Roughing it up:

Disentangling Continuous and Jump Components in Measuring, Modeling and Forecasting Asset Return Volatility

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Important Advances in Volatility Measurement and Modeling

- Volatility from parametric econometric models ("GARCH vol")
- Volatility implied by derivatives prices ("implied vol")
- Volatility from direct indicators ("realized vol")

Realized Volatility

(...Merton....)

French, Schwert and Stambaugh (1987, *JF*)

Andersen et al. (2001, *JFE*; 2001, *JASA*; 2003, *Econometrica*)

Barndorff-Nielsen and Shephard (2002, *JRSS*)

Fleming, Kirby and Ostdiek (2003, *JFE*)

Survey: Andersen et al. (2005), in Carey/Stulz (eds.), *Risks of Financial Institutions*, University of Chicago Press for NBER, forthcoming

Basic Theory

$$dp(t) = \sigma(t)\, dW(t)$$

$$r_{t,\Delta} \equiv p(t) - p(t-\Delta) = \int_{t-\Delta}^{t} \sigma(s)\, dW(s)$$

$IV_t \equiv \int_{t-\Delta}^{t} \sigma^2(s)\, ds$ (integrated vol, quadratic variation)

$RV_t \equiv \Sigma_{j=1,..,1/\Delta} \; r^2_{t-1+j\cdot\Delta,\Delta}$ (realized vol, empirical quadratic variation)

$$plim_{\Delta\to 0} \; RV_t = IV_t$$

Simple Extensions: Drift, Multivariate, **Jumps**, ...

Important Issues Remain Incompletely Resolved:

Separation of jump and diffusive movements

Why Care about Jumps, and Separating Jump from Diffusive Movements?

Improved understanding of the price discovery process
and the macro/finance interface

Improved forecasts of RV for:

- Asset pricing
- Asset allocation
- Risk management

Jump Diffusion

$$dp(t) \;=\; \mu(t)\,dt \;+\; \sigma(t)\,dW(t) \;+\; \kappa(t)\,dq(t)$$

Jump size $\kappa(t) = p(t) - p(t-)$

Counting Process $q(t)$

Time-varying intensity $\lambda(t)$

$P[dq(t) = 1] = \lambda(t)dt$

Andersen, Benzoni and Lund (2002)
Bates (2000), Chan and Maheu (2002)
Chernov, Gallant, Ghysels, and Tauchen (2003)
Drost, Nijman and Werker (1998)
Eraker (2004), Eraker, Johannes and Polson (2003)
Johannes (2004), Johannes, Kumar and Polson (1999)
Maheu and McCurdy (2004), Pan (2002)

Jump Diffusion Integrated Volatility

$$IV_{t+1} = \int_{t}^{t+1} \sigma^2(s)\, ds + \sum_{t<s\leq t+1} \kappa^2(s)$$

Jump Diffusion Realized Volatility

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} r^2_{t+j\cdot\Delta,\Delta} \rightarrow \int_{t}^{t+1} \sigma^2(s)\, ds + \sum_{t<s\leq t+1} \kappa^2(s)$$

Andersen, Bollerslev and Diebold (2004)
Andersen, Bollerslev and Diebold and Labys (2003)
Barndorff-Nielsen and Shephard (2002, 2003)

Realized Bi-Power Variation
(The Key to Everything, I)

Barndorff-Nielsen and Shephard (2004a, 2005)

Builds on earlier work on realized power variation:
Aït-Sahalia (2003)
Barndorff-Nielsen and Shephard (2003a)

$$BV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{1/\Delta} | r_{t+j\cdot\Delta,\Delta} | \; | r_{t+(j-1)\cdot\Delta,\Delta} |$$

$$\rightarrow \int_t^{t+1} \sigma^2(s)\, ds$$

The Key to Everything, II:

$$J_{t+1}(\Delta) \equiv RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \rightarrow \sum_{t<s\leq t+1} \kappa^2(s)$$

Data

DM/$ spot exchange rate, 12/1986 - 6/1999, 3,045 days
(Olsen&Associates, Zürich)

S&P 500 futures, 1/1990 - 12/2002, 3,213 days
(Chicago Mercantile Exchange)

30-Year T-bond futures, 1/1990 - 12/2002, 3,213 days
(Chicago Board of Trade)

First-Pass Practical Implementation

Sampling Frequency $\Delta \rightarrow 0$

Five-Minute Returns

$\Delta = 1/288$ for DM/$

$\Delta = 1/97$ for S&P and T-Bond

Single adjustment:

$$J_{t+1}(\Delta) = \max[RV_{t+1}(\Delta) - BV_{t+1}(\Delta),\ 0]$$

Figure 1A
Daily DM/$ Realized Volatilities and Jumps

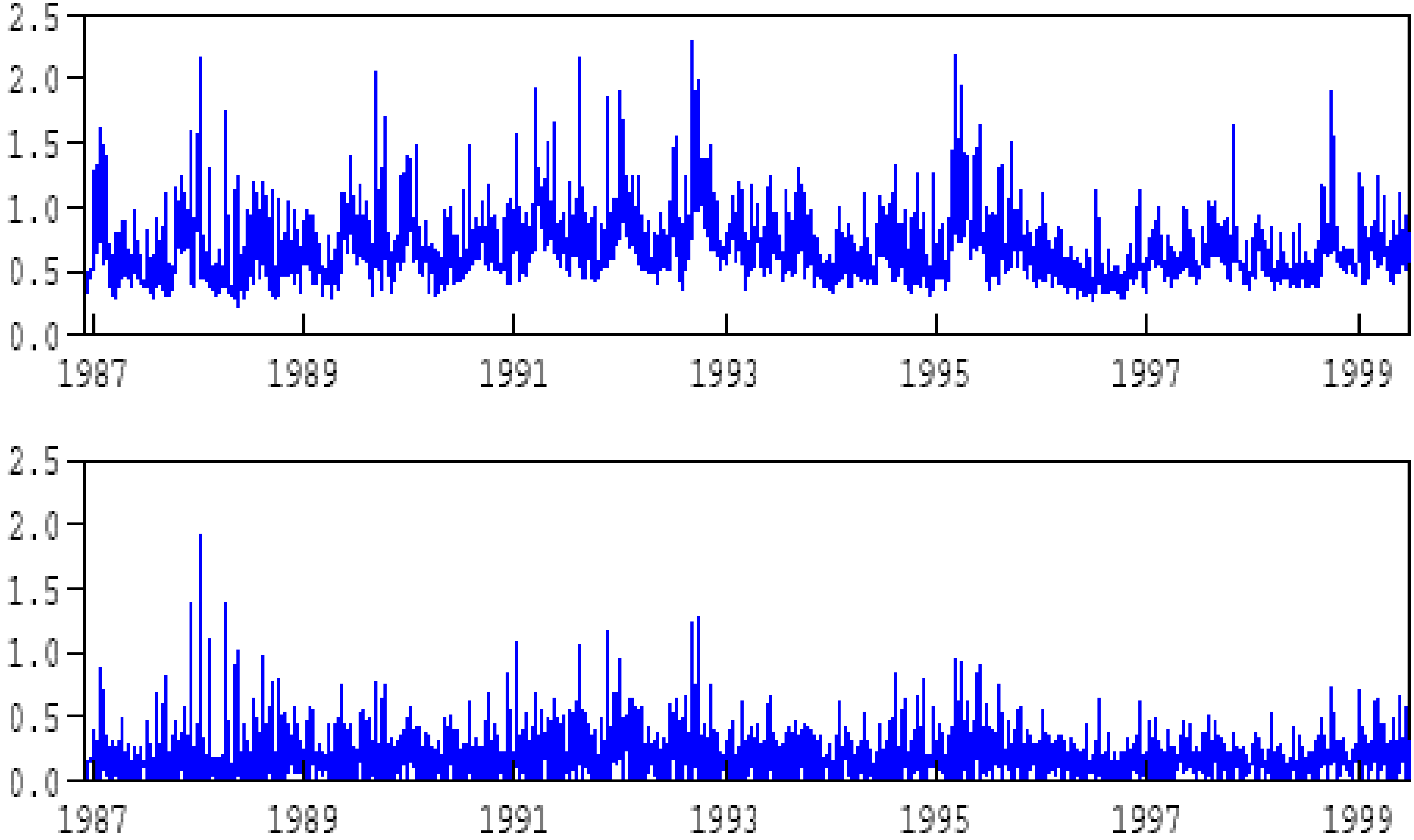
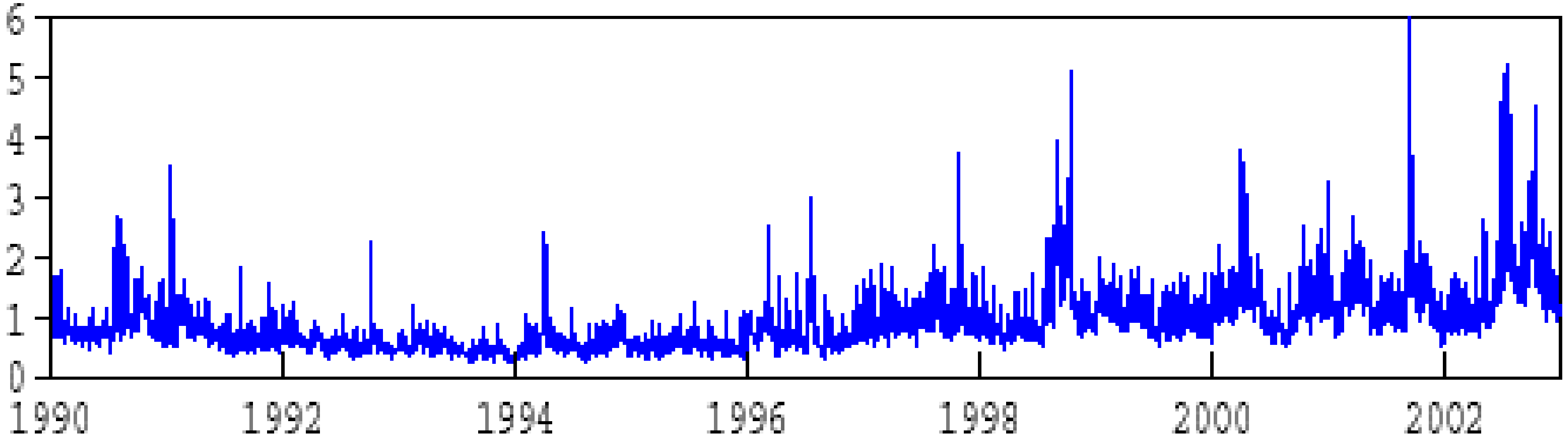


Figure 1B
Daily S&P500 Realized Volatilities and Jumps



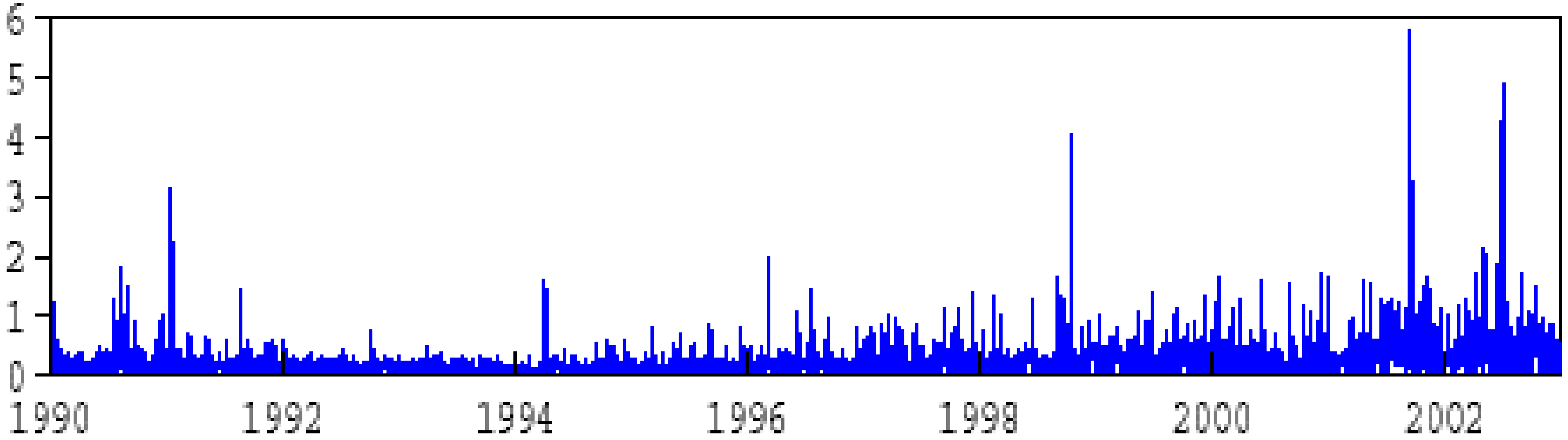
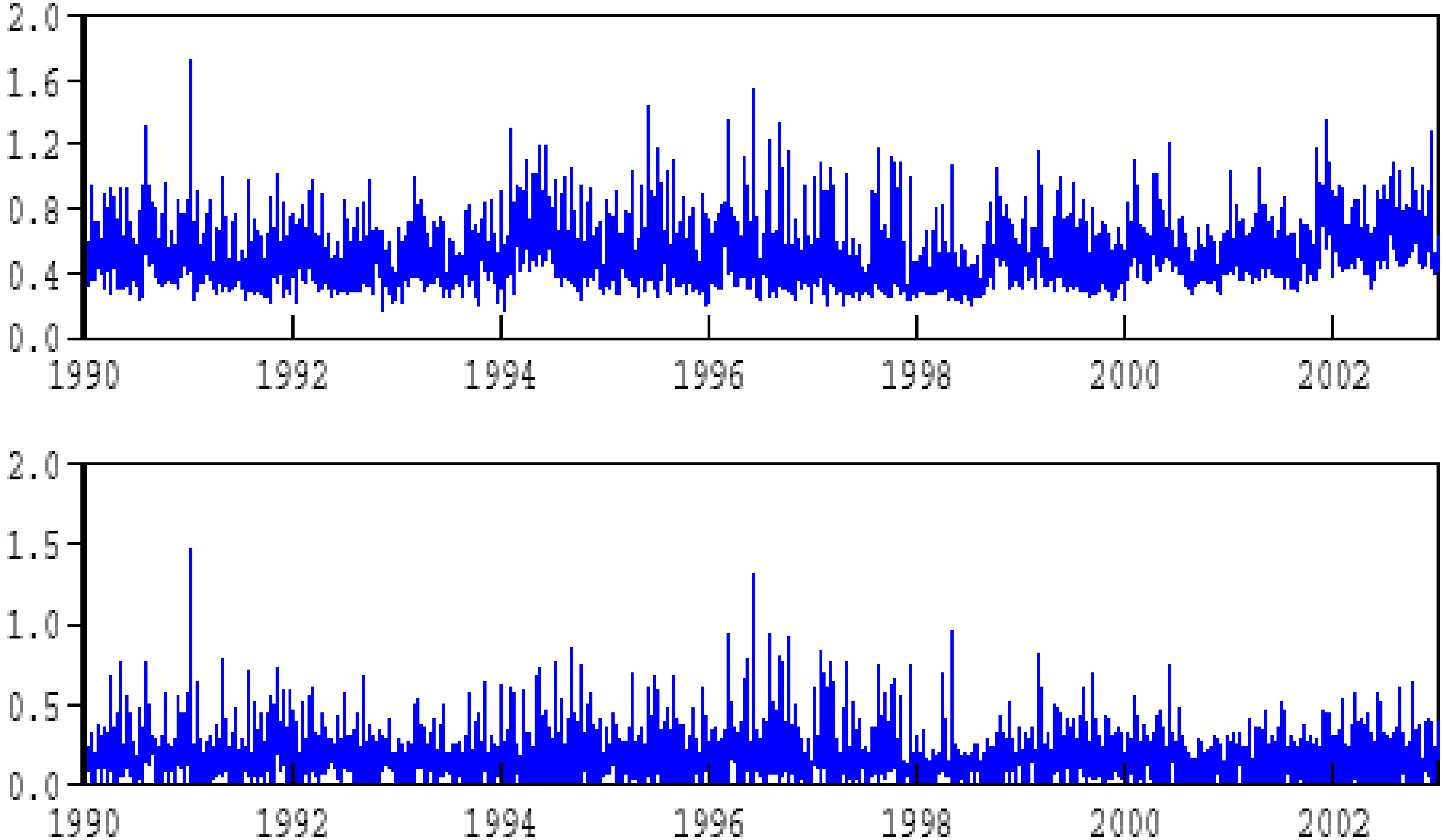


Figure 1C
Daily U.S. T-Bond Realized Volatilities and Jumps



What do we Learn?

- Realized vol mean different across markets:
  S&P > DM/$, T-Bond
  S&P realized vol U-shaped; forex and bonds flat

- Realized vol serial correlation high for all markets

- Jump vol is a significant fraction of overall realized vol
  $\Rightarrow$ Underlying price process not continuous

- Jump mean different across markets: S&P > DM/$ $\approx$ T-Bond

- Jump intensity different across markets: DM/$ $\approx$ T-Bond > S&P

- Jumps much less serially correlated than realized vol

Important Issues:

- Assessing jump “significance”
- Handling microstructure noise

Assessing Jumps I

Asymptotic ($\Delta \rightarrow 0$) distribution in the absence of jumps:

$$\Delta^{-1/2} \frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5) \int_t^{t+1} \sigma^4(s)\, ds]^{1/2}} \rightarrow N(0, 1)$$

Barndorff-Nielsen and Shephard (2004a, 2005)

Infeasible...

Realized Tri-Power Quarticity

$$TQ_{t+1}(\Delta) \equiv \Delta^{-1} \mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} | r_{t+j\Delta,\Delta} |^{4/3} | r_{t+(j-1)\Delta,\Delta} |^{4/3} | r_{t+(j-2)\Delta,\Delta} |^{4/3}$$

$$\rightarrow \int_t^{t+1} \sigma^4(s)\, ds$$

Builds on earlier work on realized power variation:
Barndorff-Nielsen and Shephard (2002a, 2004a, 2005)
Andersen, Bollerslev and Meddahi (2005)

Assessing Jumps II

$$W_{t+1}(\Delta) \equiv \Delta^{-1/2} \frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5)\, TQ_{t+1}(\Delta)]^{1/2}}$$

Feasible!

Incorporation of Variance Stabilizing Transforms and Maximum (Jensen's Inequality) Adjustment

$$Z_{t+1}(\Delta) \equiv \Delta^{-1/2} \frac{[\,RV_{t+1}(\Delta) - BV_{t+1}(\Delta)\,]\,RV_{t+1}(\Delta)^{-1}}{[\,(\mu_1^{-4} + 2\mu_1^{-2} - 5)\max\{\,1\,,\,TQ_{t+1}(\Delta)\,BV_{t+1}(\Delta)^{-2}\,\}\,]^{1/2}}$$

Barndorff-Nielsen and Shephard (2004b)
Huang and Tauchen (2005)

Very well behaved in finite samples

Huang and Tauchen (2005)

A Concise Statement

$$RV_{t+1}(\Delta) \quad = \quad C_{t+1,\alpha}(\Delta) \; + \; J_{t+1,\alpha}(\Delta)$$

$$C_{t+1,\alpha}(\Delta) \; \equiv \; I\,[\, Z_{t+1}(\Delta) \le \Phi_{\alpha} \,] \cdot RV_{t+1}(\Delta) \; + \; I\,[\, Z_{t+1}(\Delta) > \Phi_{\alpha} \,] \cdot BV_{t+1}(\Delta)$$

$$J_{t+1,\alpha}(\Delta) \quad \equiv \quad I\,[\, Z_{t+1}(\Delta) > \Phi_{\alpha} \,] \cdot [\, RV_{t+1}(\Delta) \; - \; BV_{t+1}(\Delta) \,]$$

Depends on α and Δ

Previous J_{t+1} corresponds to $\alpha = 0.5$

Theory and Practice

Theory: $\Delta \rightarrow 0$

Practice: Market Microstructure Frictions

Discreteness
Bid-Ask Bounce
Unevenly Spaced Observations

Aït-Sahalia, Mykland and Zhang (2005)
Andersen, Bollerslev, Diebold and Labys (2000)
Andreou and Ghysels (2002), Areal and Taylor (2002)
Bandi and Russell (2004a,b)
Barndorff-Nielsen, Hansen, Lunde and Shephard (2004)
Barucci and Reno (2002), Bollen and Inder (2002)
Corsi, Zumbach, Müller, and Dacorogna (2001)
Curci and Corsi (2003), Hansen and Lunde (2005, 2004a,b)
Malliavin and Mancino (2002), Oomen (2002, 2004)
Zhang (2004), Zhang, Aït-Sahalia and Mykland (2005)
Zhou (1996)

Returns Polluted by Microstructure Noise

$$r_{t,\Delta} \equiv p^*(t) - p^*(t-\Delta) + \nu(t) - \nu(t-\Delta) \equiv r^*_{t,\Delta} + \eta_{t,\Delta}$$

$p^*(t)$: "true" price

$\nu(t)$: microstructure noise

$\eta_{t,\Delta}$ is $MA(1)$

Realized Volatility Polluted by Microstructure Noise

$$E[r_{t,\Delta}^2] = E[(r_{t,\Delta}^* + \eta_{t,\Delta})^2] \neq E[(r_{t,\Delta}^*)^2]$$

Noise term dominates for $\Delta \to 0$

$RV_t(\Delta)$ formally inconsistent as $\Delta \to 0$

Strategies

Fixed $\Delta \geq \delta >> 0$; e.g. Five-Minute Returns

Andersen, Bollerslev, Diebold and Labys (2000, 2001, 2003)

Pre-Filtering and Kernel Methods

Andreou and Ghysels (2002), Areal and Taylor (2002), Barndorff-Nielsen, Hansen, Lunde and Shephard (2004), Corsi, Zumbach, Müller, and Dacorogna (2001), Hansen and Lunde (2005, 2004a,b), Oomen (2002, 2004), Zhou (1996)

Fourier Methods

Barucci and Reno (2002), Malliavin and Mancino (2002)

"Optimal" Sampling Schemes

Aït-Sahalia, Mykland and Zhang (2005), Bandi and Russell (2004a,b)

Sub-Sampling

Müller (1993), Zhang (2004), Zhang, Aït-Sahalia and Mykland (2005)

New Robust Estimators

"Standard" Bi-Power Variation

$$BV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{1/\Delta} | r_{t+j\cdot\Delta,\Delta} | \, | r_{t+(j-1)\cdot\Delta,\Delta} |$$

Staggered Bi-Power Variation

$$BV_{1,t+1}(\Delta) \equiv \mu_1^{-2} (1 - 2\Delta)^{-1} \sum_{j=3}^{1/\Delta} | r_{t+j\cdot\Delta,\Delta} | \, | r_{t+(j-2)\cdot\Delta,\Delta} |$$

"Standard" Tri-Power Quarticity

$$TQ_{t+1}(\Delta) \equiv \Delta^{-1} \mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} |r_{t+j\Delta,\Delta}|^{4/3} |r_{t+(j-1)\Delta,\Delta}|^{4/3} |r_{t+(j-2)\Delta,\Delta}|^{4/3}$$

Staggered Tri-Power Quarticity

$$TQ_{1,t+1} \equiv \Delta^{-1} \mu_{4/3}^{-3} (1 - 4\Delta)^{-1} \cdot$$

$$\sum_{j=5}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}|^{4/3} |r_{t+(j-2)\cdot\Delta,\Delta}|^{4/3} |r_{t+(j-4)\cdot\Delta,\Delta}|^{4/3}$$

Results

Figure 1A
Daily DM/$ Realized Volatilities and Jumps

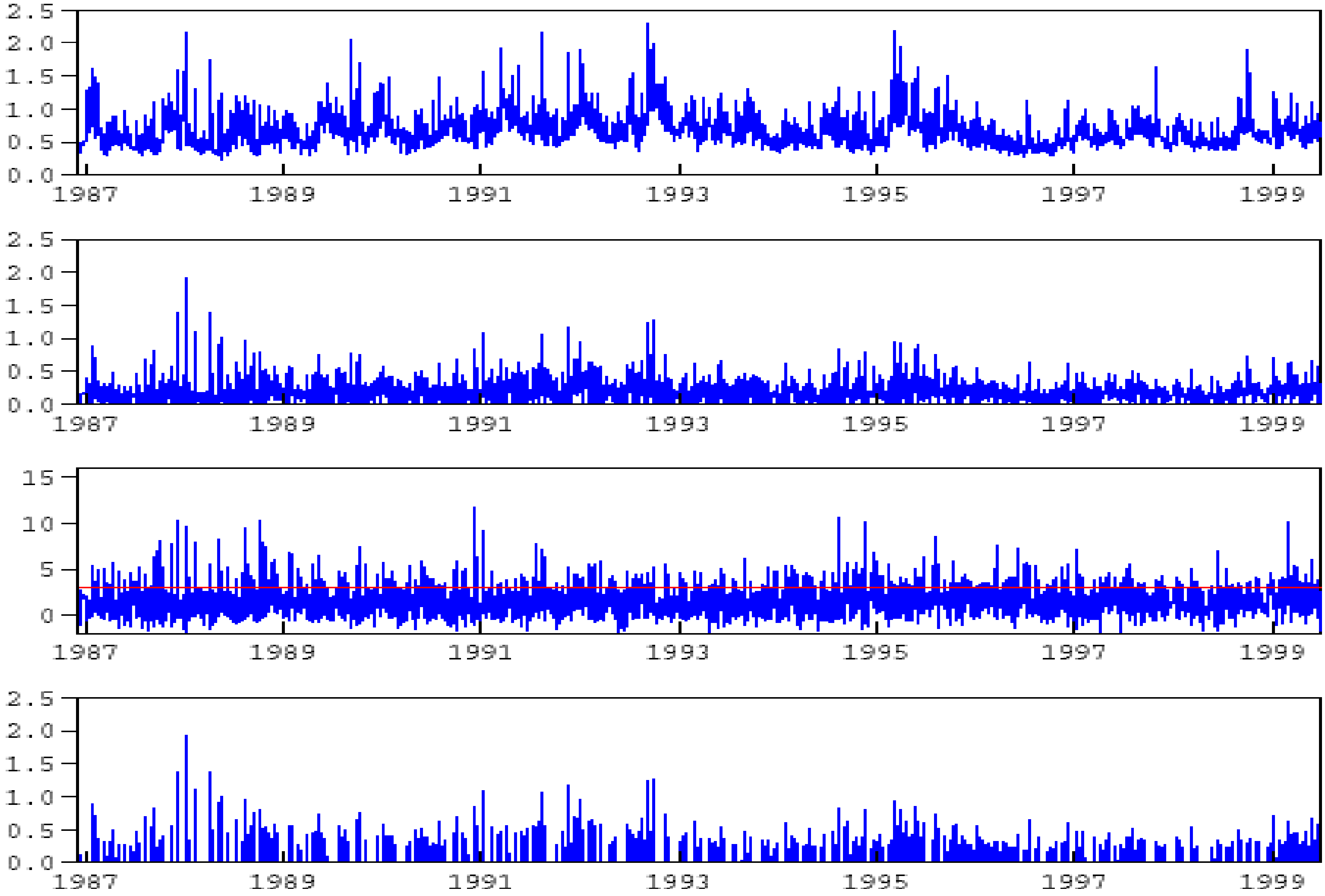


Figure 1B
Daily S&P500 Realized Volatilities and Jumps

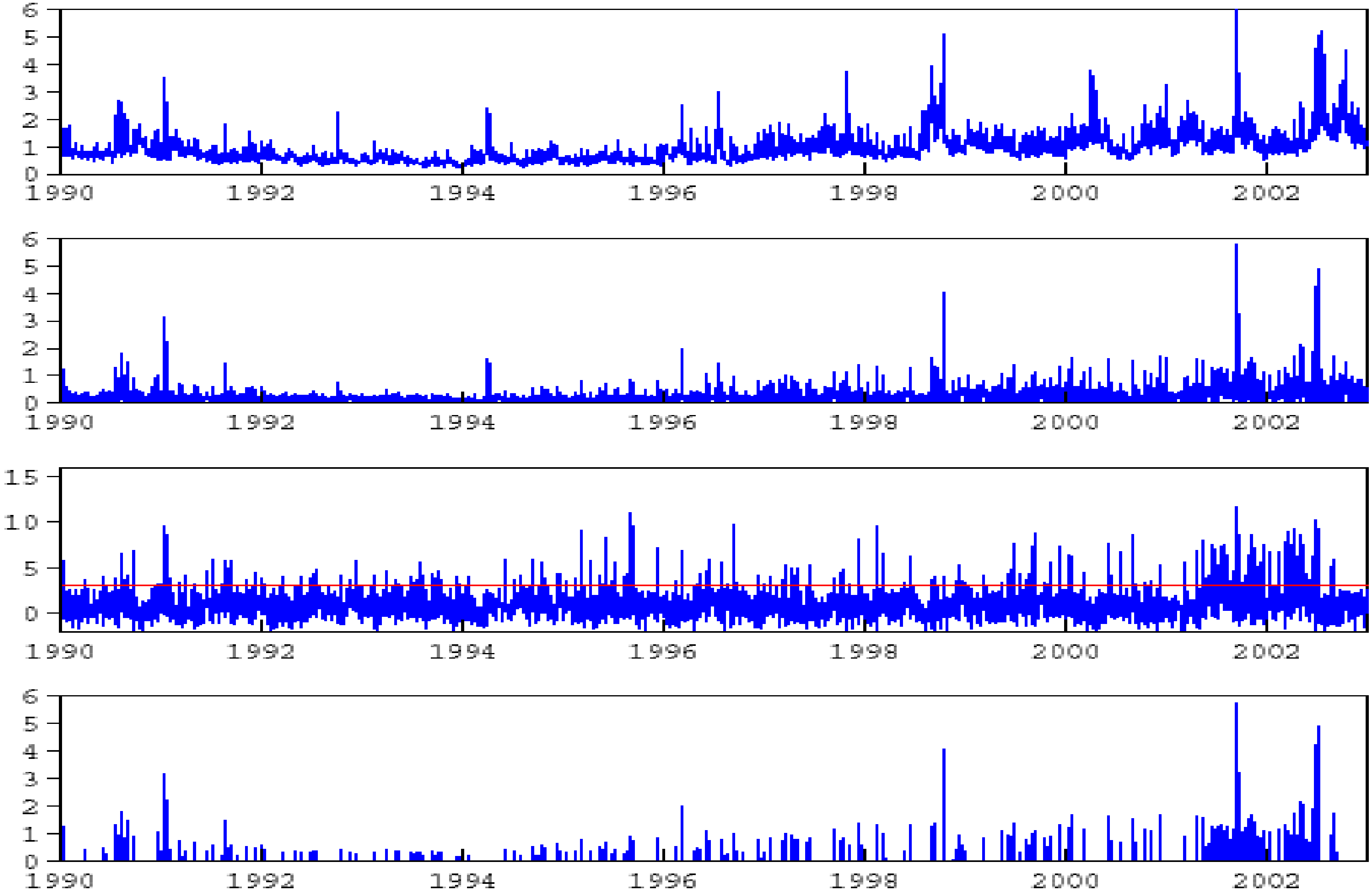
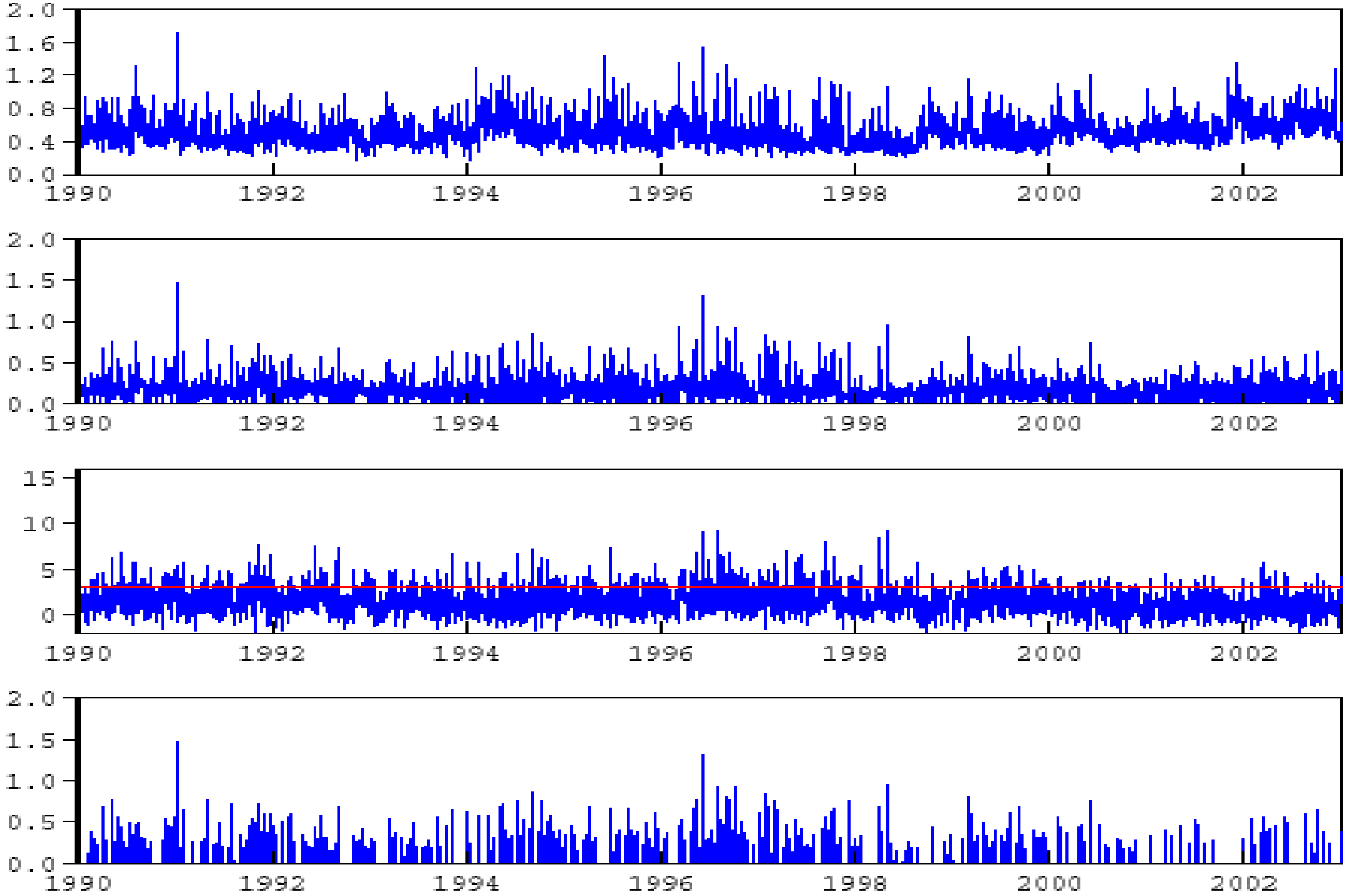


Figure 1C

Daily U.S. T-Bond Realized Volatilities and Jumps



What do we Learn?

- Jump vol is a significant fraction of overall realized vol
  $\Rightarrow$ Underlying price process not continuous

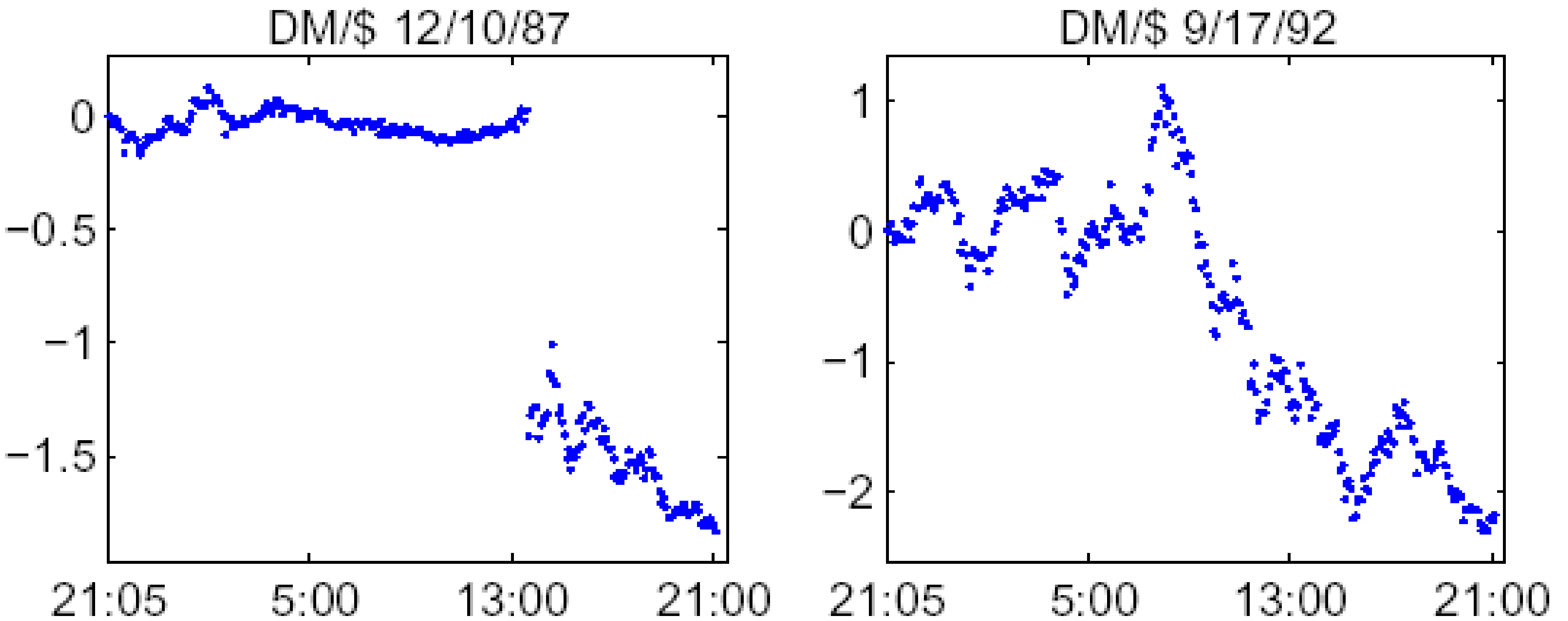
- Jump mean different across markets: S&P > DM/$ ≈ T-Bond

- Jump intensity different across markets: DM/$ ≈ T-Bond > S&P

- Jumps much less serially correlated than realized vol

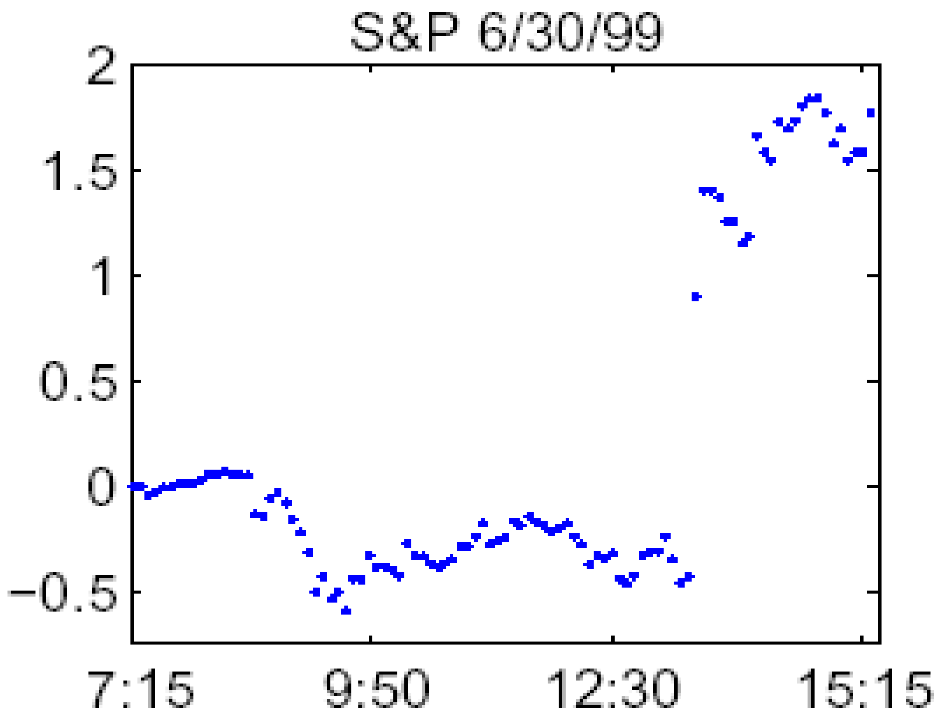
- *In general, jumps occur very frequently and would be missed without high-frequency data*

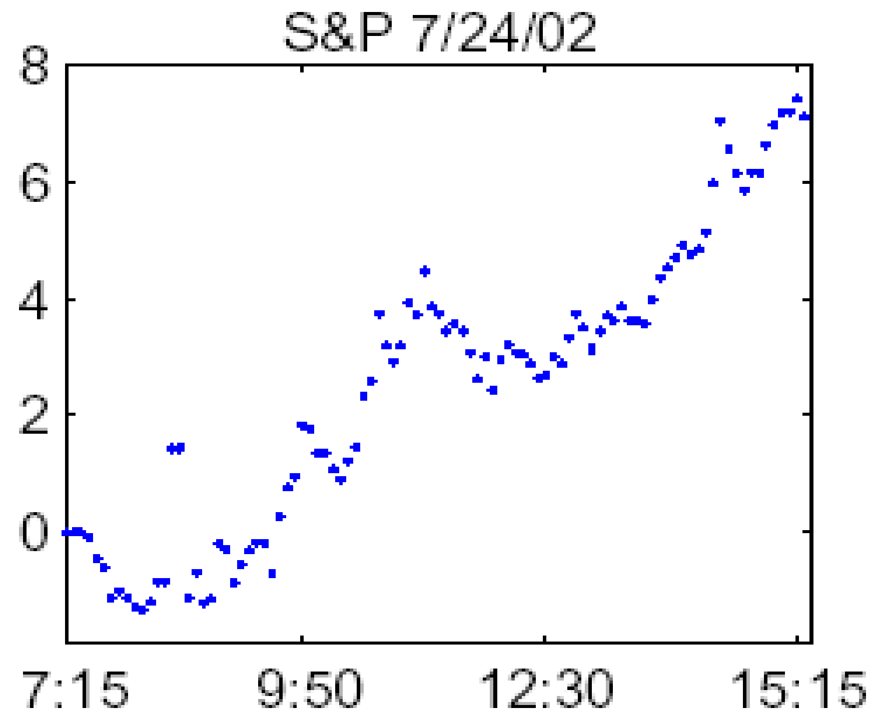
Case Study: Forex



12/10/87: Swelling U.S. trade deficit announced at 13:30GMT
9/17/92: The day following the temporary withdrawal of the British Pound from the European Monetary System

Case Study: Stocks

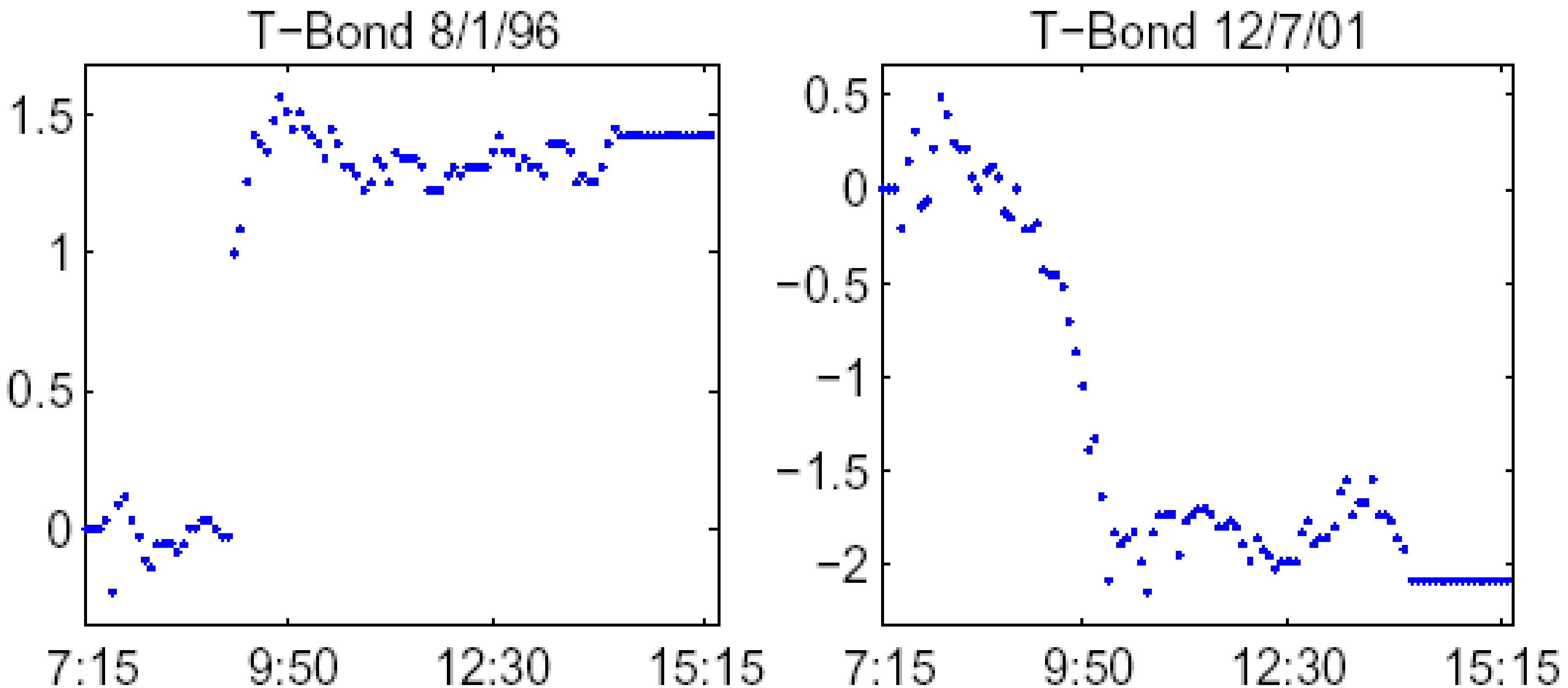




6/30/99: FED raised short rate by ¼ percent at 13:15 CST but indicated that it "might not raise rates again in the near term due to conflicting forces in the economy."

7/24/02: Record NYSE trading volume of 2.77 billion shares

Case Study: Bonds



8/1/96: Surprisingly optimistic NAPM survey released 9:00 CST
12/7/01: Rise in jobless claims and expectation that the FED will lower rates at its Board meeting on the following day

Return to a Key Motivational Issue:

How do jumps feed into
subsequent volatility movements?

Volatility Modeling and Forecasting

FIGARCH and LM-SV Models

Baillie, Bollerslev, and Mikkelsen (1996), Bollerslev and Mikkelsen (1996, 1999)
Breidt, Crato and de Lima (1998), Ding, Granger and Engle (1993)
Harvey (1998), Robinson (1991)

ARFIMA-RV Models

Andersen, Bollerslev, Diebold and Labys (2003), Areal and Taylor (2002), Deo, Hurvich and Lu (2003)
Koopman, Jungbacker and Hol (2005), Martens, van Dijk, and Pooter (2004), Oomen (2002)
Pong, Shackleton, Taylor and Xu (2004), Thomakos and Wang (2003)

Multi-Factor and Component Structures

Andersen and Bollerslev (1997), Calvet and Fisher (2001, 2002), Chernov, Gallant, Ghysels and Tauchen (2003)
Dacorogna et al. (2001), Engle and Lee (1999), Gallant, Hsu and Tauchen (1999), Müller et al. (1997)

Heterogeneous AR Realized Volatility (HAR-RV) Model

Corsi (2003)

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \varepsilon_{t+1}$$

$$RV_{t,t+h} \equiv h^{-1}(RV_{t+1} + RV_{t+2} + ... + RV_{t+h})$$

$h = 1, 5, 22$ (daily, weekly, monthly)

Approximate long-memory model

HAR-RV-J Model

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t}$$

$$+ \beta_J J_t + \varepsilon_{t+1}$$

HAR-RV-CJ Model

$$RV_{t+1} = \beta_0 + \beta_{CD} C_t + \beta_{CW} C_{t-5,t} + \beta_{CM} C_{t-22,t}$$

$$+ \beta_{JD} J_t + \beta_{JW} J_{t-5,t} + \beta_{JM} J_{t-22,t} + \varepsilon_{t+1}$$

Extensions

• Multi-Period Horizons

$$RV_{t,t+h} = \beta_0 + \beta_{CD} C_t + \beta_{CW} C_{t-5,t} + \beta_{CM} C_{t-22,t}$$

$$+ \beta_{JD} J_t + \beta_{JW} J_{t-5,t} + \beta_{JM} J_{t-22,t} + \varepsilon_{t,t+h} \qquad MA(h-1)$$

• MIDAS Regressions

Ghysels, Santa-Clara and Valkanov (2004, 2005)

• Other Volatility Transforms

$$(RV_{t,t+h})^{1/2} = \ldots, \qquad \log(RV_{t,t+h}) = \ldots$$

• Multiplicative Error Models (MEM)

Engle (2002)
Engle and Gallo (2005)

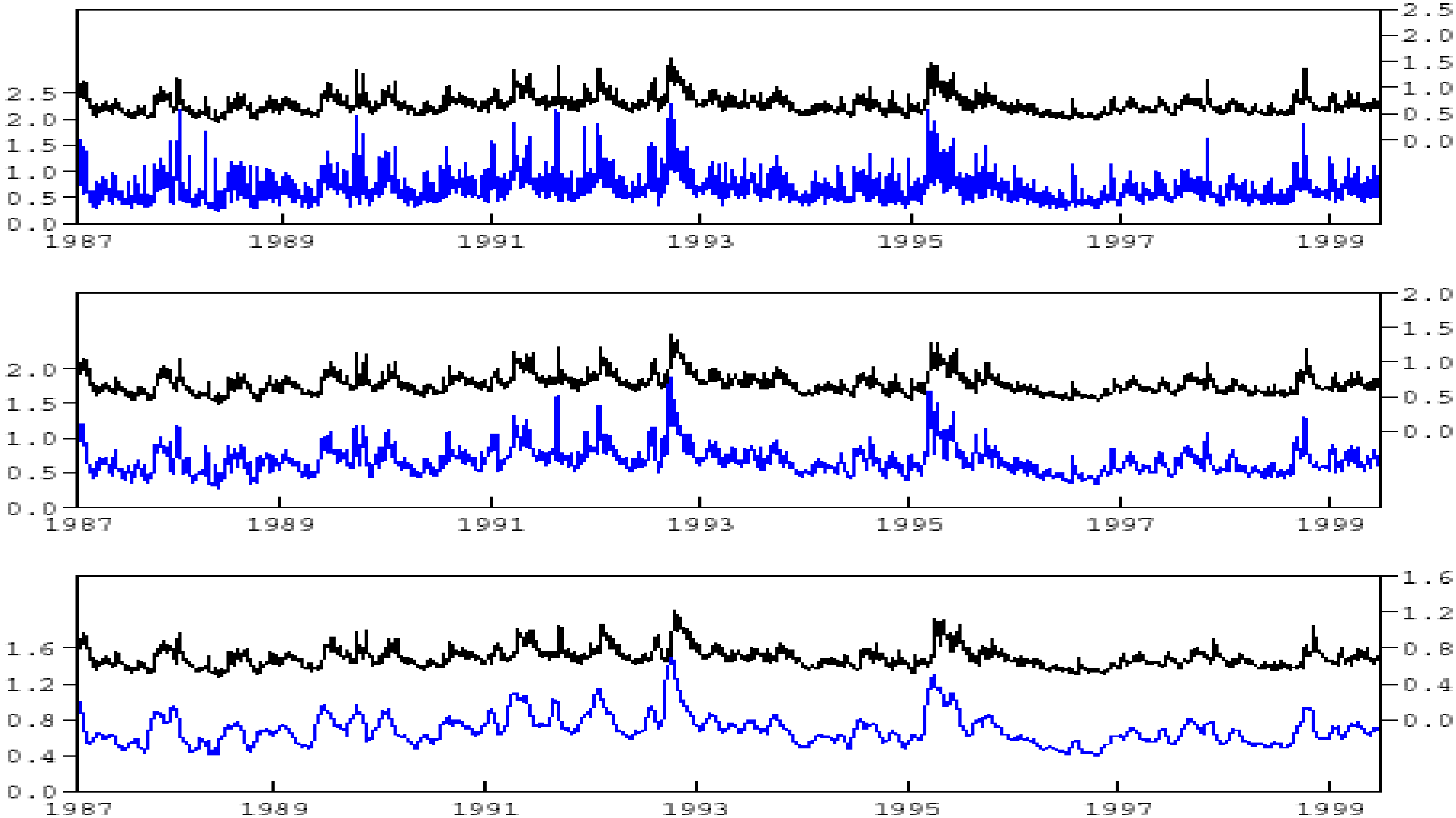
Daily, Weekly, and Monthly
DM/$ HAR-RV-J Regressions

	$RV_{t,t+h}$		
	h = 1	h=5	h=22
β_0	0.083	0.132	0.231
	(0.015)	(0.018)	(0.025)
β_D	0.430	0.222	0.110
	(0.043)	(0.040)	(0.022)
β_W	0.196	0.216	0.218
	(0.063)	(0.055)	(0.043)
β_M	0.244	0.323	0.225
	(0.061)	(0.068)	(0.062)
β_J	-0.486	-0.297	-0.166
	(0.096)	(0.070)	(0.056)
$R^2_{HAR\text{-}RV}$	0.252	0.261	0.215
$R^2_{HAR\text{-}RV\text{-}J}$	0.364	0.417	0.353

Daily, Weekly, and Monthly
DM/$ HAR-RV-CJ Regressions

	$RV_{t,t+h}$		
	h = 1	h=5	h=22
β_0	0.083	0.131	0.231
	(0.015)	(0.018)	(0.025)
β_{CD}	0.407	0.210	0.101
	(0.044)	(0.040)	(0.021)
β_{CW}	0.256	0.271	0.259
	(0.077)	(0.054)	(0.046)
β_{CM}	0.226	0.308	0.217
	(0.072)	(0.078)	(0.074)
β_{JD}	0.096	0.006	-0.002
	(0.089)	(0.040)	(0.017)
β_{JW}	-0.191	-0.179	-0.073
	(0.168)	(0.199)	(0.125)
β_{JM}	-0.001	0.055	-0.014
	(0.329)	(0.460)	(0.604)

Daily, Weekly and Monthly DM/$ Realized Vols and HAR-RV-CJ Forecasts



Summary and Directions for Future Work

- Useful to split jump from diffusive phenomena
- We have developed a nonparametric framework for doing so
- High-frequency data is key ingredient
- Assessment of “significant” jumps
- Robustness to microstructure noise
- New associated HAR-RV-J and HAR-RV-CJ models produce superior forecasts
- Future: Dynamics of Jump Intensities and Sizes

Thank you very much!