

# **The Cross-Sectional Dispersion of Stock Returns, Alpha and the Information Ratio**

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# Overview of the Study

- The cross-sectional dispersion of US stock returns and the VIX forecast the future dispersion of alpha across high- and low-performing portfolios.
- The forecasts are statistically and economically significant.
- Absolute return investors can use return dispersion and the VIX as signals to improve the tactical timing of their alpha-focused strategies.
- Cross-sectional dispersion affects relative returns and idiosyncratic risk in roughly the same order of magnitude.
  - We find that high-dispersion/high-VIX environments provide little information for relative return investors, or more specifically, IR-focused investors.

# Active Equity Managers Alpha-Hunting Record is Weak

- Elton, et al. [1993], Carhart [1997] and Bogle [1997]: mutual funds consistently underperform their benchmark indices.
- Standard & Poor's 2009 "SPIVA" scorecard (Indices vs. Active Funds) reports that, over the past 5 years:
  - 63% of large cap mutual funds,
  - 74% of mid-cap mutual funds, and
  - 68% of small-cap mutual funds
    - All underperformed their benchmark indices
- Davis [2001] and Ennis and Sebastian [2002]: small-cap managers do not add value, even though greater inefficiencies are thought to exist among small-cap stocks.

# Hedge Funds' Record is Similar

- Malkiel and Saha [2005]: Returns are “... lower than commonly supposed,” and risk is higher.
- Bogle [1998]: “Has active equity management become a loser’s game?”
- Fung, Xu and Yau [2004] report negative hedge fund alphas.
- Currency-based hedge fund strategies also destroy value.
- O’Hara [2009], writing about long-short hedge funds in Institutional Investor Magazine, asks: “If managers can’t beat the market, what purpose do they serve?”
- Statman [2004]: Investors want more than utilitarian benefits (like high returns) – they use their relationship with money management firms to express their social class and lifestyle.

# The Cross-Sectional Dispersion of Stock Volatility and Investing Performance Metrics

- We contribute to the debate over the performance of active equity managers by investigating the “cross-sectional dispersion” of stock volatility and how performance metrics like alpha and the information ratio change over time with this measure of volatility.
- This will allow us to observe how the opportunity to earn higher and lower returns relative to the market expand and contract over time.
- We conclude that the lack of manager outperformance is not due to lack of alpha opportunity, as the cross-sectional volatility of alpha is high and particularly so following high-dispersion environments.

# Cross-Sectional Dispersion in Theory

- Gorman, Sapra, & Weigand (2010) show that assuming a constant volatility and correlation matrix, cross-sectional dispersion takes the following form:

$$\sigma_{CS} = \sigma \sqrt{1 - \rho}$$

- Cross-sectional dispersion is positively related to time-series volatility and negatively related to the average correlation of stocks.
- The time-series volatility of an N-stock equally weighted portfolio is given by:

$$\sigma_P^2 = \rho \sigma^2 + \frac{\sigma_{CS}^2}{N}$$

- Cross-sectional dispersion is a component of portfolio risk.

# Idiosyncratic Risk in Theory

- Idiosyncratic risk is linearly related to the level of cross-sectional dispersion:

$$\sigma_A^2 = \sigma_{CS}^2 \sum_{i=1}^N w_{A,i}^2$$

- For example, for an N-stock equally-weighted portfolio:

$$\sigma_A = \frac{\sigma_{CS}}{\sqrt{N}}$$

- When cross-sectional dispersion is high, we would expect to see above-average levels of idiosyncratic risk.
  - All else equal, a highly-disperse return environment will naturally result in elevated levels of manager tracking error.

# Alpha and IR in Theory

- Expected relative returns are also a linear function of cross-sectional dispersion:

$$E(r) = IC \cdot \sigma_{CS} \cdot z$$

- For a given level of skill (IC), a highly disperse return environment results in a greater magnitude of expected relative returns.
- However, the Fundamental Law of Active Management prevails:

$$\frac{E(r)}{\sigma_A} = IC \sqrt{N}$$

- Changes in cross-sectional dispersion affect the numerator and the denominator of the IR in the same fashion, leaving the IR unchanged.



# Guidance from Theory

- From theory, we hypothesize:
  - Cross-sectional dispersion is positively related to time-series volatility since time-series volatility is a component of cross-sectional dispersion
  - Portfolio risk is positively related to cross-sectional dispersion since cross-sectional dispersion is a component of portfolio risk
  - Idiosyncratic risk is positively related to cross-sectional dispersion since cross-sectional dispersion is a component of active risk
  - Alpha opportunities are positively related to cross-sectional dispersion, since alpha is linear in cross-sectional dispersion
  - The information ratio is invariant to changes in cross sectional dispersion, since cross-sectional dispersion affects active return in active risk in a linear fashion

# Measuring Alpha

- Remove the effect of systematic market factors:

$$R_{i,t} - r_{ft} = \alpha_{i,t}^{FF4} + \beta_{i,1}^{FF4} (R_{MKT_t} - r_{ft}) + \beta_{i,2}^{FF4} (R_{SMB_t}) + \beta_{i,3}^{FF4} (R_{HML_t}) + \beta_{i,4}^{FF4} (R_{UMD_t}) + \varepsilon_{it}$$

- $R_{MKT_t} - r_{ft}$  is the general market factor; SMB is the small-cap factor; HML is the value factor; UMD is the momentum factor.
- Alpha is the return that is earned over and above the various systematic effects:

$$\hat{\alpha}_{i,t}^{FF4} = \bar{R}_{i,t} - \left[ \bar{r}_{ft} + \hat{\beta}_{i,1}^{FF4} (\overline{R_{MKT} - r_f}) + \hat{\beta}_{i,2}^{FF4} (\bar{R}_{SMB}) + \hat{\beta}_{i,3}^{FF4} (\bar{R}_{HML}) + \hat{\beta}_{i,4}^{FF4} (\bar{R}_{UMD}) \right]$$

- $\hat{\alpha}_{i,t}^{FF4}$  is estimated every day for every stock in the S&P 500 1981-2008.

# Measuring the Information Ratio

- The active management industry's performance metric - the information ratio - measures alpha per unit of active risk:

$$IR_{i,t} = \frac{\hat{\alpha}_{i,t}^{FF4}}{\hat{\sigma}_{\varepsilon i,t}}$$

- The IR tells us if active returns still appear large when they are scaled by the extra volatility induced by a portfolio's "activeness".
- The scaling factor is the idiosyncratic volatility from the alpha estimation regressions.
- The numerator is from the intercept of the Fama-French regressions.

# Stock Market Volatility Metrics

- Time Series Volatility of the S&P 500:

$$\sigma_{Time\ Series, t} = \left[ \sum_{t=-252}^{-1} \frac{\left( R_{S\&P500, t} - \bar{R}_{S\&P500} \right)^2}{t-1} \right]^{1/2}$$

- Cross-Sectional Dispersion of the S&P 500:

$$\sigma_{Cross\ Section, t} = \left[ \sum_{i=1}^n \frac{\left( R_{i, t} - \bar{R}_{S\&P500, t} \right)^2}{n-1} \right]^{1/2}$$

- VIX (1991-2008):
  - The CBOE’s index of “implied volatility,” widely-used by market participants as a forecast of market volatility 30 days ahead

# Performance Metric Volatility

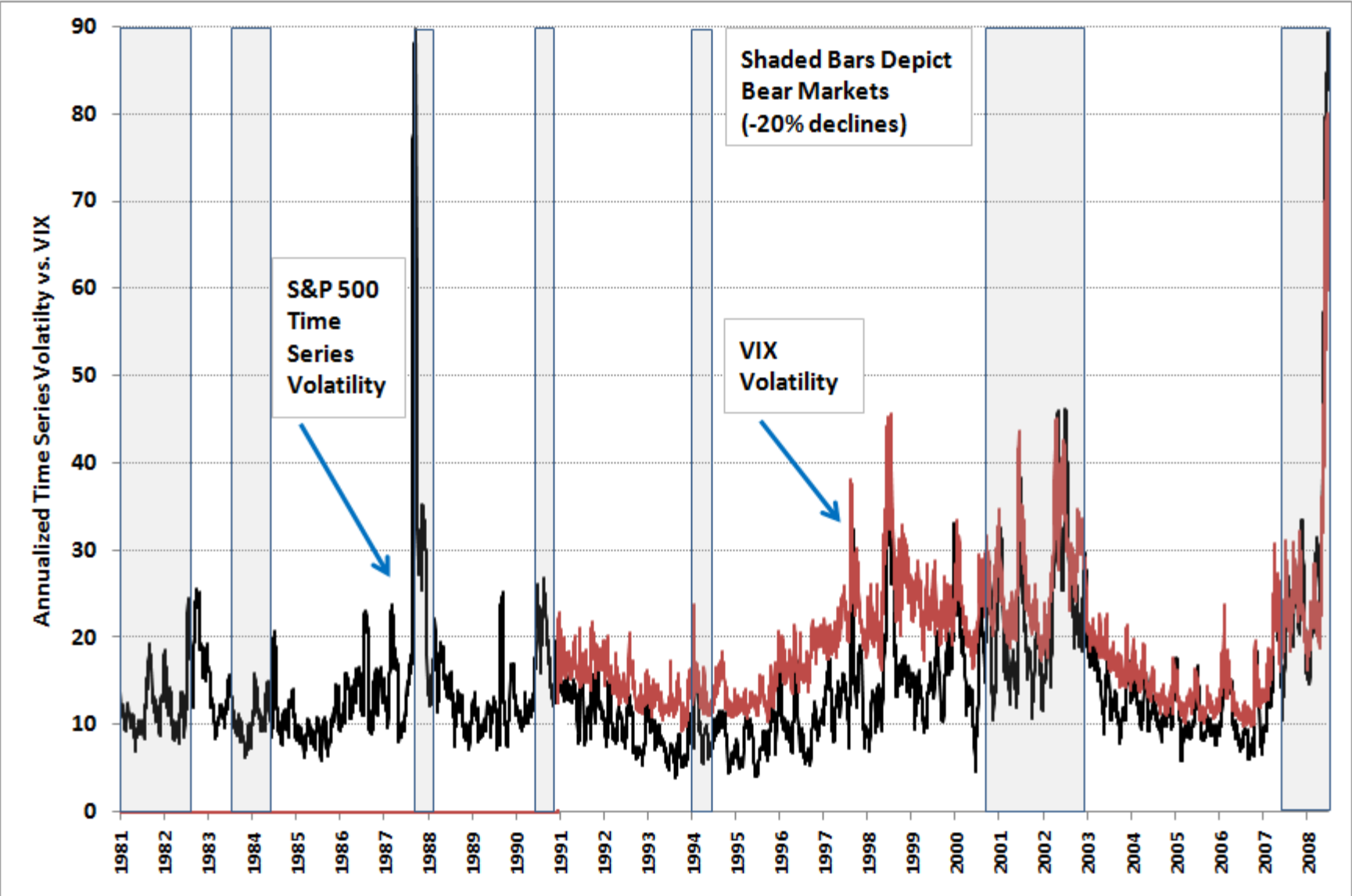
- Cross-Sectional Dispersion of Alpha:

$$\sigma_{\alpha, t} = \left[ \sum_{i=1}^n \frac{(\alpha_{i,t} - \bar{\alpha}_{S\&P500, t})^2}{n-1} \right]^{1/2}$$

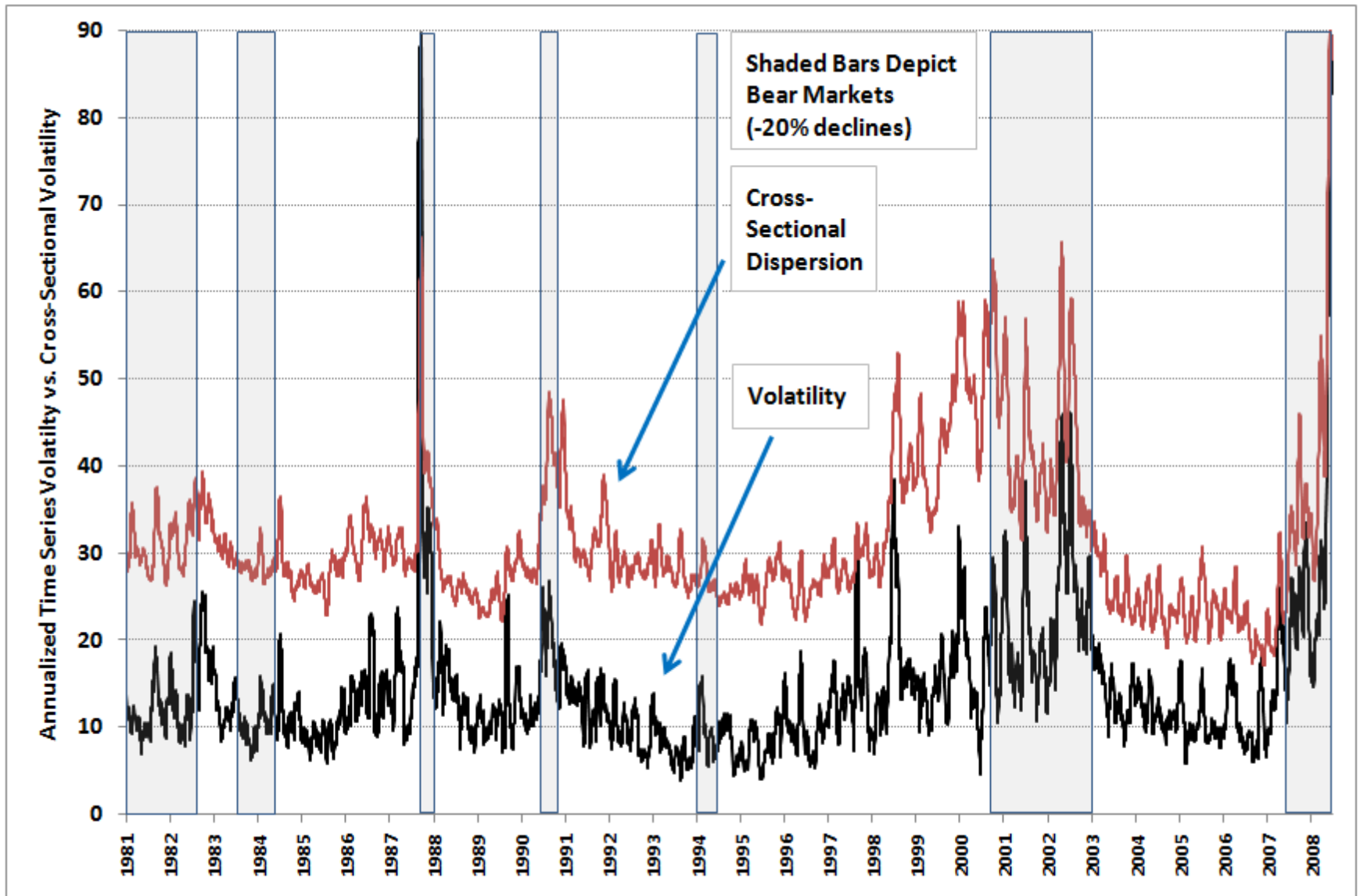
- Cross-Sectional Dispersion of the Information Ratio:

$$\sigma_{IR, t} = \left[ \sum_{i=1}^n \frac{\left( \frac{\hat{\alpha}_{i,t}}{\hat{\sigma}_{\varepsilon, t}} - \overline{IR}_{S\&P500, t} \right)^2}{n-1} \right]^{1/2}$$

# S&P500 Time Series Volatility & VIX



# Time Series & Cross-Sectional Volatility



# Dispersion and VIX Correlations

**Panel A: Contemporaneous Correlations**

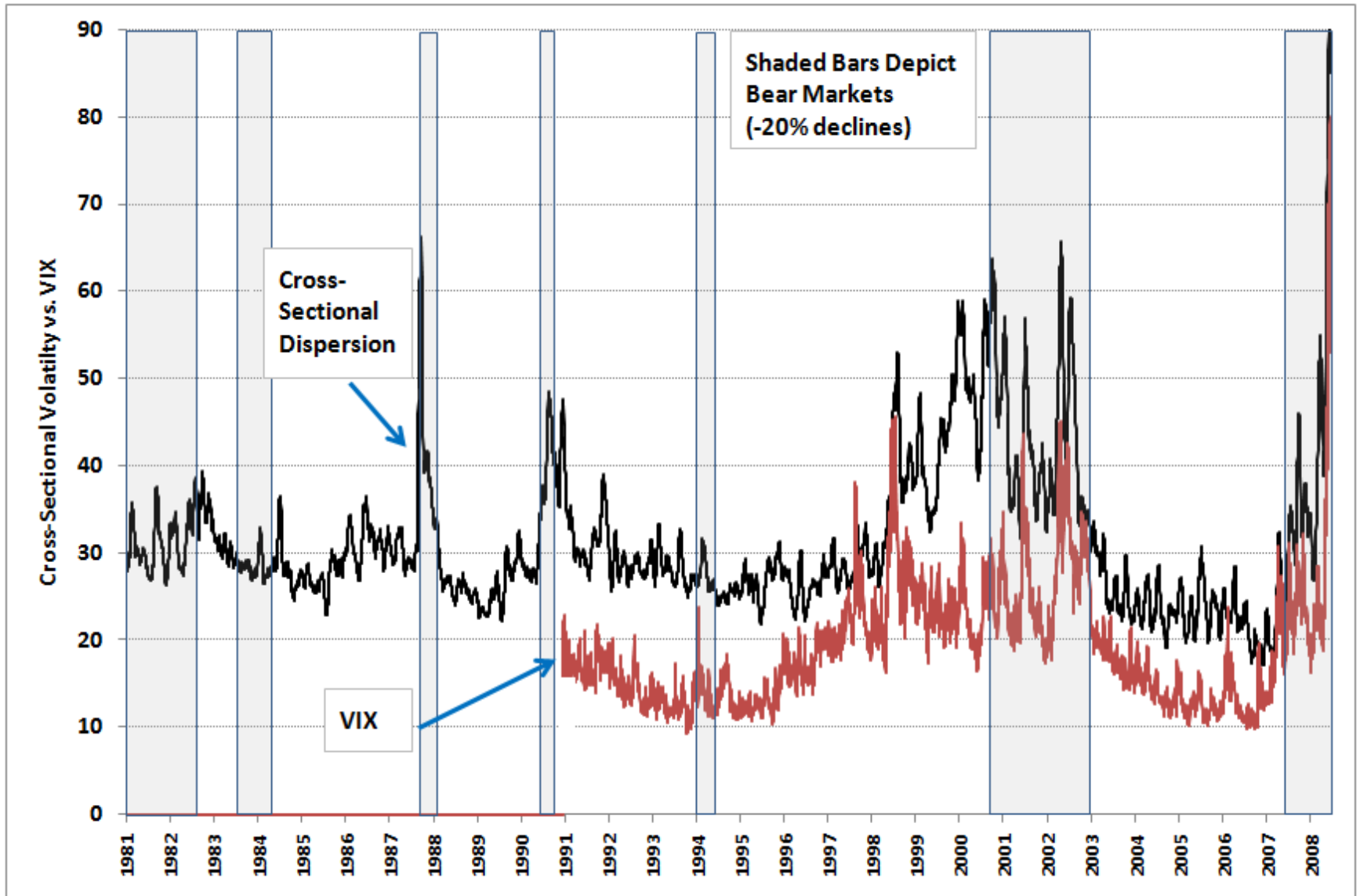
	<b>VIX</b>	<b>Time Series</b>	<b>Cross-Sectional</b>
<b>VIX</b>	1.000		
<b>Time Series</b>	0.835	1.000	
<b>Cross-Sectional</b>	0.758	0.728	1.000

**Panel B: Forward-Looking (30-Day Ahead) Correlations**

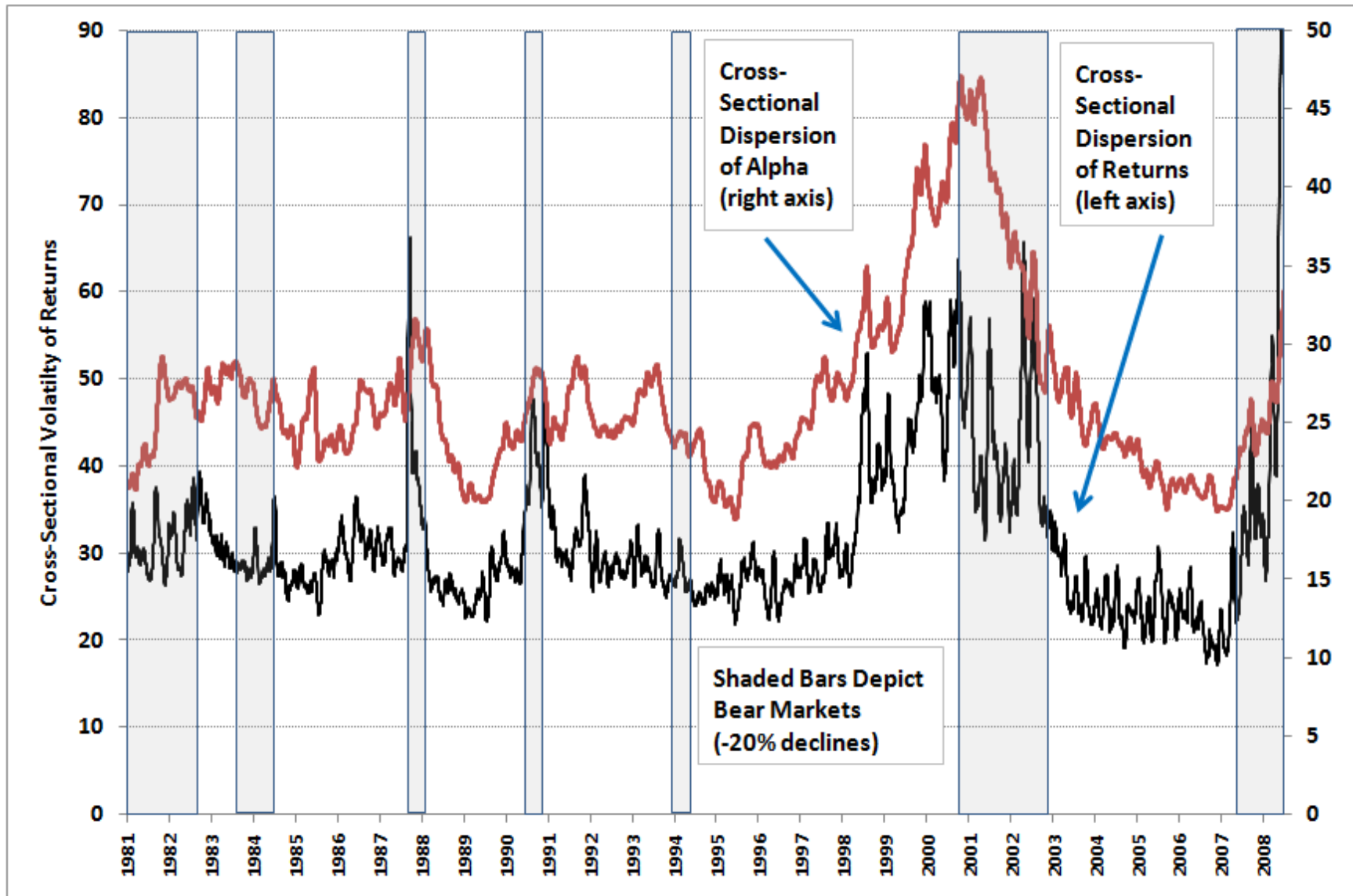
	<b>VIX</b>	<b>Time Series</b>	<b>Cross-Sectional</b>
<b>VIX<sub>30</sub></b>	0.790		
<b>Time Series<sub>30</sub></b>	0.676	0.556	
<b>Cross-Sectional<sub>30</sub></b>	0.700	0.476	0.823



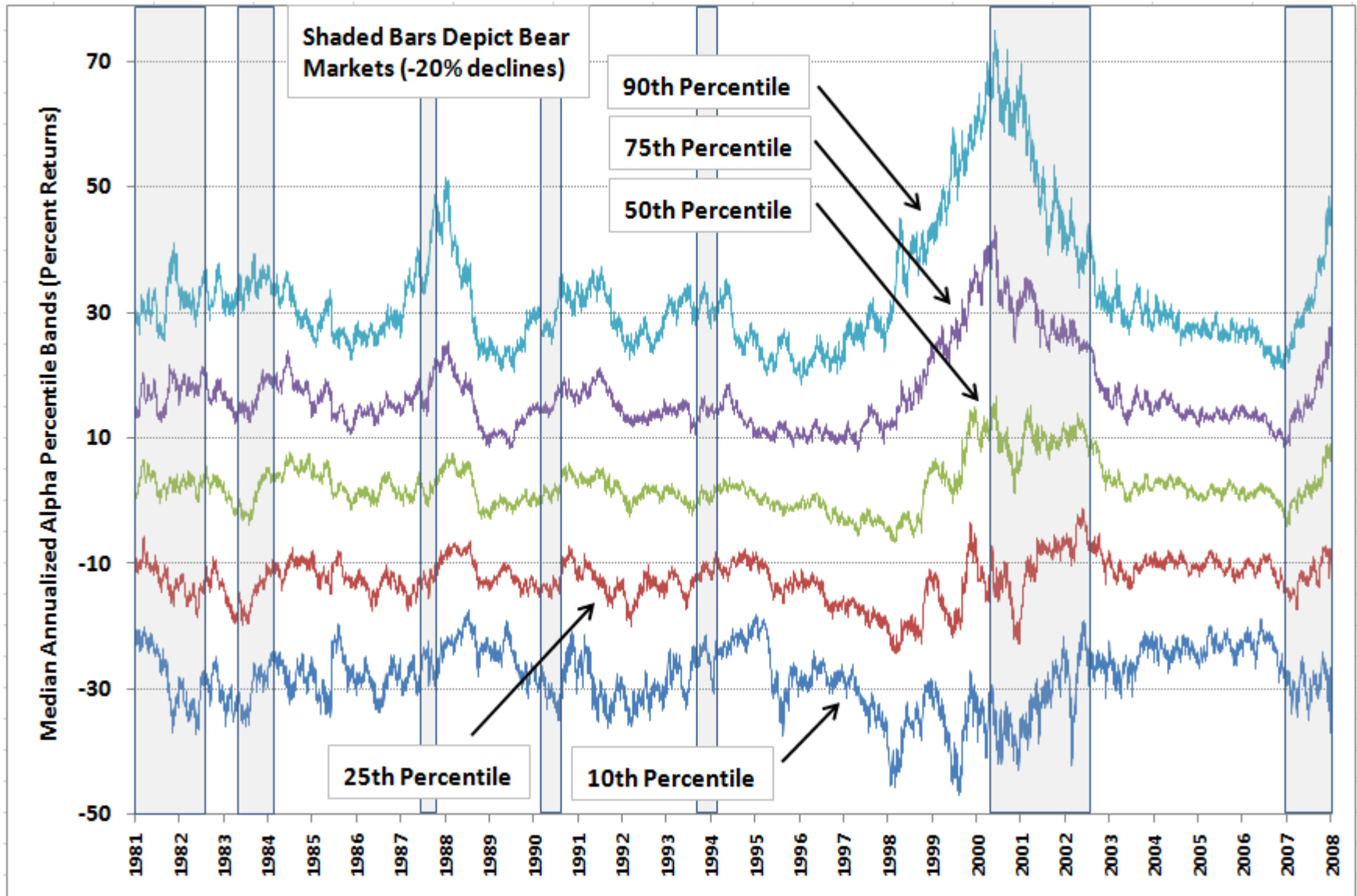
# Cross-Sectional Dispersion & VIX



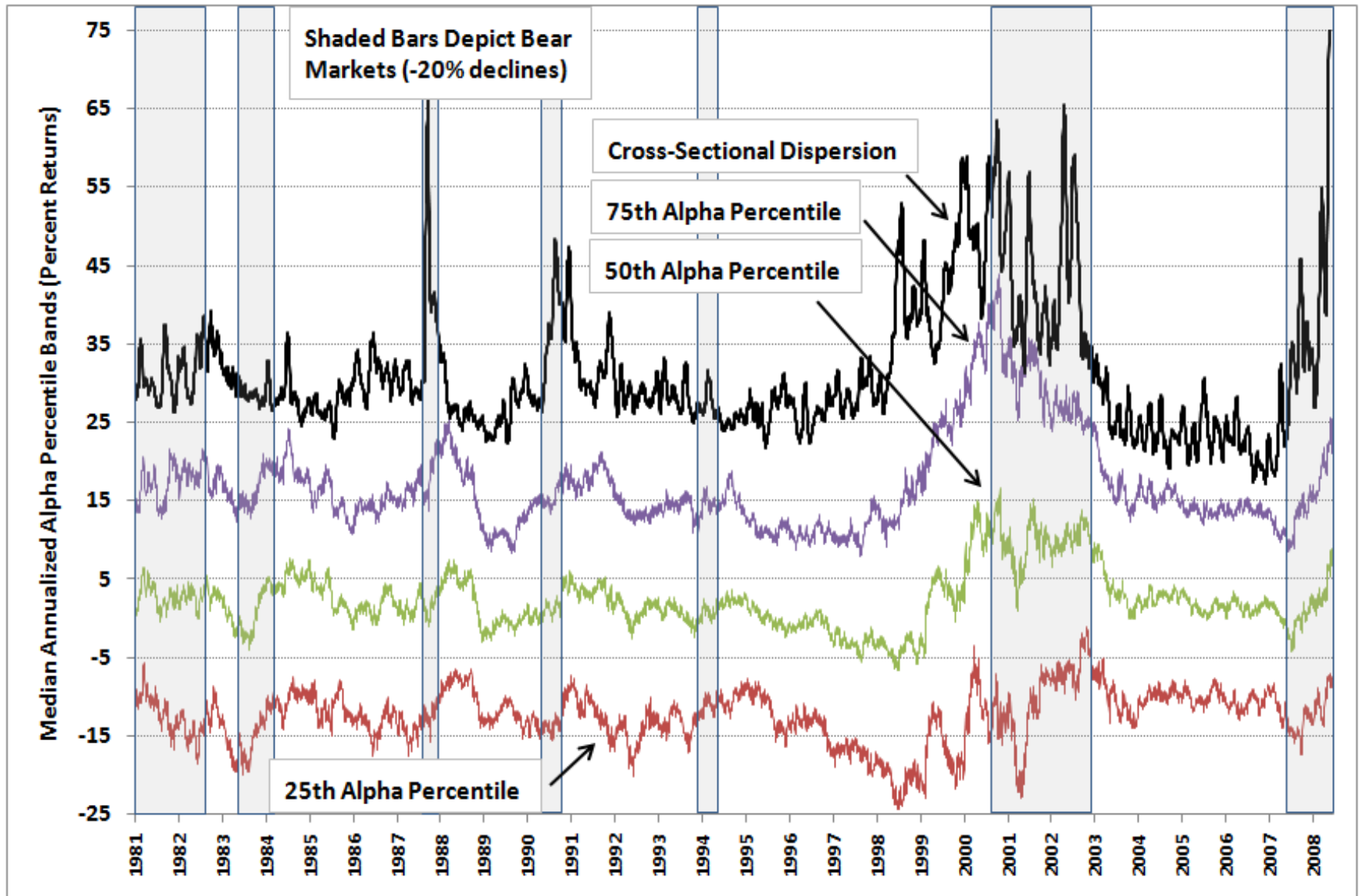
# Cross-Sectional Volatility: Returns & Alpha



# Median Alpha Percentiles



# Alpha Percentiles & Return Dispersion



# Return Dispersion is Positively Related to Future Alpha Dispersion

## Panel A: Cross-Sectional Dispersion and Alpha

Looking Ahead	10 <sup>th</sup> Alpha Percentile	90 <sup>th</sup> Alpha Percentile
63 Days	-0.43	+0.65
252 Days	-0.32	+0.62

## Panel B: The VIX and Alpha

Looking Ahead	10 <sup>th</sup> Alpha Percentile	90 <sup>th</sup> Alpha Percentile
63 Days	-0.43	+0.53
252 Days	-0.44	+0.52

# Cross-Sectional Dispersion and the VIX Predict Future Alpha Spreads

## 63-Day Forecasts (Next 3 Trading Months)

Quintile	X-Sec Vol. of Returns			Alpha 10th Percentile			Alpha 50th Percentile			Alpha 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>23.52</b>	17.16	25.73	<b>-52.33</b>	-105.38	-34.36	<b>1.31</b>	-14.35	11.47	<b>52.59</b>	39.52	76.06
2	<b>27.07</b>	25.74	28.03	<b>-59.96</b>	-134.47	-36.54	<b>0.54</b>	-15.46	22.67	<b>60.00</b>	36.53	137.30
3	<b>29.08</b>	28.03	30.43	<b>-63.20</b>	-145.32	-37.04	<b>1.78</b>	-16.75	27.32	<b>64.55</b>	44.04	153.72
4	<b>32.55</b>	30.43	35.76	<b>-64.85</b>	-128.05	-39.45	<b>2.05</b>	-15.65	28.26	<b>66.54</b>	46.47	122.23
5	<b>42.96</b>	35.76	90.66	<b>-75.65</b>	-139.02	-39.30	<b>8.09</b>	-13.02	29.88	<b>86.05</b>	46.99	136.78

Quintile	VIX			Alpha 10th Percentile			Alpha 50th Percentile			Alpha 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>11.97</b>	9.31	13.16	<b>-54.02</b>	-99.59	-34.36	<b>1.40</b>	-14.35	9.62	<b>54.49</b>	40.56	81.65
2	<b>14.73</b>	13.17	16.33	<b>-56.47</b>	-134.87	-36.54	<b>1.87</b>	-12.74	22.17	<b>55.95</b>	41.15	115.08
3	<b>18.15</b>	16.33	20.07	<b>-64.17</b>	-145.32	-39.03	<b>1.56</b>	-15.46	29.88	<b>61.96</b>	42.03	134.65
4	<b>21.98</b>	20.07	24.31	<b>-73.20</b>	-132.48	-41.38	<b>4.60</b>	-16.75	28.12	<b>81.61</b>	45.29	133.54
5	<b>28.18</b>	24.32	70.33	<b>-74.02</b>	-139.02	-44.94	<b>6.48</b>	-13.91	29.62	<b>81.40</b>	47.79	136.78

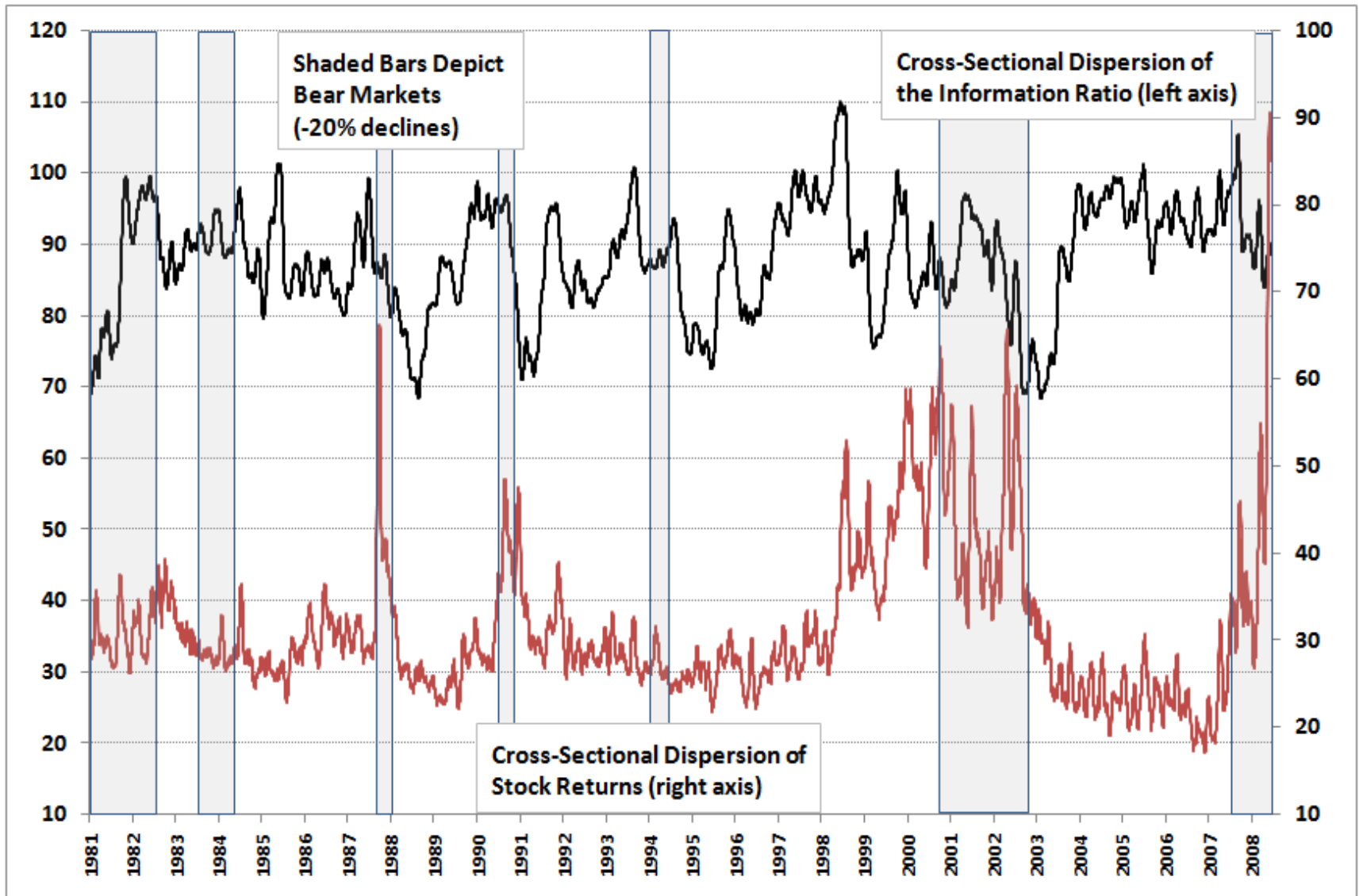
# Cross-Sectional Dispersion and the VIX Predict Future Alpha Spreads

## 252-Day Forecasts (Next Trading Year)

Quintile	X-Sec Vol. of Returns			Alpha 10th Percentile			Alpha 50th Percentile			Alpha 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>23.52</b>	17.16	25.73	<b>-26.28</b>	-40.55	-18.76	<b>0.91</b>	-5.73	8.99	<b>26.73</b>	19.65	48.53
2	<b>27.07</b>	25.74	28.03	<b>-27.11</b>	-45.60	-18.05	<b>0.59</b>	-6.69	9.04	<b>27.26</b>	18.40	46.96
3	<b>29.08</b>	28.03	30.43	<b>-27.97</b>	-45.58	-17.69	<b>1.97</b>	-6.50	10.13	<b>30.93</b>	20.29	51.55
4	<b>32.55</b>	30.43	35.76	<b>-27.61</b>	-38.39	-17.19	<b>2.21</b>	-5.89	14.74	<b>32.97</b>	20.85	65.12
5	<b>42.96</b>	35.76	90.66	<b>-31.24</b>	-46.96	-19.45	<b>5.43</b>	-4.07	16.65	<b>43.23</b>	22.44	75.79

Quintile	VIX			Alpha 10th Percentile			Alpha 50th Percentile			Alpha 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>11.97</b>	9.31	13.16	<b>-24.61</b>	-37.57	-18.09	<b>0.67</b>	-4.24	3.63	<b>26.13</b>	18.40	35.17
2	<b>14.73</b>	13.17	16.33	<b>-25.98</b>	-35.50	-18.05	<b>0.84</b>	-5.73	7.78	<b>26.95</b>	19.84	45.92
3	<b>18.15</b>	16.33	20.07	<b>-29.08</b>	-45.13	-20.53	<b>1.79</b>	-5.87	14.98	<b>28.75</b>	20.66	69.10
4	<b>21.98</b>	20.07	24.31	<b>-31.67</b>	-45.60	-19.02	<b>4.52</b>	-6.69	15.61	<b>40.03</b>	22.14	75.79
5	<b>28.18</b>	24.32	70.33	<b>-31.60</b>	-46.96	-20.24	<b>4.29</b>	-6.62	16.65	<b>41.03</b>	26.40	72.32

# Dispersion of Returns and the IR





# Return Dispersion is Inversely Related to Future IR Dispersion

**Panel A: Cross-Sectional Volatility and the Information Ratio**

<b>Looking Ahead</b>	<b>10<sup>th</sup> IR Percentile</b>	<b>90<sup>th</sup> IR Percentile</b>
63 Days	+0.33	0.00
252 Days	+0.41	+0.23

**Panel B: The VIX and the Information Ratio**

<b>Looking Ahead</b>	<b>10<sup>th</sup> IR Percentile</b>	<b>90<sup>th</sup> IR Percentile</b>
63 Days	+0.24	+0.02
252 Days	+0.31	+0.27

# Return Dispersion and Dispersion of the IR are Inversely Related

## 63-Day Forecasts (Next 3 Trading Months)

Quintile	X-Sec Vol. of Returns			Information Ratio 10th Percentile			Information Ratio 50th Percentile			Information Ratio 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>23.52</b>	17.16	25.73	<b>-2.61</b>	-4.39	-1.69	<b>0.07</b>	-0.85	0.64	<b>2.69</b>	1.82	3.50
2	<b>27.07</b>	25.74	28.03	<b>-2.66</b>	-4.39	-1.63	<b>0.03</b>	-0.65	1.10	<b>2.66</b>	1.73	4.56
3	<b>29.08</b>	28.03	30.43	<b>-2.54</b>	-4.35	-1.34	<b>0.08</b>	-0.67	1.07	<b>2.69</b>	1.84	4.52
4	<b>32.55</b>	30.43	35.76	<b>-2.50</b>	-4.04	-1.26	<b>0.09</b>	-0.65	1.18	<b>2.58</b>	1.73	4.75
5	<b>42.96</b>	35.76	90.66	<b>-2.19</b>	-4.23	-1.26	<b>0.28</b>	-0.44	1.14	<b>2.66</b>	1.79	4.65

Quintile	VIX			Information Ratio 10th Percentile			Information Ratio 50th Percentile			Information Ratio 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>11.97</b>	9.31	13.16	<b>-2.57</b>	-4.39	-1.70	<b>0.07</b>	-0.85	0.48	<b>2.68</b>	1.94	3.32
2	<b>14.73</b>	13.17	16.33	<b>-2.50</b>	-4.24	-1.76	<b>0.10</b>	-0.75	0.85	<b>2.68</b>	1.93	4.42
3	<b>18.15</b>	16.33	20.07	<b>-2.65</b>	-4.39	-1.60	<b>0.08</b>	-0.67	1.18	<b>2.71</b>	1.92	4.56
4	<b>21.98</b>	20.07	24.31	<b>-2.42</b>	-4.04	-1.59	<b>0.18</b>	-0.66	1.09	<b>2.78</b>	1.90	4.75
5	<b>28.18</b>	24.32	70.33	<b>-2.28</b>	-3.85	-1.44	<b>0.23</b>	-0.57	1.14	<b>2.64</b>	1.89	4.65

# Return Dispersion and Dispersion of the IR are Inversely Related

## 252-Day Forecasts (Next Trading Year)

Quintile	X-Sec Vol. of Returns			Information Ratio 10th Percentile			Information Ratio 50th Percentile			Information Ratio 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>23.52</b>	17.16	25.73	<b>-1.13</b>	-1.52	-0.79	<b>0.05</b>	-0.27	0.36	<b>1.25</b>	0.89	1.68
2	<b>27.07</b>	25.74	28.03	<b>-1.07</b>	-1.75	-0.65	<b>0.03</b>	-0.26	0.31	<b>1.13</b>	0.85	1.57
3	<b>29.08</b>	28.03	30.43	<b>-1.04</b>	-1.74	-0.63	<b>0.08</b>	-0.27	0.33	<b>1.18</b>	0.86	1.61
4	<b>32.55</b>	30.43	35.76	<b>-1.01</b>	-1.41	-0.49	<b>0.10</b>	-0.20	0.42	<b>1.19</b>	0.91	1.63
5	<b>42.96</b>	35.76	90.66	<b>-0.91</b>	-1.33	-0.47	<b>0.18</b>	-0.15	0.47	<b>1.31</b>	0.90	1.66

Quintile	VIX			Information Ratio 10th Percentile			Information Ratio 50th Percentile			Information Ratio 90th Percentile		
	Median	Min	Max	Median	Min	Max	Median	Min	Max	Median	Min	Max
1	<b>11.97</b>	9.31	13.16	<b>-1.07</b>	-1.46	-0.79	<b>0.03</b>	-0.23	0.18	<b>1.21</b>	0.88	1.50
2	<b>14.73</b>	13.17	16.33	<b>-1.10</b>	-1.58	-0.73	<b>0.04</b>	-0.27	0.30	<b>1.20</b>	0.85	1.53
3	<b>18.15</b>	16.33	20.07	<b>-1.06</b>	-1.75	-0.60	<b>0.08</b>	-0.25	0.42	<b>1.15</b>	0.87	1.63
4	<b>21.98</b>	20.07	24.31	<b>-0.89</b>	-1.70	-0.47	<b>0.15</b>	-0.27	0.45	<b>1.28</b>	0.93	1.67
5	<b>28.18</b>	24.32	70.33	<b>-0.92</b>	-1.72	-0.50	<b>0.16</b>	-0.26	0.47	<b>1.31</b>	0.98	1.68

# Conclusions

- We examine how aggregate alpha changes with return dispersion and the VIX.
- As theory predicts, alpha-capture opportunities expand and contract with cross-sectional return dispersion and the VIX.
- There is an economically meaningful spread in the median alpha percentiles:
  - A long-short manager who was skilled at identifying stocks in the 25<sup>th</sup> and 75<sup>th</sup> alpha percentiles should earn a raw annualized alpha of approximately 30% per year.
  - Is managers' poor aggregate performance due to a lack of skill or market competition?
    - It is clearly not due to lack of an opportunity set.

# Conclusions (continued)

- Cross-sectional dispersion and the VIX provide forecasts of expanding and contracting alpha-capture opportunity.
- Cross-sectional dispersion and the VIX also provide forecasts of expanding and contracting idiosyncratic risk.
  - Chasing alpha leaves a proportional volatility footprint that roughly equates the benefits of active equity management with its costs – at least in the aggregate.
- One of the main difficulties facing active investors in using these alpha signals arises because return dispersion, the VIX and alpha dispersion increase during bear markets.
  - Alpha-capture opportunities are best during periods when equity values are generally declining and volatility is high.
  - A time when most investors are decreasing equity allocations and trying to reduce the risk exposure of their portfolios.