Financial Intermediary Leverage and Value-at-Risk*

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The Changing Nature of Credit Markets: Risks and Opportunities
Institute for Financial Research (SIFR)
Stockholm, August 25–26, 2008

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York or the Federal Reserve System.

Motivation

- Repos are major funding source to market based financial system.
  - Haircuts constrain maximum leverage.
  - Liquidiy and Leverage: Adrian Shin (2007).
- Dislocations in repo markets can lead to spreading of crisis.
  - Adverse feedback loop (FOMC).
  - Margin spiral (Brunnermeier-Pedersen).
  - Contagion (Kyle-Xiong, Adrian-Brunnermeir *CoVaR*).
  - Extreme: haircut runs (Bear Stearns).
- In tri-party repo contract lender keeps collateral in case of bankruptcy.

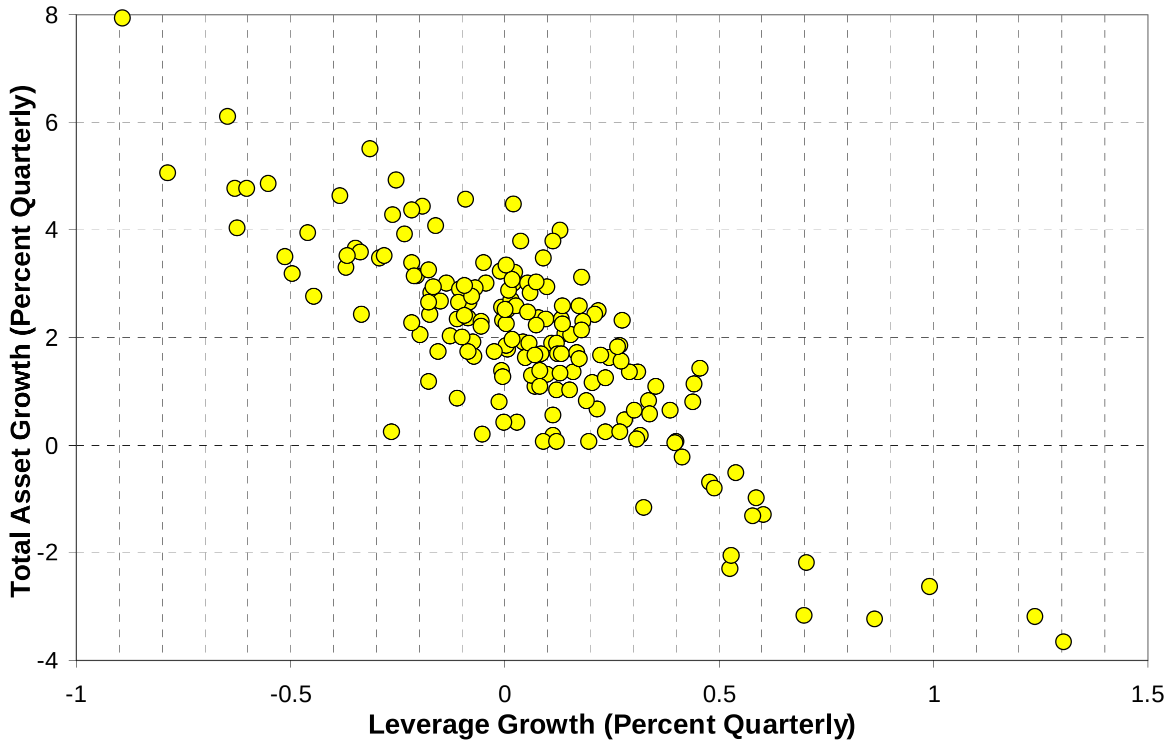
Overview

- Microeconomic foundation for the economics of repo borrowing:
  - Corporate finance of broker-dealers / market based financial system.
- Bank maximizes return on equity by taking on leverage.
  - Has incentive to choose risky projects.
  - Lender imposes constraint on bank by imposing haircut.

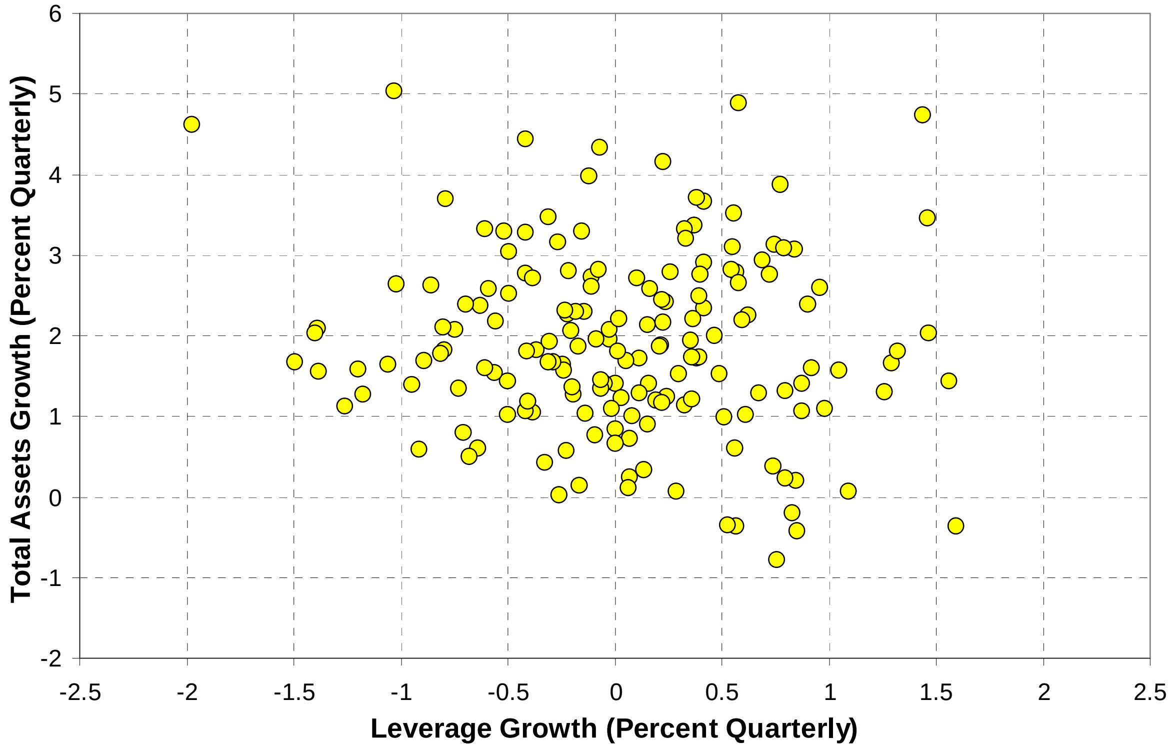
1. Result: procyclical leverage.
2. Result: VaR rule optimal in certain cases.

- KEY: Corporate finance of broker-dealers profoundly different than corporate finance of non-financials or households.

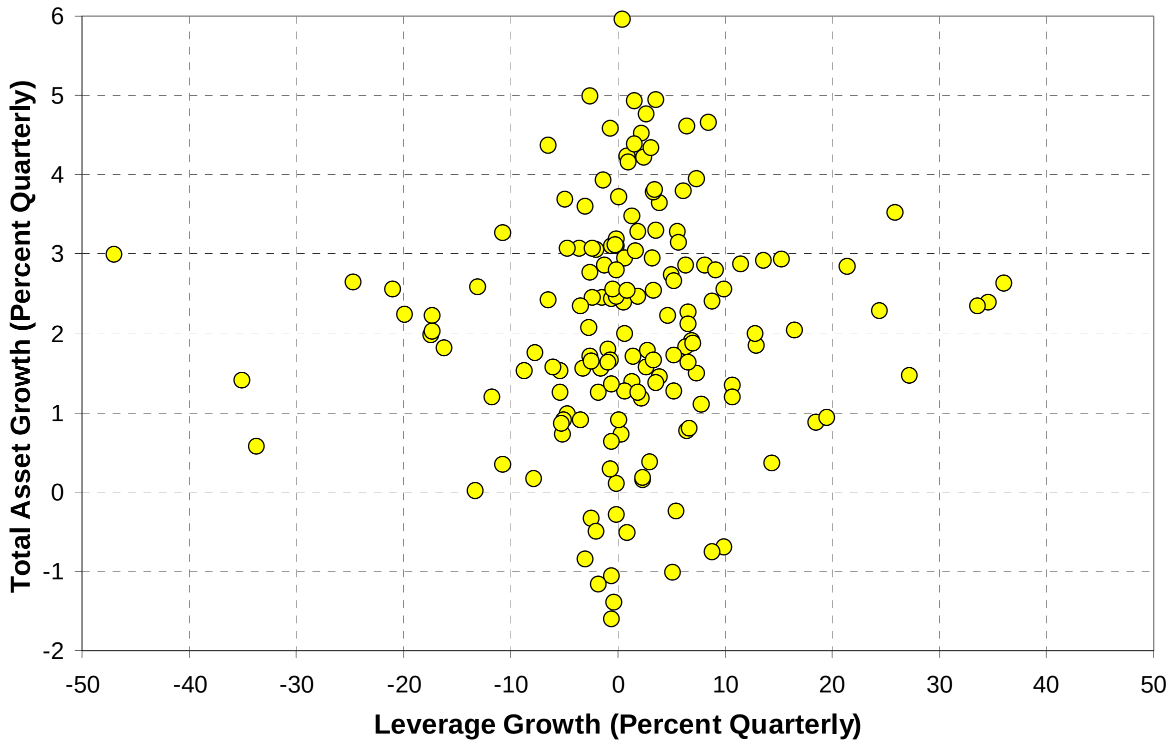
Balance Sheet Size and Leverage: Households



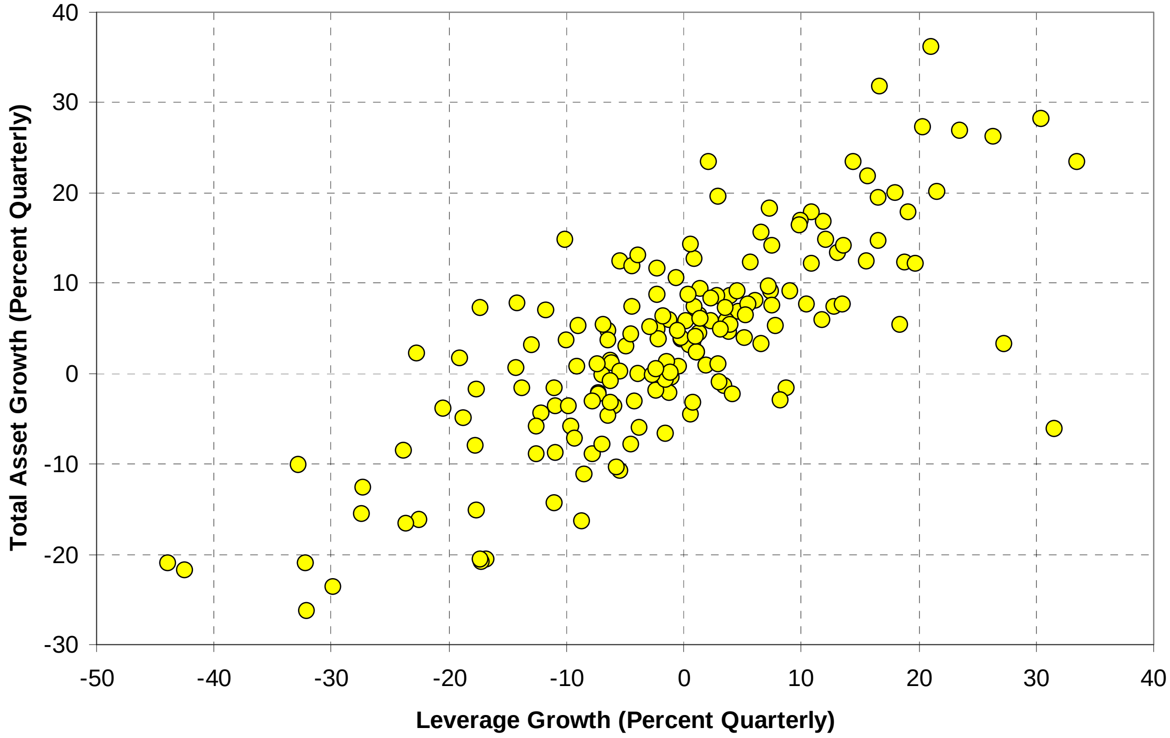
Non-Financial, Non-Farm Corporations



Commercial Banks



Security Dealers and Brokers



Model

Agent is a broker-dealer (investment bank).

Principal is creditor to the bank (e.g. money market mutual fund).

Balance sheet at date 0 in market values:

Assets	Liabilities
Assets A	Debt D
	Equity E

Balance sheet in notional (face) values:

Assets	Liabilities
Assets $A(1+\bar{r})$	Debt $\bar{D}$
	Equity $\bar{E}$

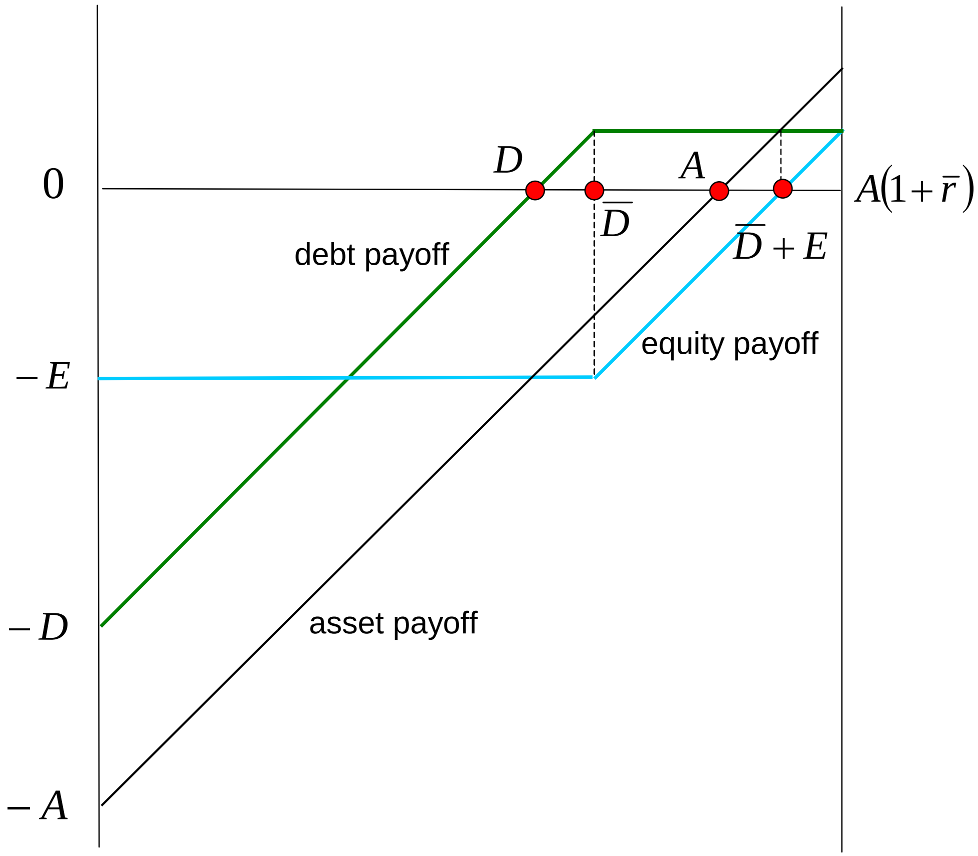
Repo transaction: Sell A worth of securities for D at date 0, repurchase for $\bar{D}$ at date 1. Then, $\bar{D} - D$ is "haircut" and $\bar{D}/D - 1$ is repo interest.

Solving for Leverage and Balance Sheet Size

Fix E. Optimal contract maximizes agent's payoff by choice of A, D and $\bar{D}$ subject to (IC) and (IR).

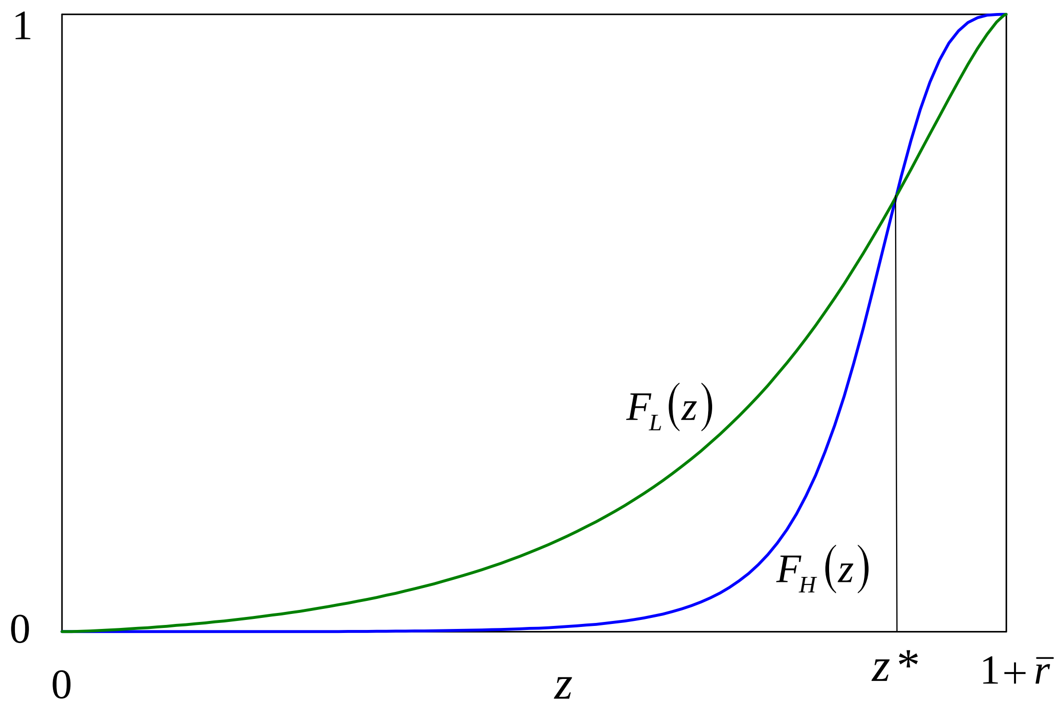
- Solve for:
  - Leverage $A/(A-D)$
  - Balance sheet size A
  - Repo spread $\bar{D}/D-1$
- Creditor's payoff has embedded short put option
- Banks' payoff has embedded long put option

Net Payoffs


0
$-E$
$-D$
$-A$
D
$\overline{D}$
A
$\overline{D}+E$
$A(1+\bar{r})$
debt payoff
equity payoff
asset payoff

Moral Hazard

Choice between two types of securities. One dollar of good security has expected payoff $1+r_H$. Outcome density $f_H(.)$. Bad security has expected payoff $1+r_L$ with density $f_L(.)$.



Creditor's Payoff

- Creditor's *net* expected payoff is

$$V(A) = \bar{D} - D - \pi_H\left(\bar{D}\right) \equiv A\left(\bar{d} - d - \pi_H\left(\bar{d}\right)\right)$$

$\pi_H\left(\bar{d}\right)$ is price of put \$1 of assets with strike $\bar{d}$ (Merton, 1974).

- The credit holder's a portfolio:
  - short put option with strike price $\bar{d}$
  - long risky asset with expected payoff $A\left(\bar{d} - d\right)$

- Participation constraint is

$$V(A) \geq 0 \tag{IR}$$

Bank Equity Holder's Payoff

The bank's equity holder is residual claimant. Net expected payoff:

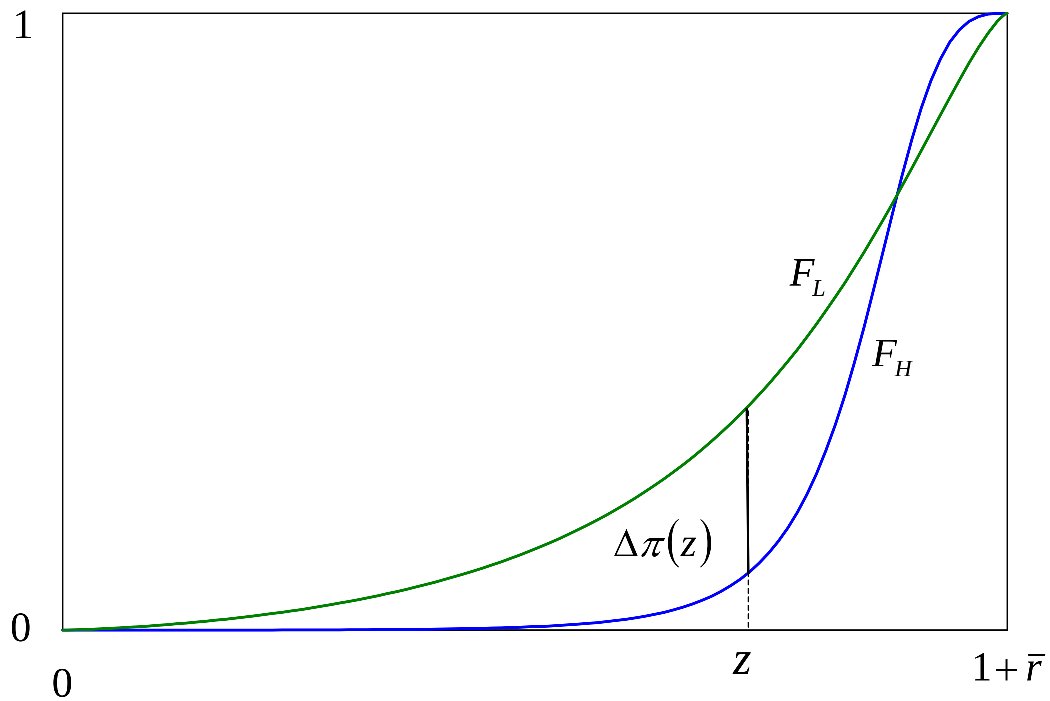
$$U(A) = A\left(r - \bar{d} + d + \pi\left(\bar{d}\right)\right)$$

- The equity holder's stake is a portfolio:
  - put option on the assets of the bank with strike price $\bar{D}$
  - risky asset with expected payoff $A\left(r - \bar{d} + d\right)$
- Incentive compatibility constraint:

$$r_H - r_L \geq \pi_L\left(\bar{d}\right) - \pi_H\left(\bar{d}\right) = \Delta\pi\left(\bar{d}\right)$$

(analogous to private benefit in Holmstrom and Tirole (1997)).

Lemma 1. $\Delta\pi(z)$ *is a single-peaked function of* z*, and is maximized at the value of* z *where* F_H *cuts* F_L *from below.*



Difference in Option Values $\Delta\pi(z)$

Leverage Constraint

Assume (IC) binds. Then optimal $\bar{d}^*$ is smallest solution to the equation:

$$\Delta\pi\left(\bar{d}\right) = r_H - r_L$$

$\bar{d}^*$ is mixes notional and market values.

From participation constraint,

$$d^* = \bar{d}^* - \pi_H\left(\bar{d}^*\right) = \int_0^{1+\bar{r}} \min\left\{\bar{d}^*, s\right\} f_H(s)\, ds$$

gives debt ratio in market values.

Solving for Balance Sheet Size

Bank equity holder's expected payoff under the optimal contract is:

$$U(A) \equiv A\left(r_H - \bar{d}^* + d^* + \pi_H\left(\bar{d}^*\right)\right)$$

Expression inside the brackets is strictly positive, since the equity holder extracts the full surplus.

Equity holder's payoff is strictly increasing in A. For

$$\lambda^* \equiv \frac{1}{1-d^*}$$

We have

$$A = \lambda^* E$$

Comparative Statics

Outcome densities parameterized by σ

Higher σ indicating mean preserving spreads.

$\pi_H(z, \sigma)$ is value of put option parameterized by σ.

$$\sigma' > \sigma \Rightarrow \pi_H(z, \sigma') > \pi_H(z, \sigma)$$

Proposition 2. *If $\Delta\pi(z, \sigma)$ is increasing in σ, then both $\bar{d}^*$ and d^* are decreasing in σ.*

Value at Risk

Value at risk (VaR) is smallest non-negative number V such that

$$\text{Prob}\left(A_1 < A - V\right) \leq 1 - c$$

Generalized extreme value distribution

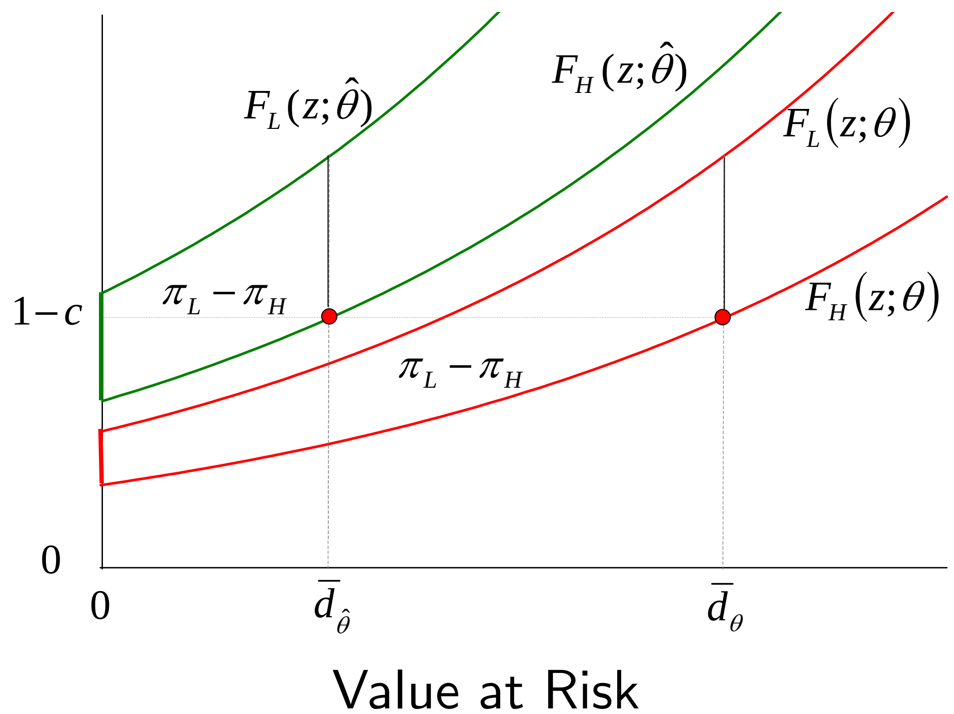
$$G\left(z\right) = \exp\left\{-\left(1 + \xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-1/\xi}\right\}$$

Take $\xi = -1$, $\sigma = 1$. Family $\{G_L, G_H\}_\theta$ parametrized by θ

$$G_L\left(z;\theta\right) = \exp\left\{z - \theta\right\} \quad \text{and} \quad G_H\left(z;\theta\right) = \exp\left\{z - k - \theta\right\}$$

where $k > 0$.

Proposition 3. *For all* $\theta \in [\underline{\theta}, \bar{\theta}]$ *suppose that* $\bar{d}^*(\theta) < \hat{z}$. *Suppose also that* $r_H - r_L$ *stays constant to shifts in* θ. *Finally, suppose that above condition holds. Then the probability that the bank defaults is constant over all optimal contracts parameterized by* $\theta \in [\underline{\theta}, \bar{\theta}]$.



Empirical Evidence

- Conditional variance of bank stock return from daily data

- Set value at risk $= 2.33 \times \sigma_s$
  - σ_s is annualized forecast standard deviation of return on bank's adjusted market assets (from volatility of market equity)
  - construct quarterly series of value at risk to coincide with accounting dates

- Quarterly regressions of (i) leverage growth (ii) asset growth and (iii) repo growth on value at risk

Panel A: Leverage in Investment Bank Panel

	VaR	Constant	R^2	Obs	Specification
coef	-0.18	28.77	be: 61%	239	FE
t-stat	-3.68	233.03	wi: 18%		
coef	-0.18	30.75	be: 64%	239	FE, controls
t-stat	-3.64	136.40	wi: 26%		

Panel B: Leverage Growth in Investment Bank Panel

	VaR Change	Constant	R^2	Obs	Specification
coef	-2.31	1.30	be: 9%	239	FE
t-stat	-3.70	0.79	wi: 6%		
coef	-2.34	1.45	be: 9%	239	FE, controls
t-stat	-3.59	0.87	wi: 8%		

Financial System Debt Capacity

Balance sheet identity of bank i in market values:

$$y_i + \sum_j \pi_{ji} x_j = e_i + x_i$$

Interpret as

$$\underbrace{x_i}_{\text{debt capacity}} = \underbrace{y_i + \sum_j \pi_{ji} x_j}_{\text{collateral value}} - \underbrace{e_i}_{\text{haircut}}$$

$$[x_1, \cdots, x_n] = [x_1, \cdots, x_n] \left[\quad \Pi \quad \right] + [y_1, \cdots, y_n] - [e_1, \cdots, e_n]$$

or

$$x = x\Pi + y - e$$

Debt capacity is recursive. Each bank's debt capacity is increasing in the debt capacity of other banks. Solve for y as

$$y = e\left(\Lambda\left(I - \Pi\right) + \Pi\right)$$

Λ is the diagonal matrix $[\lambda_i]$

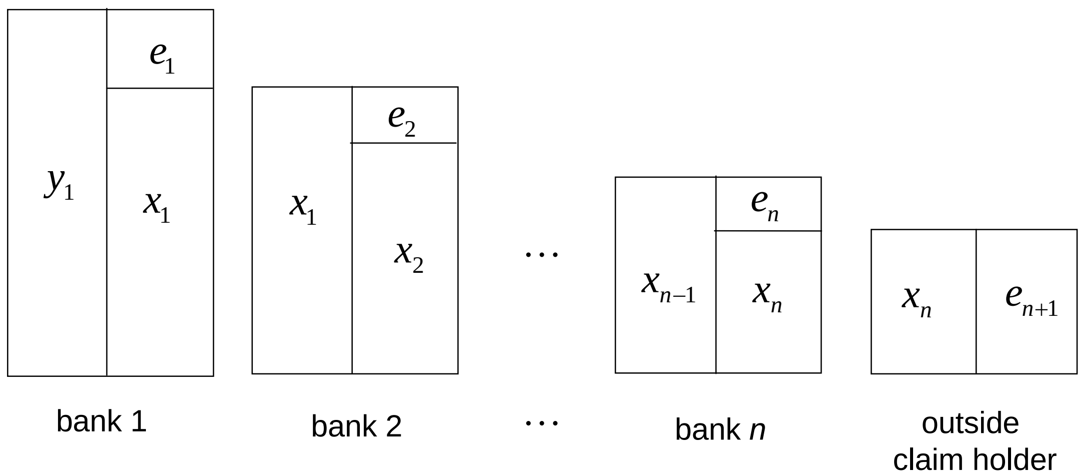
Total lending to end-users depends on

1. how much equity e there is in the banking system

2. how much leverage is permitted

3. structure of the interbank market (given by Π)

Simple Chain



$$
\begin{aligned}
y_1 &= \lambda_1 e_1 \\
&= e_1 + \lambda_2 e_2 \\
&= e_1 + e_2 + \lambda_3 e_3 \\
&= e_1 + e_2 + \cdots + e_{n-1} + \lambda_n e_n
\end{aligned}
$$

For the financial system as a whole to support debt level of y_1, all of the following inequalities must hold.

$$
\begin{aligned}
y_1 &\leq \lambda_1 e_1 \\
y_1 &\leq e_1 + \lambda_2 e_2 \\
y_1 &\leq e_1 + e_2 + \lambda_3 e_3 \\
&\vdots \\
y_1 &\leq e_1 + e_2 + \cdots + e_{n-1} + \lambda_n e_n
\end{aligned}
$$

Financial system debt capacity y^* is given by

$$
y^* = \min_i \left\{ \sum_{j=1}^{i-1} e_j + \lambda_i e_i \right\}
$$

The bank i for which $\sum_{j=1}^{i-1} e_j + \lambda_i e_i$ is the lowest among all banks is the "pinch point" in the financial system.

If $y^* < y_1$, then there is a run in the manner of Northern Rock or Bear Stearns.

Even if bank 1 has capacity to borrow, it is a bank further up the chain that chokes off lending.

The system as a whole then runs out of lending capacity.

Conclusion

- We provide a theory that ties leverage constraints imposed by haircuts to moral hazard.
- Higher risk increases option value to equity holders, so debt holders respond by increasing haircuts.
- We present empirical evidence linking risk to leverage.
- The model gives rise to procyclical leverage, which is in the data for broker-dealers, but not non-financial corporations.
- Microfoundation for the "adverse feedback loop" / "haircut spiral".
  - Model can be used in GE models for "supply of credit friction" in the transmission of monetary policy. (AS-JH)