

# Putting Economics (Back) into Quantitative Models

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I am not an Economist. I am not even a union card holder in Finance, and accidentally arrived on a Wall-Street trading desk via the back-door cracked open by the 1990 recession in science. Now that I have had some experience trading and managing money, and also observing a handful of very successful investors and some not so successful ones, and in building models that have worked and failed, I have a few ideas on what kind of models work and what kind don't work. The common thread that ties these ideas is that explicit recognition and inclusion of the economics underlying the models is critical for their quality. Its not easy, but recent research by many in academia and industry has shown that indeed we can make a fair bit of progress. (Ex)- physicists talking about putting economics back in finance - and you thought you had seen everything!

Somewhere along the way as we quants became more mathematically sophisticated and obtained faster processing power, the approximation and computational muscle that was a short-cut took on a life of its own, largely at the expense of the economic common sense that lies behind the purpose of investing - making superior excess risk-adjusted returns. The last twenty years have seen some serious financial debacles, and perhaps the next generation of models will incorporate the errors of the previous generation as part of the modeling framework. Bjorn Borg, one of the greatest tennis players of all time was his coach's exhibit on what not to do on the tennis court; especially not to use two hands on the tennis racquet; until he started to win championships. Financial models need the second hand to make them better - not using economics from the very beginning as inputs into the model is playing with one hand behind your back.

Models are assumed to operate in a theoretically ideal environment. When modeling the behavior of an elephant in the jungle, a physicist makes the assumption that the elephant can be thought of as a large sphere. Everything that follows will then depend on this assumption, and hence it can be expected to fail under a wide range of conditions when the elephant is not really a sphere.

In the words of Myron Scholes, <sup>3</sup>

We make models to abstract reality. But there is a meta-model beyond the model that assures us that the model will eventually fail. Models fail because they fail to incorporate the inter-relationships that exist in the real-world.

At a recent conference of quants in the financial markets, Jack Treynor was asked what he would recommend as a topic to the group of senior industry and academic participants

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<sup>2</sup>Presented as Plenary Session - Risk Magazines Annual Quant Congress, 2005, NY Nov. 8 and 9, 2005.

<sup>3</sup>M. Scholes, speech given at the NYU/IXIS conference on Hedge Funds, New York, September 2005

who meet twice a year to discuss important issues in modeling. Somewhat reluctantly (since Jack is a very modest person) said that he wished he had studied macroeconomics more carefully, and that “macroeconomics” as a topic would be highly welcome, because so much of actual policy and investment depends on it.

I will argue that incorporating economic principles; such as demand and supply, investor behavior, preferences etc. from microeconomics, and monetary policy, macro aggregates, deficits, trade balances etc. from macroeconomics can help make our models more flexible, and hence more robust. Incentives drive the action of market participants, investors, and quantitative modelers. This approach to modeling is not pie in the sky. Recent work by many <sup>4</sup> has shown that we can build better models without giving up the basic principles that form the foundations of arbitrage free pricing or the law of one price. Having economics as the backdrop enables “better estimates of the distribution of future outcomes than a simplistic stochastic process of an unobserved factor estimated over a period of decades”<sup>5</sup>. Models fail not because the math underlying them is wrong, but because by blindly going to the risk-neutral framework we often misprice risk-premium, and hence mis-price risk. Emanuel Derman talked about neoclassical, behavioral, physical and mathematical approaches to model building. Incorporating economics leads to an approach which is the proper blend of all these - a real-world risk based approach to pricing and investing.

Why do we need economics in our models? Doesn't risk neutral valuation make real-world probabilities irrelevant for pricing? Risk-neutral valuation argues theoretically that if you can decompose the movement of any security into continuously tradable replicating portfolios, and actually traded the replicating portfolios continuously, you can choose any probability measure you want. Its the “actual” part that causes problems and breakdown between theory and reality. There is no instantaneity in real markets. Economics, risk-aversion, preferences creep in. Even if you could overcome the technical difficulties, the approach to model building that we have been taught might lack in some key non-technical aspects; and the difference between investors who deliver returns with a good risk profile over multiple cycles and realizations of the real world, and those who don't, might actually be explainable in terms of how their approach to investment explicitly incorporates the economic environment. So, I am not simply going to argue that Black-Scholes is incomplete since it assumes the wrong distributions, static volatility etc. Incorporating economics explicitly simply makes better models without throwing away any of the gains from our well-developed theoretical arbitrage free framework. It is possible to be arbitrage free, while at the same time stay in the world of common sense. There are successful academics who have made lots of money as investors, and many of these have realized that model frameworks stuck in static reality operate in a parallel world of spherical elephants; and users of these models can be arbitrated by more realistic denizens of the financial jungle. Investment advisors still advocate standard friction-free idealized CAPM even though it has been shown to be woefully inadequate - “Investors have realized frequently that they were grossly misled by academics...for example, by advocating the holding of the market portfolio of stocks, academics were basically marketing the stocks in the New York Stock

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<sup>4</sup>The “classic” reference for the resurgent interest in this area is Ang, Andrew, and Monika Piazzesi, “A No-Arbitrage Vector Autoregression of Term-Structure Dynamics with Macroeconomic and Latent Variables.”, *Journal of Monetary Economics*, Vol. 50, No. 4(2003), p 745.

<sup>5</sup>I would like to thank Brian Sack for this comment.

Exchange.”<sup>6</sup>.

Let us begin by discussing the main purposes of quantitative models, whether they are reasonable, and how we can improve them by using economics. At the end of this discussion I will show a simple toy model that incorporates economics and arbitrage free concepts from the very beginning. I should mention that this is not the only possible implementation. Others have implemented other specifications. The value is really in the concept, not in the particular mathematical form; though having a simple mathematical form simply makes analysis more tractable.

### 1. Relevance for making money and managing risk

There are basically four (honest) ways to generate returns (we ignore the possibility of front-running or using privileged information as an option). I will call them “profit modes”:

- (a) Taking factor risk exposures (macro)
- (b) Intermediation (brokering)
- (c) Liquidity and or risk transfer (insurance)
- (d) Mispricing (arbitraging inefficiency)

Most money making enterprises are combinations of these modes. To make a model relevant to the particular combination of profit making modes, it has to be calibrated to the relevant combination of modes that is expected to yield excess returns for the investor selecting the combination. The economic environment is the backdrop within which modes for profit are evaluated and models calibrated. A good example comes from option pricing models used on derivatives trading desks. Whether used for taking factor exposures, intermediating, warehousing or arbitraging, the model development process works in the following sequence:

- (a) Assume a distribution for the fundamental variables (for the standard libor market model forward rates are assumed lognormally distributed), and estimate the parameters using history and common sense as input,
- (b) Generate probable paths of evolution,
- (c) Fit remaining free parameters in the model to traded security prices,
- (d) Price other securities with the model.

The framework as laid out is consistent for relative pricing, but because it has substantial number of underlying distributional assumptions, it might not be equipped to automatically adapt as the underlying dynamics undergo a structural shift. These shifts have to be incorporated by hand to ensure that the model is well-calibrated for what the model is designed to do. A case in point: the failure of many models as Japanese nominal rates dropped to zero over the last fifteen years. The lognormal framework would of course make it impossible for rates to fall below zero percent. But zero percent floors indeed have been marketed and traded. A model that works well

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<sup>6</sup>Sanford Grossman, speech at NYU/IXIS conference on Hedge Funds, 2005.

for long term investing and is designed to warehouse risks would fail miserably if it is used for high-frequency directional trading or arbitrage, and vice versa. Dealer desks make money through the flow business - they only need models that can be locally calibrated to today's market, because tomorrow the position will be unloaded to another customer. For shorter holding periods, it does not make a big difference if the model is improperly calibrated to the big picture. Hedge funds make money through taking residual risks - it does matter that the models they use are properly calibrated to possible episodes of missing liquidity. Mutual funds and insurance companies do not lever much, hence their risk taking necessarily reflects their ability to hold illiquid positions for longer periods. Hedge funds and mutual funds seemingly operate in the same mode, but differ fundamentally in their ability to obtain funds with lower haircuts and for longer commitment periods. This financing difference makes all the difference on how holding period returns are computed. Between the time of putting a position on and taking it off the world can change; recalibration shocks should not force an unwind of the position. Dealer desks and hedge funds operations are suited to risk-neutral and physical measures respectively because that's the most appropriate measure for their investment objective.

It is critical that the fundamental variables in the model reflect observables, or executables. It is elegant to build models that can be described in terms of a small set of factors (principal components for interest rate models), but if a portfolio manager cannot quickly translate the change in the market into simple combinations of market instruments, the models become irrelevant to practical investing.

## 2. Stability, analytical tractability and the ability to stress test

Exact solvability has forced many term structure and credit modelers to restrict their attention to a subclass of models called affine models; with a limited set of prices of bonds and swaps as inputs, but not their options as inputs, these models typically do not replicate the underlying dynamics of the yield curve and its volatility very well. Partial technical victory is assumed to subsume reality. While these models provide a nice sandbox within which to do thought experiments, and also spawn Ph.D. theses, they lack fundamentally in the ability to capture the important nuances of the market. For example, the evolution of the covariance of the term structure is largely undescrivable in this framework. Good models would have rates and volatilities described simultaneously since practitioners know that the shape of the yield curve also reflects premium due to uncertainty.

Even if we could surmount the technical limitations of affine models using more sophisticated, albeit numerical models, there are other assumptions that most analytical models make which are harder to get around. Analytically tractable models are based on assumptions of continuity, but the world is full of structural breaks. Most models lean very heavily on the notion of stability by assuming stable distributions for the dynamics. After a hundred years of stochastic calculus this is all we really have to work with mathematically, since stable distributions and processes are easy to model and solve. They are the harmonic oscillator of finance. But markets and policies undergo rapid structural breaks. No avalanche is allowed in typical models until it happens. Since most term structure models are based on the principle that long rates

are time averages of short rates plus risk premium, there is no way to account for the possibility of structural breaks unless it is done right from the building block level. Again, as Scholes puts it - “shocks occur that stop time, and this gives us a chance to recalibrate our old models to new data, or to incorporate new thinking into old models; human capital becomes relevant.” I find it unsatisfactory that human capital has to bail out our models as frequently as it has.

To pursue this further, let’s discuss the basic building block for interest rate models - the short rate. Note that the Taylor rule (widely argued to be the Fed’s rate setting rule during the Greenspan era, at least) is specified as:

$$i_t = (r^* - \theta_1 \pi^*) + \theta_1(\pi_t) + \theta_2(u^* - u_t) + \pi_t. \quad (1)$$

where  $i_t$  is the short rate,  $r^*$  is the equilibrium real rate,  $\pi_t$  is the inflation rate at time  $t$ , and  $u_t$  is the unemployment rate at time  $t$ .  $\pi^*$  and  $u^*$  are respectively the target inflation and unemployment rates, and  $\theta_1$  and  $\theta_2$  are coefficients to be estimated. Any other rate in the yield curve can be obtained by integrating the time evolution of this rate and computing the log expectation. Fed behavior in setting monetary policy has seen some major structural breaks (e.g. when transitioning from the pre-Volcker era to the Volcker-Greenspan era)<sup>7</sup>, and perhaps more recently with LTCM in 1998. If the ultimate provider of money, the Central Bank, changes its structural framework, it seems reasonable that asset pricing models should be flexible enough to capture these structural shocks as well. To see the kind of changes, let us econometrically estimate equation 1; the estimate of the constant term tells us the relationship between the equilibrium real rate and the target inflation rate. As table 1 illustrates, there is

Period	$c$	$\theta_1$	$\theta_2$	$R^2$
Burns (1970Q3-1978Q1)	-2.118 (1.835)	0.555 (0.326)	2.737 (0.483)	0.54
Volcker (1979Q3-1987Q2)	1.295 (1.120)	0.561 (0.178)	0.623 (0.280)	0.72
Greenspan (1987Q3-2005Q1)	-0.435 (0.295)	1.107 (0.110)	1.995 (0.128)	0.859
Greenspan (1987Q3-1998Q3)	1.109 (0.188)	0.540 (0.059)	1.940 (0.069)	0.96
Greenspan (1998Q4-2005Q1)	1.379 (0.546)	0.161 (0.339)	2.494 (0.093)	0.96
(1960Q1-2005Q1)	1.279 (0.372)	0.330 (0.090)	0.562 (0.136)	0.58

Table 1: Estimation of the relationship between target real rate and inflation rate for different Fed regimes using equation 1. Here the constant  $c$  is given by the relationship  $c = r^* - \theta_1 \pi^*$ .

large amounts of variation in the constants as well as the weights on inflation and growth during the tenure of different Fed chairmen. In the absence of some measure of the possible variability of this response function, it is impossible for the model to be robust to structural shocks. If one takes the approach of modeling the yield curve directly without reference to the Taylor rule, and simply fitting yields to some non-linear function, we run the risk of running afoul of what fundamentally is the

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<sup>7</sup>Clarida, Richard, Gali, Jordi and Gertler, Mark. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory”, *Quarterly Journal of Economics* 113, pp 147-180, 2000.

driving dynamics for the short rate. Figures 1,2 show the structural break, and figure 3 shows the tradeoff between coefficients during the Greenspan years. Figure 4 shows the difference in coefficient tradeoffs during different Fed chairmen tenures.

Let's take a look at models for structured credit. For ease of computation, the current state of the art of credit modeling assumes a copula function to describe the joint behavior of loss; quants know the limitation of the approach, but as of today there is no common benchmark model to improve upon the gross shortcomings of the copula approach. The Gaussian copula is to structured credit what the lognormal assumption is to option valuation. The base correlation skew of copula based models is like the vol skew of options. The key simplification in the copula approach is provided by the assumption of a default correlation parameter which is an input. However, the default correlation that goes into the model is not directly observable. When Northwest and Delta entered bankruptcy within minutes of each other in September, we saw default correlation in practice, and it was 1! No estimation devoid of ex-ante input would have this as an input. When describing the reason for the simultaneous defaults, most analysts would use macroeconomic variables as the cause - underfunded pensions, falling revenues, competition, increased oil prices. It seems hard, if not impossible, to include the macro inputs as complex as these into one parameter. So we settle for charge for the uncertainty by fitting prices to imply the value of the parameters. We even trade tranches assuming that they can be packaged together in an essentially arbitrage free way, which is just a wrong assumption unless liquid recovery products can be traded. Figure 6 shows the actual and realized delta adjusted return of tranches in the Spring. Not only was the hedge not a hedge in magnitude, for many tranches, the sign was wrong!

At the very least, if the copula model for trading default correlation has any value, it should be possible to attribute portions of the default correlation to real life economic factors. In other words, if the model is required to possess any depth, what is exogenous has to be made endogenous. Numerous smart proprietary trading desks make their living by constructing structures that take advantage of the lack of uniformity in modeling standards. Some recent progress has been made in this direction without giving up analytical tractability; for example, state contingent joint distribution of returns to describe comovement, instead of unconditional correlations, where the states depend on macro scenarios.

Perhaps the most important reason for using simple, parametric models is that these models can be solved and the parameters stress tested to obtain hedge ratios. For trading desks trained under the risk-neutral approach, the local hedge ratios, mark-to-market constraints, and running "flat" books can take a life unto itself. This approach works well when markets are liquid and price discovery is easy. However, locally risk-neutral models fail horribly when securities are illiquid or markets are under stress. The limitations of using risk-neutral pricing become all too obvious. The most perverse outcome is that the hedge ratios take on a life of their own; to hedge, underlying securities are traded. When the market hedge ratios are large enough, the act of dynamic hedging can lead to the modification of the prices of the hedging securities in an amplifying feedback. When large enough in magnitude,

these models can actually lead to systemic market distortions, such as the numerous mortgage debacles of the last decade, and perhaps even the crash of 1987. Anecdotally, the rapid rise of quantitative credit models has coincided with numerous proprietary “capital structure arb” desks and dedicated hedge funds. With Merton like modeling framework applied to credit and equity, a new theoretical connection has become all-too practical. Is falling equity volatility that has been accompanied with tightening credit spreads (see figure 8) a coincidence or a consequence of massive scale application of these sort of models?

But doing away with risk-neutrality is not a cure-all. Simple actual world Value at Risk models are notorious for failing when you need them most and for working great when you don’t need them. The reasons are many - first, since most VaR models are based on historically estimated covariance matrices, so they simply cannot see stresses unless the covariance matrix is taken to its mathematically extreme values. To systematically take covariance matrices to logical extreme values, economic priors and portfolio risks under those outcomes have to be built in right from the beginning.<sup>8</sup> For example, a VaR model based on history alone would almost never capture episodes like bull flattening, where yields and the steepness of the curve fall simultaneously. As many mortgage investors recently found out, valuation and risk models did very badly in the 2004/2005 bull flattening episode. Hurricanes in Canada? Possible if you allow for parameters to take values under structural shocks.<sup>9</sup> The risk model has to be simple enough to enable translation of envisioned outcomes into ranges of outcomes for the risk factors.

### 3. Freedom from arbitrage

Models are approximations; demand and supply considerations are largely ignored. When we are unable to estimate the impact of clientele effects, we resort to the assumption of freedom from arbitrage to make the models work. With this approach, we have seen some major technical breakthroughs in our time. Black-Scholes for options is one - and it depends on some simplifying assumptions regarding the underlying distributions. The copula approach is another, increasingly used for credit correlation products, and also depends on some simplifying assumptions regarding the form of the copula. However, even armed with the best arbitrage free quantitative models possible, experts can and do disagree. For example, in table 2 we show the OAS and duration numbers from a survey of the top MBS dealers on Wall Street for the same security.<sup>10</sup> A levered derivative structure like an IO would magnify these errors even further. Is it a wonder then that there are routine blowups in the mortgage derivative markets when there are large movements in underlying markets? It is often argued that you can reconstitute the collateral from the IO to PO, so given two of the three, the remaining one should trade at fair value. The problem is that liquidity, risk aversion and transaction constraints make it largely impossible for investors to reconstitute and arbitrage away the mispricing. So given the model risk, both IO’s

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<sup>8</sup>See, for example, V. Bhansali and M.B. Wise, Forecasting Portfolio Risk in Normal and Stressed Markets, *Journal of Risk* 4, 2001.

<sup>9</sup>Hurricane Hazel hit Southwestern Ontario on October 14, 1954.

<sup>10</sup>I am grateful to the Pimco MBS desk for this data.

and PO's should trade with a risk-premium. The magnitude of this risk-premium of course cannot be modeled, but we do observe that it increases in periods of economic uncertainty and risk. What also stands out is that the research desks and trading desks don't usually agree - the trading desk effectively adds on a risk correction to the research models. Since risk-takers are the ones who have the most to lose from bad models, they supplement the research models with their economic priors via a haircut applied to the model numbers. It would be nice if such expectations of changing economic conditions could be systematically incorporated into prepayment models, if for nothing else but for calibrating the model risk-premium; and indeed some progress has been made. Theoretically, we can also construct a corporate bond with a credit default swap and risk-free instrument, but the equivalence only holds true if gross assumptions that have no hope of holding in real markets is made. Traders routinely trade the cash bond vs. CDS basis for profits. Even portfolio products like the CDX indices of default swaps (which are PV01 weighted averages of 125 five year single name CDS), have traded portfolio spreads very different than intrinsic values computed from the single names.

For fixed income instruments, we can think of any yield over a benchmark instrument as arising from an embedded option. Even more yield from duration extension is the price for the option to reinvest at a higher rate than that implied in the forward curve. Investors do not all value options efficiently; and even if they did, under different economic conditions they do not have the will or the means to exercise these options in the most optimal fashion. This can lead to what appear grossly inefficient markets from the viewpoint of arbitrage free models unless principles of economics - demand and supply are imposed endogenously.

A striking example from recent memory comes from the interest rate futures markets. The treasury futures market is one of the largest and most liquid derivative markets anywhere. When the contract was designed, the open interest in the contract was expected to be comparable to the size of the cheapest to deliver securities. So one could use good, arbitrage free derivatives models to value the delivery option and trade the cash versus futures basis. As time passed, the ease of use of futures increased the participation in the futures contract, whereas the size of the treasury bond market dwindled (due to various reasons, including the fact that the treasury started a buy-back program in 1999-2000). The mismatch in the size of the deliverable to the open interest in the contract (approximately 20 BN in the CTD T 4.375 of August 2012) to approximately 160 BN in the December futures contract has made application of arbitrage free theoretical models largely relevant for rich-cheap valuation. Squeezes have become a lot more prevalent (see figure 5).

#### 4. The investor's behavior and preference is irrelevant to the model

Risk premium in incomplete markets cannot be hedged, and unhedgeable risk premium comes with unhedgeable risk. Risk-premium is where the value is; successful investors know this. Security prices reflect the prices of risk-transfer, and in the real world, investors are not identical, so it is impossible that all securities can be modeled without taking investor risk preferences/behavior into consideration. However, most pricing taught in schools assumes that investors can always be made locally risk-neutral; the

market is assumed to be complete and risk-aversion is assumed to be irrelevant because one can always transform to the risk-neutral measure. This is a huge assumption though. As time and time again we have witnessed, large, unhedgeable moves in the market are routine, and even the best structured relative value trades expose their absolute risks.<sup>11</sup>

Locally hedgeable portfolios frequently cannot be created - as a matter of fact, locally hedgeable portfolios are the exception rather than the rule (forget about globally hedgeable portfolios; the only way to create them is to be a broker and lay off the risk completely or sell your position). The infamous CDX mezzanine versus equity tranche trade of the summer of 2005 illustrates this clearly. The correlation based model assumes that tranches can be reconstituted to make up the underlying index, and comes up with a theoretical value based on this. However, when the underlying economic environment suffered a shock with a possible auto default, securities behaved in exactly the opposite way than their theoretical deltas would have us believe they should, due to what would be called “irrational” deleveraging. So in practice we are left with the problem of how to deal with risk-premium. Risk-premium is not constant. Far from it. Theoretically, the risk-premium is proportional to the covariance of the investor’s utility with the economic state of the world. In other words, assets that lose their value when you are poor suffer a penalty (since when you are poor you value the marginal dollar more), and assets that gain in value in bad states of the world carry a premium (such as treasury bonds). In practice most consumers probably do not do the life-cycle consumption maximization exercise, but rather look at the economy they can observe and expect. This is also why asset classes can quickly influence each other and lead to simultaneously lower expected returns across the cross-section. As Greenspan has remarked recently: “... low risk-free long-term rates worldwide seem to be one factor driving investors to reach for higher returns, thereby lowering the compensation for bearing credit risk and many other financial risks over recent years.”<sup>12</sup>. See figure 8 validating the assertion that uncertainty across markets can have a high degree of correlation to lower rates of expected return. Market volatility has dived to low levels, but so has the volatility of economic variables such as inflation and GDP (figure 7). The Fed now moves in smaller steps. So to understand the sources of risk, and risk-premium, it is essential that the economics of the market environment is understood. Assuming that assets will covary according to historical correlations in all environments is a dangerous assumption - it throws out the understanding that asset performance comes eventually from the need for the asset in a portfolio, not from a stand-alone pure value. This portfolio need for assets changes as the economic

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<sup>11</sup>The key idea of risk-neutral pricing is that if a locally hedgeable portfolio can be created, then you can replace the risk premium term with zero (or equivalently the discount rate with the risk-free rate). We can write the risk neutral probability in terms of the risky probability as arising from shifting the means

$$\tilde{q} = N(N^{-1}(q) + (\tilde{\mu} - \mu)\sqrt{T}) \tag{2}$$

where  $N$  represents the cumulative normal distribution and the physical probability is  $\tilde{q}$  with risk-neutral probability  $q$ .

<sup>12</sup>Remarks by Chairman Alan Greenspan Central Bank panel discussion To the International Monetary Conference, Beijing, Peoples Republic of China June 6, 2005

environment changes. The academic community tries to explain risk premium in the default swap market by comparing the risk-neutral reduced form intensities in the default swap market with the s-called “actual” probabilities implied by Merton or Leland-Toft models that have been industrialized by KMV and others. However, this approach is limited in its scope because it ignores that economic considerations (such as the increased supply of investable cash), has gone preferentially into corporate bonds rather than equities.

#### 5. The modeler should be irrelevant to the model

The variation in the modeler’s preferences on what short-cuts to take or what logic to follow in the creation of models, i.e. the value of the modeler’s human capital is assumed to be largely irrelevant. It is no secret that in recent years the single largest impact on fixed income markets has been the growth of mortgages as an asset class, and the commoditization of prepayment risk. Since the right to prepay is an option for the homeowner, the investor in mortgages is short the prepay option. Now assume that you are a large pension fund or agency trying to figure out how much your interest rate risk can change as prepayments change. The way you go about solving the problem is to build a model to value the prepay option. Here’s the rub - the prepay option is priced using econometric inputs. In other words, homeowners do not prepay efficiently (and this is what presumably makes selling the option worth it to the investor), and the inefficiency of the prepay option exercise is quantified using the behavioral response of the collective homeowners in the past. But the behavior in the past depends on a multitude of actual economic conditions (rate levels, credit conditions, demographics, unemployment rates, availability of housing, tax rates and breaks, etc.). The hedge ratios for mortgages are thus related to economic inputs directly, and the model driven hedging activity, if large enough (as in Fall 2002), can impact the prices of related securities and lead to a vicious feedback mechanism (need for duration lead to buying of treasuries and falling yields lead to higher prepayments and more need for duration). People’s behavior changes as a response to the economic environment, so the models have to be able to capture this. How will a collapse of housing in California, were it to happen, affect risk-premia given that homeowners are buying homes essentially on their credit card? <sup>13</sup> Experts can disagree! We stress tested a number of models on their essential inputs for the same econometric variables, and found that the response of the models to the same set of inputs was very different. In other words, two modelers working across the street from each other, with the same inputs and same training in Finance come up with very different models. There is more art than science.

#### 6. Price should reflect Value

This assumption is frequently violated. The following quote from none other than Adam Smith jumps to mind:<sup>14</sup>

The things that have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value

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<sup>13</sup>Recent structures such as option ARMs has made this metaphor a reality.

<sup>14</sup>Adam Smith, “The Wealth of Nations”, Book 1, Ch. 4 (1776).

Dealer	Treasury OAS	Libor OAS	OAD (Research)	OAD (Trading Desk)
Lehman	26	3	4.8	4.6
Goldman	81	34	4.6	4.0
Greenwich Capital	36	17	4.0	4.0
CSFB	52	18	6.1	4.4
Salomon	51	16	5.2	4.4
Morgan Stanley	52	29	5.0	4.2
Bank of America	64	25	4.2	4.5
UBS	44	20	4.9	4.7
Countrywide	107	50	5.1	4.1
JP Morgan	54	23	6.1	4.1
Merrill	52	21	5.0	4.1
Bear	65	21	5.3	4.3
Average	57	23	5.0	4.3
Range	81	47	2.1	0.6
Min	26	3	4.0	4.0
Max	107	50	6.1	4.7

Table 2: FNMA 5.0% 30 Year Passthrough Option Adjusted Spreads and Option Adjusted Durations as of May 05, 2003. Source: PIMCO survey.

in exchange have frequently little or no value in use. Nothing is more useful than water; but it will purchase scarce any thing: scarce anything can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it.

It is relatively easy to create a model to justify the price of a security without reference to the economic environment, since in equilibrium demand and supply match. It is harder to create a model to justify the value of a security. Even for the most liquid securities, there is one price, but different values for different investors. On September 26th, Goldman Sachs issued 30 year tax exempt liberty bonds at a yield of approximately 4.59 % when 30 year treasuries were at 4.50 %. For a taxable investor this yield may or may not be absolutely attractive (grossed up yield of almost 7%) given that we were at historically low levels in yields globally. On a relative basis, versus treasuries, this implied a higher than 100 % ratio, i.e. an implied term tax rate of zero percent without adjusting for credit. So on a relative basis one would say that the deal was immensely attractive for a crossover taxable buyer. But implicit in the valuation, both absolute and relative, is a view - for absolute buyers that such a low yield is justified because it locks in a 4.59 % yield for 30 years on a tax-exempt basis, which would be justified if inflation falls. Since the tax risk cannot be hedged, if taxes go up this would also turn out to be a good investment relative to a government bond at a lower yield. But people are buying taxable treasuries over the munis at the same time. Does the buyer of the muni who is short the tax option deserve so

much higher a pre-tax yield? How much is the tax option really worth? Since we cannot really hedge the tax risk a relative value statement on munis versus taxables requires us to incorporate economics in the model right from the beginning; taxes are an economic variable! Perhaps option pricing was more accurate before Black-Scholes came around, because it matched buyer and seller with an explicit recognition of their ability to take risk; and did not simply lean on the framework that we have come to believe in so deeply where all unwanted risks are assumed hedgeable. This so-called muni puzzle has been around for decades, and in some cases it is blatantly in violation of arbitrage principles, i.e. that forward implied tax rates can go negative! To take advantage of the arbitrage opportunity, you would need to short a muni bond, which is notoriously hard to do. You can, of course, lock-in the potential of sure gains over some finite horizon by buying munis and shorting taxables according to some model based hedge ratio, but then you are subject to volatile markets while the arbitrage relationship converges security prices to fair values. The difference between price and value is captured by using risk-neutral versus risky distributions. In the world of credit, reduced form models such as Duffie-Singleton are pricing models, while the Merton model is a valuation model. Regressions that forecast ten year bond yields based on macroeconomic variables are valuation models, while term-structure models such as HJM, BGM, BDT + etc. are pricing models. We cannot always connect the two approaches once they have been laid out - it is important to build them right from the beginning.

#### 7. Prices of securities are irrelevant for the evolution of the state of the world

Our friction-free models assume that today's price of a traded security should not impact the state of the world in the future. But as discussed by many in the press and in journals, we are in the middle of a great conundrum - the ten year note yield has remained very low by any standards, and has led to a boom in housing and perhaps other long term asset prices that depend upon low funding rates. Eventually, the boom will deflate either because of policy or the sand-pile effect. Whatever the mechanism, it is hard to argue with the fact that current prices can impact tomorrow's economic environment, and tomorrow's environment will impact tomorrow's prices...and so on. As discussed earlier, the price of credit risk is lower now than in a couple of decades, and can be traced to the same common economic sources. Prices of securities do not simply indicate value in transaction; they also have an important signalling effect, which cannot be evaluated without putting the prices in the correct economic context. When the Treasury started to tinker with the long bond, the immediate impact was the loss of signaling ability of that part of the yield curve. There is a belief that purchases of treasuries by foreigners has in general impeded the information content of the yield curve.

Incorporating the economic environment can only add to the accuracy of our pricing models. It is fair to say that recent financial research is seeing a resurgence of an approach where it is clear from the outset that economic variables can affect price, value and risk, and price, value and risk can affect economic variables. This direction of research is bringing to the forefront of our profession what we already know intuitively: the economic state of the world *does* matter.

So, how do we properly put economics (back) into models? I propose one, not the only possible, framework. Arguably, the framework is better adapted for models that are used for extracting systematic alpha. We begin by translating economic priors into possible economic scenarios and the realization of the factors. For example, typically, but not always, low inflation and low GDP is associated with low interest rate levels and relatively flat yield curves. When pricing a credit risk-free security, the arbitrage free price can be compared or supplemented with pricing obtained by the risk premia of these factors on the security. If under shocks of the factors a so-called arbitrage free package yields non-zero excess returns, it has to hold true that the risk-neutral price is wrong, or there are hidden factors, or there are risk-free profit opportunities.

- We can start by defining risk exposures in terms of specific factors. For instance, duration, curve exposure, spread exposures of various kinds, and structural components such as the magnitude of embedded volatility sale and the static aspects, such as roll-down in the curve. For each of these factor exposures, we obtain factor risk premia. For packages that allow factor risk-premia to be hedged out, we can price using the arbitrage free approach. Our model is general enough to have the economic content incorporated right from the beginning.
- Depending on our outlook of the world and the expected variation in risk premia, certain sources of risk are better than others at different times. Whenever we expect to be able to earn high risk premia in a sector, we overweight that sector/security. For example, the term-premium is the premium that is earned by increasing duration. To quantify the term-premium content of the yield curve, we build models that can simultaneously extract the term premium from the whole yield curve.
- To enable relative pricing, we need to be able to create an arbitrage free model for the discount factor; to do this, we need to have the proper forward rates in the risk neutral measure and also the physical measure. To connect the physical measure forward rates to the economic variables, we can build up an economic model for the short rate. The Taylor rule connects the short rate to the deviation of inflation and output from targets. Since the Fed appears to follow the Taylor rule in setting the short nominal rate, this input is crucial if we want to extract the economic content embedded in the yield curve.
- Once economic variables are included, one can investigate the impact of changing demand and supply dynamics. For example, has the valuation of the treasury curve changed due to the participation of foreign central banks; and if so, by how much? Research by using a macro-term structure model by ex Fed Governor Bernanke and his coauthors shows that this is of the order of 50 basis points. Given a macro term structure model, the direction of causality can also be inferred.

Much research along these lines has already happened. To show that such models are viable, we can build well-motivated simple toy models with latent factors without economic variables, estimate the latent variables, and then regress the latent variables on economic variables to see if we can find a relationship.

Let us build a quantitative term-structure model with economic content. Our approach will be to build a well-motivated term structure model that is based in economics, estimate the parameters in the model using market data (in an arbitrage free way), and then pick relevant economic variables that are related to the latent variables by regression. Figure 10 shows the key economic variables we will use - the inflation rate, real GDP, current account deficit as a percent of GDP, and the difference in the ten year and one year treasury rates as a proxy for the term premium:

A general form of a two factor latent variable model is given by

$$\begin{aligned} dx &= -\mu_x(x - \theta_x) dt + \sigma_x dw_x \\ dy &= -\mu_y y dt + \sigma_y dw_y \\ dz &= k(x + y - z) dt \end{aligned}$$

We can think of  $z$  as the instantaneous short rate, with fit results in figure 11. We can think of  $x$  as related to inflation, and  $\theta_x$  as the long term target for this variable.

Since the change in  $z$  is related to the difference from the sum of  $x$  and  $y$ , it is tempting to identify these variables with the inflation and GDP gaps as in the Taylor rule and the difference equation for  $z$  as the Taylor rule. The benefit of this specification is that we can now bring in the machinery for solving for zero coupon bond prices and yields using:

$$P(t, T) = E_t[e^{-\int_t^T z(\alpha) d\alpha}] = e^{-y(T-t)} \quad (3)$$

which can be solved in closed form. Now that we have an analytical model for the discount factors, we can fit the model to the term structure every day. Going back to 1960, we do this exercise and obtain the estimated values of the parameters and variables of interest (see figure 12). Now we can try combinations of economic variables that correlate well with these estimated parameters. By construction, the structural changes in the ‘‘Taylor rule’’ are transmitted across the yield curve via the latent factors  $x$  and  $y$ . Figure 13 shows that  $\theta_x$  fits well with the inflation rate (plus some premium), and figure 14 shows the dependence of the factor  $y$  on the slope of the yield curve (curve risk -premium).<sup>15</sup> Since the inflation risk factor is best expressed in terms of duration risk, and the curve risk-premium is best expressed in terms of yield curve steepening or flattening exposure, now we have a methodology to connect macro variables with risks of securities to these macro factors in an arbitrage free way (we simply compute the sensitivity with respect to the factors for every credit risk free bond using the model).

We can also investigate the impact of the mushrooming US current account deficit by adding in the current account deficit as a percent of GDP to the regression for the long term inflation rate  $\theta_x$ . As expected and shown in figure 15, we find that the market implied long term inflation expectations have been kept much lower than they would have; For very long horizons, long term inflation expectations could be almost two percent higher (difference of fitted residuals in the model with and without the current account deficit as an explanatory variable).

Now that we see that economic variables have good explanatory power for the latent variables, there is no reason why we should not build the economic variables into the model

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<sup>15</sup>I would like to thank Wendong Qu of Pimco for collaboration on the implementation.

right from the beginning; in other words, one could argue that before fitting to the yield curve and security prices, we should constrain the ranges of the parameters and factors using our economic priors. In a sense, when we specify the model to have two sources of uncertainty, plus policy, we are already imposing economics, i.e. we know from experience that yield curve fluctuations can be described by at most three factors, and in most circumstances, by two factors. We also know that credit free fixed income securities are predominantly determined by inflation (and inflation expectations), Fed behavior and risk-premium. So a simple, economically well-specified model is better suited to fit the market than a very general model that has little to do with the real-world in its specification. An advantage of such a model is also that the effect of the yield curve on the macroeconomy, as well as the effect of the macroeconomy on the yield curve can be estimated. The model is efficient enough to stress test with. We have also explored an extension where credit risk, prepayment risk and tax risk is introduced into the model explicitly from the start with stochastic intensities correlated to macro factors. We can throw in all relevant economic variables into the mix right from the beginning and estimate a no arbitrage vector autoregression in the spirit of Ang and Piazzesi (and others). Minimization of fitting errors then picks out the relevant economic variables. Due to the simultaneity of the market and economic variables, the impulse response functions show the impact of each set of variables into subsequent realizations of the others. The results appear promising.

We have systematically tried to put some economics back into models, at least a little bit of economics. Much remains to be done. I would like to thank readers of initial drafts of this paper, and participants at Risk's Quant Congress (2005) for listening to the talk and their followup questions.

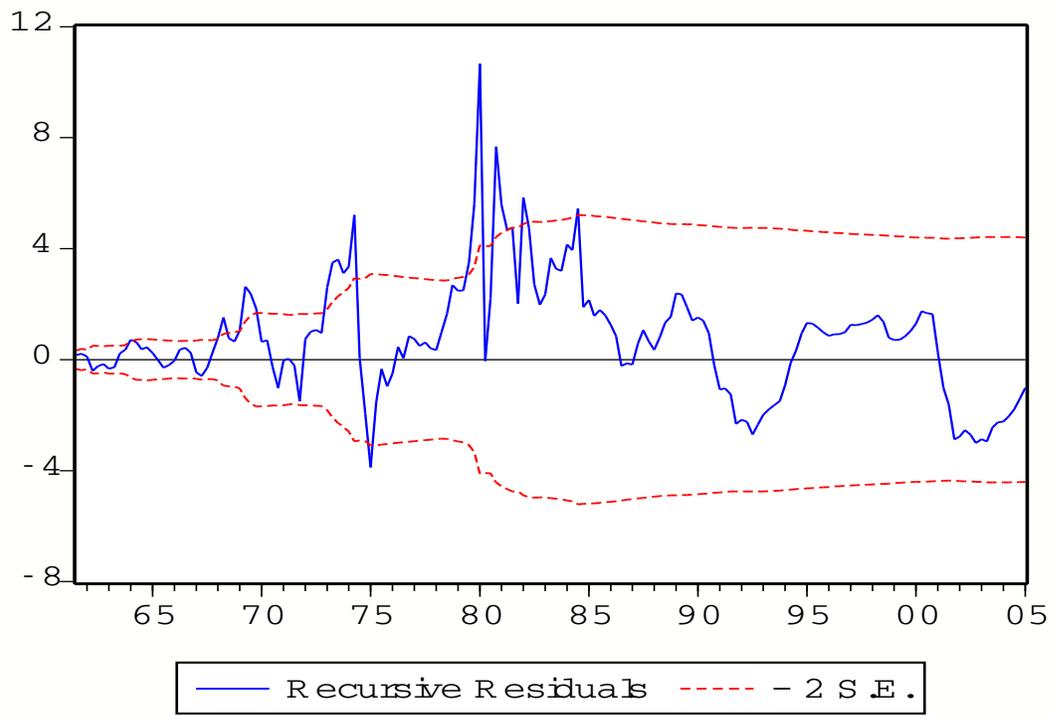


Figure 1: Recursive residuals. Demonstrates possibility of structural breaks in Taylor Rule in the mid 70s to mid-80s. Source: PIMCO.

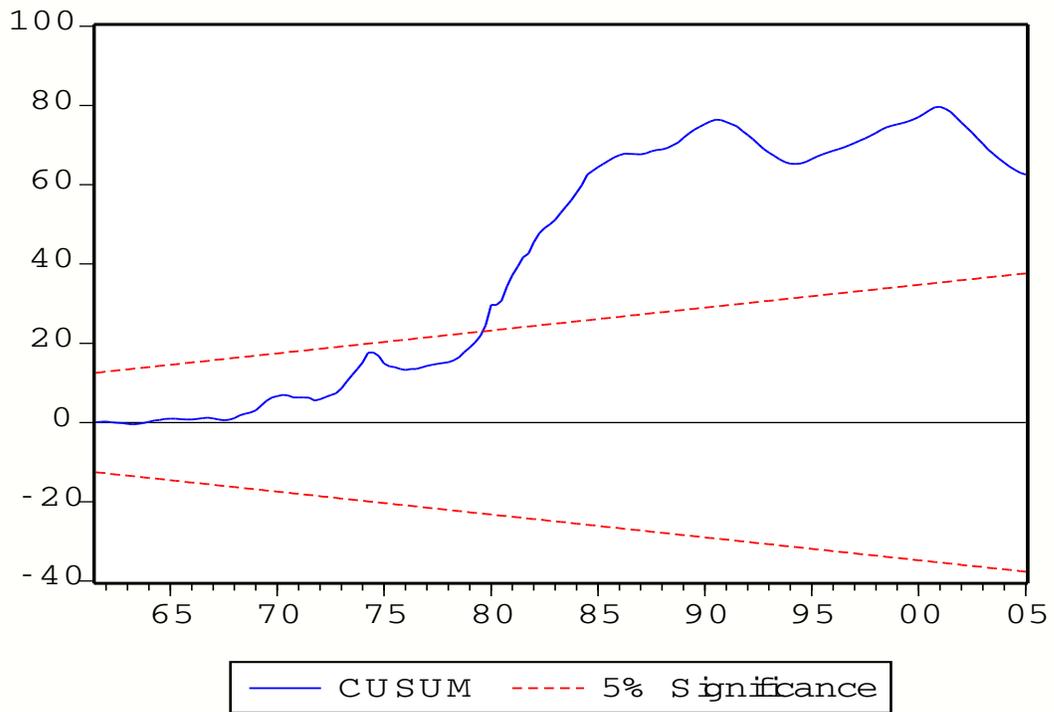


Figure 2: Cumulative Sum Test for testing for structural breaks. Demonstrates structural break in Taylor rule in 1980. Source: PIMCO.

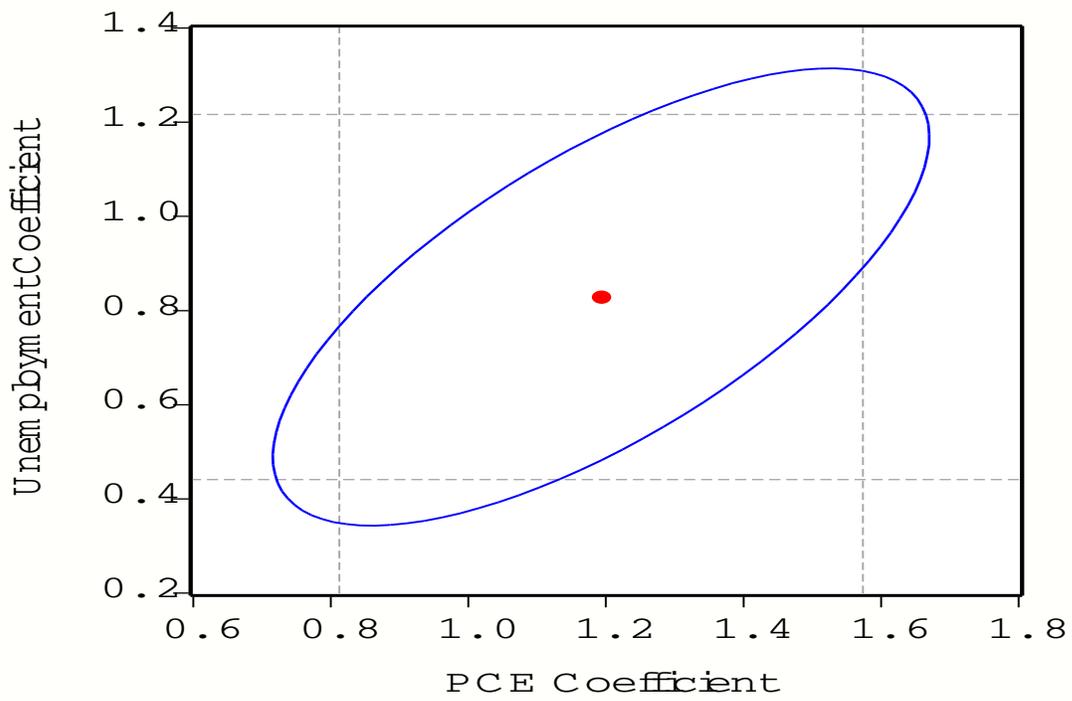


Figure 3: Coefficient tradeoffs in Greenspan years estimated for 1984-2005 period at 95% confidence level. Source: PIMCO.

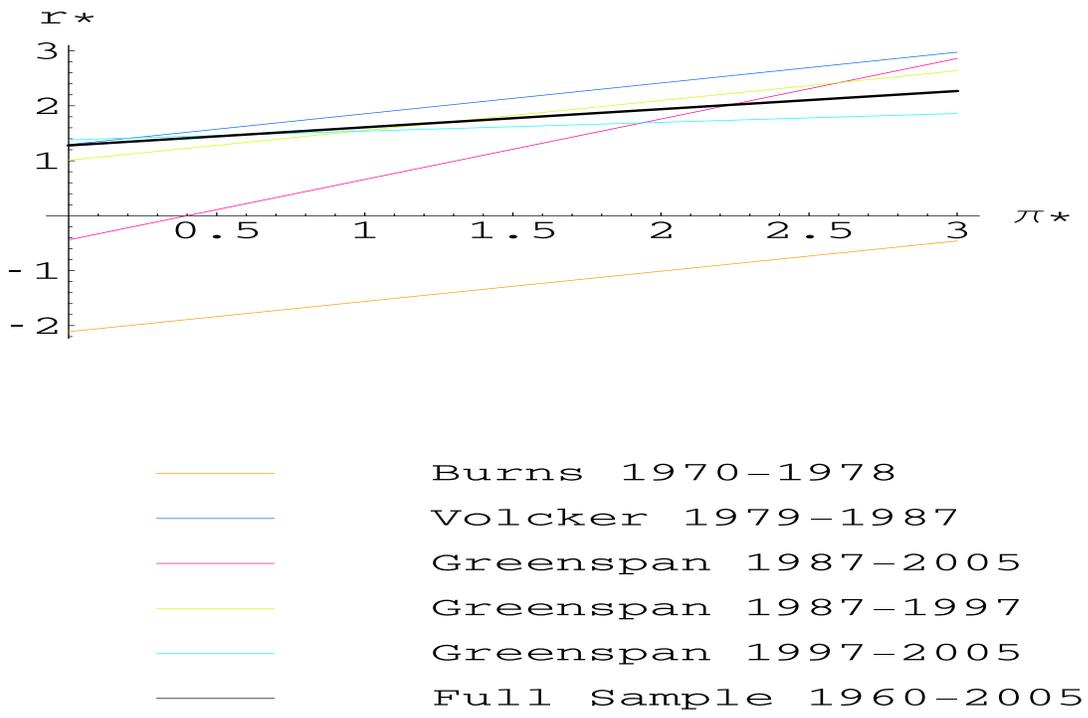


Figure 4: Tradeoff of PCE target and Real Rate Target in Era of Different Fed Chairmen.  
Source: PIMCO.



Figure 5: Richness/cheapness of futures contracts using a theoretical model for pricing the cheapest to deliver option. Source: PIMCO.

## Delta-Hedged Tranche Price Changes in May (bp)

	5Y		10Y	
	US	Euro	US	Euro
0-3% (points upfront chg)	4.0pts	3.9pts	1.5pts	-2.5pts
3-7%	(84)	(49)	133	28
7-10%	(38)	(24)	(85)	(19)
10-15%	(15)	(11)	(37)	(55)
15-30%	(2)	(1)	(14)	(6)

Note: Delta-hedged positions involve selling protection on the tranche versus buying protection a delta-adjusted notional amount of the underlying index.

Source: Morgan Stanley

Figure 6: Delta adjusted performance of IG4 tranches. Source: Morgan Stanley, June 2005.

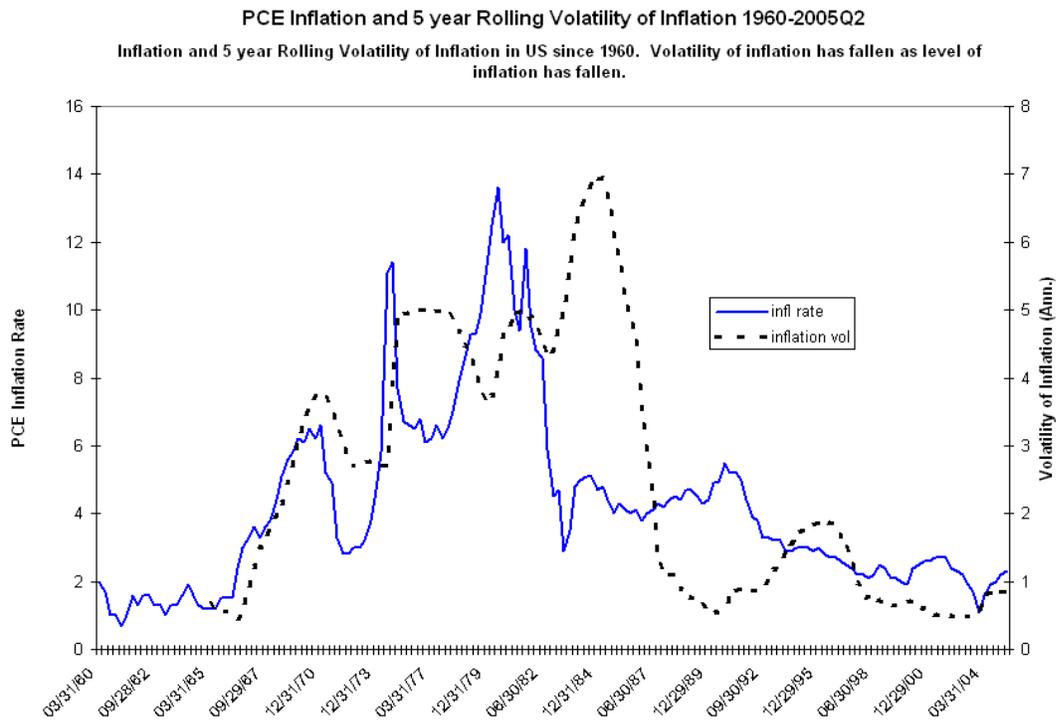


Figure 7: Inflation Rate and Inflation Volatility. Source: PIMCO.

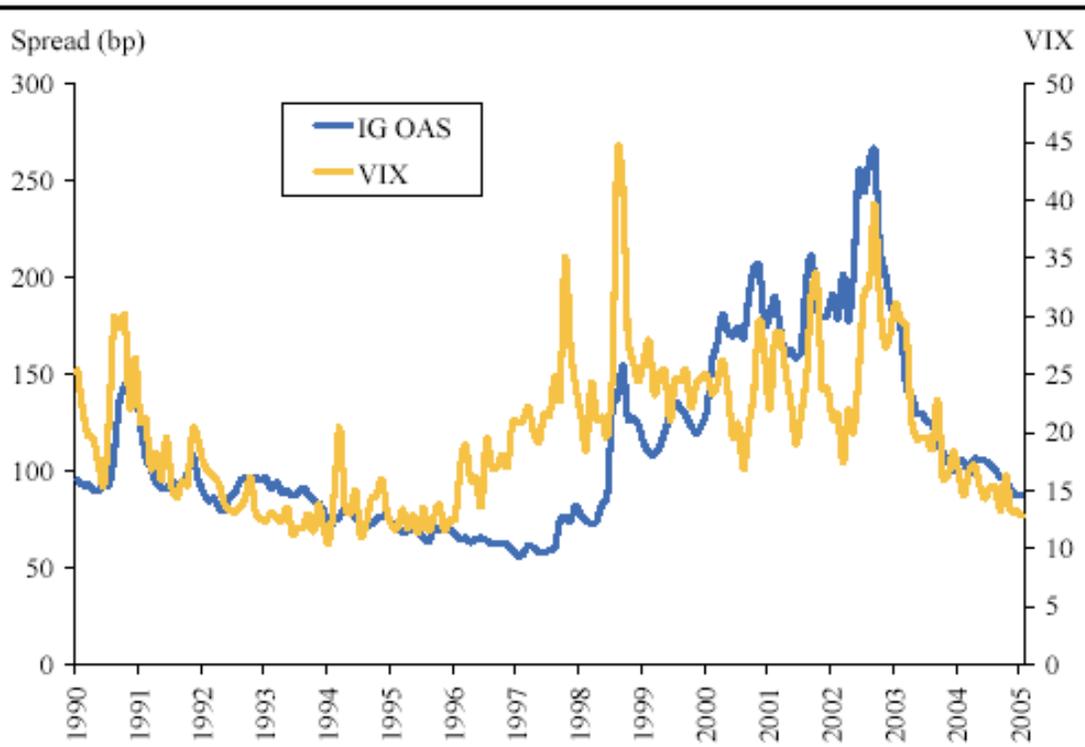


Figure 8: VIX and Corporate Spreads. Source: Lehman Brothers.

Asset Return Comparisons ( Rebalanced to LBAG Duration Monthly, LIBOR Financing )											( 07/91 07/05 )			
Date	LBAG	MBS	TSY	AGCY	CORP	CMBS	TIPS	Muni	EM	HY	ED5	FV	TY	US
MTD	(0.91)	(1.17)	(1.00)	(1.01)	(0.63)	(1.16)	(1.12)	(0.17)	(0.06)	1.59	(1.58)	(1.38)	(1.25)	(0.91)
YTD	1.58	1.09	1.93	1.46	1.60	1.20	1.01	2.17	3.91	2.50	(0.97)	0.31	1.68	3.25
0.25	0.71	0.30	0.59	0.59	1.13	0.49	(0.26)	0.86	3.40	5.06	(0.18)	(0.10)	0.36	1.10
0.5	0.95	0.39	1.29	1.03	0.94	0.66	0.98	1.53	3.39	2.61	(0.51)	0.39	1.26	2.15
1	4.79	4.66	4.54	4.25	5.27	4.58	5.07	4.81	12.82	10.43	0.96	3.14	4.72	6.61
2	4.81	5.21	4.06	4.26	5.20	4.73	5.97	4.31	11.33	11.48	2.14	4.37	5.00	5.42
3	5.02	4.10	3.96	4.28	6.14	5.07	6.10	3.74	14.95	14.85	4.58	4.84	4.82	4.60
5	7.01	7.27	5.90	7.13	7.08	8.16	7.93	4.93	9.55	6.58	10.62	8.21	7.26	5.53
7	6.21	6.23	5.59	6.18	6.11		7.38	4.75	9.60	4.43	8.12	7.27	6.70	5.42
10	6.75	6.92	6.25	6.75	6.69						8.07	7.48	7.22	6.32
14	7.29	7.24	6.89	7.26	7.46						9.15	8.28	7.85	6.78
<b>Information Ratio Comparison ( Annualized Excess Return over LBAG/Annualized Standard Deviation of Excess Return, monthly data)</b>														
Date	LBAG	MBS	TSY	AGCY	CORP	CMBS	TIPS	Muni	EM	HY	ED5	FV	TY	US
0.5	0.00	(1.82)	1.37	0.62	(0.01)	(1.28)	0.05	0.72	1.55	0.46	(2.37)	(1.01)	0.61	5.60
1	0.00	(0.21)	(0.52)	(1.29)	0.72	(0.29)	0.19	0.01	3.03	1.06	(1.98)	(1.32)	(0.08)	2.87
2	0.00	0.44	(1.56)	(1.31)	0.66	(0.10)	0.60	(0.27)	2.18	1.43	(1.04)	(0.26)	0.21	0.77
3	0.00	(0.30)	(1.55)	(1.36)	1.32	0.05	0.52	(0.62)	2.02	1.64	(0.16)	(0.09)	(0.15)	(0.50)
5	0.00	0.10	(1.35)	0.14	0.05	1.04	0.44	(1.01)	0.34	(0.05)	1.07	0.56	0.17	(1.30)
7	0.00	0.01	(0.65)	(0.03)	(0.07)		0.50	(0.68)	0.22	(0.20)	0.53	0.50	0.31	(0.59)
10	0.00	0.09	(0.62)	0.00	(0.05)						0.42	0.39	0.34	(0.36)
14	0.00	(0.03)	(0.55)	(0.06)	0.17						0.57	0.54	0.42	(0.43)
<b>Excess Return Correlation vs. LBAG</b>														
Date	LBAG	MBS	TSY	AGCY	CORP	CMBS	TIPS	Muni	EM	HY	ED5	FV	TY	US
0.5	1.00	0.45	0.18	0.43	(0.35)	0.93	(0.55)	(0.94)	0.03	(0.44)	0.94	0.51	0.54	0.43
1	1.00	0.10	0.07	0.34	(0.22)	0.74	(0.47)	(0.95)	0.08	(0.43)	0.74	0.58	0.52	(0.10)
2	1.00	0.32	(0.30)	0.48	(0.28)	0.73	(0.41)	(0.92)	0.24	(0.45)	0.77	0.75	0.73	(0.48)
3	1.00	0.39	(0.32)	0.28	(0.27)	0.59	(0.18)	(0.93)	(0.02)	(0.46)	0.38	0.56	0.62	(0.21)
5	1.00	0.42	(0.13)	0.36	(0.33)	0.49	(0.24)	(0.87)	(0.20)	(0.38)	0.51	0.55	0.58	(0.17)
7	1.00	0.38	(0.14)	0.27	(0.26)		(0.32)	(0.82)	(0.13)	(0.38)	0.47	0.51	0.52	(0.18)
10	1.00	0.32	(0.11)	0.24	(0.22)						0.44	0.52	0.55	(0.09)
14	1.00	0.20	(0.07)	0.14	(0.12)						0.53	0.57	0.59	(0.13)

Figure 9: Asset Excess Return Performance and Correlation to overall bond market as represented by the Lehman Brothers Aggregate over five years. Source: PIMCO. Hypothetical example for illustrative purposes only. No representation is being made that any account, product, or strategy will or is likely to achieve profits, losses, or results similar to those shown. Hypothetical or simulated performance results have several inherent limitations. Unlike an actual performance record, simulated results do not represent actual performance and are generally prepared with the benefit of hindsight. There are frequently sharp differences between simulated performance results and the actual results subsequently achieved by any particular account, product, or strategy. In addition, since trades have not actually been executed, simulated results cannot account for the impact of certain market risks such as lack of liquidity. There are numerous other factors related to the markets in general or the implementation of any specific investment strategy, which cannot be fully accounted for in the preparation of simulated results and all of which can adversely affect actual results.

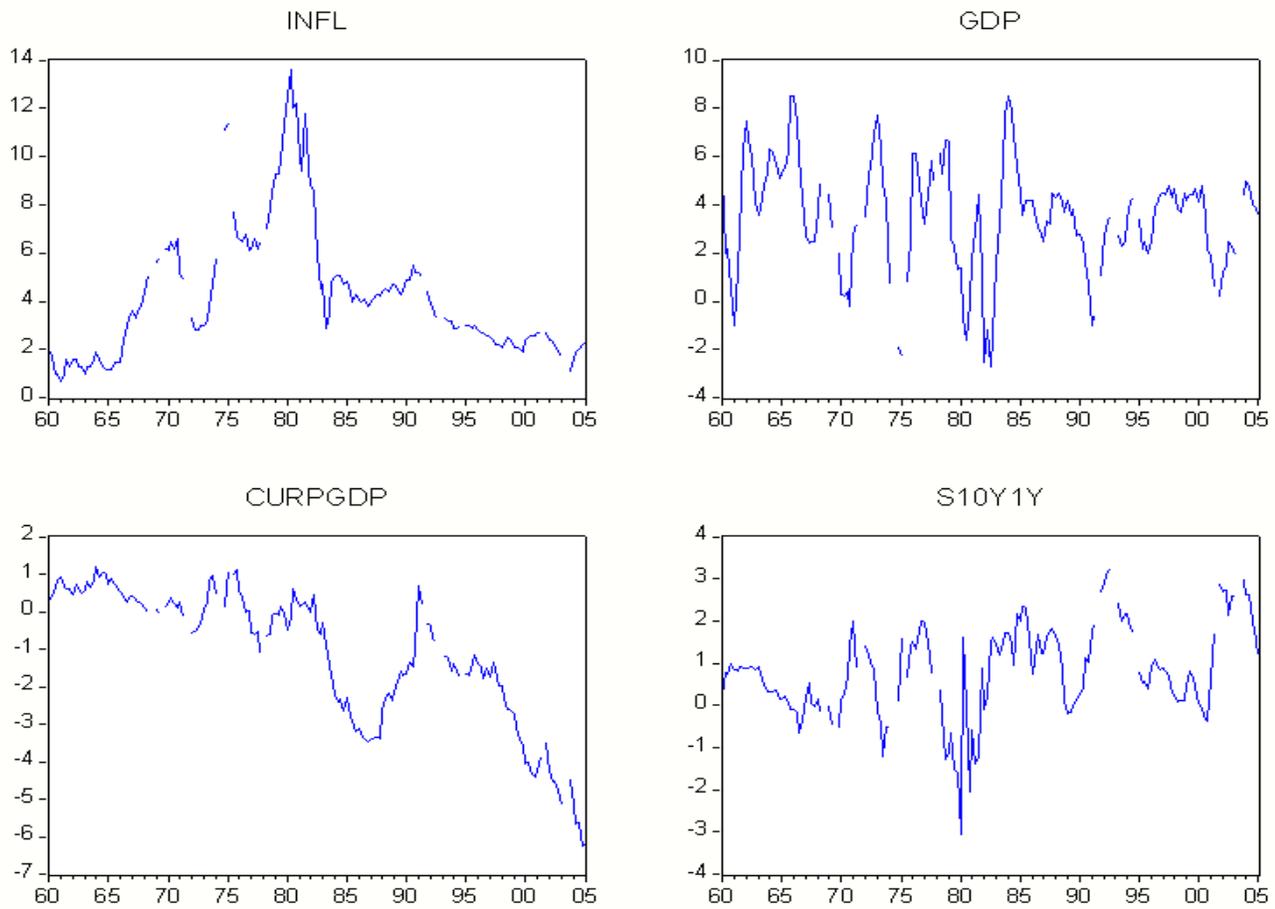


Figure 10: Macro Economic data series used for fitting two-factor model. Source: PIMCO.

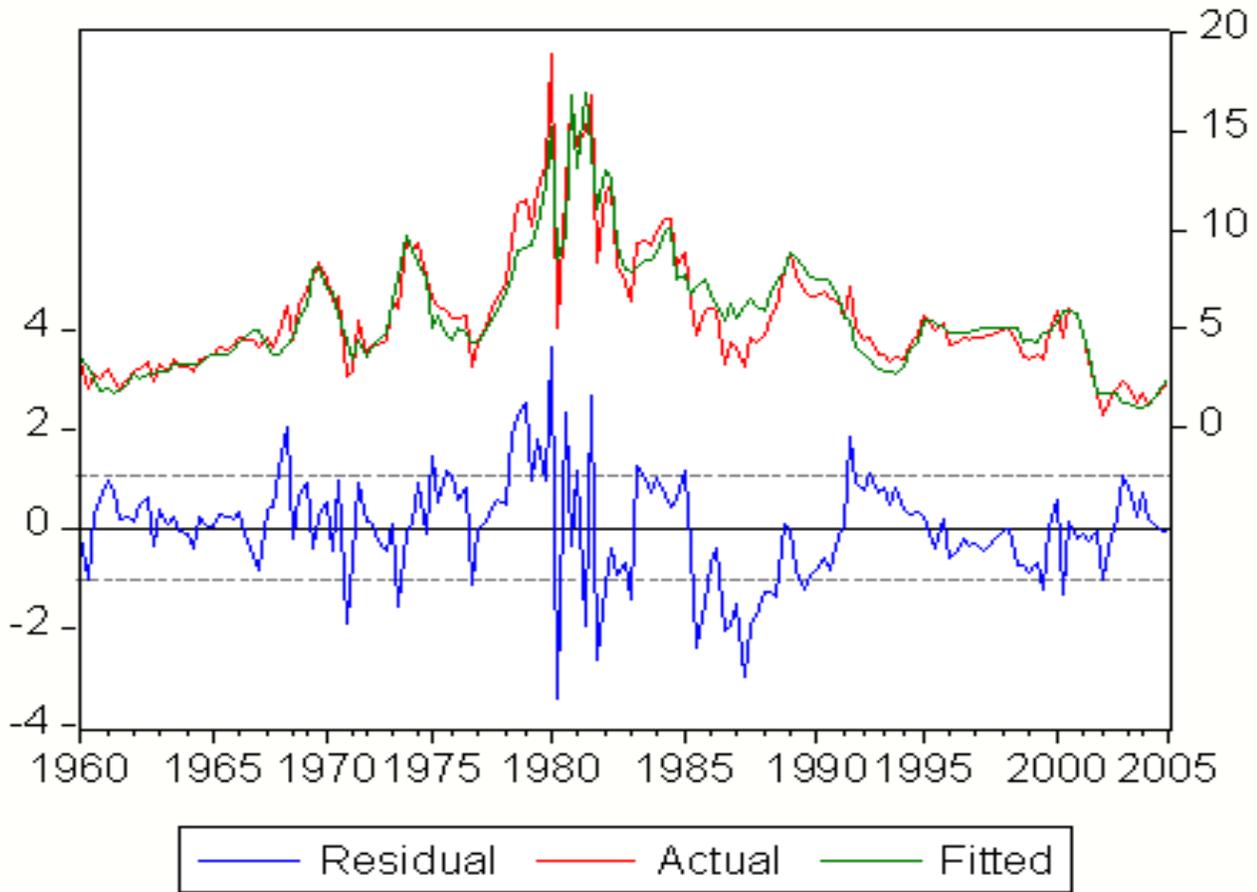


Figure 11: Fed Funds Rate and  $z$ . Source: PIMCO.

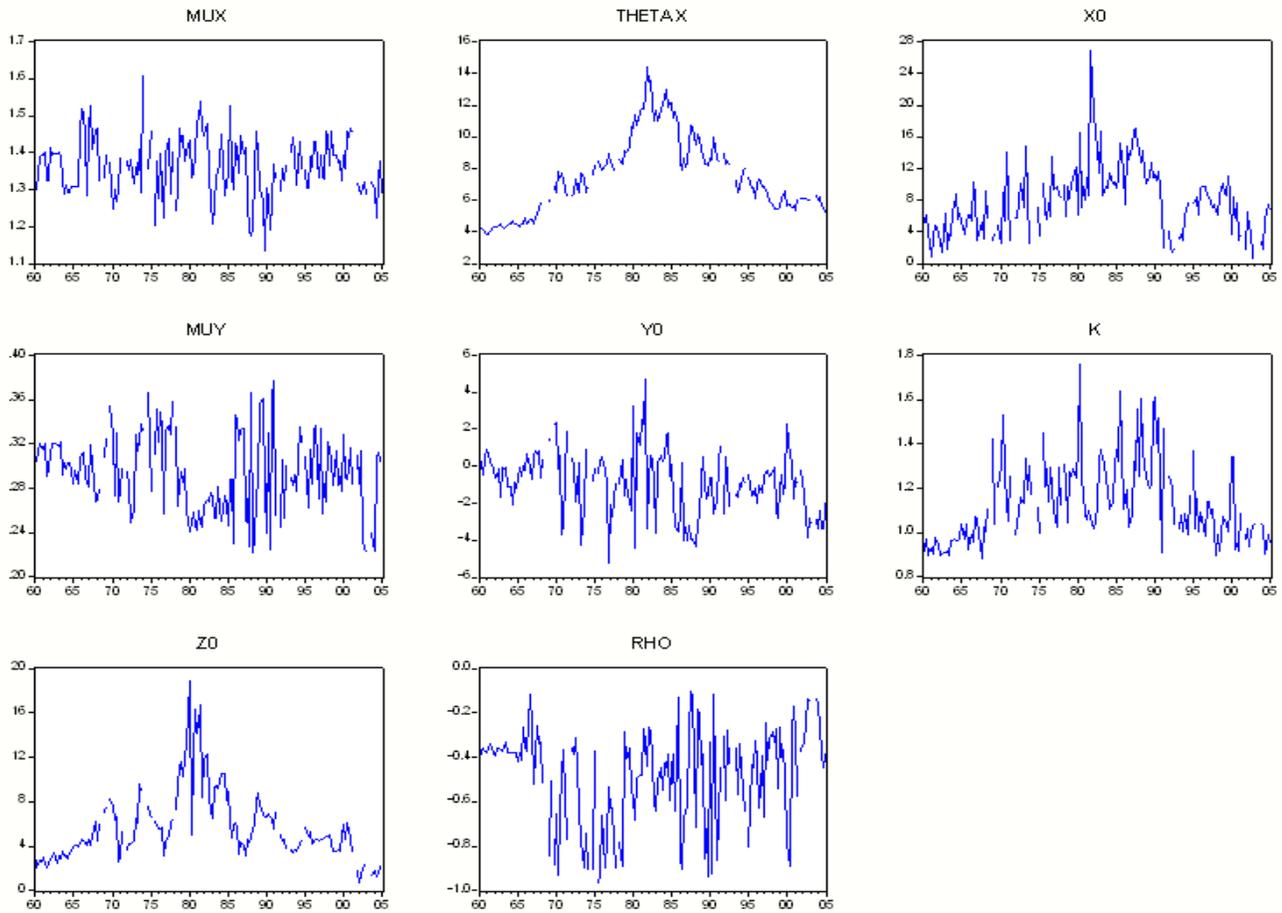


Figure 12: Model Fits using only 1y, 3y, 5y, 10y treasuries as input. Source: PIMCO.

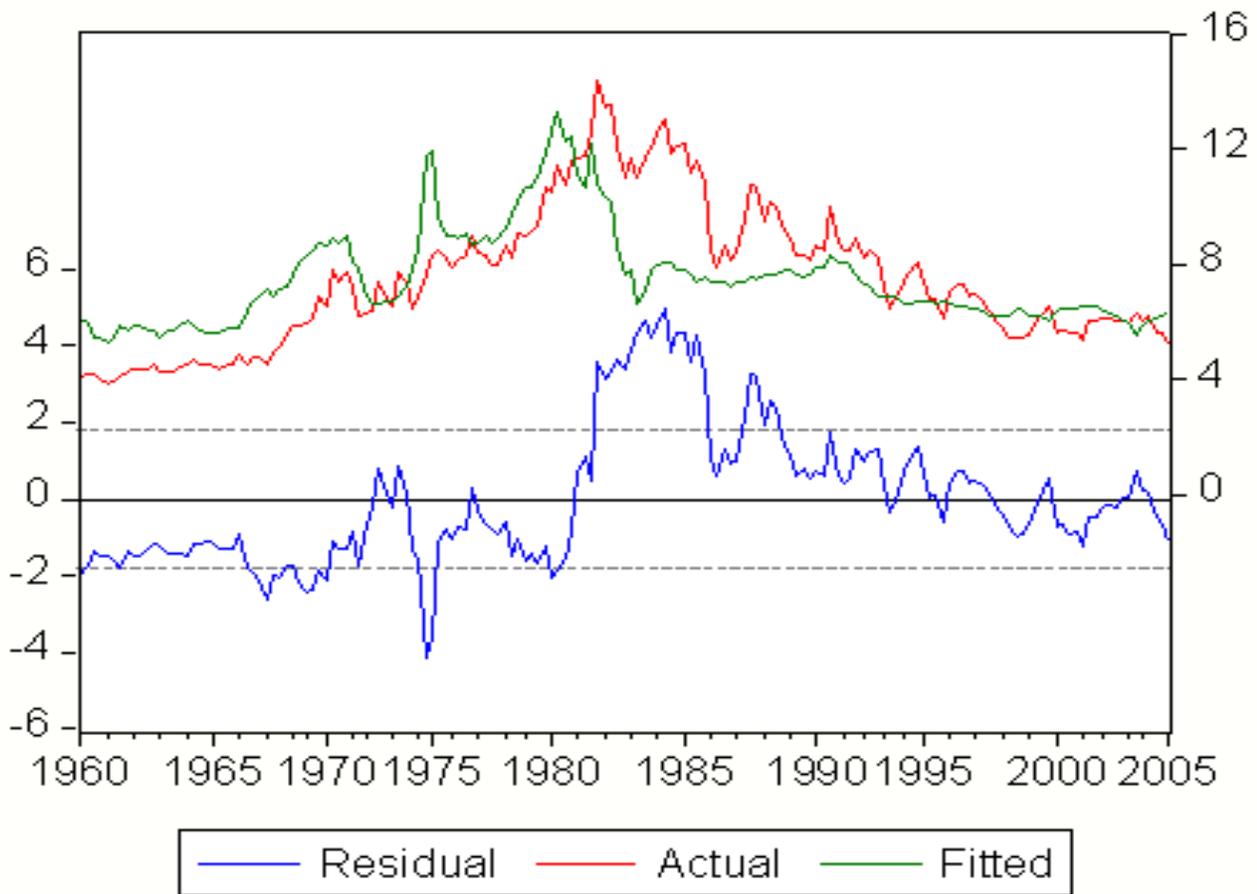


Figure 13: Model implied long term inflation rate regressed against actual inflation. Regression equation  $\theta_x = 4.868 + 0.619 * INFL$  yields  $r^2$  of 0.46, with infl t-stats = 11.9. Source: PIMCO.

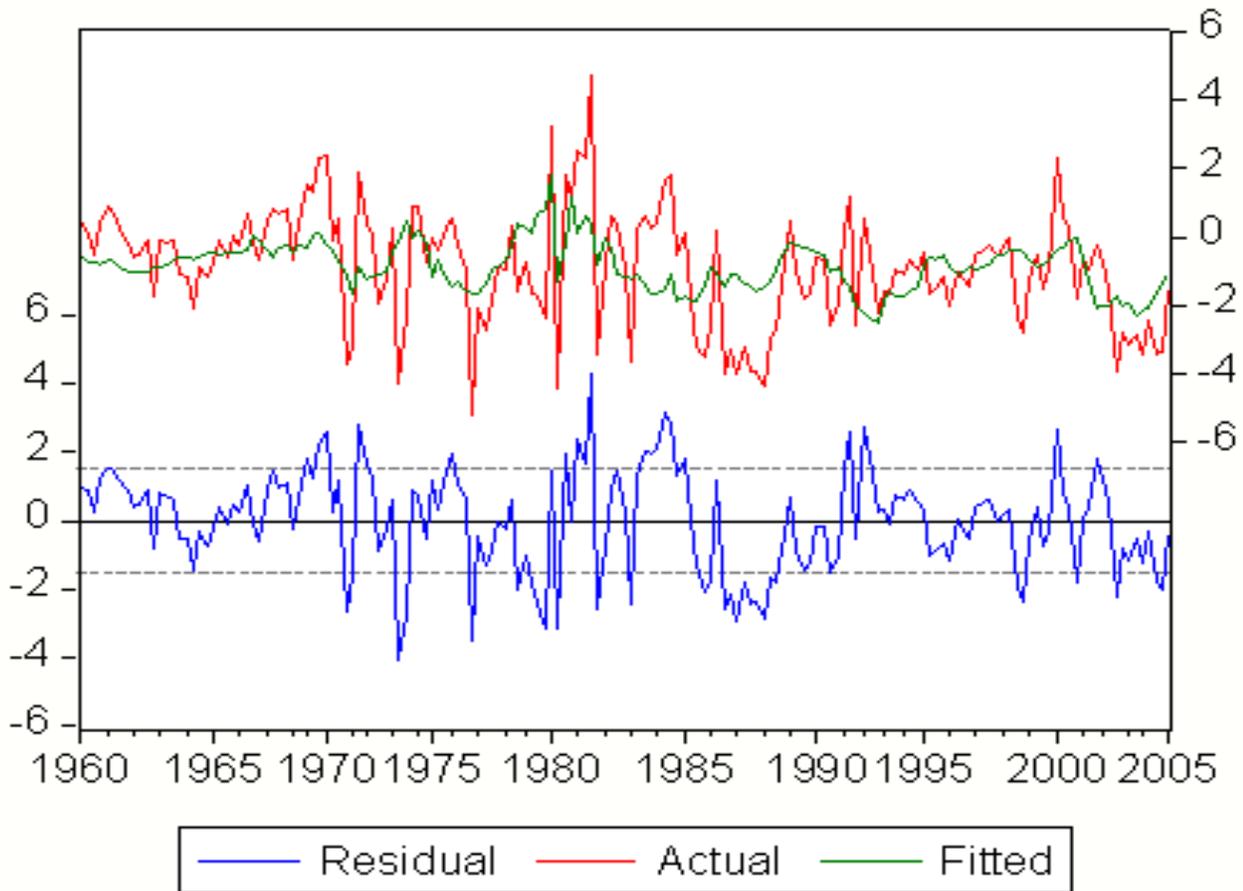


Figure 14: Second factor  $y$  versus economic and market variables. Regression equation  $y_0 = -0.14 - 0.0422 * GDP - 0.00579 * INFL - 0.667 * (10Y - 1Y)$ . Only the slope factor  $10Y - 1Y$  is significant. This indicates that the strongly mean reverting second factor might be more correlated to risk-premia and preferences than to economic cycles. Regressions against market volatility indicators also show a high correlation of the second factor. Source: PIMCO.

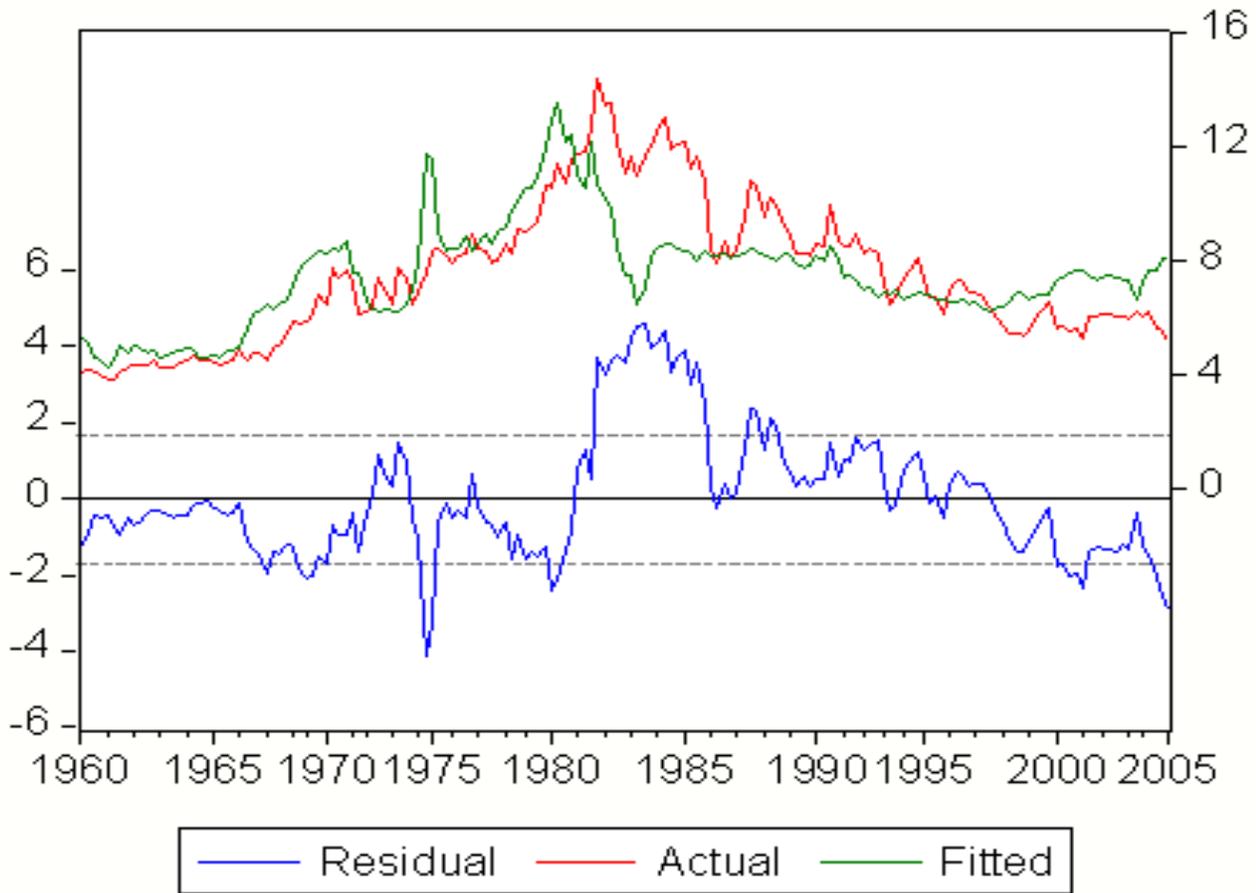


Figure 15: Long term inflation expectations implied by term structure model regressed against actual inflation rates and current account deficit. Regression equation  $\theta_x = 4.13 + 0.687 * INFL - 0.387 * CURPGDP$ , where t-stats on inflation rate equal 13, and on current account deficit equal 5, so both variables are significant. Regression  $r^2 = 0.54$ . Source: PIMCO.