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Portfolio Choice with Illiquid Assets

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Harvard Endowment: A Cautionary Example

“Liquidating Harvard” Columbia Case available from http://www.gsb.columbia.edu/caseworks

Harvard Endowment

- Performance of Harvard endowment June 2008 to June 2009: -27.3%. Fund shrank from $36.9 billion to $26.0 billion [Note S&P500 performance was -30% during this period]
- At June 2008, endowment distributions totaled $1.2 billion, representing 34% of the University's $3.5 billion revenue. For some schools, the reliance on the endowment was even higher:

Radcliffe	83%
Faculty of Arts and Sciences	52%
Law	37%
Business	20%

- Spending rate (payout rule) is variable, but it is smooth and at June 2008 was 4.8%

Harvard Endowment

- Harvard was an early adopter of the "endowment" model based on diversification concepts extended to illiquid assets (thanks to Swensen, Leibowitz, and others)

Harvard Endowment Asset Allocation Pre-Financial Crisis

	Policy Portfolio	Actual Portfolio	
Liquid	36%	27%	Dev Mkt Equity, Liquid Commodities, Govt Bonds
Semi-Liquid	33%	35%	Emg Mkt Equity, High-Yield Bonds, Hedge Funds
Illiquid	31%	39%	Private Equity, Timber/Land, Real Estate
Total	100%	100%	

- The losses from the financial crisis mean Harvard's budget has to shrink by approximately 20%, not including the massive cash outflows Harvard is taking from its swap positions. Harvard found out it can't "eat" illiquid assets!

Portfolio Choice with Illiquid Assets

Classic Portfolio Allocation

- Standard asset allocation models assume that investors have the ability to freely rebalance their portfolios at any time (sometimes at a cost)
- However, some assets cannot be traded, at any price, for significant lengths of time:
  - Direct real estate investments
  - Private equity and venture capital
  - Certain securitized fixed income and structured credit products
- How does illiquidity affect asset allocation?

Model

Assets:

- Riskless bond, interest rate *r*, freely tradeable
- Liquid risky asset [equity], freely tradeable
- Illiquid risky asset. Tradeable only at random times $\tau \sim$ Poisson (λ). The expected waiting time between rebalancing is $1/\lambda$. More illiquid assets have lower λ.

Notation: W = total wealth, X = illiquid asset wealth

Model

Preferences:

- CRRA utility over consumption

Outputs:

- Optimal asset holdings: liquid and illiquid asset holdings, risk-free holdings
- Optimal consumption

- Value function $F(W_t, X_t) = \max E\left[\int_0^\infty e^{-\beta t} \frac{C_t^{1-\gamma}}{1-\gamma}\right] \Rightarrow W_t^{1-\gamma} G\left(\frac{X_t}{X_t + W_t}\right)$

- When a trading time arrives, there is a jump in the agent's continuation value as he rebalances $F \to F^*$

$$F^*(W_t, X_t) = \max_{I \in [-X_t, W_t]} F(W_t - I, X_t + I) = K_\lambda (W_t + X_t)^{1-\gamma}$$

- Three unknowns: $G(\cdot), K_\lambda, \left(\frac{X_t}{X_t + W_t}\right)^*$

- Three conditions:
  - Hamilton-Jacobi-Bellman
  - Value matching. At $\left(\frac{X_t}{X_t + W_t}\right)^*, F = F^*$
  - Smooth pasting. At $\left(\frac{X_t}{X_t + W_t}\right)^*, F_W = F_W^*$

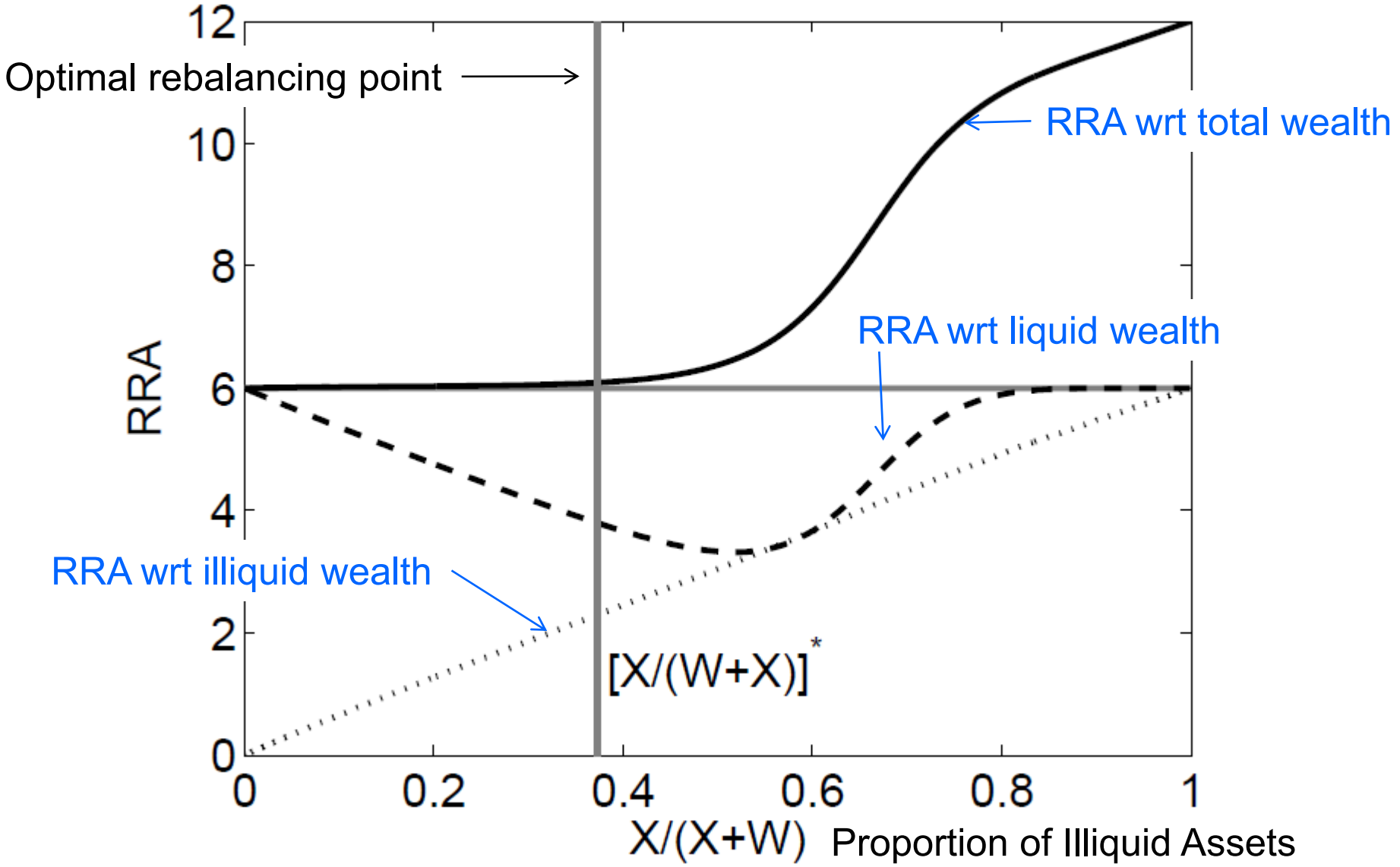
Illiquidity-Induced Endogenous Risk Aversion

- The presence of illiquidity induces time-varying, endogenous risk aversion
- Intuition:

  In a standard Merton problem where both assets are always tradeable, an agent only cares about total wealth. The risk is that total wealth goes to zero and the agent cannot consume.

  The agent can only consume out of liquid wealth. Therefore, with illiquid and liquid assets he also cares about the risk of liquid wealth going to zero.
- The ratio of liquid to total wealth becomes a state variable. That is, effective risk aversion depends on liquidity solvency ratios.

Effective Relative Risk Aversion (RRA)

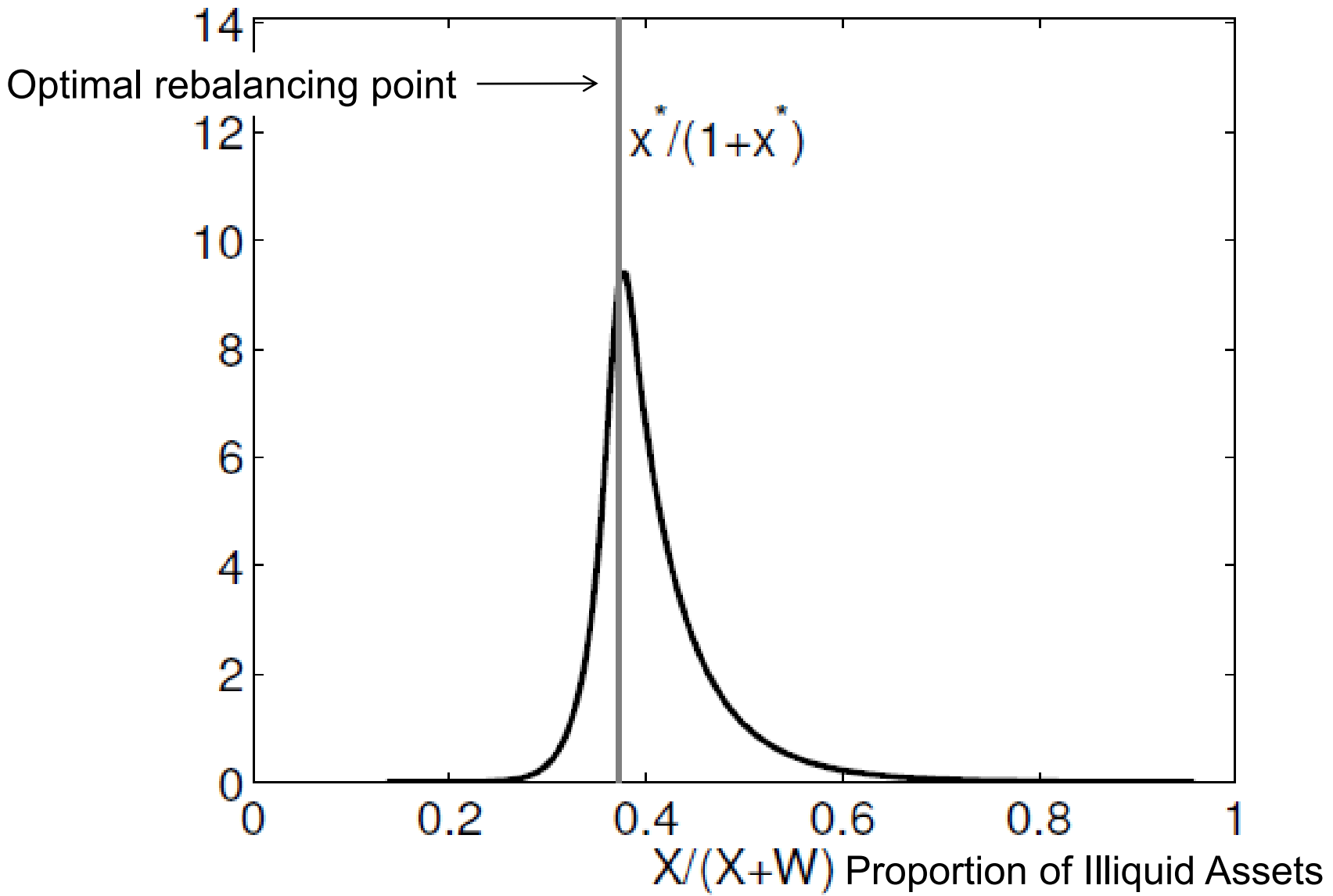


Illiquid Asset Holdings

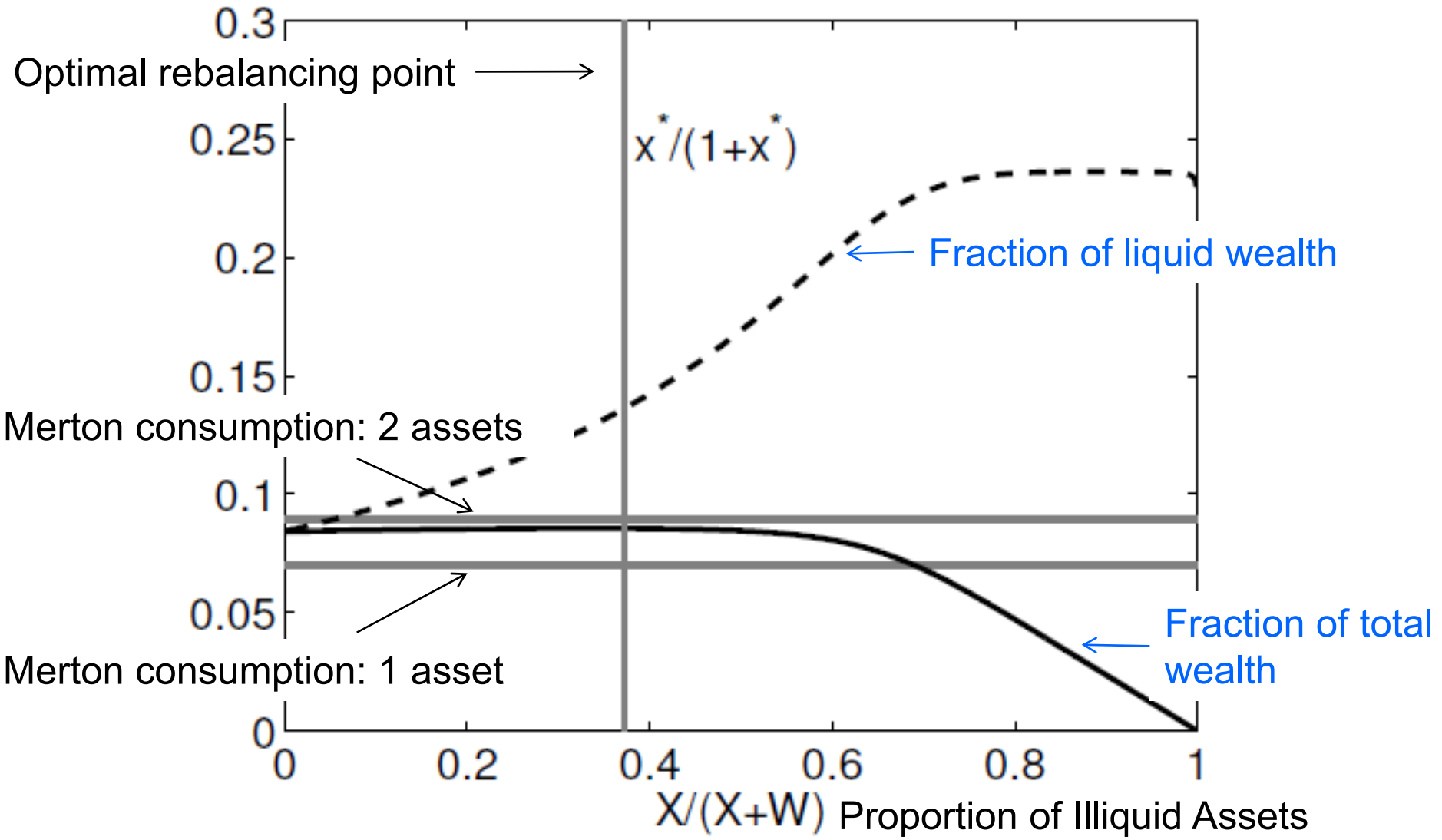
- Illiquidity markedly reduces optimal holdings relative to the Merton benchmark. Furthermore, illiquid asset holdings are very skewed.

Average Turnover	λ	Optimal Rebalance Value
10 years	0.1	0.05
5 years	0.2	0.11
2 years	0.5	0.24
1 year	1.0	0.37
½ year	2.0	0.44
Continuously	∞	0.59

Distribution of Illiquid Holdings

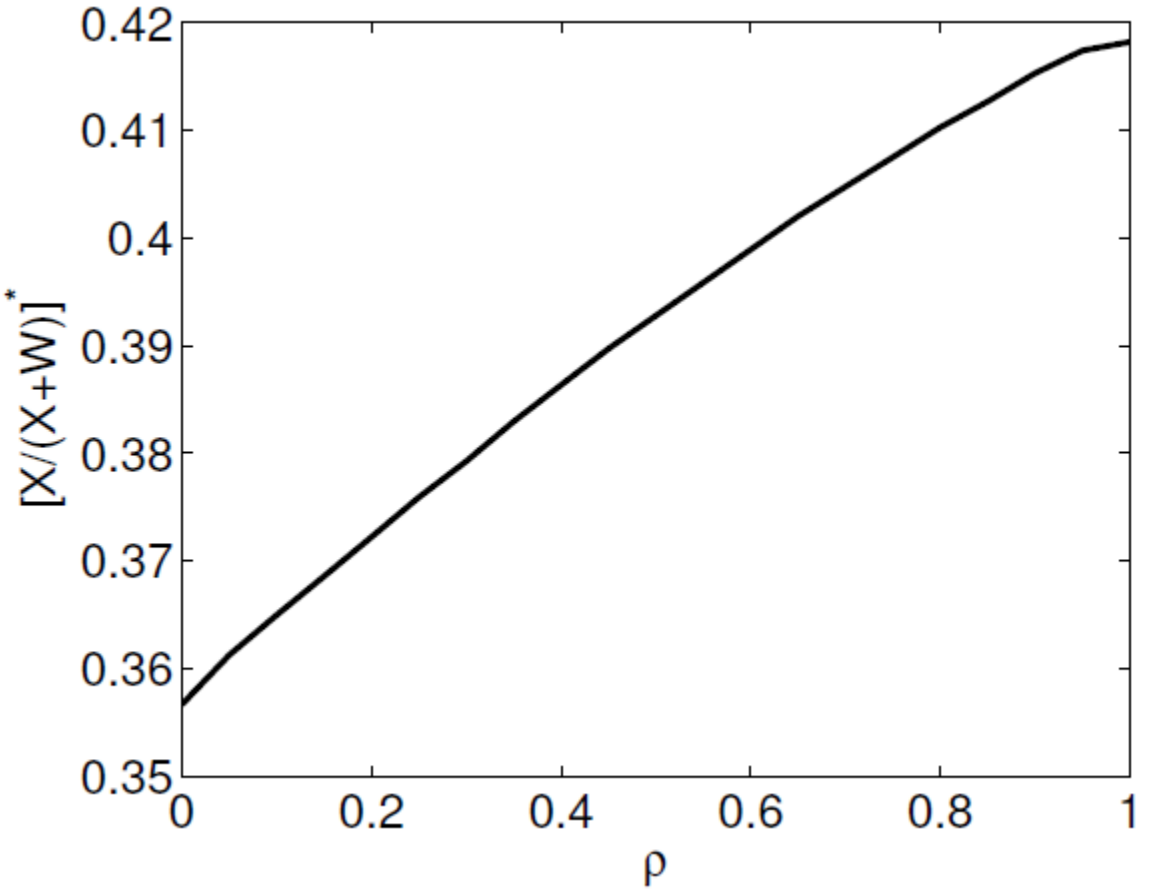


Optimal Consumption



Correlation

- In the presence of illiquidity, “near-arbitrage” opportunities arising from high correlations are not exploited. There is no “arbitrage” because illiquid and liquid assets are not close substitutes
- Note: Merton positions at $\rho = 1$ are +/- infinity



Illiquidity Premiums

- How much does an investor need to be compensated for illiquidity? To be able to trade the illiquid asset continuously, an investor requires liquidity premiums of:

Average Turnover	λ	Illiquidity Premium
10 years	0.1	0.060
5 years	0.2	0.043
2 years	0.5	0.020
1 year	1.0	0.009
½ year	2.0	0.007

Conclusion

- The periodic inability to trade (illiquidity) has a large impact, for realistic parameters, on portfolio choice
  - Lower holdings of both the liquid and illiquid assets relative to the Merton benchmark of continuous trading for all assets
  - Consumption is lower than Merton
- Illiquidity risk induces time-varying risk aversion which is greater than the constant risk aversion coefficient of utility. This is because illiquid assets cannot be used to fund immediate consumption.
- Extensions: Time-varying illiquidity risk (regimes); Correlated frequency of trading with asset returns; Investors pay a cost to increase liquidity (optimal transactions costs).