Persistence, Predictability and Portfolio Planning

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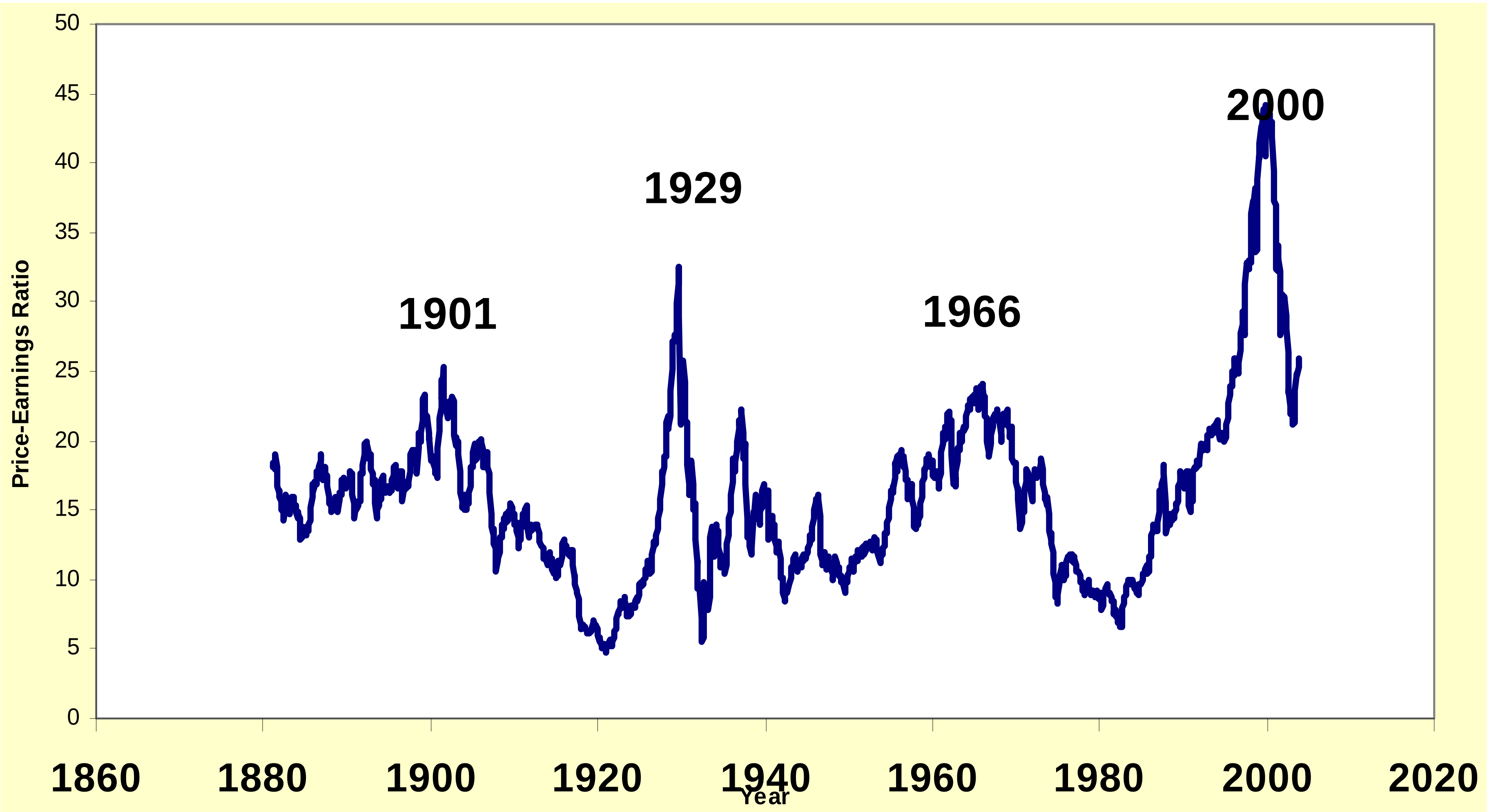
Stocks are sometimes cheap and sometimes dear. Important for long run investors?
Price/10 year Average Earnings 1880-2003



Background

- Academic studies have found:
  - stock returns predictable by such variables as
    Dividend yield, B/M, interest rates etc
  - But virtually no out of sample *return* predictability
- Does this mean that investors should ignore time variation in returns and behave as though expected returns constant?
- **NO!**

We show that:

- Return predictability that is of first order importance to long run investors will be:
  - associated with large price variation.
  - hard to detect using standard regression framework even when a perfect signal is available
  - hard to *estimate* for portfolio planning purposes

- A promising alternative to popular *academic* predictors is forward looking forecasts of *long run* returns from DDM
  - Convert long run forecasts to short run

A Simple Model of Return Predictability

$$\frac{dP}{P} = \mu dt + \sigma_P dz_P$$

$$d\mu = \kappa(\bar{\mu} - \mu)dt + \sigma_\mu dz_\mu$$

Mean reverting expected return

Parameters for simulations chosen so that:

- Unconditional distribution of μ is **fixed** at $N(\bar{\mu} = 9\%, \nu = 4\%)$
  - varies a lot: 1 sigma interval (5% to 14%)
- Consistent with a 14% annual stock return volatility
- Risk free rate is **constant** at 3%, implying 6% equity premium.
- *Nine* scenarios from the combination of

$$\kappa = 0.02, 0.10, 0.5, \text{ and } \rho = corr\left(dz_P, dz_u\right) = 0.0, -0.5, -0.9$$

Strategy

Use this (simulation) model to show this amount of expected return variability

- Implies big variability in *prices*
- Little short run *return predictability* an dis hard to detect
- Possibly strong long run return predictability

Later we will show:

- The data consistent with this amount of predictability
- How to exploit it

P/D Ratios implied by the scenarios

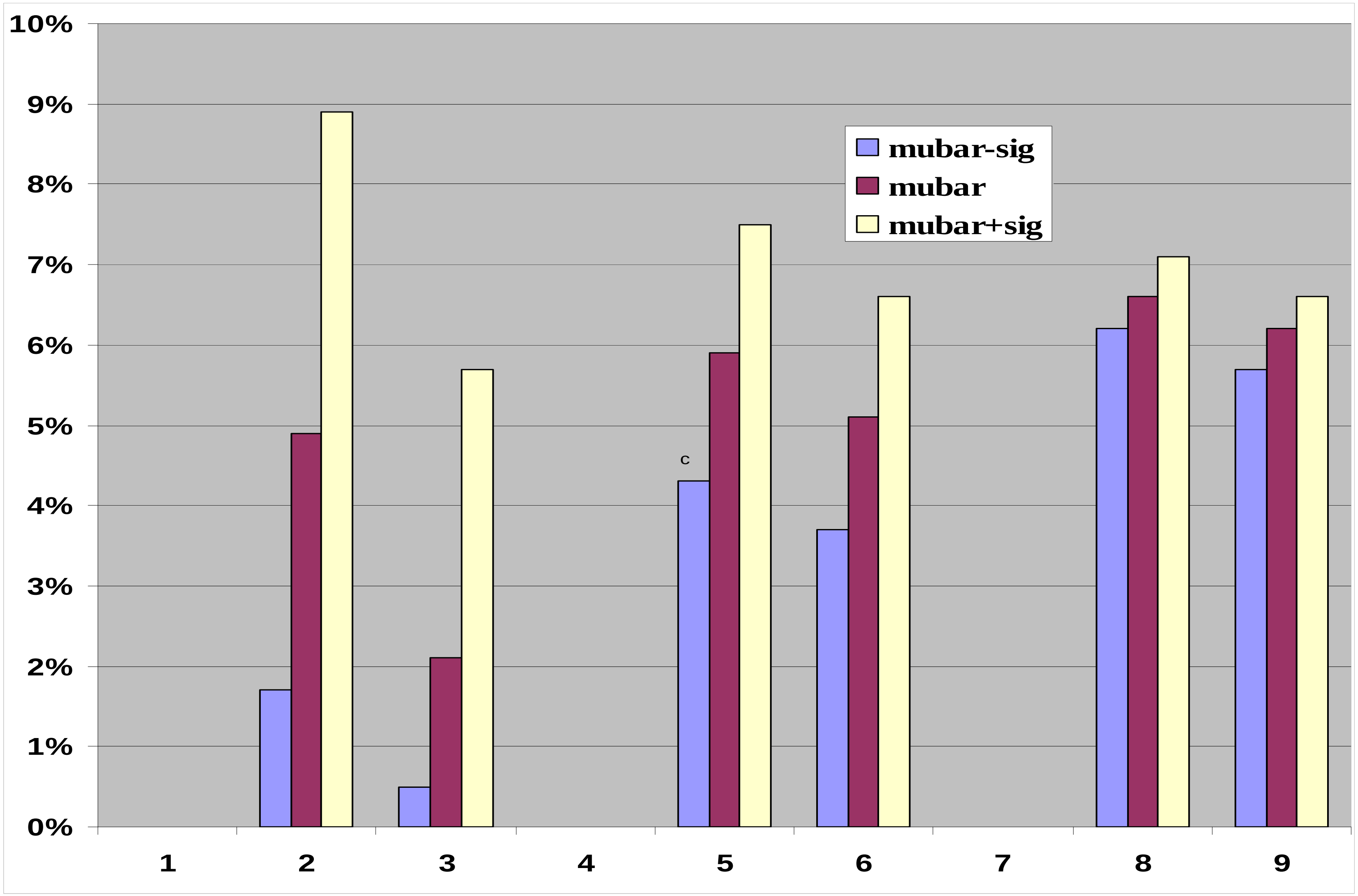
- Dividend Process: $$\frac{dD}{D} = g\,dt + \sigma_D dz_D$$
- Expected Rate of Return:

$$E\left[\frac{D\,dt + dV}{V}\right] = \mu\,dt \qquad d\mu = \kappa(\bar{\mu} - \mu)dt + \sigma_\mu dz_\mu$$

- Differential equation allows us to solve for price as a function of dividend growth rate:

$$P(D,\mu) = Pv(\mu)$$

Dividend yields can vary a lot as μ changes even though dividend growth assumed constant (g = 1.85%)

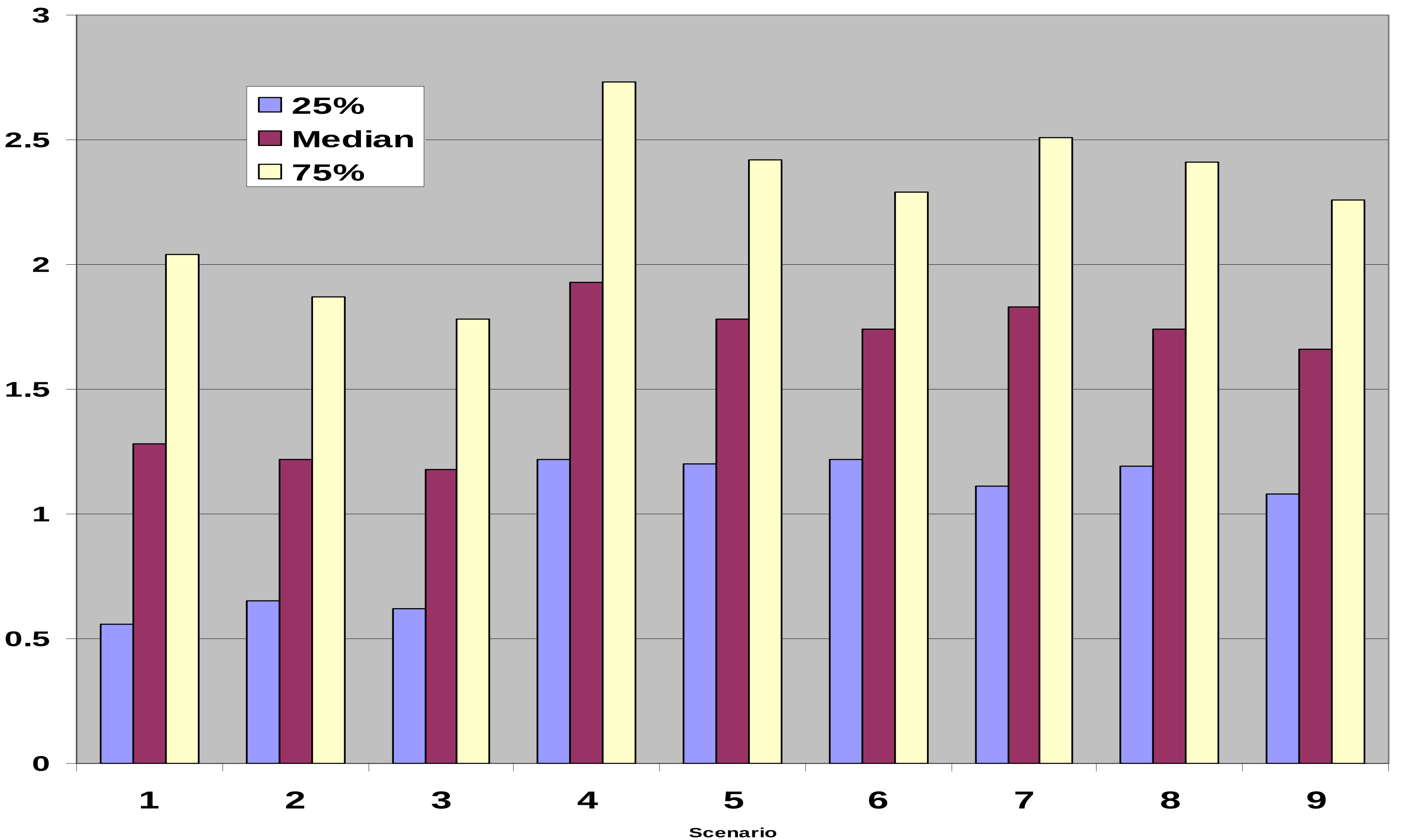


- Under our scenarios
  - Prices vary a lot
  - Expected returns vary a lot (5%-13%)

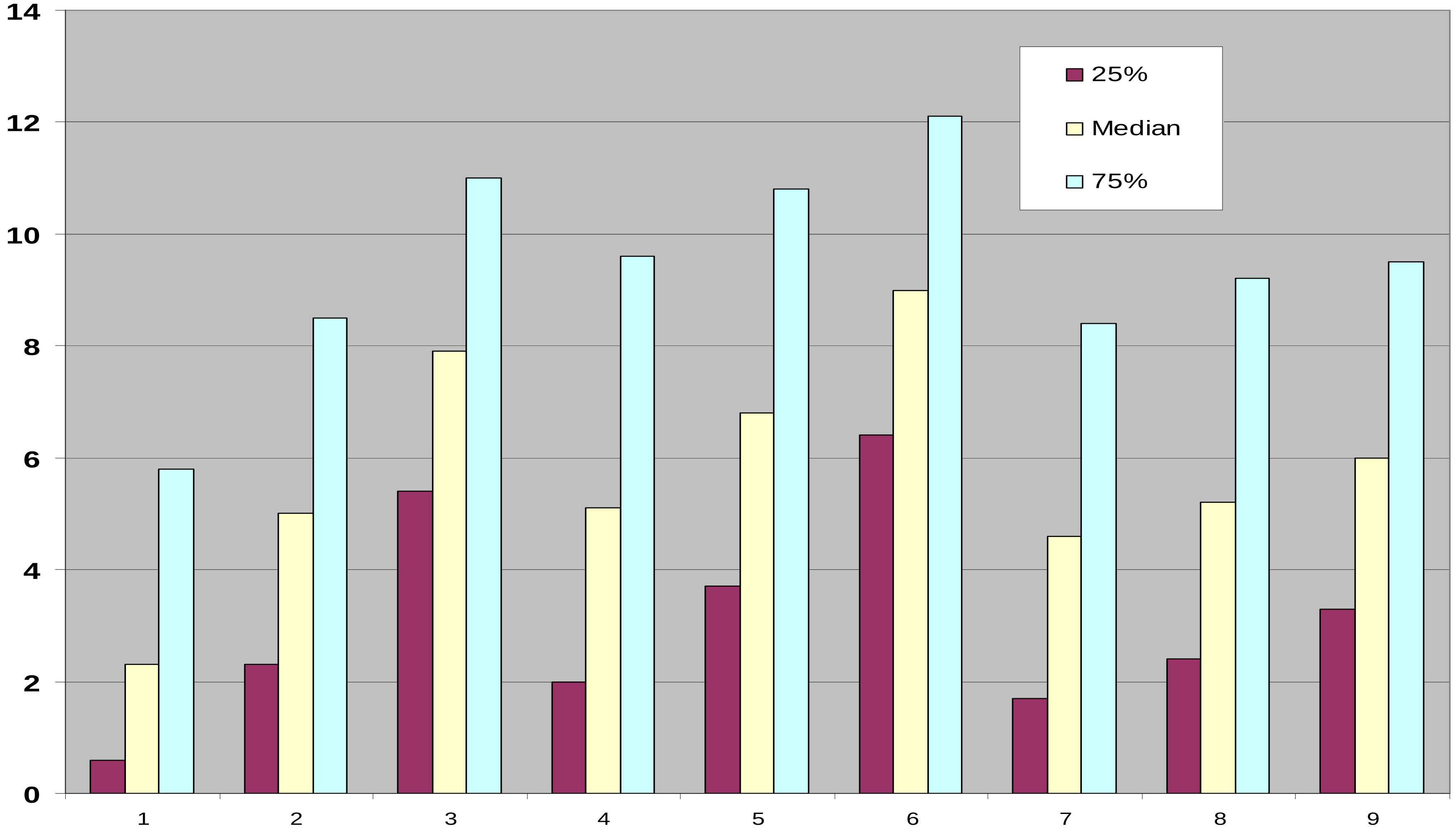
- Are we likely to detect this predictability by regressing returns on 💻 (or proxies for 💻)?

$$R(t,t+\tau) = a + b\mu_t + \varepsilon_{t,t+\tau}$$

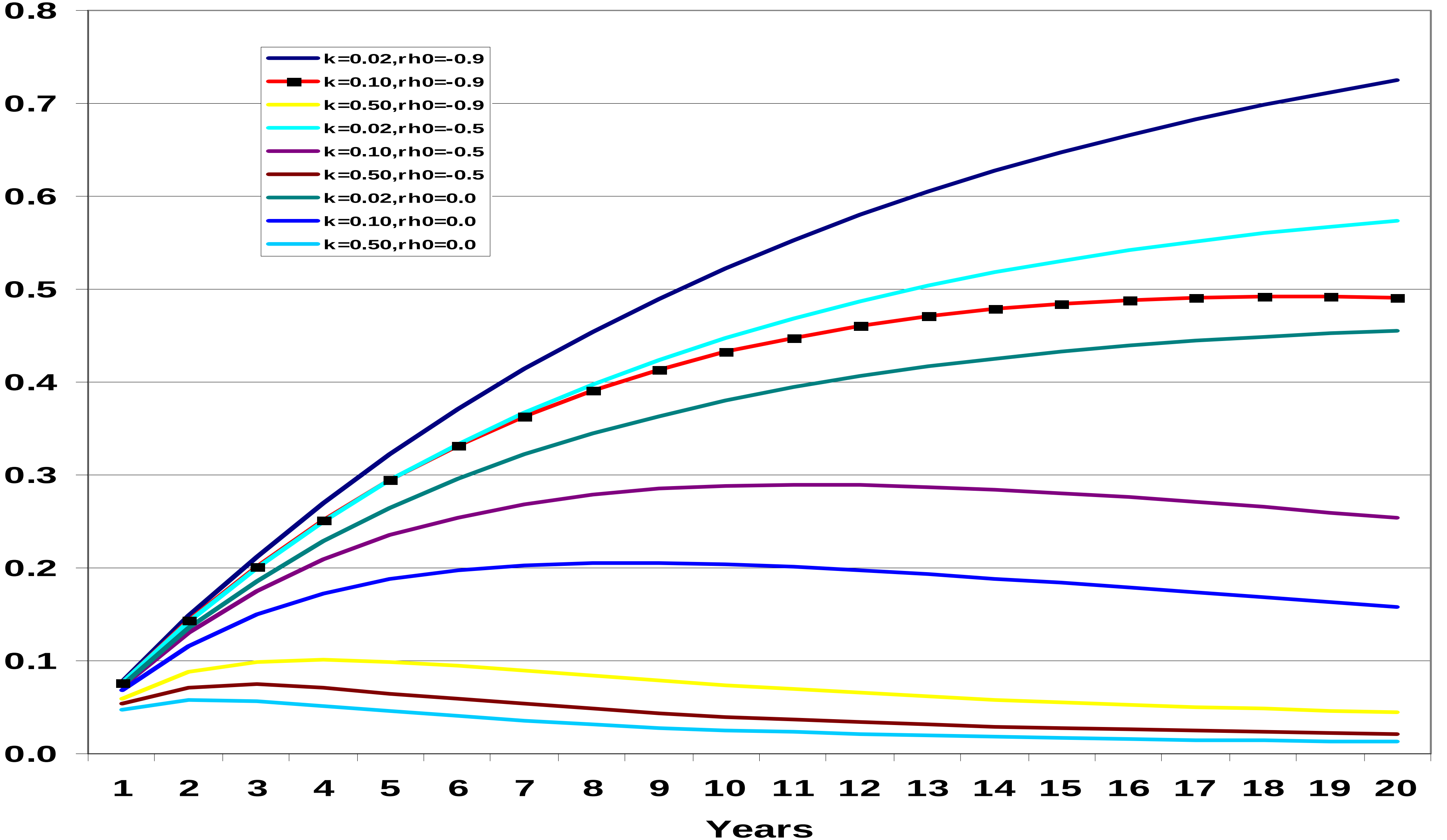
Distribution of corrected *t*-ratios on the predictor using 70 years of simulated monthly returns



R^2 (%) in an Annual Return Predictive Regression (70-years simulated returns)



R^2 as a function of horizon for different values of κ and ρ



- Short run predictability is hard to detect and measure.

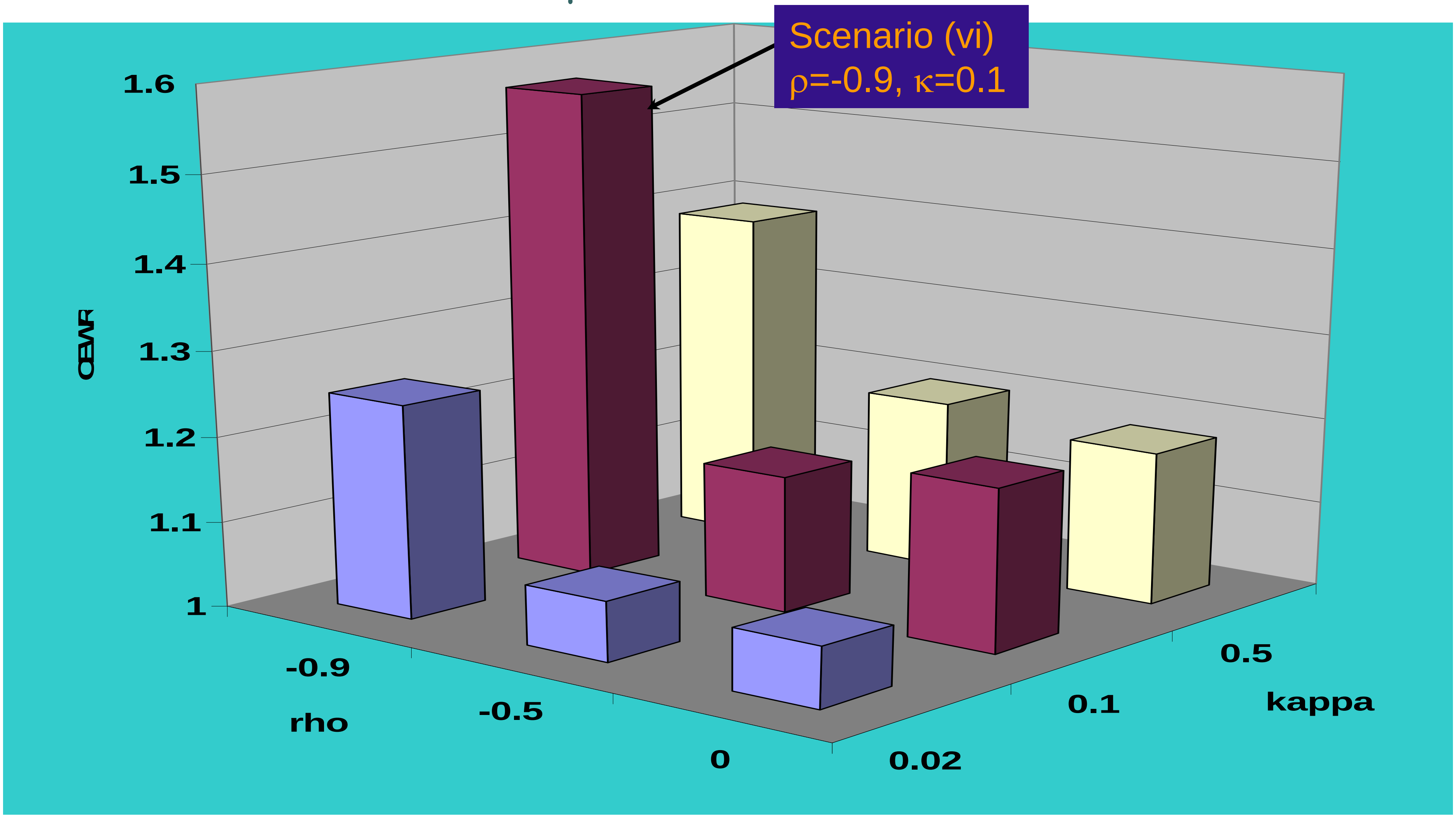
- Would it be valuable if we could detect it?

  e.g observe

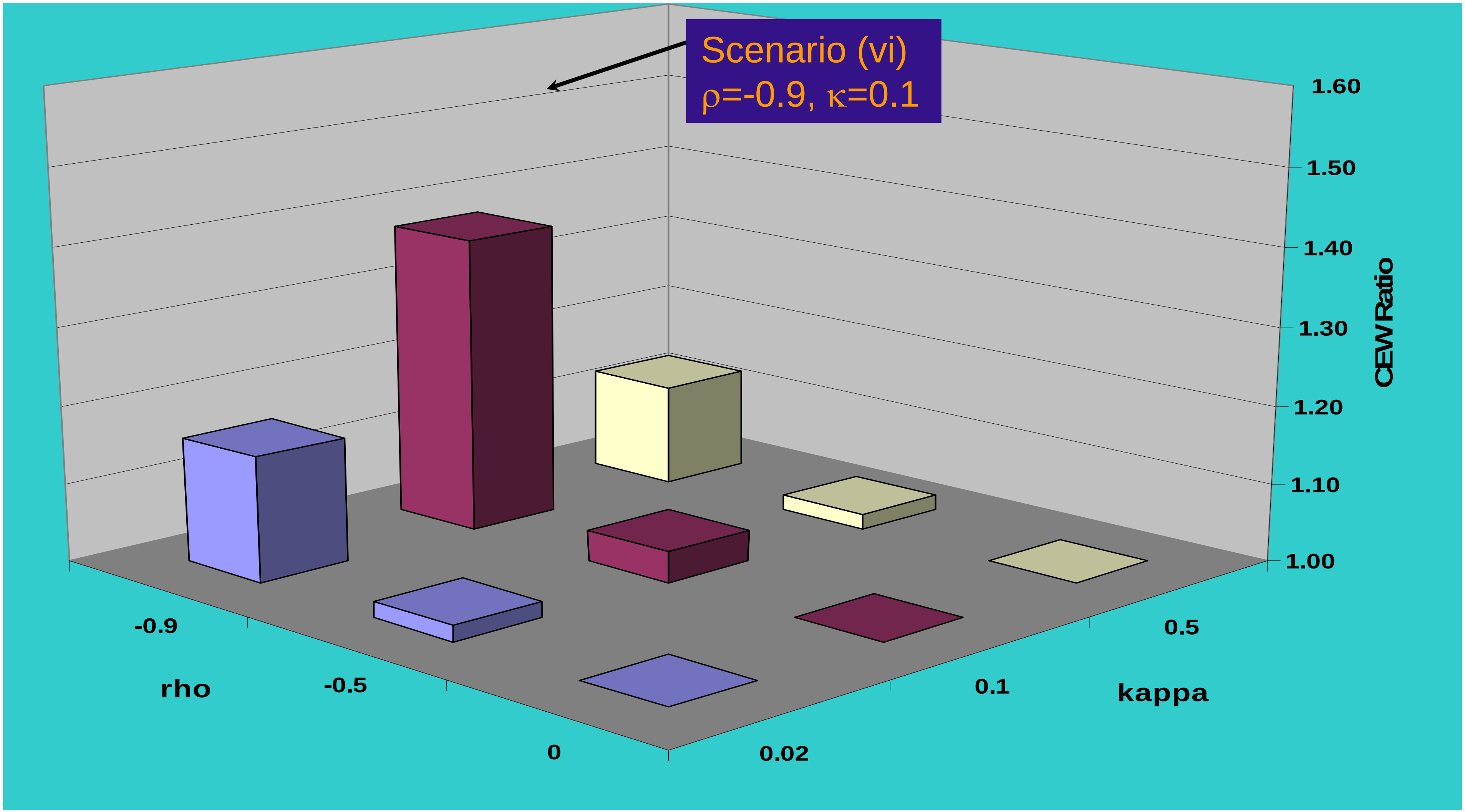
Economic Value of Market Timing

- Investor is assumed to maximize expected CRRA utility (RRA = 5) over terminal wealth.
- Measure economic value using certainty equivalent wealth ratio between different strategies (CEWR)
  - Optimal dynamic strategy
  - Myopic strategy
  - Unconditional strategy

Value of (optimal) dynamic strategy relative to unconditional strategy: CEW(O)/CEW(U) ($T=20$, $\sigma_P=0.14$, $\sigma_\mu=0.04$, μ = 9%=mubar)

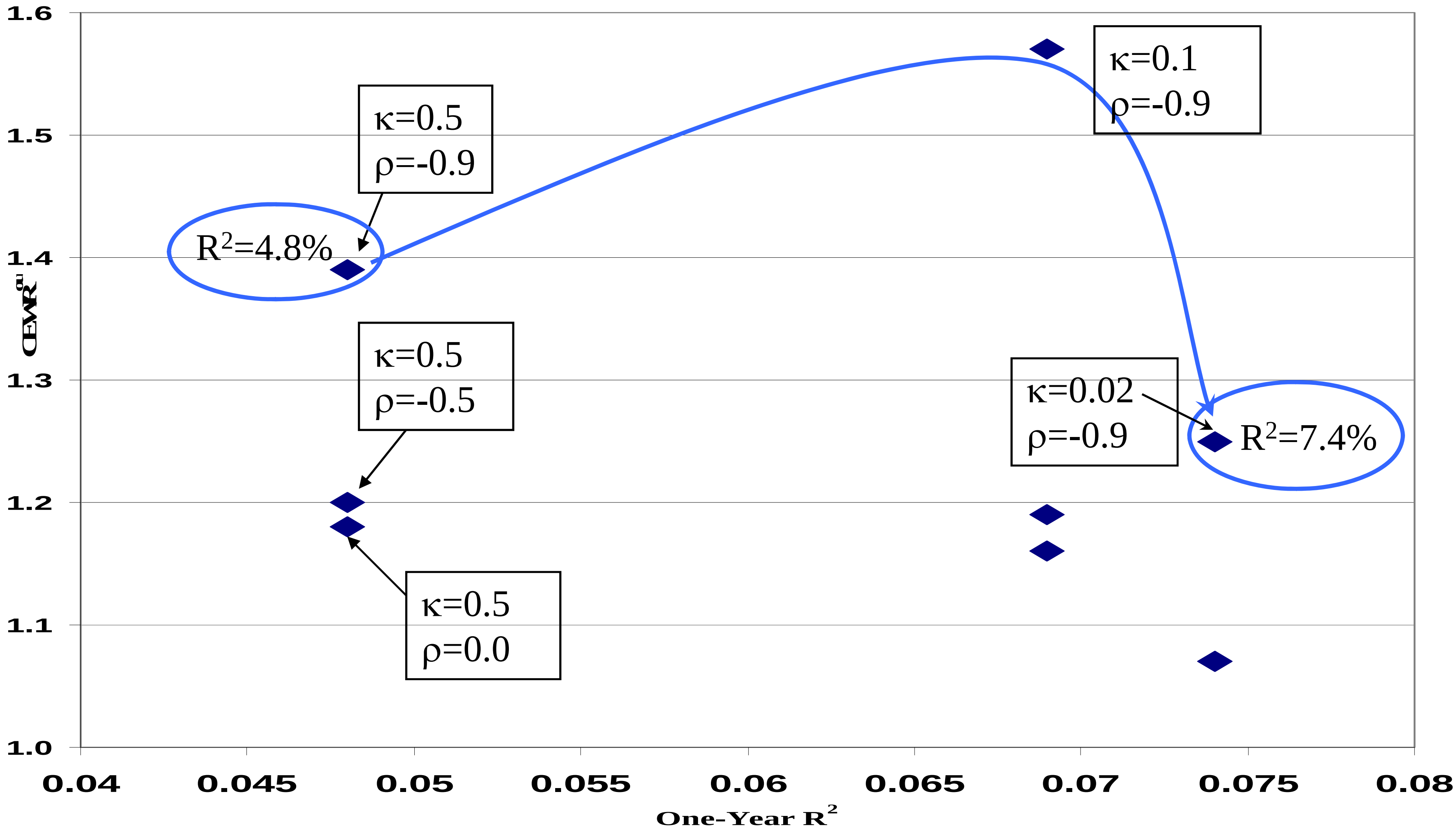


Value of (optimal) dynamic strategy relative to myopic strategy: CEW(O)/CEW(M) (T=20, σ_P=0.14, σ_μ=0.04, μ = 9%=mubar)



Value of Market Timing (CEWR^{ou}) for Investors with 20-year horizon

1 year R^2 tells us nothing about value of timing



- Market timing valuable if we could observe 💻.
- But in practice we can only rely on proxies for 💻 (dividend yields) and badly estimated regression coefficients.
- A better approach is to rely on *direct estimates* of 💻 that do not rely on regression estimates

A Forward-Looking Method: DDM Model

- DCF approach

$$P_t = \sum_{\tau=1}^{\infty} \frac{E_t\left[D_{t+\tau}\right]}{\left(1+k_t\right)^{\tau}}$$

- Forecasts of future dividends provided by analysts yield current estimates of *long run* expected returns on stocks, k_t
- Problem: How to map k into short run expected rate of return μ

$$d\mu = \kappa(\bar{\mu} - \mu)dt + \sigma_{\mu} dz_{\mu}$$

DDM approach to estimating 💻

- If we know the parameters of the Vasicek interest rate model we can infer the short rate, r, from the long rate, *l*.

- In same way, if we know the parameters of

$$d\mu = \kappa(\overline{\mu} - \mu)dt + \sigma_{\mu} dz_{\mu}$$

  we can infer 💻 from k_t

- Iterative procedure for inferring 💻 and updating parameters

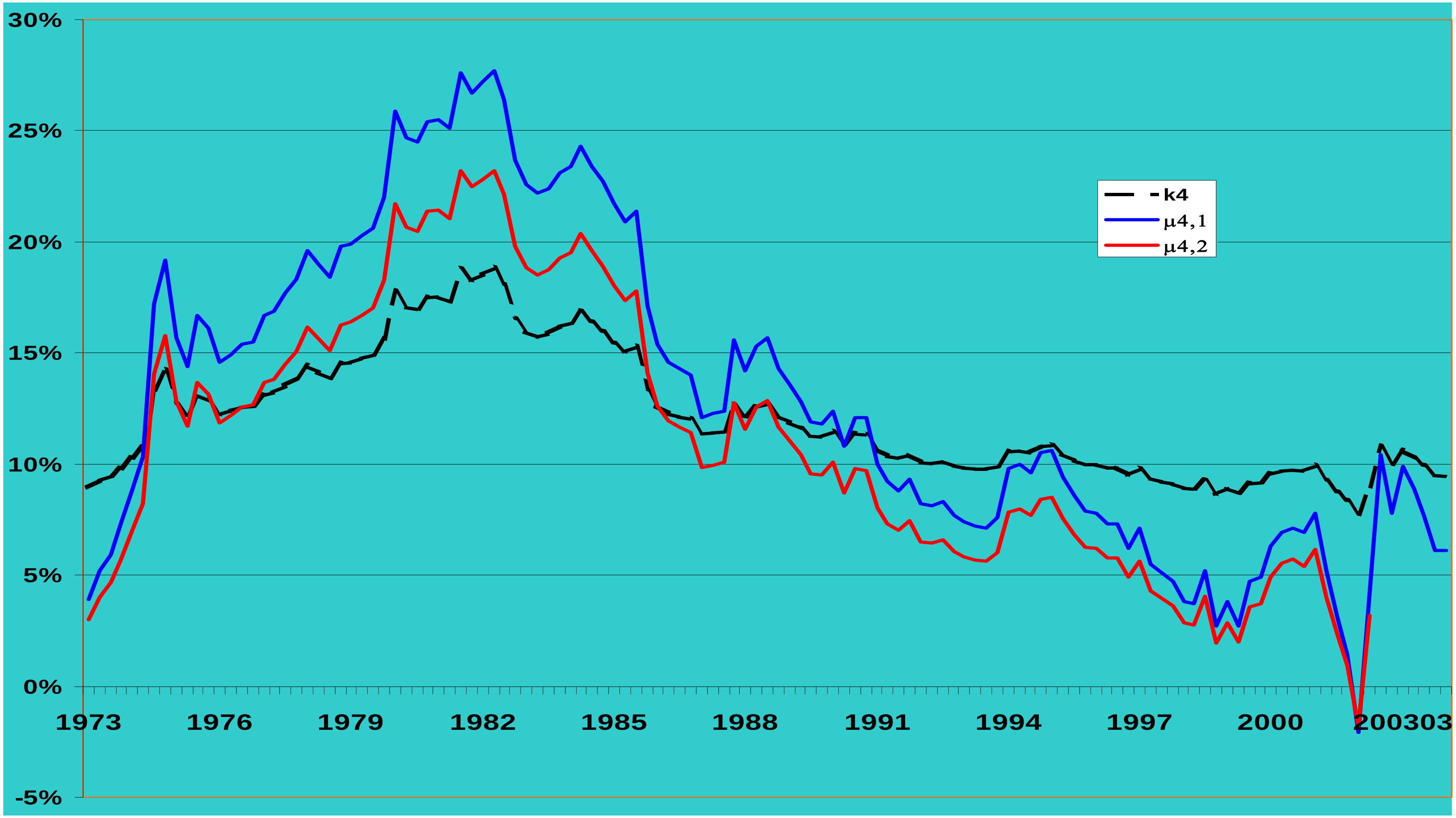
- Also estimate model in which dividend growth rate follows O-U process:

Four DDM *k* series

- Arnott & Bernstein (A&B) (2002), and Ilmanen (IL) (2003)
  - 1950.1 - 2002.2 quarterly
  - real, ex-post (back-casted)
  - based on smoothed GDP growth rate

- Barclays Global Investors (BGI) and Wilshire Associates (WA)
  - 1973.1 to 2002.2 monthly – converted to quarterly
  - nominal, ex-ante (real time)
  - using I/B/E/S consensus estimates

Estimates of μ from the WA DDM k series (1973.Q1 to 2002.Q2)



Estimated μ process parameters

- Similar across 4 models and Close to scenario (vi)

	Real		Nominal		Scenario
	(A&B)	(IL)	(BGI)	(WA)	(vi)
κ_μ	0.085 (0.083)	0.122 (0.115)	0.091 (0.085)	0.122 (0.095)	0.1
σ_μ	0.017 (0.017)	0.0196 (0.022)	0.024 (0.021)	0.034 (0.027)	0.018
ν_μ	0.042 (0.042)	0.040 (0.047)	0.056 (0.052)	0.068 (0.061)	0.04
$\rho_{\mu P}$	-0.98 (-0.98)	-0.88 (-0.89)	-0.81 (-0.66)	-0.68 (-0.71)	-0.9

Statistical Significance: In-Sample Quarterly Predictive Regressions

- Regression:

$$R(t, t+0.25) = a_0 + a_1 \left[\frac{1.0 - e^{-0.25\kappa}}{\kappa} \right] \mu_t^{i,2} + \varepsilon_t$$

$$i = 1\,(\text{A \& B}),\, 2\,(\text{IL}),\, 3\,(\text{BGI}), 4\,(\text{WA})$$

- Theoretical value: $\boldsymbol{a_1 = 1.0}$

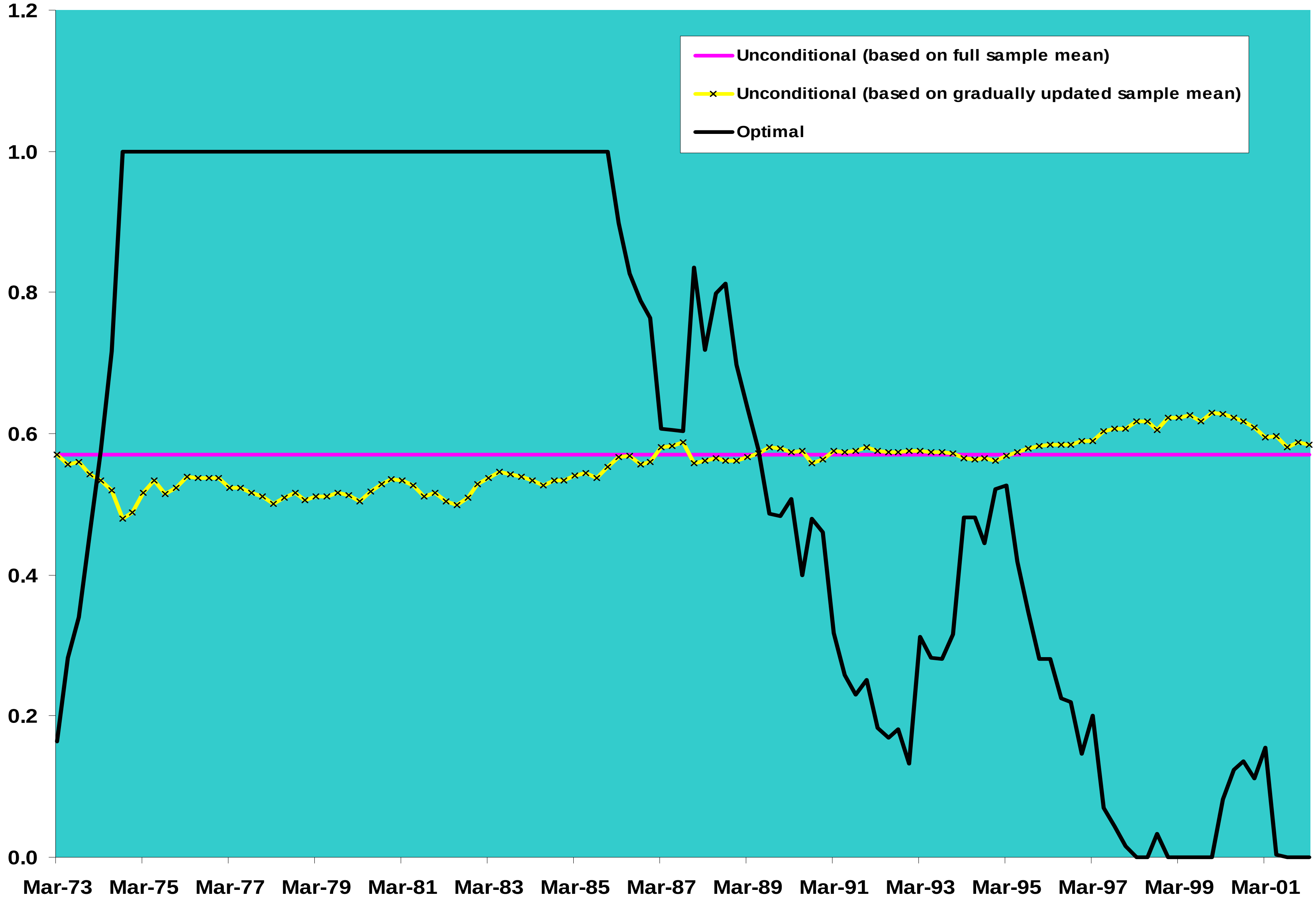
In-Sample Quarterly Predictive Regressions Results

Predictor	a_1 H0: a_1=1	R^2 (%)	N
$\mu^{1,2}$	0.874 [1.60]	1.43	209 (1950.Q2 – 2002. Q2)
$\mu^{2,2}$	0.701 [1.27]	1.12	209 (1950.Q2 - 2002. Q2)
$\mu^{3,2}$	1.026 [1.69]	2.89	117 (1973.Q2 to 2002.Q2)
$\mu^{4,2}$	0.924 [1.66]	2.74	117 (1973.Q2 to 2002.Q2)

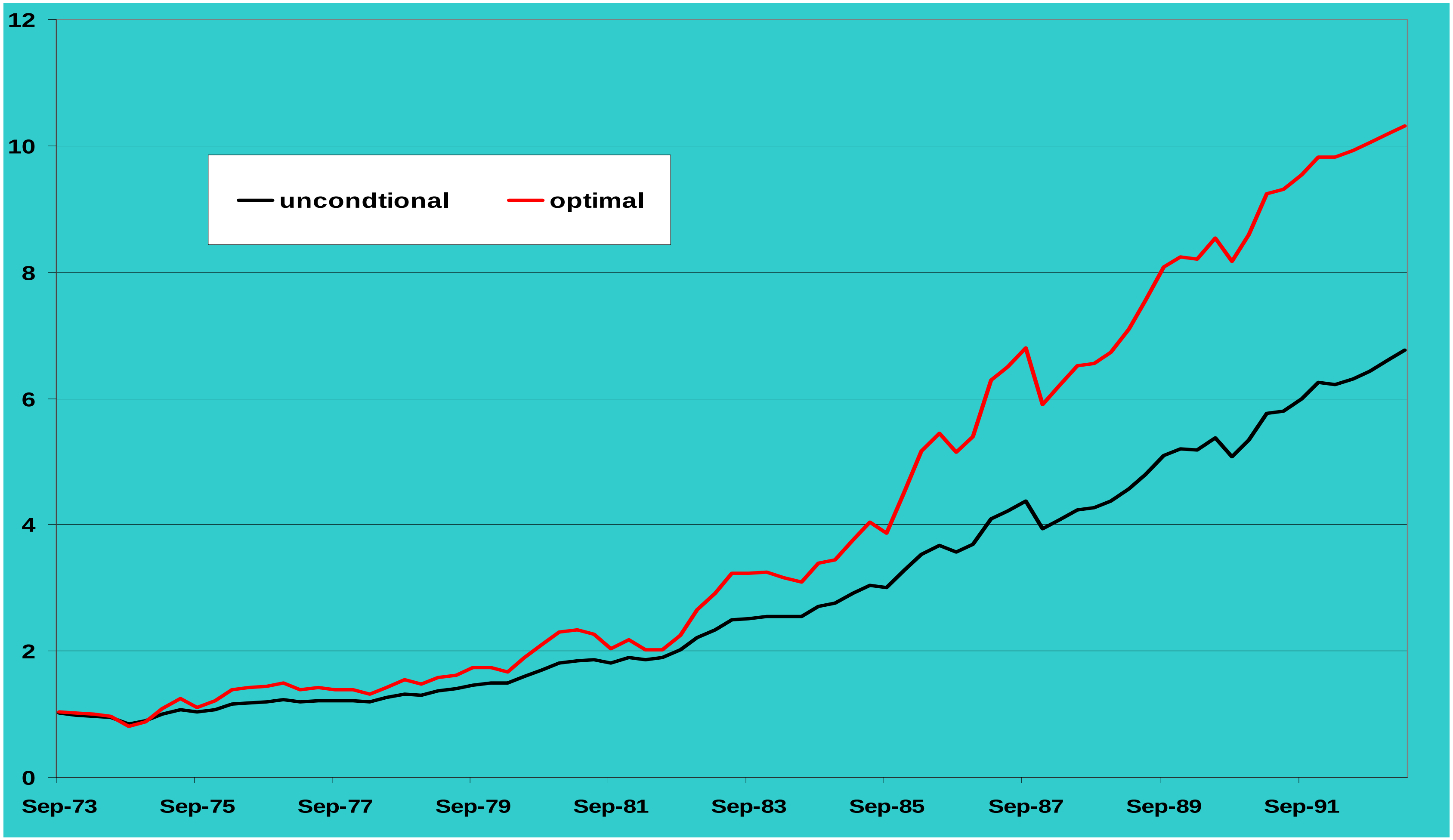
Economic Importance: Simulation of Market Timing and Unconditional Strategies

- RRA = 5
- Unconditional Strategy: $$x^* = \frac{\overline{\mu} - r}{\gamma \sigma_P^2}$$
- Optimal Market Timing: $$x^* = \frac{\mu_t - r}{\gamma \sigma_P^2} \quad plus \quad hedging \quad terms$$
- Risky asset: S&P500
- Riskless asset: 30 day T-Bill

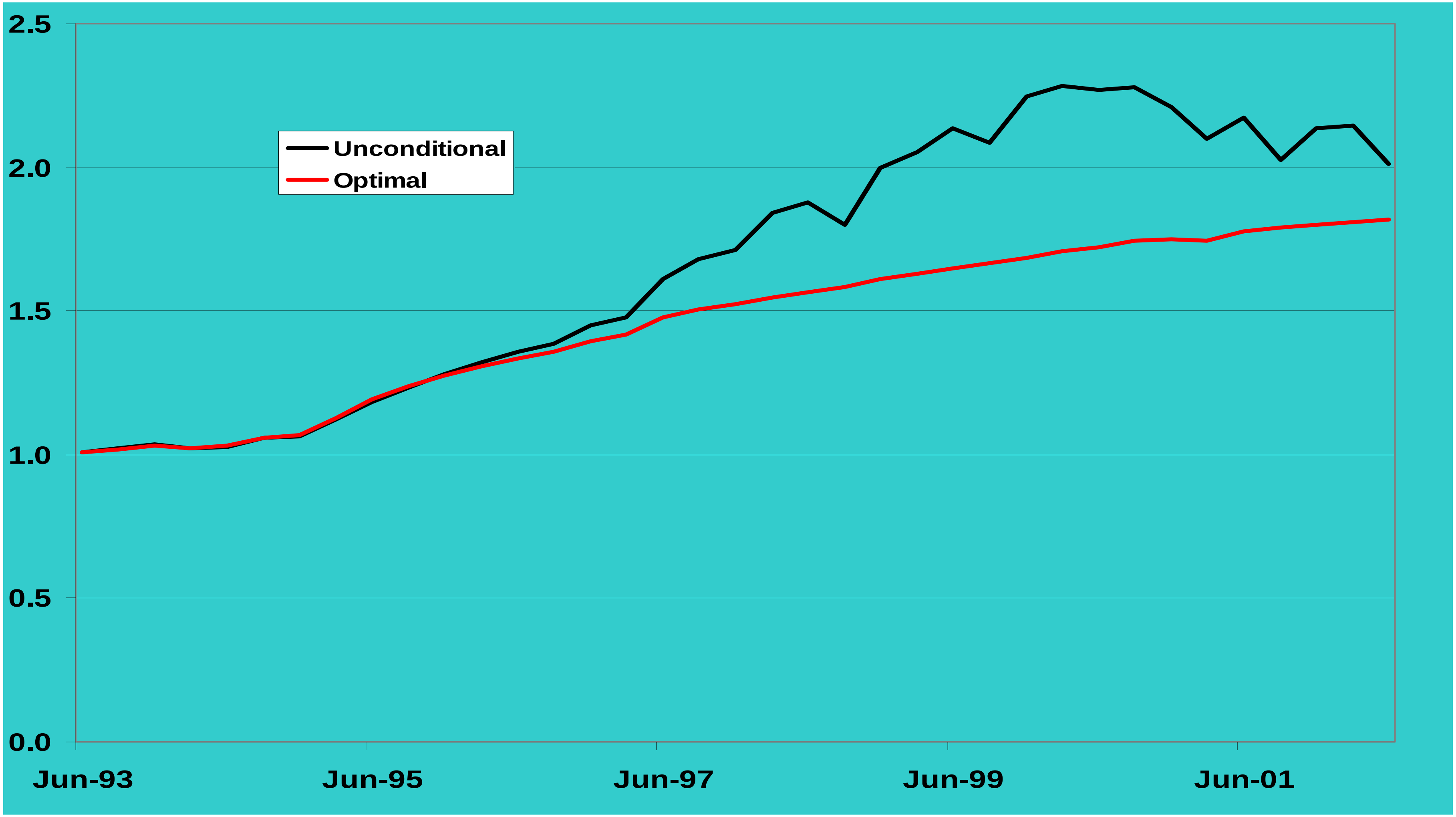
Proportion of Wealth Invested in Stocks



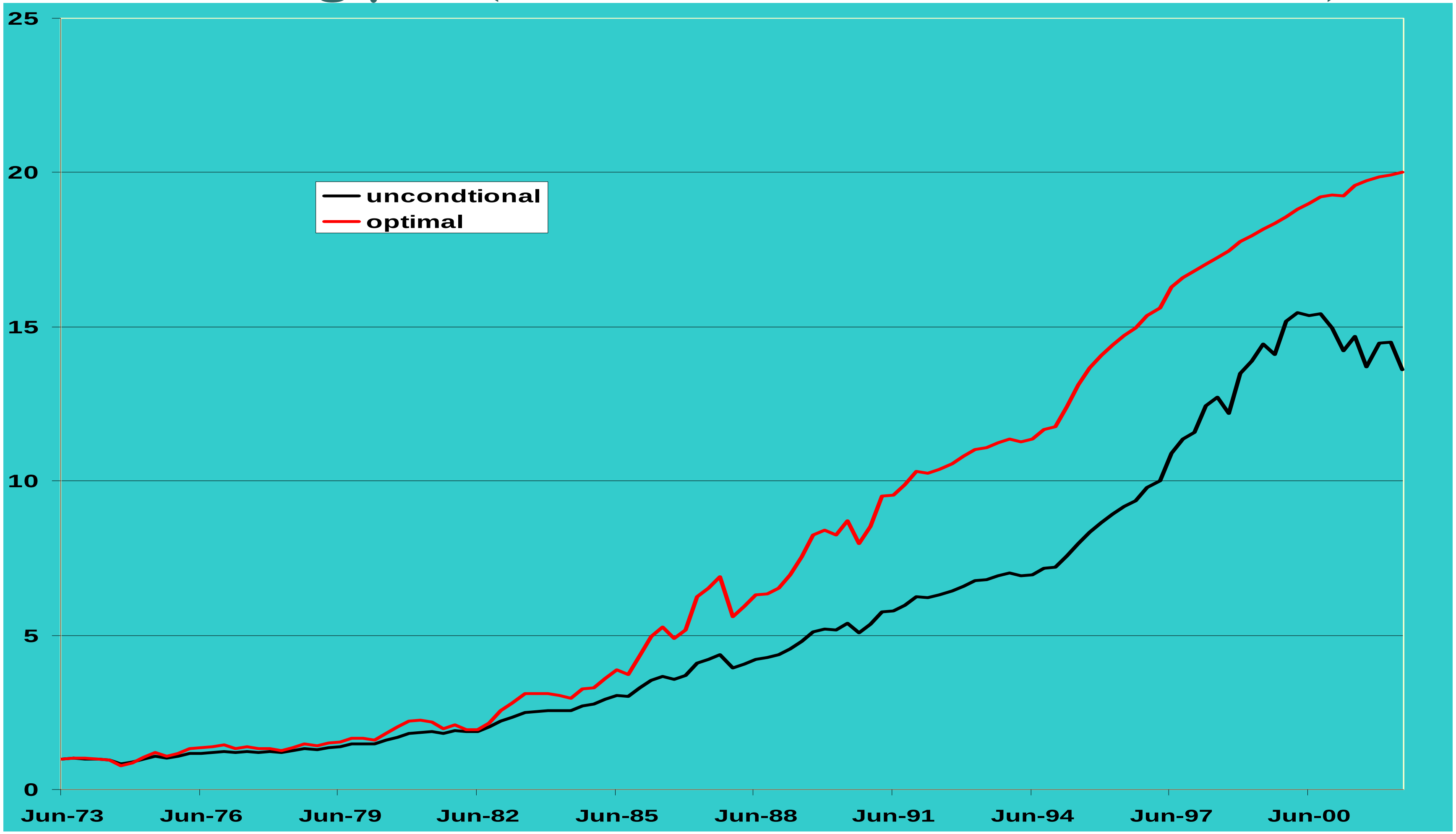
Wealth under optimal and unconditional Strategies for a 20-year horizon constrained investor using $\mu^{4,2}$ (RRA = 5, 1973.Q2 -1993.Q1)



Wealth under optimal and unconditional Strategies for a 9-year horizon constrained investor using $\mu^{4,2}$ (RRA = 5, 1993.Q2 - 2002.Q2)



Wealth under optimal and unconditional Strategies for a 29-year horizon constrained investor using $\mu^{4,2}$ (RRA = 5, 1973.Q2 – 2002.Q2)



Conclusion

- Time-varying expected returns economically important, even though
  - Hard to detect, measure
- **Substantial** benefit from the optimal strategy

- DDM discount rates are a useful input for long run investor.

- Long run investors (pension, insurance) should *hedge* against changes in investment opportunities.