12}$ is fund j's expense ratio in month $t-12$. The results are very similar to those in Table 5, so we report them only in the Internet Appendix.

as its counterpart computed across all funds. Among low-f funds, the high-minus-low-δ difference is also negative, but it is small and insignificant. The high-minus-low-f difference between the two high-minus-low-δ differences is large and statistically significant ($t = -2.59$). Across all ten portfolios in the double sort, the lowest return is earned by the high-f-high-δ portfolio, and the highest return by the high-f-low-δ portfolio. To summarize, funds with higher excess size underperform those with lower excess size, especially among expensive funds. These results provide strong support for our model.

5. Conclusions

We make three sets of contributions. First, we model and document strong tradeoffs among the most salient mutual fund characteristics—fund size, expense ratio, turnover, and portfolio liquidity. We find empirically that active funds with smaller size, higher expense ratios, and lower turnover tend to hold less liquid portfolios. They also tend to hold more diversified portfolios. All of these findings are predicted by our equilibrium model, in which the key fund characteristics are jointly determined. Additional model predictions also hold in the data: larger funds are cheaper, and funds that trade less are larger and cheaper. These results provide strong new evidence of decreasing returns to scale in active management.

Second, we introduce the concept of portfolio liquidity. We show that a portfolio's liquidity depends not only on the liquidity of its holdings but also on its diversification. We derive simple measures of portfolio liquidity and diversification. Based on these measures, we find that active mutual funds' portfolios have become more liquid over time, mostly as a result of becoming more diversified. We also find that the components of portfolio liquidity are substitutes: funds holding less liquid stocks tend to diversify more.

Third, we use portfolio liquidity to identify and analyze capital misallocation to mutual funds. We find that misallocation gets corrected over time but persists for years. Excess fund size predicts performance: funds that are too big underperform those that are too small, consistent with decreasing returns to scale. This relation is stronger among more expensive funds, as predicted by the model.

Our empirical analysis focuses on U.S. equity mutual funds. Future research can apply our concepts and measures to portfolios held by other types of institutions, such as hedge funds, private equity funds, fixed income mutual funds, and pension funds. More research into relations among fund characteristics also seems warranted. Finally, it could be useful to analyze what determines misallocation of capital to funds, both theoretically and empirically.

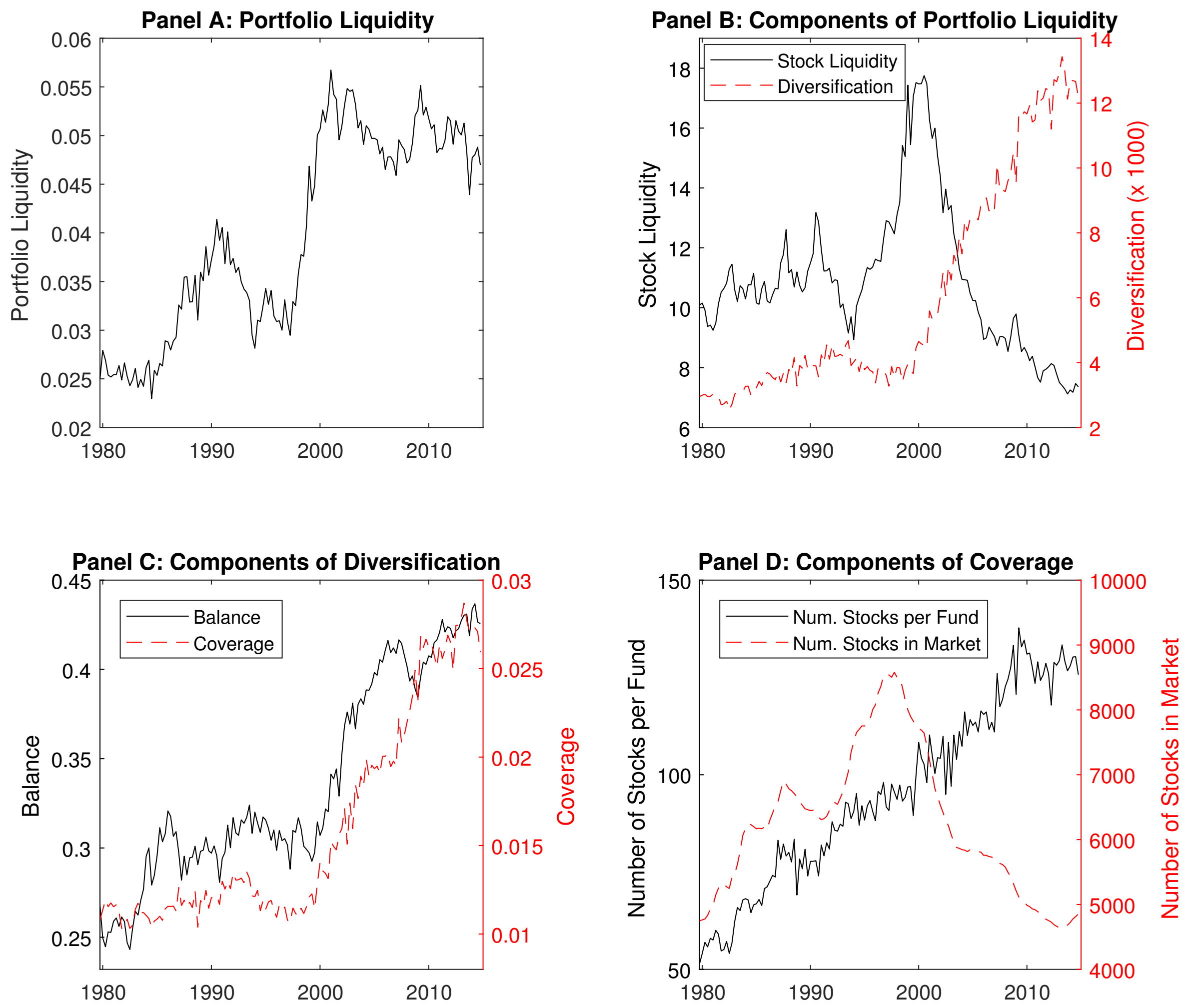

Figure 1. Time Series of Average Portfolio Liquidity and Its Components. This figure plots the quarterly time series of the cross-sectional means of portfolio liquidity, stock liquidity, diversification, coverage, balance, and the number of stocks held by each fund. Liquidity, diversification, and its components are computed with respect to the market benchmark. In Panel D we also plot the number of stocks in the market portfolio.

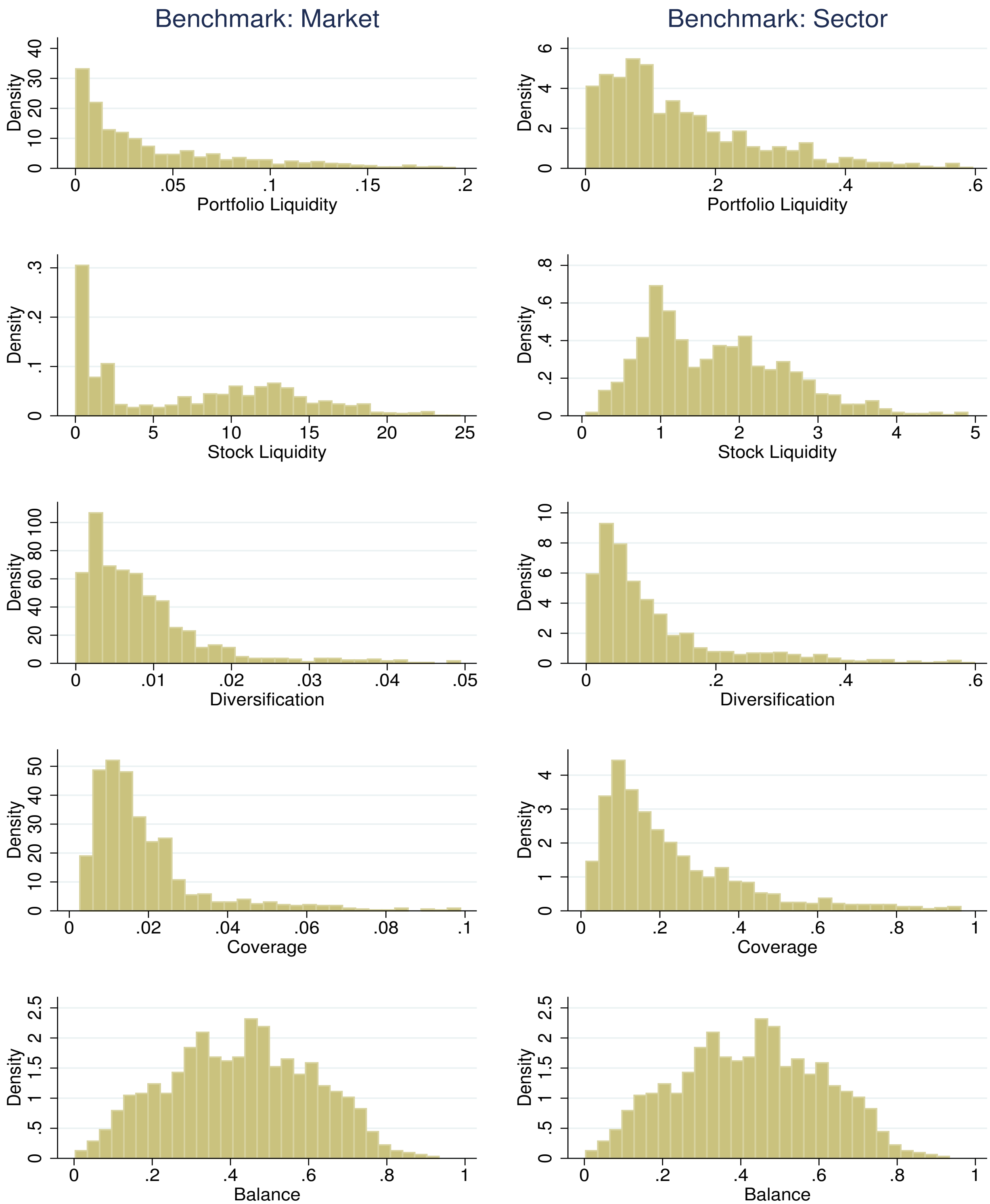

Figure 2. Cross Section of Portfolio Liquidity and Its Components. This figure plots histograms of portfolio liquidity, stock liquidity, diversification, coverage, and balance across all funds at the end of our sample (2014Q4).

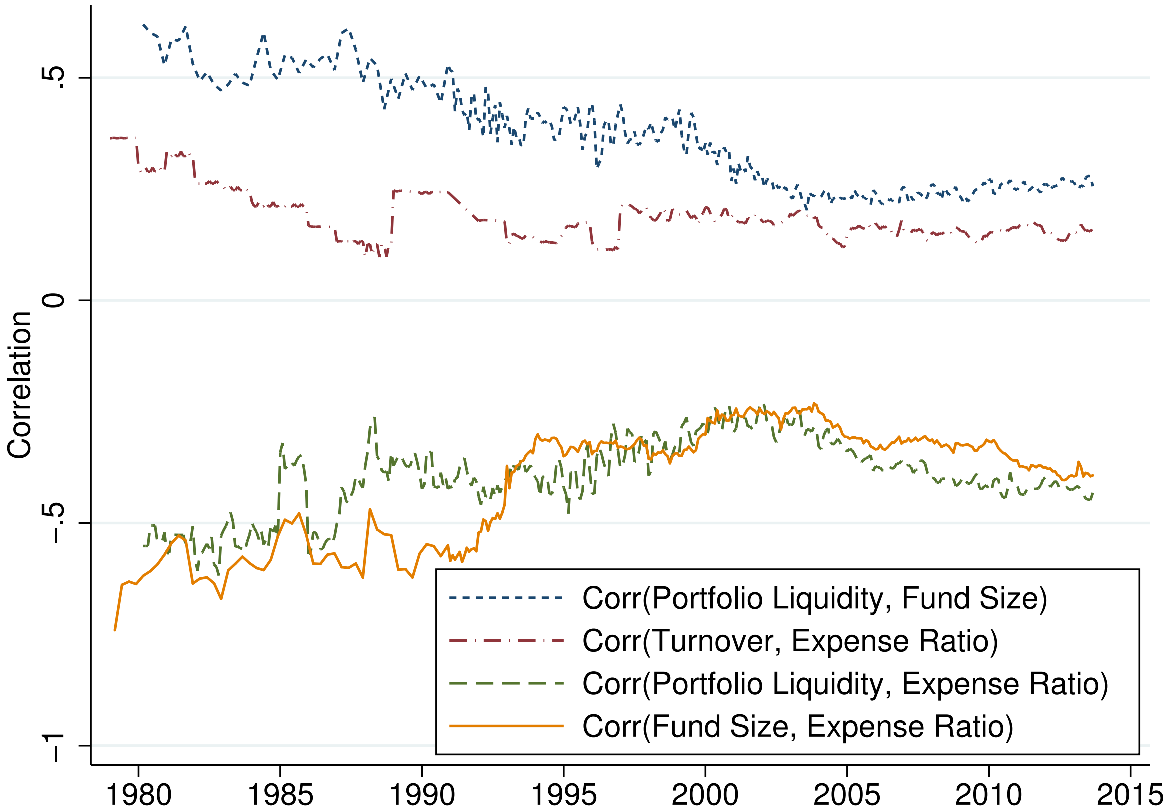

Figure 3. Cross-Sectional Correlations Over Time. This figure plots monthly time series of the cross-sectional correlation between the two variables noted in the legend. All variables are measured in logs. For each correlation, we drop months with fewer than 30 observations. To convert portfolio liquidity from a quarterly to a monthly variable, we take portfolio liquidity from the current month or, if missing, from the previous two months. Portfolio liquidity is computed with respect to the market benchmark.

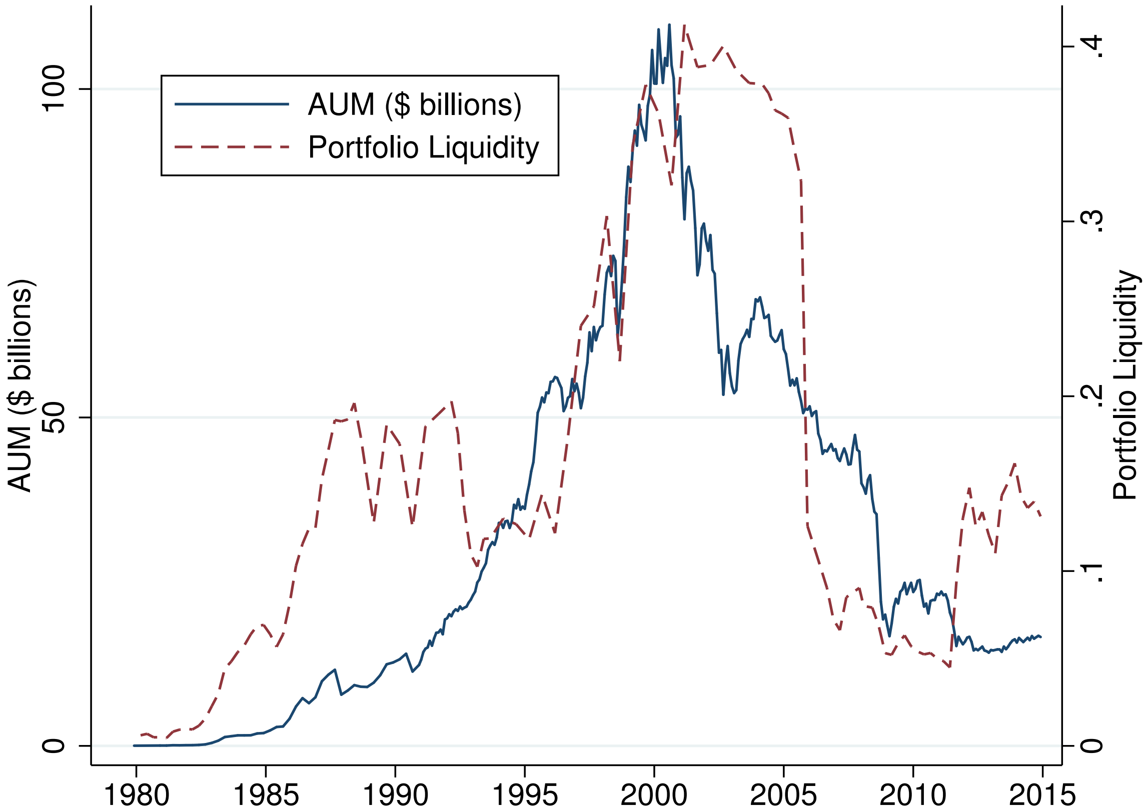

Figure 4. Fidelity Magellan Fund. This figure plots Magellan's assets under management (AUM) and portfolio liquidity, computed with respect to the market benchmark.

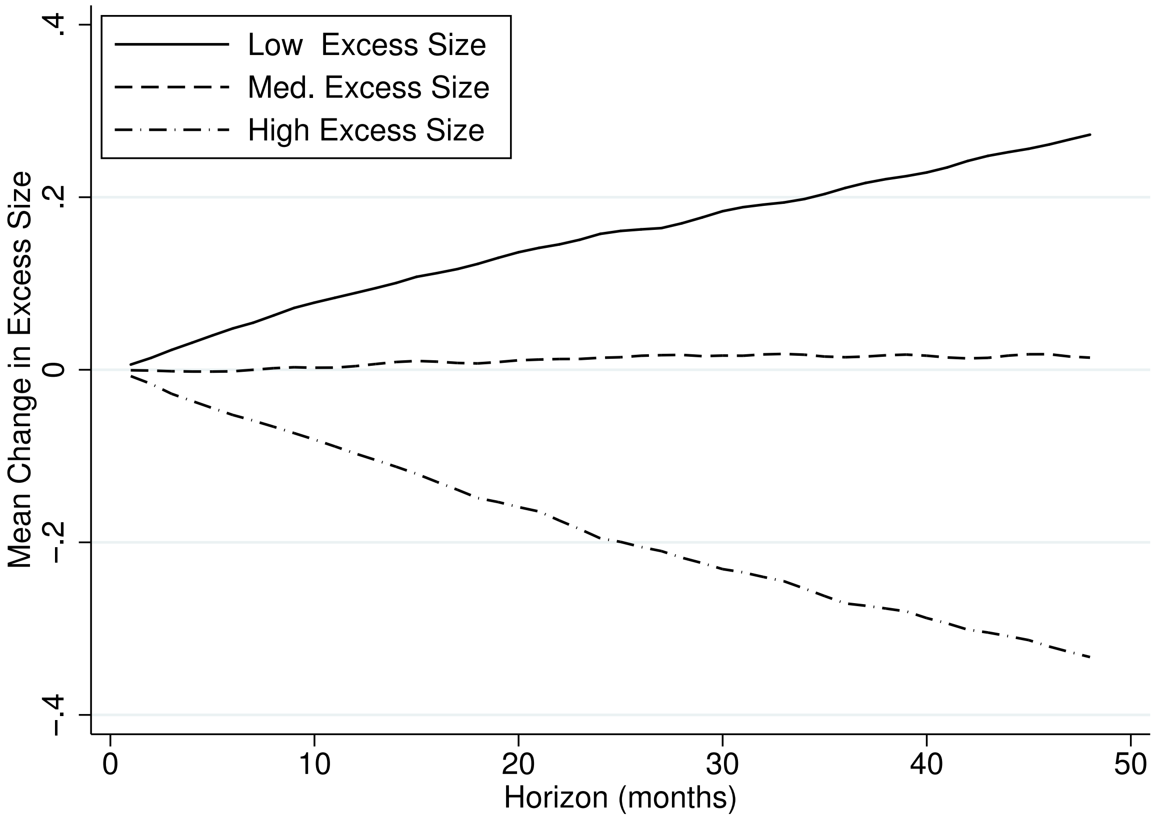

Figure 5. Does Excess Fund Size Adjust Over Time? At the end of each month t, we sort funds into quintiles within sectors based on $\delta_{j,t-12}$, which is the estimated residual for fund j and month $t-12$ from a rolling monthly panel IV regression that uses data from 1979 through month $t-12$. The IV specification corresponds to column (1) in Table 4. For each horizon τ between 1 and 48 months, and for each quintile, we compute the average of $\delta_{j,t+\tau} - \delta_{j,t}$ across months t and funds j in the quintile, weighting each fund/month observation equally. This figure plots these averages against horizon τ for quintiles 1, 3, and 5. We use a ten-year burn-in period for estimating the IV regressions.

Table 1
Correlations Between Portfolio Liquidity and Its Components

Panel A reports raw correlations, which are computed from panel data without any de-meaning. Panel B reports cross-sectional correlations computed by first de-meaning each variable using the mean across all observations from the same quarter, then computing the full-sample correlation between the two de-meaned variables. Panels C and D are the same as Panel B except that they replace quarter with quarter×sector (Panel C) or with fund (Panel D). All variables are measured in logs.

	Portfolio Liquidity	Stock Liquidity	Diversi-fication	Coverage	Balance
Panel A: Raw Correlations					
Portfolio Liquidity	1				
Stock Liquidity	0.712	1			
Diversification	0.300	-0.456	1		
Coverage	0.288	-0.339	0.826	1	
Balance	0.177	-0.386	0.748	0.244	1
Panel B: Cross-Sectional Correlations					
Portfolio Liquidity	1				
Stock Liquidity	0.744	1			
Diversification	0.282	-0.432	1		
Coverage	0.270	-0.307	0.805	1	
Balance	0.157	-0.362	0.731	0.183	1
Panel C: Cross-Sectional Correlations Within Sectors					
Portfolio Liquidity	1				
Stock Liquidity	0.228	1			
Diversification	0.798	-0.406	1		
Coverage	0.650	-0.302	0.797	1	
Balance	0.544	-0.311	0.703	0.132	1
Panel D: Time-Series Correlations					
Portfolio Liquidity	1				
Stock Liquidity	0.400	1			
Diversification	0.724	-0.342	1		
Coverage	0.524	-0.240	0.718	1	
Balance	0.547	-0.266	0.761	0.096	1

Table 2
Explaining Portfolio Liquidity and Its Components

This table presents results from five OLS panel regressions with dependent variables noted in the column headers. All regressors are measured contemporaneously with the dependent variable. All variables are measured in logs. The unit of observation is the fund/quarter. All regressions include sector×quarter fixed effects (FEs) and cluster by fund. The R^2 values in the penultimate row include the FEs' contribution. The last row contains the R^2 values from the regression of the dependent variable on the FEs alone. t-statistics are in parentheses.

	(1) Portfolio Liquidity	(2) Diversification	(3) Coverage	(4) Balance	(5) Stock Liquidity
Fund Size	0.124	0.134	0.0940	0.0452	0.0122
	(13.76)	(15.00)	(12.08)	(7.54)	(2.35)
Expense Ratio	-0.608	-0.622	-0.408	-0.238	-0.132
	(-11.26)	(-11.00)	(-9.33)	(-6.95)	(-5.26)
Turnover	0.101	0.122	0.102	0.0247	-0.0146
	(4.93)	(5.96)	(6.37)	(1.92)	(-1.32)
Stock Liquidity		-0.621	-0.337	-0.308	
		(-21.61)	(-14.21)	(-14.90)	
Balance			-0.0447		
			(-2.08)		
Coverage				-0.0343	
				(-2.09)	
Diversification					-0.264
					(-24.49)
Observations	76928	76928	76928	76928	76928
R^2	0.652	0.465	0.336	0.286	0.882
R^2 (FEs only)	0.598	0.240	0.163	0.172	0.857

Table 3
Correlations Among Fund Characteristics

This table reports correlations among the given fund characteristics, all measured in logs. Panel A reports correlations across funds within sector-quarters. Starting with our full panel dataset, we first de-mean each variable using the mean across all observations in the same sector and quarter, then we compute the full-sample correlation between the two de-meaned variables. Panel B reports time-series correlations within funds, which we compute analogously except that we de-mean each variable using each fund's time-series mean. Fund size is scaled by total stock market capitalization. Portfolio liquidity is defined with respect to the market benchmark. t-statistics are computed clustering by fund.

	Fund Size	Expense Ratio	Portfolio Liquidity	Turnover	Activeness
Panel A: Cross-Sectional Correlations Within Sectors					
Fund Size	1				
Expense Ratio	-0.315	1			
	(-15.30)				
Portfolio Liquidity	0.285	-0.291	1		
	(17.90)	(-13.39)			
Turnover	-0.105	0.130	0.039	1	
	(-6.01)	(6.35)	(1.92)		
Activeness	-0.231	0.255	-0.440	0.880	1
	(-13.27)	(13.58)	(-24.35)	(93.41)	
Panel B: Time-Series Correlations					
Fund Size	1				
Expense Ratio	-0.251	1			
	(-17.61)				
Portfolio Liquidity	0.308	-0.118	1		
	(18.23)	(-6.87)			
Turnover	-0.147	0.105	-0.109	1	
	(-12.17)	(7.57)	(-6.85)		
Activeness	-0.268	0.136	-0.540	0.895	1
	(-17.60)	(9.20)	(-40.14)	(127.68)	

Table 4
Table 2 Reestimated by Using Instrumental Variables

This table is analogous to Table 2, except that we estimate the models by IV instead of OLS. We instrument for the natural logarithm of fund size by using the natural logarithm of a variable equal to family size plus $15 million.

	(1) Portfolio Liquidity	(2) Diversification	(3) Coverage	(4) Balance	(5) Stock Liquidity
Fund Size	0.338	0.368	0.270	0.130	0.0423
	(13.53)	(14.88)	(14.00)	(7.42)	(2.77)
Expense Ratio	-0.289	-0.278	-0.166	-0.137	-0.0985
	(-4.52)	(-4.17)	(-3.24)	(-3.66)	(-3.50)
Turnover	0.130	0.156	0.129	0.0410	-0.00878
	(5.89)	(7.00)	(7.33)	(3.00)	(-0.75)
Stock Liquidity		-0.575	-0.321	-0.307	
		(-19.39)	(-13.10)	(-15.07)	
Balance			-0.103		
			(-4.57)		
Coverage				-0.0802	
				(-4.16)	
Diversification					-0.277
					(-23.34)
Observations	76828	76828	76828	76828	76828

Table 5
Predicting Fund Returns with Excess Fund Size

This table contains results from panel regressions of benchmark-adjusted fund returns in month $t+\tau$ on funds' excess size in month $t-12$ and sector$\times$month fixed effects. The values of τ are shown in the table's top row. The regressor, $\delta_{j,t-12}$, is the estimated residual for fund j in month $t-12$ from a rolling IV panel regression that uses data from 1979 to month $t-12$. The IV specification corresponds to column (1) in Table 4. We allow a ten-year burn-in period for the IV regressions. All returns are in percent per month. We report t-statistics clustered by sector$\times$month.

	Forecasting Horizon (τ, in months)						
	1	3	6	12	24	36	48
Excess Size	-0.0360	-0.0349	-0.0282	-0.0283	-0.0236	-0.0177	-0.0118
	(-4.00)	(-3.95)	(-3.31)	(-3.51)	(-2.97)	(-2.38)	(-1.57)
Observations	199,311	195,765	190,612	180,763	161,449	143,412	127,440

Table 6
Predicting Fund Returns Out of Sample: Portfolio Sorts

In the "All Funds" row, this table reports average benchmark-adjusted returns, in percent per month, on portfolios formed based on funds' excess size. At the end of each month t we sort funds within sectors into five equal-weighted portfolios based on their values of $\delta_{j,t-12}$, which is the estimated residual for fund j in month $t-12$ from a rolling IV panel regression that uses data from 1979 to month $t-12$. The IV specification corresponds to column (1) in Table 4. We allow a ten-year burn-in period for the IV regressions. We record the portfolios' returns in month $t+1$ and report their averages in this table. We drop months with fewer than five funds in any portfolio. The portfolios' benchmark-adjusted returns are computed by averaging funds' net returns in excess of Morningstar's designated benchmark return. The table also reports analogous results for high- and low-expense-ratio subsets of funds, which are created based on funds' lagged expense ratios relative to the month's median. t-statistics are in parentheses.

	Excess Fund Size					
	Low	2	3	4	High	High−Low
All Funds	-0.028	-0.055	-0.062	-0.071	-0.082	-0.053
	(-0.78)	(-1.91)	(-2.26)	(-2.73)	(-3.02)	(-1.32)
High Expense Ratio	-0.006	-0.078	-0.094	-0.081	-0.115	-0.109
	(-0.14)	(-2.29)	(-3.26)	(-2.77)	(-4.18)	(-2.47)
Low Expense Ratio	-0.040	-0.038	-0.037	-0.052	-0.046	-0.006
	(-0.98)	(-1.23)	(-1.16)	(-1.72)	(-1.48)	(-0.13)
High−Low	0.034	-0.040	-0.057	-0.029	-0.069	-0.103
	(0.91)	(-1.26)	(-2.08)	(-1.08)	(-3.23)	(-2.59)

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APPENDIX—FOR ONLINE PUBLICATION

Appendix A. Proofs

Proof that the most liquid portfolio is the benchmark portfolio:

Starting from equation (3), we solve the following constrained minimization problem:

$$\min_{\{w_i\}} \sum_{i=1}^{N_M} \frac{w_i^2}{m_i} \qquad \text{subject to } \sum_{i=1}^{N_M} w_i = 1 \ . \tag{A1}$$

The problem is convex, so the first-order conditions describe the minimum. Denoting the optimal portfolio weights by $\tilde{w}_i$ and the Lagrange multiplier by ζ, the first-order conditions are $\frac{2\tilde{w}_i}{m_i} - \zeta = 0$, so that $\tilde{w}_i = \frac{\zeta m_i}{2}$. Substituting into the constraint yields $\sum_{i=1}^{N_M} \frac{\zeta m_i}{2} = 1$, which implies $\frac{\zeta}{2} \sum_{i=1}^{N_M} m_i = 1$, which in turn implies $\zeta = 2$, which then gives $\tilde{w}_i = m_i$.

A different proof, which is instructive in its own right, relies on a perturbation argument. Consider a portfolio with liquidity L. We perturb this portfolio by buying a bit of stock i and selling a bit of stock j, so the new portfolio weights are $w_i^* = w_i + u$ and $w_j^* = w_j - u$, where $u > 0$ and all other weights remain the same. The portfolio's illiquidity changes to

$$\begin{aligned} \left(L^{-1}\right)^* &= \sum_{n \notin \{i,j\}} \frac{w_n^2}{m_n} + \frac{(w_i + u)^2}{m_i} + \frac{(w_j - u)^2}{m_j} \\ &= L^{-1} + 2u \left(\frac{w_i}{m_i} - \frac{w_j}{m_j}\right) + u^2 \left(\frac{1}{m_i} + \frac{1}{m_j}\right) \ . \end{aligned} \tag{A2}$$

If the original portfolio is the benchmark portfolio, for which $w_i/m_i = w_j/m_j = 1$, it follows immediately that any perturbation increases portfolio illiquidity: $(L^{-1})^* > L^{-1}$.

Proof of equation (10):

First, define $m = \sum_{i=1}^{N} m_i$ and note that

$$m = \sum_{i=1}^{N} m_i = \sum_{i=1}^{N} \frac{M_i}{M} = \frac{\sum_{i=1}^{N} M_i}{\sum_{i=1}^{N_M} M_i} = \frac{N}{N_M} \times \frac{1}{N} \sum_{i=1}^{N} \frac{M_i}{\frac{1}{N_M} \sum_{j=1}^{N_M} M_j} \ . \tag{A3}$$

Second, rearrange the inverse of portfolio liquidity from equation (3) as follows:

$$\begin{aligned} L^{-1} &= \sum_{i=1}^{N} \frac{w_i^2}{m_i} = \frac{1}{m} \sum_{i=1}^{N} \frac{w_i^2}{m_i^*} = \frac{1}{m} \sum_{i=1}^{N} m_i^* \left(\frac{w_i}{m_i^*}\right)^2 = \frac{1}{m} \mathrm{E}^* \left\{ \left(\frac{w_i}{m_i^*}\right)^2 \right\} \\ &= \frac{1}{m} \left[\left(\mathrm{E}^* \left\{ \frac{w_i}{m_i^*} \right\} \right)^2 + \mathrm{Var}^* \left(\frac{w_i}{m_i^*} \right) \right] = \frac{1}{m} \left[1 + \mathrm{Var}^* \left(\frac{w_i}{m_i^*} \right) \right] \ , \end{aligned} \tag{A4}$$

where E^* is the expectation with respect to the m^* measure. Combining equations (A3) and (A4) yields equation (10).

Proofs of statements from Section 3.4:

First, we prove that the cross-sectional correlation between fund size and expense ratio is negative as long as $\beta_{\mu,f} < 1/2$. Take logs in equation (17), so that

$$\log(A) = -\log(f) + 2\log(\mu) + \text{constant} \ , \tag{A5}$$

and note that

$$\begin{aligned} \text{Cov}(\log(A), \log(f)) &= \text{Cov}(-\log(f) + 2\log(\mu), \log(f)) \\ &= 2\text{Cov}(\log(\mu), \log(f)) - \text{Var}(\log(f)) \ . \end{aligned} \tag{A6}$$

For this covariance to be negative, we need $\text{Cov}(\log(\mu), \log(f))/\text{Var}(\log(f) < \frac{1}{2}$, or $\beta_{\mu,f} < \frac{1}{2}$.

Second, we prove that the correlation between $TL^{-\frac{1}{2}}$ and f is positive as long as $\beta_{\mu,f} < 1$. Take logs in equation (18), so that

$$\log(TL^{-\frac{1}{2}}) = \log(f) - \log(\mu) + \log(2) \ , \tag{A7}$$

and note that

$$\begin{aligned} \text{Cov}(\log(TL^{-\frac{1}{2}}), \log(f)) &= \text{Cov}(\log(f) - \log(\mu), \log(f)) \\ &= \text{Var}(\log(f)) - \text{Cov}(\log(\mu), \log(f)) \ . \end{aligned} \tag{A8}$$

For this covariance to be positive, we need $\text{Cov}(\log(\mu), \log(f))/\text{Var}(\log(f) < 1$, or $\beta_{\mu,f} < 1$.

Finally, we prove that the correlation between $TL^{-\frac{1}{2}}$ and A is negative as long as $\beta_{\mu,A} < 1$. Take logs in equation (14), so that

$$\log(TL^{-\frac{1}{2}}) = \log(\mu) - \log(A) + \text{constant} \ , \tag{A9}$$

and note that

$$\begin{aligned} \text{Cov}(\log(TL^{-\frac{1}{2}}), \log(A)) &= \text{Cov}(\log(\mu) - \log(A), \log(A)) \\ &= \text{Cov}(\log(\mu), \log(A)) - \text{Var}(\log(A)) \ . \end{aligned} \tag{A10}$$

For this covariance to be negative, we need $\text{Cov}(\log(\mu), \log(A))/\text{Var}(\log(A) < 1$, or $\beta_{\mu,A} < 1$.

Proofs of statements from Section 4:

The fund chooses $TL^{-\frac{1}{2}}$ according to equation (14), so that $TL^{-\frac{1}{2}} = \mu M/(2cA)$. Since the equilibrium values satisfy $\bar{T}\bar{L}^{-\frac{1}{2}} = \mu M/(2c\bar{A})$, equation (24) implies that $TL^{-\frac{1}{2}} = \bar{T}\bar{L}^{-\frac{1}{2}}/\left(1+\delta\right)$. From equation (18), we have $\mu = 2f/(\bar{T}\bar{L}^{-\frac{1}{2}})$. Substituting this for μ in equation (14) yields $T^2L^{-1} = fM/(Ac\left(1+\delta\right))$. Taking logs and approximating $\delta \approx \log\left(1+\delta\right)$, we obtain equation (25). We use the equation $T^2L^{-1} = fM/(Ac\left(1+\delta\right))$ again when substituting for T^2L^{-1} in equation (15). We also substitute $\mu = 2f/(\bar{T}\bar{L}^{-\frac{1}{2}}) = 2f/(TL^{-\frac{1}{2}}\left(1+\delta\right))$ in equation (15). That equation then quickly turns into equation (26).

Appendix B. Alternative Trading Cost Functions

We now make more general assumptions about the trading cost function. In equation (5), the cost per dollar traded increases linearly with the ratio of the dollar amount traded to market capitalization. We now replace the linearity in equation (5) by nonlinearity:

$$C_i = \tilde{c}\left(\frac{D_i}{M_i}\right)^{\gamma} , \tag{A11}$$

where $\gamma > 0$. The proportional trading cost function then becomes

$$\mathrm{E}\left(\frac{C}{A}\right) = \left(\frac{A}{M}\right)^{\gamma} T^{1+\gamma} c \underbrace{\left(\sum_{i=1}^{N} \frac{w_i^{1+\gamma}}{m_i^{\gamma}}\right)}_{L^{-1}} , \tag{A12}$$

where $c = \tilde{c}\,\mathrm{E}\left\{(1+\varepsilon_i)^{1+\gamma}\right\}$, so that portfolio liquidity is given by

$$L = \left(\sum_{i=1}^{N} \frac{w_i^{1+\gamma}}{m_i^{\gamma}}\right)^{-1} . \tag{A13}$$

For the baseline case of $\gamma = 1$, which we use throughout the paper, equations (A11), (A12), and (A13) simplify to equations (5), (7), and (3), respectively. Under this alternative definition of L, we still have $L \in (0, 1]$, and the maximum value of $L = 1$ is still achieved by the benchmark portfolio.

This alternative measure of portfolio liquidity can also be decomposed into stock liquidity and diversification, as in equation (10), but the formulas are a bit more complicated:

$$L = \underbrace{\left(\frac{1}{N}\sum_{i=1}^{N} \mathcal{L}_i\right)^{\gamma}}_{\text{Stock Liquidity}} \times \underbrace{\left(\frac{N}{N_M}\right)^{\gamma} \left[\mathrm{E}^*\left\{\left(\frac{w_i}{m_i^*}\right)^{1+\gamma}\right\}\right]^{-1}}_{\text{Diversification}} . \tag{A14}$$

Our equilibrium analysis from Section 3.2 generalizes easily. Equation (12) becomes $a = \mu T\, L^{-\frac{1}{1+\gamma}}$. Combining this profit function with the cost function in equation (A12), we obtain net profit, which the fund maximizes by choosing $TL^{-\frac{1}{1+\gamma}}$. Setting $\alpha = 0$, we obtain

$$A = \frac{\mu^{\frac{1+\gamma}{\gamma}}}{c^{\gamma} f} \frac{\gamma M}{(1+\gamma)^{\frac{1+\gamma}{\gamma}}} . \tag{A15}$$

which is the counterpart of equation (17). Continuing as in the baseline case, we obtain

$$\log(L) = \gamma \log(A/M) - \log(f) + (1+\gamma)\log(T) + \log(\gamma) + \left(1 - \gamma - \gamma^2\right)\log(c) . \tag{A16}$$

Comparing this equation to equation (20), we see that all of the empirical predictions that we test in Table 2 continue to hold in this alternative framework. In the regression of portfolio liquidity on fund size, expense ratio, and turnover, the slopes on all three regressors retain the same signs as in the baseline case of $\gamma = 1$. When we reestimate our results from Table

2 for the alternative measure of L with values of γ ranging from 0.1 to 0.9, we find similar results, as we show in the Internet Appendix.

Another possible modification involves introducing stock-specific values of c_i and turnover. Define stock i's turnover as $t_i = V_i/M_i$, where V_i is total dollar volume for stock i. Given this stock-level heterogeneity, it makes sense to modify equations (5) and (6) into

$$C_i = \tilde{c}_i \frac{D_i}{V_i} \tag{A17}$$

$$D_i = ATt_i w_i(1+\varepsilon_i) \; . \tag{A18}$$

The idea behind these modifications is that cost per dollar traded is increasing in the fraction of the stock's volume traded, and that each fund trades more of stocks that have higher turnover. The cost function from equation (7) then becomes

$$\mathrm{E}\left(\frac{C}{A}\right) = \left(\frac{A}{M}\right) T^2 \underbrace{\left(\sum_{i=1}^{N} \frac{c_i t_i w_i^2}{m_i}\right)}_{L^{-1}} , \tag{A19}$$

where $c_i = \tilde{c}_i\,(1+\sigma_\epsilon^2)$. In this case, our portfolio liquidity measure generalizes as indicated in equation (A19). While modeling stock-level turnover t_i is beyond the scope of this paper, it seems reasonable to assume that t_i is lower for stocks with higher transaction costs c_i. In fact, at the fund level, this is a result in our model: recall that funds with less liquid portfolios optimally choose to trade less. If $c_i t_i$ is constant across stocks in the benchmark, then the liquidity measure from equation (A19) simplifies to the one from equation (3).

Finally, we combine the above extensions and assume that

$$C_i = \tilde{c}_i \left(\frac{D_i}{V_i}\right)^{\gamma} \tag{A20}$$

along with equation (A18). The cost function then becomes

$$\mathrm{E}\left(\frac{C}{A}\right) = \left(\frac{A}{M}\right)^{\gamma} T^{1+\gamma} \underbrace{\left(\sum_{i=1}^{N} \frac{c_i t_i w_i^{1+\gamma}}{m_i^{\gamma}}\right)}_{L^{-1}} , \tag{A21}$$

where $c_i = \tilde{c}_i \mathrm{E}\left\{(1+\varepsilon_i)^{1+\gamma}\right\}$. Our portfolio liquidity measure then generalizes as indicated in equation (A21). If $c_i t_i$ is constant across stocks in the benchmark, then this liquidity measure simplifies to the one in equation (A13).

Appendix C. Data

To construct our sample of actively managed U.S. domestic equity mutual funds, we begin with the 1979–2011 dataset constructed by Pástor, Stambaugh, and Taylor (2015), which combines and cross-validates data from CRSP and Morningstar. A detailed description of the dataset is in the online Data Appendix to that paper. We expand the dataset by merging it with the Thomson Reuters dataset of fund holdings and adding data from 2012 through 2014. We restrict the sample to include fund-month observations whose historical

Morningstar category falls within the traditional 3×3 style box (small-cap, mid-cap, large-cap interacted with growth, blend, and value). This restriction excludes non-equity funds, international funds, and industry-specific funds. We also exclude funds identified by CRSP or Morningstar as index funds, funds whose name contains the word "index," and funds classified by Morningstar as funds of funds. We exclude fund-month observations with expense ratios below 0.1% per year since they are extremely unlikely to belong to actively managed funds. Finally, we exclude fund-month observations with lagged fund size below $15 million in 2011 dollars. We aggregate share classes belonging to the same fund.[17]

When computing portfolio weights w, we drop all fund holdings that are not included in our definition of the market portfolio, which is guided by the holdings of Vanguard's Total Stock Market Index fund. A fund's holding can fall outside the market if its CUSIP cannot be linked to the CRSP database (1.0% of the Vanguard fund's holdings), or if the security is in CRSP but outside our definition of the market (e.g. ADRs, units, limited partnerships, etc., which make up 0.1% of the Vanguard fund's holdings). These holdings mainly represent cash, bonds, and other non-equity securities. For the median (average) fund/month observation in our sample, 2.3% (3.5%) of holding names and 1.9% (3.1%) of holding dollars are outside the market.

When computing fund size, we cross-verify monthly AUM between CRSP and Morningstar as described in Pástor, Stambaugh, and Taylor (2015). Annual data on expense ratios and turnover are from CRSP. Turnover is the minimum of the fund's dollar buys and sells during the fiscal year, scaled by the fund's average total net assets. Following Pástor, Stambaugh, and Taylor (2017), we winsorize turnover at the 1st and 99th percentiles. Monthly fund returns, net of expense ratio, are from CRSP and Morningstar. Following Pástor, Stambaugh, and Taylor (2015), we require that CRSP and Morningstar agree closely on a fund's return; otherwise we set it to missing.

For any fund-level variable requiring holdings data, we set the variable to missing if there is a large discrepancy in a fund's AUM between our CRSP/Morningstar database and the Thomson Reuters holdings database. We compute the ratio of the fund's AUM according to CRSP/Morningstar to the fund's AUM obtained by adding up all the fund's holdings from Thomson Reuters. If this ratio exceeds 2.0 or is less than 0.5, we set all holdings-based measures to missing. This filter drops the holdings-based variables for 3.4% of fund/quarter observations. We suspect that some of these large discrepancies are due to poor links between Thomson Reuters and CRSP/Morningstar.

[17] Many mutual funds offer multiple share classes, which represent claims on the same underlying assets but have different fee structures. Different share classes of the same fund have the same Morningstar FundID. We aggregate all share classes of the same fund. Specifically, we compute a fund's size by summing AUM across the fund's share classes, and we compute the fund's expense ratio, returns, and other variables by asset-weighting across share classes.

Table A1
Summary Statistics

This table presents summary statistics of the fund-level variables used in the empirical analysis. Portfolio liquidity and its components (the first five variables) are defined in Section 2. These variables are measured quarterly as they require holdings data. The remaining variables, which are measured monthly, are defined in this Appendix. Fund size and family size are measured as fractions of the total stock market capitalization. Expense ratio and turnover are in units of fraction per year. Adjusted return is the fund's return in excess of the return on the Morningstar-designated benchmark. Both returns are in units of fraction per month.

	N	Mean	Stdev.	P1	P25	P50	P75	P99
Portfolio Liquidity	93,366	0.0461	0.0636	0.0006	0.0075	0.0227	0.0619	0.2949
Stock Liquidity	93,366	10.68	10.12	0.15	1.63	9.18	16.67	42.31
Diversification	93,366	0.0080	0.0190	0.0002	0.0020	0.0042	0.0084	0.0585
Coverage	93,366	0.0191	0.0332	0.0029	0.0077	0.0121	0.0194	0.1312
Balance	93,366	0.3711	0.1835	0.0389	0.2271	0.3584	0.5052	0.7838
Fund Size$\times 10^4$	351,243	0.955	3.472	0.011	0.052	0.170	0.594	14.319
Family Size$\times$100	377,842	0.439	1.217	0.000	0.007	0.066	0.296	6.393
Expense Ratio	365,301	0.0123	0.0044	0.0034	0.0095	0.0117	0.0146	0.0250
Turnover	336,006	0.83	0.70	0.03	0.34	0.64	1.10	3.89
Return	394,203	0.0081	0.0525	-0.1494	-0.0190	0.0123	0.0386	0.1309
Adjusted Return	384,508	-0.0006	0.0219	-0.0602	-0.0100	-0.0009	0.0083	0.0628