Diversification Across Time

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Buying stock on margin when young combined with reduced equity exposure when older can reduce lifetime portfolio risk. For example, an initially-leveraged portfolio can produce the same mean accumulation as a constant 74% stock allocation with a 21% smaller standard deviation. Since the means are equal, the reduced volatility doesn't depend on the equity premium. A leveraged lifecycle strategy also allows investors to come closer to their utility-maximizing equity allocation. Monte Carlo simulations show that the gains continue even with equity premia well below historical levels.

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> [The businessman] can look forward to a high salary in the future; and with so high a present discounted value of wealth, it is only prudent for him to put more into common stocks compared to his present tangible wealth, borrowing if necessary for the purpose . . . . [This point] does justify leveraged investment financed by borrowing against future earnings. But it does not really involve any increase in relative risk-taking once we have related what is at risk to the proper larger basis.
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> – Paul Samuelson (1969)

This paper shows that a leveraged lifecycle strategy, one which starts with a leveraged allocation in stock and then gradually decreases leverage and ultimately becomes unleveraged near retirement, produces a substantial improvement in expected utility. The gain comes from two factors. The first is improved diversification. Even if investors are well diversified across assets, they are insufficiently diversified across time. They have too much invested in stock late in their life and not enough early on. An initially-leveraged portfolio can produce the same mean accumulation with a 21% smaller standard deviation.

The second source of gain comes from coming closer to the utility-maximizing investment level. The recommendation from Samuelson (1969) and Merton (1969, 1971) is to invest a constant fraction of wealth in stocks (what we call the Merton-Samuelson share). The mistake in translating this theory into practice is that young people invest only a fraction of their *current* savings instead of their discounted lifetime savings. For someone in their 30's, investing even 100% of current savings is likely to be less than 10% of their lifetime savings and typically far less than the utility-maximizing Merton-Samuelson share.

In the Merton-Samuelson framework, all of a person's wealth for both consumption and saving was assumed to come at the beginning of the person's life. That isn't the situation for a typical worker who starts with almost no savings. Thus, the advice to invest even 50% of the present value of future savings in stocks would imply an investment well more than what would be currently available. This leads to our prescription: *buy stocks using leverage when young*. More specifically, following Merton-Samuelson, we analyze a strategy that targets investing in stock a constant percentage of the present value of lifetime savings. This strategy calls for leveraged investing in the early years of life when the present value of future contributions is large relative to current savings and then reduced leverage and finally unleveraged investing as current savings grow and the present value of future contributions decline.

For this strategy to be effective, it requires the ability to borrow at low cost. As we will show, the implied interest cost for 2:1 leverage is remarkably low—it was below 1.0% in August 2010. Our leveraged strategy still falls short of the investment target in initial years because the

incremental cost of borrowing to achieve more than 2:1 leverage increases rapidly and outweighs the benefits. Hence, we analyze leveraged lifecycles which initially allocate to stock a maximum 200% of current savings and then ramp down over time toward the Merton-Samuelson share as the investor nears retirement (and has no future expected saving contributions).

To motivate what follows, imagine that a worker has to choose between investing in (i) a fund that allocates a constant 74% of the current portfolio to equities and the remainder to government bonds or (ii) a leveraged lifecycle fund that begins by allocating 200% of the current portfolio value to stock but ramps down to a 50% allocation in stock as the worker nears retirement. Most people instinctively feel that the leveraged strategy is riskier. They focus on the fact that the leveraged lifecycle begins by exposing a much higher proportion of portfolio value to stock risk; however, the leveraged strategy ends with a lower stock allocation when the absolute size of the portfolio is larger. Based on historical stock returns (from 1871 through 2009), we show that investing in the leveraged lifecycle would have produced the same mean with substantially lower volatility in retirement accumulations than investing in a constant 74% equity fund. We chose the constant 74% equity allocation strategy precisely because it produces the same mean retirement accumulation as the leveraged lifecycle that goes from 200% down to 50%. Thus our result does not depend on a greater exposure to the equity premium. The leveraged lifecycle strategy leads to a 21% reduction in the standard deviation of retirement wealth. It reduces volatility because it better diversifies the exposure to stock risk across time.

Figure 1 demonstrates this result graphically. It shows the mean along with the 1.96 standard deviation range for the constant 74%-equity strategy compared to a leveraged strategy with the same return.

Figure 1: 95%-Confidence Intervals
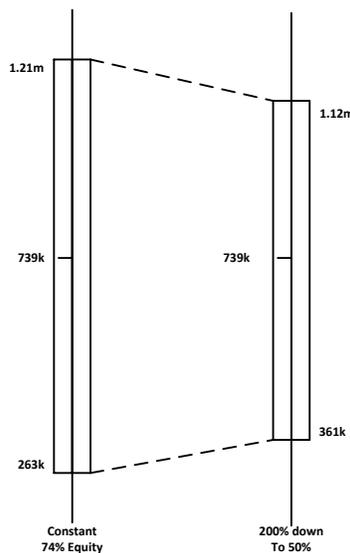
The underlying intuition is quite simple. Consider a three-period model with exogenous savings of $100 in each period where the investor puts a constant 74% of his savings in equities. His exposure to equities would grow over time, something like $74, $148, $222. This is poorly diversified in that the investor is much more exposed to market risk in period 3 compared to period 1. With the use of leverage in the first period and a reduced equity allocation in the third period, the investor can achieve $148, $148, $148, the same total exposure but better spread out over time and thus lower risk. If the returns in the three periods were independent, the equal investment approach has a $1/7^{th}$ lower variance. In practice, the actual exposure is even more lopsided as expected stock returns are positive and savings contributions rise over time, and thus the potential risk reduction is even greater.

A leveraged strategy when young also allows investors to come closer to their utility-maximizing allocation strategy. Consider a conservative young investor who, if not liquidity-constrained, would put just 50% of his assets into equities. Because most of his wealth is tied up in his human capital, this investor only has only a small fraction of his wealth in liquid form. Absent leverage, he may be limited to putting 5 percent of his wealth in equities. With the use of 2:1 leverage, he could put 10 percent into equities. As shown in Section 4, Table VI, this can lead to a substantial improvement in expected utility compared to the traditional unleveraged target-date strategy. Given historical returns, a 50% target is appropriate for someone with a relative risk aversion of 4, and this limited use of leverage leads to an 11.5 percent improvement in the certainty equivalent of retirement savings. Here, leverage gives the investor a larger lifetime exposure to equities, but one that he desires given his level of risk tolerance.

In our U.S. data (going back to 1871), we find that equities returned 8.83% (6.61% real), while the cost of margin was 5.01%. This 3.82% equity premium was the source of the increased returns of our leveraged lifecycle strategy in this second comparison, where we didn't compare strategies with equal expected returns.[1] As Barberis (2000) observes, this equity premium is based on relatively limited data and just one sample path; thus investors should not count on the equity premium persisting at historical levels. Shiller (2005a,b, 2006) goes further to suggest that the U.S. equity performance is unlikely to be repeated.[2] In our Monte Carlo analysis, we show that even with the equity premium reduced to half its historical level (or with a higher margin rate) there is still a gain from employing leverage while young. Our basic results are also robust to using historical FTSE and Nikkei data.

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[1] Cornell, Arnott, and Moroz (2010) estimate the equity premium at between 4.0 and 4.5%.
[2] The high equity premium may also be an artifact of survivorship bias (see Brown, Goetzmann, and Ross (1995)).

Our focus is on investment allocation during working years. We do not consider how the portfolio should be invested during the retirement phase, although results from Fontaine (2005) suggest that standard advice may be too conservative here as well.[3] Nor do we take on the difficult and interesting question of how much people should optimally save over the course of their lives. Instead, we focus on the allocation between stocks and bonds taking the savings rate as exogenously given.

The assumption of exogenous savings is reasonable. Many people save money for retirement via automatic payroll deduction (Poterba and Samwick (2001)). There are tax advantages to putting aside money in a relatively illiquid 401(k) plan and these contributions are often matched by the employer. Due to employer matching and tax advantages, even young workers who are constrained in terms of consumption might still choose to put something away toward retirement. Whether savings are optimal or not, the Merton-Samuelson logic suggests that any retirement savings that do occur should initially be invested on a leveraged basis so that more than 100% of the net portfolio value is in equities.

With the shift away from defined benefits to defined contribution pensions, much of early savings comes from tax-advantaged and employer-matched 401(k) plans. Thus our advice is especially relevant for the allocation of stocks inside a 401(k) plan. However, current regulations effectively prevent people from leveraging and investing at our prescribed rate with regard to their 401(k) investments. The reason is that an employer could lose its safe-harbor immunity for losses if any one of its plan offerings is later found by a court to not be a prudent investment. Allowing employees to buy stocks on margin is not yet considered prudent.[4]

Our core analysis ignores the impact of human capital or housing investments on the optimal retirement investment. There is a large literature that considers how to translate future earnings into the initial wealth and the impact that has on current investment; see Bodie, Merton, and Samuelson (1992); Heaton and Lucas (1997); Viceira (2001); Campbell and Viceira (2002); Benzoni, Collin-Dufresne, and Goldstein (2007); and Lynch and Tan (2004). As emphasized by Viceira (2001) and Campbell and Viceira (2002), many people, especially the self-employed, are already heavily invested in the market via human capital. To the extent that human capital is correlated with the market, some individuals might already be fully invested in equities. At the extreme, Benzoni, Collin-Dufresne, and Goldstein (2007) show that a risk-averse (γ=5) young worker may actually want to *short* equities. The reason is the high cointegration of the labor and

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[3] This asset allocation during retirement can be avoided through the purchase of annuities, which also solves the problem of an uncertain lifetime.

[4] It is possible to create the equivalent to leveraged positions in self-directed IRAs and Keogh plans by investing in options on stock indexes; see www.cboe.com/institutional/irakeogh.aspx.

equity markets.

The degree of correlation is an empirical question that varies by profession. In academia where budgets are somewhat insulated from short-term market forces by endowments and government grants, the salaries of professors are much less volatile than the stock market. Thus, even taking human capital exposure to stock market risk into account, assistant professors and many others should still invest on margin when young.

Data from Heaton and Lucas (2000) shows that most people's wages do not have a strong positive correlation with stock returns. Based on a 1979–1990 panel of individual tax returns, they find that for one-third of their sample, the correlation between wages and the market is nearly zero (between –0.25 and 0.25). Almost another third had wages that were even more negatively correlated with the market and only 10 percent had a positive correlation above 0.50. Thus, even after accounting for the correlation of human capital with stock risk, leveraged investments would remain relevant to a broad group of young investors.

We are not claiming that the leverage strategy is optimal for everyone. The right strategy depends on the individual's risk aversion, his expectation of the equity premium and market risk, and the correlation between his future income and the market; see Milevsky (2008). The same is equally true for the standard target-date fund that starts at 90% and ends at 50%. There is no one size fits all portfolio allocation. For a typical 45-year old tenured professor, Milevsky suggests a 280% allocation to equities. This falls to 170% for a bankruptcy lawyer, 125% for a mechanical engineer, and 60% for an investment banker. Fewer than half the people in Heaton and Lucas (2000) can achieve their Merton-Samuelson equity exposure without the use of leverage. Our results challenge the standard orthodoxy that counsels *everyone* against buying stock on margin.

In this paper, we are not trying to determine the optimal savings rate, consumption rule, and portfolio allocation, all while taking into account Social Security. While some, such as Gomes, Kotlikoff, and Viceira (2008), have taken on that challenging task, they place a restriction that the allocation to stock could never exceed 100% of current savings. Instead, we relax the restriction on leverage in order to show the two values of employing leverage: diversification and utility-maximizing allocation. We use historical and Monte Carlo simulation to show the potential superiority of leveraged lifecycle strategies to constant equity allocations and target-date funds.

Demonstrating that a leverage life cycle strategy is superior to a traditional target-date fund is particularly significant because the later is widely adopted and growing. Poterba, Rauh, Venti, and Wise (2005a) report that in 2000, target-date funds held $5.5 billion. By 2009, their assets had grown to $256 billion (Brady, Holden, and Short (2010)). Vanguard estimates that 79 percent of all 401(k) plans offer target-date funds and that adoption rate is 48 percent (Vanguard (2010)).

Target-date funds will continue to grow in importance as the Department of Labor now permits them to be used as a qualified default investment alternative for defined contribution plans.

There is the old saying: Don't count your chickens before they hatch. We agree. But you can (dis)count your chickens before they hatch. Instead of ignoring your future retirement savings, individuals should calculate the present value of expected saving contributions and use leverage to invest some of those contributions in the stock market today, especially when those future savings are more like a bond than a stock.

I.   Connection to the Literature

The theory approach to lifecycle portfolio allocation begins with Samuelson (1969) and Merton (1969). They demonstrate that with constant relative risk aversion the allocation between equities and bonds should be constant over the lifecycle. The allocation depends only on the degree of risk aversion and the return on equities, not age. Samuelson was responding to the view that young investors should take more risks because they have more years with which to gamble. This is the intuition used to support investment advice such as the "110 − Age" rule.[5] The surprising aspect of the Merton and Samuelson results is that for an investor who has all of his wealth upfront, there is no reason to ramp down exposure to equities over time.

The fact that young investors have a longer horizon over which they can adjust their consumption or work effort to unanticipated market returns might suggest there is less risk to early investments. However, in the case of constant relative risk aversion, there is no advantage from this extra flexibility. While consumption does adjust, the investor would choose the same asset allocation for all income levels. Thus the amount invested changes, but not the percentage.

Moving outside the world of constant relative risk aversion offers a potential motivation for changing the equity allocation over time. The later period allocations can respond to changes in wealth. The early allocation might then respond to the fact that later allocations can adjust. This flexibility increases the attractiveness of investing, but whether it increases the marginal attractiveness when young is less clear.

An implicit recommendation from the Merton-Samuelson model is that stock investments should be made as a fraction of lifetime wealth. In contrast, the target-date funds base investments on current savings, not on lifetime wealth. This is the most significant departure of practice from theory. For young workers, lifetime wealth is likely to be a large multiple of current

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[5] For example, Malkiel (2003) proposes a portfolio that is starts at 75% equities (including real estate), ramps down to 65% in the late 30s/early 40s, reduces to 57.5% exposure in the mid 50s, and falls to 40% at retirement. This is close to a 110 − Age rule. The Vanguard and Fidelity funds go from 90% at age 20 down to 50% at age 65, but they fall more slowly at first making them closer to 120 − age than 110 − age.

savings. Thus, the way to maximize expected utility and follow the Merton-Samuelson prescription is to invest using leverage.

In Merton and Samuelson, this issue is partially obscured, as wealth is given exogenously up front. With constant relative risk aversion, the wealth, W, can be factored out of the utility function. It is generally left out of the calculation because it doesn't matter. In our case, it does matter, even with constant relative risk aversion. It matters because it determines the goal. Consider the case where our investor has future wealth with a present value of W but only has access to liquid savings of S. For purposes of illustration, assume that absent any constraints he would invest a share λ of (S+W) in equities. If S < λ(S+W) and the investor can borrow at the bond rate, then he is better off employing some leverage to get closer to λ(S+W). If 2S < λ(S+W), then he should go all the way to 2:1 leverage to get closer to λ(S+W). In theory, more leverage would get him even closer to λ(S+W), but the ability to borrow at near bond rates disappears once leverage starts exceeding 2:1.

To evaluate an allocation rule, we look at its historical performance along with the results from Monte Carlo simulation. Poterba, Rauh, Venti, and Wise (henceforth PRVW) (2005a,b) use simulation to compare several portfolio allocation strategies, including comparing a target-date fund to a constant equity strategy with the same average equity exposure. As theory would predict (due to improved diversification across time), they find that the target date fund does better; for risk aversion of 4, the gain in certainty equivalent is between 3.2% and 6%, with the larger percentage gains going to the individuals with the highest education. Because this gain is small, they conclude that maintaining a constant percentage in equities can mostly replicate the results of a target-date fund.

We interpret their results a little differently. A 3–6% increase in certainty equivalent is if not a free lunch at least a free hors d'oeurves and one worth taking. This is just the gain from improved time diversification and only part of what's possible since the initial investments are limited to 90% of savings compared to 200% at the start of our leveraged lifecycle. In Table VIII, we show that a leverage lifecycle with the same equity exposure as the traditional target date fund can provide an additional 3.5% improvement in certainty equivalent over the target date fund. Furthermore, the early leverage allows the investor to get closer to his optimal level of equity exposure. Adding this gain, the total improvement from leveraged lifecycle investing over the original constant equity share increases to 7.3%.

It is also worth noting that our simulation approaches are somewhat different. PRVW take thirty five draws with replacement from the 76 years of stock returns between 1926 and 2002. In our historical simulation, we use a series of overlapping 44-year histories of actual stock

performance from 1871 through 2009. (In our Monte Carlo simulation, we take independent draws from a log-normal distribution.) The result is that the worst-case scenarios in PRVW experience multiple cases of the negative 36% return in 1931. In their bottom 1% cases, the all-equity portfolios end up roughly 95% below the all-bond portfolios.

Shiller (2006) considers a conservative lifecycle strategy, such as what might be used for private social security accounts. The allocation to equities starts at 85 percent and ramps down to 15 percent at retirement age. This is much less exposure to equities than Vanguard and Fidelity lifecycle funds, which fall to 50 percent equities at retirement. Shiller finds that investing 100 percent of current savings in stock throughout working life produces higher expected payoffs and even higher minimum payoffs than his conservative lifecycle strategy.

Others have recognized the potential value of leverage. Viceira (2001) considers the investment allocation in a model where consumption and investment are both optimally chosen. His approach is based on finding a steady-state allocation. Thus a "young" worker is one who has a small (but constant) chance of retiring each period. The allocation for older workers is the steady-state solution where the retirement probability is increased. The steady-state solution avoids the issue of workers having to build up savings from zero (which is the focus of our results). In Viceira's framework, the margin rate equals the bond rate. In a calibrated example where wages and equities are uncorrelated, he finds that "young" workers with low risk aversion (constant relative risk aversion of 2) will want to invest 292 percent of their wealth in equities. This falls to 200 percent when the worker only has an expected 22 years left in the workforce or if risk aversion were to rise to almost 3.[6]

Closest to our work is Willen and Kubler (2006), who quantify the potential gain from investing on a leveraged basis. Using similar parameters, they find that the ability to buy stocks with 2:1 leverage only leads to a 1.2 percent gain in certainty-equivalent relative to the case where no leverage is possible.[7] While the magnitude of their findings appears to be quite different, the results are not as divergent as this statistic suggests. Willen and Kubler look at the present discounted value of lifetime consumption. For comparison, our expected gain of 11.5 percent vs. a traditional target-date strategy in certainty equivalent of retirement wealth is measured at age 67. The improvement for total lifetime utility would be smaller because the gain is only during the years of retirement and the gains are delayed until the future, which would be discounted.

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[6] When the correlation between wages and equities rises to 25 percent, the young worker's allocation to equities falls by about 13%.
[7] This is with a 2:1 maximum leverage on margin accounts, a constant relative risk aversion parameter of 3, and a 4% equity premium for stocks over the margin rate; see Willen and Kubler (2006, Table 8).

Even so, our gain is still substantially larger than the estimate of Willen and Kubler. This difference is due to different modeling assumptions. Willen and Kubler emphasize the value of smoothing lifetime consumption. The high cost of borrowing against future income for consumption (10 percent in their model) means that most people consume too little when young. As a result, their investors do not even begin to invest in stocks until their early 50s, and this reduced period of investing substantially shrinks any gains from leverage. In our calculations, we have people saving a constant 4 percent of their income each year. While we appreciate that young consumers want to smooth consumption, putting aside some savings may be desirable when they are tax-advantaged and matched by an employer. In our model, most of the gains from leverage arise from investments made prior to age 50.

II.   The Leverage Lifecycle Strategy

Our leveraged lifecycle strategy is an extension of the Samuelson (1969) and Merton (1969) result to take into account the fact that investors do not start with all of their wealth upfront. As in Merton-Samuelson, we assume that the investor's utility period function has constant relative risk aversion, $U(x) = x^{1-\gamma}/(1-\gamma)$, where $\gamma > 0$ so that the individual is risk averse.[8] With these preferences and all wealth provided upfront, the optimal portfolio choice is independent of wealth or investor age. In addition, the optimal allocation can be calculated without knowing the consumption rule, assuming only that consumption is chosen optimally (or independently of retirement savings).

Most investors do not have all of their wealth upfront and thus are liquidity constrained when young. For simplicity, we assume that future income is non-stochastic and that unleveraged equity investment is limited by liquid savings. This leads us to consider leverage and the relevant opportunity cost of buying equities. When investors use leverage, the relevant forgone interest is the margin rate (as the investor could have paid down the debt); when investors invest without leverage, then the relevant foregone interest is the bond rate. Initially, we assume that these two rates are the same, and then extend the investment rule to the case where the margin rate is higher than the bond rate.

As in Merton-Samuelson, we consider a two-asset world where stocks are the risky assets and bonds are the safe asset. The extension to include investing on margin is straightforward. We consider two interest rates, r_m the real margin rate, and the risk-free real rate, $r_f \leq r_m$. For simplicity, we assume that the distribution of real stock and bond returns are i.i.d. over time and

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[8] Note that for $\gamma = 1$, the utility is defined as $U(x) = \ln(x)$.

hence leave out a subscript for the time period. Following Merton (1969), the optimal or target equity allocation $\lambda(z, r)$ is determined by

$$\lambda(z, r) = (z - r) / [\text{Volatility} * \text{Risk Aversion}], \qquad (1)$$

where z is the expected future return of stock, r is the relevant interest rate (either r_f or r_m), Volatility is the expected variance of future stock returns, and Risk Aversion is a specific investor's constant relative risk aversion. When an investor is investing on an unleveraged basis, the relevant opportunity cost of investing in stock is the foregone return on risk-free bonds, r_f. But when an investor is investing on a leveraged basis, the relevant cost is margin rate, r_m. The higher the rate, the lower the equity premium and the lower the allocation to stocks. Thus the investor will have a lower target allocation when paying the margin rate compared to when the relevant interest rate is the risk-free rate.

The investor's liquid savings are represented by S, and the present value of future saving contributions is represented by W. The investor's target proportion in stock is $T_m = \lambda(r_m)*(S+W)$ during periods when the investor is investing on levered basis and $T_f = \lambda(r_f)*(S+W)$ when investing on unlevered basis.

The margin collateral rule requires that the investor put up m dollars of collateral for each dollar of equity. Thus the person with S of liquid assets is limited to buying S/m dollars of equities. We assume that S is initially zero. The investor starts out with no savings. Savings are built up from the 4 percent of income that is allocated to savings each period. Thus, initially, the investor will be constrained by the margin rule and unable to invest T_m in stock. The person will invest the maximum possible amount, S/m. We show (in Table I) that even when legal regulation does not bind, the marginal cost of borrowing increases with the degree of leverage to an extent that investors would not choose to borrow more than two hundred percent of their current portfolio value.

This targeting rule produces a 4-phase path. Initially, the investor would like to be at $\lambda(r_m)$, but is unable to reach this allocation due to limits on the maximum leverage ratio. Thus the investor employs maximum leverage until $\lambda(r_m)$ is achieved (phase 1). The investor then deleverages her position while maintaining the $\lambda(r_m)$ allocation (phase 2). Once fully deleveraged, the new target is $\lambda(r_f)$. The investor allocates 100 percent of her available wealth in equities until this target is reached (phase 3). Finally (phase 4), the investor maintains the $\lambda(r_f)$ allocation, rebalancing the portfolio based on changes in wealth.

In sum, what we will call the "two-target" investment strategy consists of four phases:

In phase 1: $\lambda < \lambda(r_m)$. All liquid wealth is invested at maximum leverage. In this phase,

10

the amount invested in stock is $2S < T_m$.

In phase 2: $\lambda = \lambda(r_m)$. The investor deleverages until $\lambda = \lambda(r_m)$ is achieved without leverage. In this phase, the amount invested in stock is $T_m \leq 2S$.

In phase 3: $\lambda(r_m) < \lambda < \lambda(r_f)$. The investor puts all his or her liquid wealth into equities. The amount invested is S.

In phase 4: $\lambda = \lambda(r_f)$. The investor maintains the optimal Merton-Samuelson allocation. The amount invested is T_f.

The optimal investment is the product of the percentage target and the present discounted value of wealth (S+W). Leverage is required when the desired investment exceeds S and the investor is liquidity constrained when this exceeds 2S.

Declines in S caused by declines in the stock market can cause an investor to revert to earlier phases of investment. Consider, for example, an investor with a leverage percentage target $\lambda(r_m)$ of 50 percent who has saved (S) $100,000. If she has a present value of future savings (W) of $280,000, she will be in phase 2 because she can meet her target T_m of $190,000 = $\lambda(r_m)*(S+W)$ with only partial leverage. If the stock price declines by 10 percent, the investor's S will fall to $81,000 and the investor will revert to phase 1 investing. Even though her target T_m is $180,500 =.5*(81+280), she will only be able to invest $162,000 because of the margins requirement (S/m).

This 4-phase strategy has the advantage that it is characterized by just two percentage targets, $\lambda(r_m)$ and $\lambda(r_f)$. Furthermore, a person can get started on the optimal path without knowing the initial target. A young investor who starts with little liquid assets will take several years to reach the first target, even when investing all liquid assets fully leveraged. In our simulations, we find that a person who saves 4 percent of her income remains fully leveraged until sometime between age 28 and 36 (95% confidence interval). Thus she can start down the optimal path even without knowing precisely W, the present value of her future saving contributions.

The level of the margin rate relative to the risk-free rate and the expected stock return has a large impact on the optimal investment strategy. If the margin rate equals the risk-free rate, i.e., if investors could borrow at the risk-free rate, then $\lambda(r_m) = \lambda(r_f)$ and the third phase vanishes. Investors maintain a constant Merton-Samuelson percentage of wealth in stocks as soon as $\lambda(r_m)$ is reached. This single-target, three-phase strategy is relevant because, as an empirical matter, current margin rates are close to the risk-free rates and thus the two targets are also close. Therefore, we find that even with the simpler single-percentage target, the three-phase strategy performs almost as well as the four-phase approach. For simplicity, our simulations focus on the

11

single percentage target.[9]

While our strategy is a translation of the Merton-Samuelson approach, we should emphasize that the Merton-Samuelson proof of optimality no longer holds. The reason is that the utility function is no longer multiplicative in wealth. Specifically, the margin constraint is not multiplicative in S+W. If the person's total wealth is doubled, but the liquid assets remain constant, then the person will not be able to double her investment in equities. Another way of seeing this is that if the stock return is very negative, the person may end up liquidity constrained in the next period. Thus the investment choice tomorrow is no longer independent of the decision made today. We show that the leveraged lifecycle strategy improves upon a wide range of traditional investment strategies in both historic and Monte Carlo simulations.

III. Data and Methodology

We begin by simulating the returns from alternative investment strategies using long-term historical market data covering the years 1871–2004 collected by Shiller (2006) and updated through 2009 using Global Financial Data. In order to include the returns to leveraged investment strategies, we add historical data on margin rates to the Shiller tables. We assume that the maximum leverage on stocks is 2:1, pursuant to Federal Reserve Regulation T.[10]

A. Margin Rates.

For the margin rates, we use the broker "call money" rates plus 30 basis points.[11] This assumption may seem controversial because many major brokers currently charge margin rates that are substantially higher than the current call money rate. For example, in May 2010, low-cost brokers such as Vanguard and Fidelity charged margin rates respectively of 7.25% and 8.5% on small-balance margin loans, a rate that far exceeds the then-prevailing 2% call money rate. Fortune (2000, Table 3) and Willen and Kubler (2006) also reported margin interest rates from

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[9] There is an important class of investors where the opportunity cost of borrowing is higher than r_m. Investors who have outstanding loans with after-tax interest rate r_l who choose to use savings to buy stock on a levered basis instead of paying down their outstanding debt, in effect must bear an average cost of (r_m + r_l)/2. Somewhat surprisingly, investors who would not invest marginal savings in an unlevered purchase of stock may find it worthwhile to invest in stock on a leveraged basis. This result suggests that young investors with student loans might want to invest on a levered basis (instead of paying off their loans), even if r_l is roughly equivalent to the expected return of stocks.

[10] The law independently limits the ability of individuals to invest savings on leveraged basis. Mutual funds offered inside and outside of defined contribution plans are limited in their ability to purchase stock on margin. Under the Investment Company Act, mutual funds registered as investment companies are prohibited to purchase "any security on margin, except such short-term credits as are necessary for the clearance of transactions." 15 U.S.C. § 80a-12(a)(1).

[11] According to Fortune (2000), the broker call money rate is commonly used as the base lending rate; see Global Financial Data monthly series for the call money rate series.

traditional brokers on small margin loan balances which were more and sometimes much more than 300 basis points above the call money rate. But far more competitive margin rates are available. Also in May 2010, Interactive Brokers charged rates of 1.71% on loans below $100,000, 1.21% on the amount borrowed between $100,000 and $1,000,000, and 0.71% on the next tranche of borrowing up to $3 million. ProShares offers an ETF which provides twice the daily performance of the S&P 500 index. The fees associated with this fund are 0.95% plus their cost of leverage, which is close to the interbank rate. Barclays offers an ETN which provides a return of twice the total return of the S&P 500 and one with three times the total return of the S&P 500. In May 2010, the interest cost associated with the Barclays products was 91 basis points, 75 basis points above the then 16 basis point 91-day U.S. Treasury Bill rate.[12]

As Davis, Kubler, and Willen (2006) point out, the typical cost of consumer debt is well above the risk-free rate and even above the expected return on equity.[13] There is, however, a large difference between the interest charged to finance consumption (e.g. credit card debt) and that charged to finance leveraged equity investment. A margin loan should be one of the most secured loans. It starts with 50% collateral and (unlike a mortgage) the lender has the ability to liquidate the portfolio in the event that the remaining equity falls below the maintenance requirement of 25%. Thus the loss rate on margin loans should be near zero. The security of these loans explains why competitive lenders allow investors to gain leverage at rates only slightly above Treasury and even below the call money rate.

Stock index derivatives provide another route that allows investors to take on the equivalent of leveraged positions at implicit interest rates that are below the call money rate. Based on forward and spot market data from 2000 to 2008, the implicit margin rate for the S&P 500 futures has averaged only 4.56% which was just 129 basis points above the average 1-month LIBOR rate for the same time period and 40 basis points below the "call money" rates for the same time periods.

The purchase of deep-in-the-money LEAP call options offer an additional low-cost route to obtaining leverage. For example, on Sept. 3, 2010, when the S&P 500 index was trading at

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[12] Note that the Barclay's products (BXUB and BXUC) are not rebalanced so that the leverage will change over time. Furthermore, the ETN is an unsecured debt obligation of Barclays Bank PLC and so the buyer is exposed to their credit risk.

[13] As Davis, Kubler, and Willen (DKW) (2006) also point out, leverage is not desirable once the borrowing cost exceeds the expected stock return. DKW argue that the high interest rates on consumer debt explain the low rate of equity ownership. As they observe, if investors could borrow at rates close to the risk-free rate, they should be investing much more in equities, especially when young. Our paper is prescriptive: investors should be investing more in equities since when they borrow to buy equity and use their existing equity as collateral, the interest rate is close to the risk-free rate. Indeed, an unsung benefit of buying equities is that they can be used as collateral and thereby help obtain low-interest margin loans.

$1,104.51, a 15-month LEAP call option on the S&P index with a strike price of $550 was priced at $544.20. This contract provides almost 2:1 leverage. It allows the investor, in effect, to borrow $560.31 (the savings compared to buying the actual S&P index). At the end of the contract, the investor has to pay $550 to exercise the contract. Compared to buying an S&P mutual fund, the index holder will have also sacrificed $20.10 in foregone dividends (for holding the index rather than the stocks). Thus the true cost of buying the index is $575.72. The total cost of paying $575.72 fifteen months after borrowing $560.31 produces an implied interest of 2.15% which is just 15 basis points over the contemporaneous call money rate (and 178 basis points over a Treasury note with equal maturity). Table I derives the implied interest of thousands of LEAP call options for fourteen years of option data.

Table I : Implied Interest Rates for 1 Yr. S&P Calls 1996--2009					
Range of "Leverage" Ratios	Contracts Observed	Average Implied Interest Rate	Mean Spread Over Call-Money Rate	Mean Spread over 1-Year Treasury Note	Marginal Interest Rate at Mean Spread
1.5-2.5	2,202	4.64%	0.26%	1.89%	
2.5-3.5	3,347	5.99%	1.10%	2.79%	8.01%
3.5-4.5	3,071	7.54%	2.47%	4.19%	11.40%
4.5-5.5	2,559	8.65%	3.47%	5.20%	12.53%
5.5-6.5	2,236	9.87%	4.67%	6.42%	15.38%
Option close prices from CRSP used for LEAPS on the S&P 500, between 11 and 12 months to maturity. Implied leverage ratios and implied interest rates were calculated for each contract and then grouped by leverage ratio.					

We find that the implied interest for deep-in-the-money call options that produce effective leverage around 2:1 averaged less than two percent above the contemporaneous 1-year Treasury note. Moreover, the average implicit interest rate on these calls was 26 basis points above the contemporaneous call money rate. LEAPs also have the advantage that there is no potential for a margin call.

Given the low cost of leverage and the absence of margin calls, it might appear that young investors should consider taking on even greater amounts of leverage. However, Table I also shows that the implied interest increases with the degree of leverage. As can be seen in the far-right column, the implied *marginal* interest rate associated with additional leverage rapidly approaches (and then exceeds) the return on equity. The marginal interest rate associated with the incremental borrowing required to move from 2:1 to 3:1 leverage is 8.01%. This is substantially higher than the 4.64% implied marginal interest rate for 2:1 leverage. The marginal cost of increasing leverage rises sufficiently fast that it is not cost effective to invest at leverage of more

than 2:1 via option contracts.[14]

From Table I we can see that the derivative markets have made it inexpensive to invest 200 percent of current saving accumulations in the stock market. Whether or not investors had ready access to the broker call money rate in the past, our assumption of low-cost money going forward is particularly reasonable given the advent of competitive on-line brokers (such as InteractiveBrokers), leveraged ETNs (such as Barclays's BXUB), ETFs (such as ProShares Ultra S&P500), and options to implicitly borrow through derivative markets.

B. Investor Cohorts.

We construct 96 separate draws of a worker's 44-year experience in the markets, spanning the period from 1871 through 2009. Each of the draws represents a cohort of workers who are assumed to begin working at age 23 and retire at 67. The first cohort relates to workers born in 1848 who start work in 1871 and retire in 1915. (Later, in the Monte Carlo simulations, we allow the worker to randomly experience 44 years of returns based on a log-normal distribution of stock returns.) Following PRVW (2005b) and Shiller (2006), we assume that workers save a fixed percentage of their income. In our simulations, we use Shiller's 4% number. Thus the saving accumulations depend only on the history of 4% contributions and prior-year returns.

Although the percent is constant, the actual contributions depend on the wage profile. We assume a hump-shaped vector of annual earnings taken from the Social Security Administration's "scaled medium earner." Wages rise to a maximum of $58,782 at age 51 (generating a saving contribution in that year of $2,351) and then fall off in succeeding years.[15] However, we multiply this wage vector by a factor of 2.35 so that the income in the worker's final year is $100,000. Therefore at age 51, the worker has a peak income of $138,264 (generating a savings contribution in that year of $5,531). For a new worker at age 23, the future saving stream has a present value of $111,884 (when discounted at a real risk-free rate of 2.56%). Given this flow of saving contributions, the simulation assesses how different investment strategies fare in producing retirement wealth.

C. Deriving the Percentage Targets.

We derive percentage targets for specific levels of constant relative risk aversion (CRRA) γ by substituting into equation (1) historical parameters for the equity premium, and market volatility. Monthly data on stock and bond returns from 1871 to 2009 are provided by Global

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[14] The implied interest rate is increasing because the option not to exercise has value separate from the implicit loan component.

[15] See Shiller (2006), Clingman and Nichols (2004), www.ssa.gov/OACT/NOTES/ran3/an2004-3.html, Table 6 (scaled factors); www.ssa.gov/OACT/TR/TR04/lr6F7-2.html (average wage).

Financial Data and Shiller.[16] Table II shows summary statistics for the nominal financial returns. Stocks over this period had an average nominal return of 8.83% percent. On a monthly basis, the maximum positive return was 51.4% in July, 1932, whereas the maximum negative return was −26.2% in October, 1929.[17]

We calculate the historical equity premium to be 4.15% by taking the geometric mean of stock returns (including dividends) less the 10-year Treasury bond rate. To derive the leveraged equity premium of 3.82%, we subtract the call-money rate (plus 30 basis points) from the geometric mean of stock returns. Finally, we estimate the average volatility over the historical sample to be 14.3%.

Table II: Summary Statistics of Nominal Financial Returns				
	Geometric Mean	St. Dev.	Max	Min
Stock	8.83%	14.29%	51.35%	-26.19%
Call Money Rate	4.71%	1.62%	12.27%	0.02%
Government Bond	4.68%	5.18%	12.86%	-7.92%
Inflation	2.08%	3.72%	7.04%	-6.58%
Sources: Global Financial Data (S&P 500 Composite Price Index (w/GFD extension), USA Broker's Call Rate, and USA 10-year Bond Constant Maturity Yield). S&P dividends and US CPI from Shiller (ie_data.xls).				

The optimal allocation target is based on equation (1). For a CRRA of 4, the optimal leveraged and unleveraged percentage targets are 51.5% and 47.5% respectively. While we expect the unleveraged percentage target to be higher than the leveraged percentage, these two percentage targets are very close. This is because (as seen in Table II) the average call money rate (plus 30 basis points) in our data is only slightly higher than the average bond rate, 5.01% versus 4.68%. Thus the loss from picking a target between the two levels (50%) will be of second order as it will be close to optimal for each phase. This leads us to evaluate a single-target or three-phase strategy, which invests a constant 50% of wealth, subject to the 200% maximum leverage constraint.

Our core simulations compare our temporally diversified strategy to a constant percent strategy and a target-date strategy. For convenience we label all three strategies by the percentage of initial and ending savings allocated to stock:

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[16] See http://www.econ.yale.edu/~shiller/data.htm.
[17] Our simulations are based on real returns and real interest rates. However, when we consider the potential impact of margin calls, we employ nominal returns as margin calls depend on the nominal change in equity prices.

1. 200/λ Strategy. This strategy sets the equity percentage target at a constant percentage (λ) of discounted savings. The 200 is the cap on leverage and the target λ is chosen based on risk aversion along with anticipated returns and volatility. Initially, the worker invests her entire liquid savings on a fully leveraged basis of 2:1 and remains fully leveraged until doing so would create stock investments exceeding the target percentage. From then on the worker invests on a partially leveraged or unleveraged basis. If the unleveraged portfolio value exceeds the target percentage, then stocks are sold and the excess amount is invested in government bonds. The percent of the portfolio invested in stock is contingent on the prior years' realized returns as this impacts the current portfolio value.

2. λ/λ Strategy. Under this benchmark strategy, the worker invests a constant percentage (λ) of her liquid savings in stock. In some cases, we look for the constant percent strategy that has the same mean return as 200/λ, so as to isolate the gain that comes from risk reduction. We also look at constant percentage strategies where the investor is shortsighted. The investor has the same target as in 200/λ but only applies that λ to his liquid savings and does not take into account the present value of his future contributions.

3. 90/50 Strategy. Under this target-date strategy the worker invests 90% of portfolio value in stock at age 23 and the percentage invested in stock falls linearly to 50% by age 67. This approximation of a typical target-date strategy may be thought of as a step in the direction of 200/50. When young, the investor allocates more than his desired 50% allocation to stock in order to take account of future savings. Over time, as human capital converts to financial capital, the investor converges to his desired goal of 50%. While 50% will not be the optimal allocation for all investors, it is close to the final allocation used by target-date funds.

We limit our comparison set to these two investment strategies because they highlight the potential gain from employing leverage. The λ/λ strategy fails to make any adjustment for future savings and the 90/50 strategy makes only a partial adjustment. PRVW (2005b) and Shiller (2006) have simulated the risk and return of more than a dozen traditional investment strategies, including 100% TIPS, 100% bonds, (110 − Age)% in stocks and a variety of alternative lifecycle strategies.

IV.   Results of the Cohort Simulation

A.	Mean-Preserving Reduction in Risk

We begin our analysis of the 96 historic investment cohorts by simulating the investment outcomes of the 200/50 strategy. This 50% target is the optimal allocation for an investor with $\gamma=4$ and who expects the historical level of equity premium and market volatility. For this initially leveraged strategy, the investor in the median cohort is maximally leveraged until age 31 (range 28 to 36) and continues to have some degree of leverage until age 38 (range 32.5 to 44). We look for the λ/λ strategy that produces the same mean accumulation as the 200/50 strategy. As shown in Table III, a 74/74 strategy produces the same average accumulation ($739,000) as the 200/50 strategy.[18] Table III provides strong evidence that a leverage lifecycle strategy can reduce retirement risk: the 200/50 strategy has a 21% lower standard deviation in the retirement accumulation.

The equalized means insures that the 74/74 constant allocation leveraged lifecycle strategy has the same effective exposure to the stock market as 200/50 strategy. The reduced variance shown in Table III is not driven by exposing investors to more of the historically high equity premium. Rather the mean-preserving reduction in volatility is caused by spreading the same lifetime exposure to the equity premium more evenly across time.

Table III: Comparison of Alternative Investment Strategies Based on Optimal Investment Targets on Monthly Data			
	200/50 Strategy	**74/74 Strategy**	Improvement of 200/50 over 74/74
Max % inv.	200%	74%	
Min % inv.	50.0%	74%	
Mean	$738,684	$738,684	0.0%
St. Dev.	$192,720	$242,833	-20.6%
Min	$299,208	$306,060	-2.2%
10th pct	$478,734	$440,792	8.6%
25th pct	$621,543	$552,661	12.5%
Median	$747,344	$686,328	8.9%
75th pct	$871,931	$914,909	-4.7%
90th pct	$981,637	$1,128,575	-13.0%
Max	$1,166,663	$1,229,855	-5.1%
Note: 74/74 strategy obtained by equilibrating mean return for target strategy with that of constant percentage strategy			

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[18] The precise allocation is 73.8%.

Another way to see the risk reduction is to look at the final accumulation for each of the 96 cohorts. As shown in Figure 2, the 200/50 and 74/74 strategies have the same mean accumulation. However, when the accumulation was above average (as was the case from 1954 through 1972), the 200/50 was typically beaten by 74/74 and, conversely, when accumulations were below average (as was the case from 1972 through 1996) the 200/50 strategy typically outperformed the 74/74. If this were the case for every observation, then 200/50 would have second-order stochastic dominance over 74/74. In the historical data, we find that in 70 out of 96 cohorts (73%), the 200/50 strategy was closer to the mean than the 74/74. Our leverage approach reduces volatility by having higher lows and lower highs.

Figure 2: Final Accumulation by Cohort Retirement Year
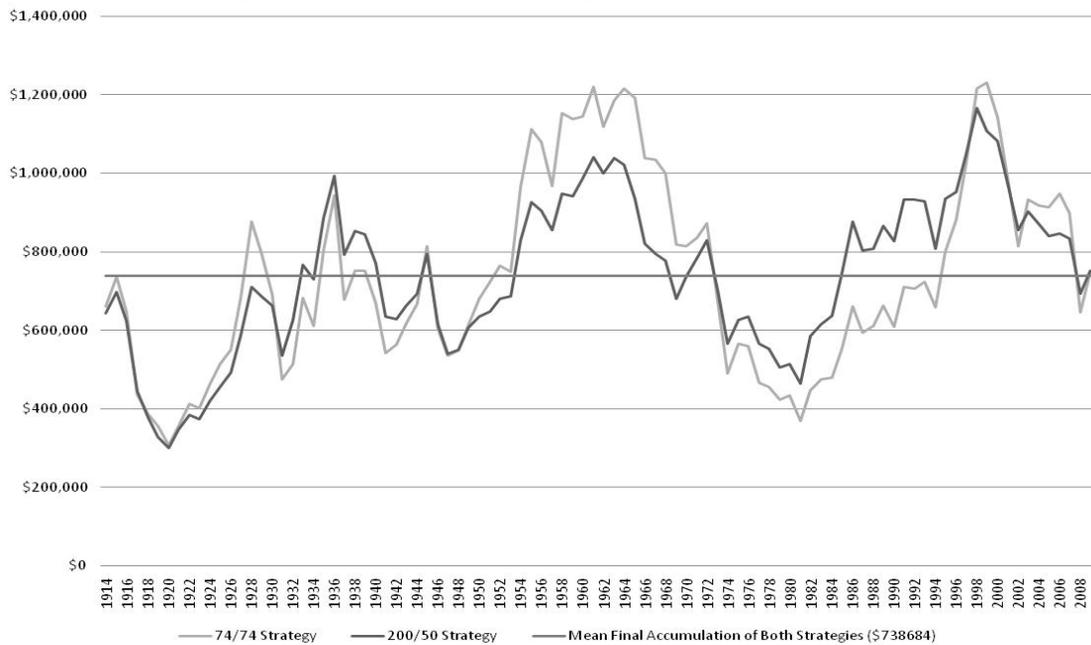
Table IV shows the mean-preserving reduction in risk for different levels of risk aversion. In each row, we first calculate the CRRA-optimal Merton-Samuelson share and then find the λ/λ strategy that has an equal mean. Note that the reduction in standard deviation is even larger (27%) when the investor is less risk averse. This is because the investor seeks more exposure to stocks and thus the gain from time diversification is larger. Even if the investor is sufficiently risk averse that only a 40% allocation is optimal, there is still a 12.5% lower standard deviation.

19

		Table IV: Risk Reduction for Leveraged Lifecycle Strategy Relative to Mean-Preserving Constant Strategy for Different Levels of Risk Aversion					
	200/λ Strategy	Mean Retirement Wealth	Standard Deviation	Constant % Stock Strategy	Mean Retirement Wealth	Standard Deviation	St. Dev. Change from Constant % to 200/λ
CRRA = 2	200/100	$1,496,382	$689,216	128/128	$1,496,382	$946,152	-27.2%
CRRA = 3	200/65	$931,746	$275,432	91/91	$931,746	$377,644	-27.1%
CRRA = 4	200/50	$738,684	$192,720	74/74	$738,684	$242,833	-20.6%
CRRA = 5	200/40	$625,244	$162,116	61/61	$625,244	$185,340	-12.5%

Note: In each row, the 200/λ strategy is derived from the CRRA-contingent Merton-Samuelson share (Equation 1), and the associated λ/λ strategy is the constant strategy that produces the same mean accumulation.

The 200/λ strategies are better diversified because they better spread the exposure to stock across time. We can assess the extent of this spreading by measuring the temporal concentration of different strategies exposure to stock market risk. The reciprocal of the Herfindahl-Hirshman Index (HHI) is a heuristic measure of the effective number of diversification years. Just as the inverse of the HHI in antitrust indicates the effective number of equally-sized investors in an industry (Ayres (1989)), the inverse of the HHI here indicates the amount of diversification that could be achieved by investing equal dollar amount in separate years. HHI estimates indicate that the average worker using the 74/74 strategy effectively takes advantage of about 23 of her 44 investments years. In contrast, the 200/50 strategy takes advantage of 31 years.

In Table V, we measure the gain from diversification through the improvement in expected utility, as measured by the gain in the certainty equivalent. An investor with CRRA of 4 who follows the 200/50 rule has a certainty equivalent of $609,000, which is 3.4% higher than one who follows the 74/74 rule. To translate this into something more tangible, our representative worker would have to increase his savings by 3.4% in order to achieve the same certainty equivalent of retirement wealth under the 74/74 rule: Better time diversification is like getting an extra 3.4% increase in savings for free.

We provide a similar comparison for investors with relative risk aversions of 2, 3, and 5. In each case, we calculate the optimal Merton-Samuelson share and then find the λ/λ strategy that has an equal mean. Given that we are comparing equal means, any gains here are coming from risk reduction rather than the equity premium. The gains in certainty equivalent are between 3.4% for $\gamma = 4$ up to 9.9% for $\gamma = 2$. A higher risk aversion leads to lower exposure to stocks, which would reduce the gain but also create greater value from the risk reduction that does occur.

Table V: Utility Gain from Time Diversification for Different Levels of Risk Aversion					
	200/λ Strategy	Cert. Eq. Wealth	Constant % Stock Strategy	Cert. Eq. Wealth	Improv. Of 200/λ over Constant %
CRRA = 2	200/100	$1,193,081	128/128	$1,085,252	9.9%
CRRA = 3	200/65	$783,261	91/91	$748,006	4.7%
CRRA = 4	200/50	$609,431	74/74	$589,107	3.4%
CRRA = 5	200/40	$501,269	61/61	$484,051	3.6%
Note: Table adjusts relative risk aversion and presets improvement in real return of target strategy over constant percentage strategy.					

PRVW (2005b) showed that a traditional target date strategy provides a small improvement over investing a constant fraction of current savings in stock market. The results above show that a 200/λ strategy that starts with leverage can similarly do a better job of diversifying over time than investing a mean-preserving constant fraction of savings in equities. Buying stock on margin when young is part of a strategy that significantly reduces the expected volatility in retirement accumulations.

B.   Expected Utility Gains from Approaching Optimal Investment Target

While the 200/50 strategy lowers volatility relative to the 74/74 strategy, it channels the all incremental benefits of better temporal diversification to risk reduction. Utility-maximizing investors will use leverage to come closer to their ideal Merton-Samuelson allocation throughout their lives. Accordingly, Table VI reports results of investors following their utility-maximizing 200/λ leveraged-lifecycle and compares this to individuals who never allocate more than λ to stocks. These myopic individuals (who play the λ/λ strategy) make the mistake of ignoring future savings and only look at current savings. We look at the optimal allocation rule for investors with a CRRA of 2, 3, 4, and 5. In each case, we pick a leveraged strategy that is between the ideal unleveraged and leveraged target. We also compare our approach to the 90/50 strategy which approximates the approach taken by traditional target-date funds.

As can be seen in the penultimate column of Table VI, the 200/λ strategy consistently outperforms the λ/λ strategy. This is to be expected. The 200/λ strategy brings the investor closer to his utility-maximizing allocation. The surprise is the extent of the improvement: the certainty equivalent of retirement wealth is between 35% and 39% higher.

Table VI: Comparison of Certainty Equivalent Wealth for CRRA-Optimal Leveraged Lifecycle Relative to Myopic Merton-Samuelson and Target-Date Strategies for Different Levels of Risk Aversion							
	200/λ Strategy	Certainty Equivalent Wealth	Constant % Stock Strategy	Certainty Equivalent Wealth	Traditional Lifecycle Strategy	Certainty Equivalent Wealth	Improvement of 200/λ over Constant %	Improvement of 200/λ over 90/50
CRRA = 2	200/100	$1,193,081	100/100	$873,018	90/50	$598,733	36.7%	99.3%
CRRA = 3	200/65	$783,261	65/65	$562,765	90/50	$572,631	39.2%	36.8%
CRRA = 4	200/50	$609,431	50/50	$444,226	90/50	$546,634	37.2%	11.5%
CRRA = 5	200/40	$501,269	40/40	$371,378	90/50	$521,853	35.0%	-3.9%
Note: In each row, the 200/λ strategy is derived from the CRRA-contingent Merton-Samuelson share (Equation 1), and the associated λ/λ strategy assumes investors myopically invest the CRRA-contingent Merton-Samuelson share through their working lives.							

For the case of a CRRA of 4, the 200/λ strategy has the same final allocation of 50% as the more traditional 90/50 target-date strategy. Starting at 200% rather than 90% leads to an 11.5% improvement in certainty equivalent. In fact, a paired cohort-by-cohort comparison shows that the 200/50 strategy produces higher final retirement accumulations than either the 90/50 strategy or the 50/50 stock strategy for each of the 96 cohorts.

In Table VI, the certainty equivalent improvements over the 90/50 strategy are even larger for risk aversion parameters of 2 and 3. However, the target-date approach did best the 200/λ strategy for γ=5. To explain this reversal, we note that the optimality of the 200/40 strategy is based on independence of stock returns over time. In our historical sample, crashes were followed by recoveries and this negative correlation increased the utility of holding stocks. It turned out that 90/50 had higher average holdings than 200/40 because the investor had much greater wealth when investing 50% rather than 40% and that more than offset the lower investments in the early years. In our historical sample, it would have been (slightly) better to allocate 50% of savings to stock rather than 40%, even if the investor had γ=5.

C.   Margin Calls

Even though it is possible to implement leverage strategies with deep-in-the-money LEAPS, this section analyzes the potential impact of margin calls on the foregoing results. In our monthly data the stock market has never declined sufficiently to wipe out the preexisting investments of any cohort adopting a 200/50 strategy. The worst case arose in October 1929, where the 200/50 strategy would have produced negative returns of 53% for young investors who were fully leveraged (2:1). Leveraged strategies expose workers to a much larger probability of incurring a substantial negative monthly return sometime during their working life. Under a 200/50 strategy, roughly one-eighth of the cohorts (11 out of 96) would have lost more than 40% in at least one month. However, the minimums reported in Table III and Figure 2 show that the risk of a substantial monthly loss does not translate to a risk of substantial loss to accumulated retirement savings.

Even without wipeouts, the prevalence of substantial market declines has a potentially devastating impact on strategies that incorporate leveraged stock purchases. A natural reality check is look at the results for worker cohorts who lived through the depression years. The real stock returns on the S&P 500 in 1929, 1930 and 1931 were −8.8%, −16.0%, and −36.5%.[19] How can it be that investors following leveraged strategies did as well as reported in Tables III–VI? The answer is that workers who retired just after the crash were not severely hurt because the 200/50 strategy had already eliminated their leverage. For example, workers retiring in 1932 following the 200/50 strategy would have had 50% of their portfolio invested in the market when the market lost more than a third of its value. Because of the success of their investments in previous years, they would still have accumulated a retirement wealth of $625,335, larger than they would have earned following a 74/74 strategy ($514,621), a 90/50 strategy ($552,645) or a 50/50 strategy ($477,816).

Individuals adopting the 200/50 strategy who began working just before the depression would have done even better. Those who entered the labor force in 1931 would have immediately experienced a 73% loss in their first investment year.[20] But this is a large percentage of a small amount, and the 200/λ strategy responds by keeping these workers fully leveraged until they hit the target. By the time of their retirement in 1974, these workers following the 200/50 strategy would have accumulated $565,369, which is $75,870 over the result with 74/74 and $91,846 better than the returns to the 90/50 allocation rule. Figure 2 showed that the 200/50 strategy produced the lowest accumulations for workers retiring in 1920 ($299,208). For these workers, enduring the double-digit market declines in 1893, 1903, 1907, 1917, and 1920 was more limiting than the more severe, but compact, declines of the depression.

These examples (and the analysis underlying Tables III–VI) do not consider interim margin calls that would occur if there was a substantial intramonth decline in the market. But the possibility of intramonth margin calls does not alter our results. Investors can implement the leveraged strategy via the purchase of deep-in-the-money LEAPs. With LEAPs, there are no margin calls. While the LEAP market was not available during most of our simulation period, it is available going forward. Furthermore, margin calls don't necessarily hurt performance. They merely force the investor to sell some of his or her stock. Being forced to sell will reduce returns

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[19] The stock market "crash" in October 1929 had been preceded by sizable increases so that the year-end nominal loss was only 8.8%.

[20] Although the market fell by 44.2%, the maximally-leveraged investors didn't lose 88.4%. The reason is that as the market fell, these investors rebalanced their portfolio. To maintain a maximum 200 percent leverage requires selling shares in response to a market decline and this provides some downside protection. In contrast, traditional target date funds rebalance via buying stock as the price falls, and this adds to their poor performance in a crash.

if the market rebounds later in the month. But being forced to sell can also increase returns if the market further deteriorates.

With 2:1 leverage and monthly rebalancing, margin calls would have been rare to nonexistent. Our estimation assumes that all leveraged positions were rebalanced at the beginning of each month. The major stock exchanges (per NYSE Rule 431 and NASD Rule 2520) require a maintenance margin of 25% on long positions. Some brokers require higher maintenance margin of 30% or 35% (Fortune (2000)). With a maintenance margin requirement of 25%, margin calls would force investors to start selling their positions if the market lost a third of its value.[21]

To determine the prevalence of margin calls, we took daily S&P returns from 1928-2009 (from Global Financial Data) and calculated the number of months that would have experienced margin calls given the cumulative interim daily returns between our monthly rebalancing of the portfolio. Under the stock exchange 25% margin maintenance requirement, there were no margin calls for a 2:1 leveraged strategy. Even under the most conservative 35% broker requirement, there were only six months with interim margin calls (Oct. 1929, Sept. 1931, Mar. 1938, May 1940, Oct. 1987, Oct. 2008).

V.   Robustness

This section considers alternative assumptions to test the robustness of the advantages to leveraged investing. We first consider simulations based on foreign stock returns. We then redo our analysis using Monte Carlo simulations where investors can experience a random collection of 44 years of returns based on a log-normal distribution of stock returns. We consider the effect of lower stock returns as well as higher volatility.[22] We find that our results are robust to a variety of assumptions. As long as the expected return on stock exceeds the net cost of maintaining a margin position, it will be optimal to employ leverage early in life. As the premium narrows, the scale and value of leverage declines, as does the optimal stock allocation of equation (1).

A.   Foreign Returns

We investigate how the 200/50 strategy would have fared in other parts of the world relative to the 74/74 strategy, the 50/50 strategy, and the 90/50 strategy. Here we have not reoptimized to take into account that the optimal Merton-Samuelson share might be different than 200/50 for someone investing in foreign equities. This is a harder test in that the investor doesn't have the

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[21] If an investor buys $200 of stock using $100 of capital and the market drops by 33.3%, then the portfolio would be worth $133 and the equity behind it would be $33.3 or 25%.

[22] Reduced stock returns is much like higher margin costs. The main difference is that higher margin interest only impacts the expected return of leveraged strategies, while lower stock returns also impacts the expected accumulation of unleveraged investment strategies.

benefit of hindsight to optimize his allocation to equities. We compare the results of 200/50 to 74/74 as the two strategies are equally aggressive based on US data. We compare the results of 200/50 to 50/50 as the two strategies have the same desired equity allocation, but the 200/50 allows the investor to get closer to the goal at an earlier age. And we compare the results of 200/50 to 90/50, as the latter approximates the traditional target-date fund.

Table VII reports the results of our historical cohort exercise using monthly returns on the Nikkei (1956–2009) and the FTSE (1937–2009). For the Nikkei Index, the leveraged strategy outperformed all three alternatives; the certainty equivalent gain (for γ=4) ranged from 26% over the 90/50 strategy to 47% over 74/74. For the FTSE All-Shares Index, we find that across the thirty cohorts, the 200/50 strategy handily beat all three alternative; the certainty equivalent gain (for γ=4) ranged from 18% over the 74/74 strategy to 55% over 50/50.

Table VII: Comparison of 200/50 Strategy to 3 Traditional Strategies Invested in English and Japanese Equities							
11 Nikkei 225 Cohorts (1956 - 2009) (in 100s Yen)							
	200%/50% Lifecycle strategy	90%/50% Strategy (Birthday Rule)	Constant 74%/74% Strategy	Constant 50%/50% Strategy	200/50 Improvement Over 90/50	200/50 Improvement Over 74/74	200/50 Improvement Over 50/50
Max % inv.	200%	90%	74%	50%			
Min % inv.	50.0%	50%	74%	50%			
Mean	¥66,154,903	¥49,890,018	¥45,584,137	¥40,113,979	32.60%	45.13%	64.92%
St. Dev.	¥33,794,530	¥21,208,742	¥21,305,413	¥12,278,965	59.34%	58.62%	175.22%
Min	¥35,008,794	¥27,056,403	¥22,216,969	¥25,130,798	29.39%	57.58%	39.31%
10th pct	¥38,165,700	¥30,803,412	¥25,385,949	¥27,726,040	23.90%	50.34%	37.65%
25th pct	¥43,281,734	¥34,317,473	¥29,711,951	¥30,718,952	26.12%	45.67%	40.90%
Median	¥51,807,633	¥39,831,052	¥38,051,375	¥36,174,717	30.07%	36.15%	43.22%
75th pct	¥74,911,987	¥56,957,418	¥52,248,423	¥45,143,533	31.52%	43.38%	65.94%
90th pct	¥111,268,034	¥82,258,110	¥79,340,443	¥59,548,896	35.27%	40.24%	86.85%
Max	¥154,153,476	¥95,644,253	¥88,764,744	¥62,800,897	61.17%	73.67%	145.46%
Certainty Equivalent (CRRA = 4)	¥49,538,693	¥39,205,097	¥33,768,991	¥34,731,055	26.36%	46.70%	42.64%
30 FTSE All-Shares Cohorts (1937 - 2009)							
	200%/50% Lifecycle strategy	90%/50% Strategy (Birthday Rule)	Constant 74%/74% Strategy	Constant 50%/50% Strategy	200/50 Improvement Over 90/50	200/50 Improvement Over 74/74	200/50 Improvement Over 50/50
Max % inv.	200%	90%	74%	50%			
Min % inv.	50.0%	50%	74%	50%			
Mean	£591,223	£483,905	£543,146	£404,052	22.18%	8.85%	46.32%
St. Dev.	£128,481	£121,513	£149,940	£103,227	5.73%	-14.31%	24.46%
Min	£331,588	£251,046	£260,363	£205,032	32.08%	27.36%	61.72%
10th pct	£405,028	£303,145	£330,573	£249,951	33.61%	22.52%	62.04%
25th pct	£533,824	£407,786	£466,132	£341,575	30.91%	14.52%	56.28%
Median	£592,006	£499,923	£548,509	£414,580	18.42%	7.93%	42.80%
75th pct	£677,071	£582,586	£652,502	£488,605	16.22%	3.77%	38.57%
90th pct	£750,829	£617,616	£732,984	£518,894	21.57%	2.43%	44.70%
Max	£831,811	£688,049	£847,159	£579,721	20.89%	-1.81%	43.48%
Certainty Equivalent (CRRA = 4)	£522,410	£408,447	£444,194	£337,928	27.90%	17.61%	54.59%

C. Monte Carlo Simulations

An advantage of the cohort simulations is that they tell what actual investors might have achieved in the past if they had pursued our proposed investment strategies. But the 96 cohorts analyzed in Tables III–VI are clearly not independent of each other. The returns of any two adjacent cohorts massively overlap—so that our effective number of independent observations is closer to 3 [(2009−1871)/44]. An alternative approach is to use the historic returns as the basis for a Monte Carlo simulation in which workers randomly draw returns. We use a lognormal distribution that matches the historical 6.61% real return on stock (geometric mean) and 14.29% standard deviation. We then estimate the distribution of returns using 10,000 trials, each time

picking 44 annual returns at random from a lognormal distribution. This approach produces returns that are independent and identically distributed—even though it is not clear that stock returns are in fact independently distributed across time (see Poterba and Summers (1988)).

	90/50 Strategy	65/65 Strategy	200/43 Strategy	Improvement of 200/43 over 90/50	Improvement of 200/43 over 65/65
Table VIII: Monte Carlo Comparison of Target-Date (90/50) Strategy with Mean-Preserving Constant and Leveraged Lifecycle Strategies					
Mean	$700,857	$700,857	$700,857	0.00%	0.00%
St. Dev.	$305,570	$309,800	$270,414	-11.51%	-12.71%
Certainty Equivalent (CRRA = 4)	$520,299	$510,528	$538,790	3.55%	5.54%
Note: Based on 10,000 Monte Carlo draws from a log-normal distribution with mean 6.61% and st.dev of 14.29%					

The first row of Table VIII is the Monte Carlo analog of our historical simulation results in Table V. To eliminate the effect of the equity premium, we compare three strategies that have equal mean returns. We start with 90/50, the traditional target-date allocation. A constant strategy of 65/65 and a lifecycle strategy of 200/43 both lead to the same expected accumulation of $700,857. However, the 200/43 achieves this result with an 11.5% lower standard deviation than the target-date fund and a 12.7% lower standard deviation than the 65/65 constant strategy. The end result is a 3.5% utility gain over the target-date fund and a 5.5% utility gain (for γ=4) over the constant allocation rule. This gain is entirely due to improved risk diversification as all the strategies have the same mean. Given the expected risk and return, the optimal Merton-Samuelson share is 50% and the resulting 200/50 rule leads to a certainty equivalent of $547,717 (see Table IX) and a 7.6% utility gain over 65/65.

Many commentators have noted that the historical equity premium may not be repeated. Shiller (2006) has suggested several reasons why the success of U.S. stocks in the 20th century will not be replicated in the 21st century. He shows that the returns on stock in other countries have been 2.2% lower than the stock returns in the U.S.. Jorion and Goetzmann (1999) find an even larger shortfall. Moreover, a 2005 *Wall St. Journal* survey of prominent economists at Wall Street brokerages reports an expected real stock return of just 4.6%, which is 2.2% lower than the return found in the historic (1871–2007) data.

Leveraged strategies will be less attractive if the expected return on stocks is lower. This led us to consider the success of our proposed strategies in the event that the future equity premium is reduced. In doing this test, we could readjust the Merton-Samuelson share to reflect the diminished return. Doing so would nearly guarantee that the expected utility would be higher, as

the investment strategy is closer to the utility-maximizing allocation. Instead, we chose a harder test. We held the investment strategy constant and considered how well it did across different possible returns. Investors today don't know what the equity premium will be in the future. Thus our experiment provides the answer to how well investors will fare if they make allocations based on historical returns but face a future with lower returns.

The first row of Table IX provides the baseline results. Here the 50% allocation target is optimal given the equity premium, volatility, and risk aversion. The next four rows of Table IX provide the results when the equity premium is reduced by 50 basis points, 1%, 1.5%, and 2%. We continue to compare the 200/50 strategy (which is no longer optimal) against the constant strategy that had the same mean and the target-date strategy. All certainty equivalents are for an investor with γ=4.

Table IX: Monte Carlo Comparison of Certainty Equivalent Wealth Across Varying Mean Real Returns					
Real Stock Return	Certainty Equivalent of 90/50	Certainty Equivalent of 74/74	Certainty Equivalent of 200/50	Improvement of 200/50 over 90/50	Improvement of 200/50 over 74/74
6.61%	$520,299	$513,816	$547,717	5.3%	6.60%
6.11%	$479,362	$467,563	$496,697	3.6%	6.23%
5.61%	$442,208	$426,025	$450,506	1.9%	5.75%
5.11%	$408,459	$388,691	$408,723	0.1%	5.15%
4.61%	$377,777	$355,104	$370,918	-1.8%	4.45%
Note: Based on 10,000 Monte Carlo draws from a log-normal distribution with st.dev of 14.29% and varying means. Constant strategy set such that mean final accumulation is equal to that of the 200/50 leveraged-lifecycle strategy for 6.61% mean stock return.					

As expected, the lower returns reduce all of the certainty equivalents. While the advantages of the leverage strategy relative to the traditional strategies are reduced, they are largely robust. Even if the equity premium is 2% lower, the 200/50 strategy beats the 74/74 by 4.5%. Since the lifetime exposure to equities is similar, most of this gain is coming from improved diversification over time. The 200/50 also beats the 90/50 target-date strategy for the cases with the expected equity return reduced 0.5%, 1%, and 1.5%. However, by the time the stock return is 2% lower— which implies the equity premium is reduced by half—the 90/50 target-date strategy comes out on top. With this reduced equity premium, the Merton-Samuelson allocation would be 25%, so that a 200/50 rule is far too aggressive and that explains how the 90/50 rule wins. Even with this reduced target, the optimal strategy would start at 200%, since 200% of a small number will still be well below 25% of lifetime savings. Thus the 200/50 rule has better allocation early on, even if it is overly exposed to equities later in life. Indeed, the reoptimized 200/25 rule would have produced a certainty equivalent that is almost 13% larger for γ=4 than that produced by the 90/50 investment rule. Thus, as theory predicts, the optimal 200/λ rule beats 90/50 as is largely robust to choosing a target based on the historical equity, even if that number ends up being too high.

We perform a similar exercise in Table X where we now hold the equity premium constant and increase the stock volatility. In the simulations below, the volatility increases from the historical 14.29% up to 29.29%. Due to the better diversification across time, the 200/50 continues to beat the 74/74 strategy. Now, however, the 90/50 target-date approach comes out on top. The reason is that the 74/74 strategy is equally aggressive and so the improved diversification helps even with greater volatility. But the 90/50 strategy is less aggressive and this is closer to the optimal allocation as volatility increases. In fact, even a small increase in the standard deviation from 14.29% to 19.29% reduces the Merton-Samuelson share from 50% down to 27%, roughly the same as a 2% reduction in the equity premium. Because the volatility term is related to variance, small increases in the standard deviation have large impacts on the ideal share. By the time volatility is up to 20.29%, the ideal share is only 12%.

Table X: Monte Carlo Comparison of Certainty Equivalent Wealth Across Varying Volatility of Real Returns					
Stock Standard Deviation	Certainty Equivalent of 90/50	Certainty Equivalent of 74/74	Certainty Equivalent of 200/50	Improvement of 200/50 over 90/50	Improvement of 200/50 over 74/74
14.29%	$520,299	$513,816	$547,717	5.3%	6.60%
19.29%	$424,462	$386,385	$418,198	-1.5%	8.23%
24.29%	$338,624	$281,938	$300,647	-11.2%	6.64%
29.29%	$268,228	$204,238	$211,690	-21.1%	3.65%
Note: Based on 10,000 Monte Carlo draws from a log-normal distribution with mean of 6.61 and varying st. devs. Constant strategy set such that mean final accumulation is equal to that of the 200/50 leveraged-lifecycle strategy for 14.29% standard deviation of stock return.					

This suggests that the advantage of a leveraged-lifecycle strategy over a target-date fund is sensitive to using the right volatility in setting the share goal. If the share is optimally adjusted to the change in volatility, it continues to outperform the target-date fund. For example, with σ=19.29%, the optimized 200/27 rule has a certainty equivalent of $453,406, which is 6.8% above the 90/50 rule. With σ=24.29%, the optimized 200/17 rule outperforms the target-date fund by 22.4% and with σ=29.29%, the optimized 200/12 rule outperforms the target-date fund by 47.2%

Fortunately, while investors can only forecast the equity premium, they can see the contemporaneous market volatility through the VIX. Thus investors don't have to forecast volatility, but they do have to adjust their strategy accordingly. This is less important when starting out, but matters once the investor has enough savings as to be in danger of overshooting the optimal Merton-Samuelson share. We also note that our comparisons have only considered futures that were worse than historical experience. If the future turns out better, than the 200/50 rule would have performed even better by comparison.

VI.   Conclusion

This paper puts into practice Samuelson and Merton's original insight that people with constant relative risk aversion should invest a constant percentage of their lifetime wealth each period in stock. For young workers, wealth exceeds liquid assets. Thus implementation of the Merton-Samuelson rule requires leveraged purchases when young.

Our recommended investment strategy is simple to follow. An investor who targets a Merton-Samuelson share follows three phases of investment. The person begins by investing 200% of current savings in stock until a target level of investment is achieved. In the second phase, the person maintains the target level of equity investment while deleveraging the portfolio and then maintains that target level as an unleveraged position in the third and final phase.

The potential gains from such leveraged investments are striking. In historic simulations, leveraged strategies produced mean-preserving reductions in risk above 20%. These reductions in risk improved the certainty equivalent of retirement wealth by 3.4% to 9.9%. Investors who applied the 200/λ (the Merton-Samuelson share) rule achieved retirement accumulations with certainty equivalents 35% higher than those produced by the myopic λ/λ rule.

Of course, the historical equity premium may not continue going forward and the optimal Merton-Samuelson allocation depends on the equity premium and volatility. Even so, both theory and Monte Carlo simulations show that as long as the expected stock return exceeds the margin rate, utility-maximizing investors will want to invest with leverage when young. Monte Carlo simulation shows that even using an inflated Merton-Samuelson share, one based on an expected premium that turns out to be 1.5% too high, still beats the traditional target-date approach. And no matter what the equity premium, there is still reduced risk through better diversification across time: for an investor with CRRA of 4, a traditional target date fund has 3.5% lower certainty equivalent than a leveraged strategy with equal mean (200/43).

Our estimation does not take into account the impact of non-portfolio wealth, such as housing and human capital. Workers with non-portfolio wealth that is correlated with the stock market already have some exposure to stock market risk. Thus the target level of equity holdings should include the human capital exposure to the market. The relevance of this issue will vary across professions and suggests that all investment strategies—including target-date funds—should be different by profession, reflecting the different indirect exposure to equities via human capital.

Our estimation also does not take account of potential general equilibrium effects that might occur if a substantial segment of investors began adopting leveraged lifecycle strategies. However, for several reasons we would still expect a solution with leverage and increased investor utility. First, to the extent that investors use leverage to produce mean-preserving

reductions in risk, there would not be a net increase in the demand for stock. Overlapping generations of investors would continue to buy the same amount of stock at any one time: younger investors would buy more, and older investors would buy less. To the extent that investors use leverage to move toward their utility-maximizing Merton-Samuelson share, we would see a net increase in the overall demand for stock. Myopic λ/λ investors who move toward $200/\lambda$ strategies buy more stock in their early years. Similarly, target date investors with λ near 50 increase their holdings in going from 90/50 to $200/\lambda$. But the general equilibrium effects of this increased demand for stock are likely to be self-limiting. If the increased demand lowers the expected return on stock (or raises the cost of borrowing on margin), the λ of all Merton-Samuelson investors will fall, thus reducing the overall demand for stock. In this lower λ equilibrium, young investors will still invest on a leveraged basis but will ramp down at the end of their working life to a lower ultimate allocation. As a historical matter, we note that economies have successfully adapted to leveraged investments in both housing and education. The recent housing crisis suggests that housing leverage of 50:1 or greater may lead to bubbles and excess volatility. But post World War II, the U.S. moved from 3:1 to 10:1 housing leverage without undermining market stability.[23]

Finally, our results have implications for legal reform. The natural places to engage in disciplined leveraged purchases are IRA and 401(k) accounts. Yet, with the exception of the index options, leveraged and derivative investments inside these accounts are prohibited. An employer who offered workers the option of investing in a fund implementing a $200/\lambda$ strategy might risk losing its statutory safe harbor. Approximately two-thirds of 401(k) plans allow employees to borrow against their plan balances to fund present consumption; in stark contrast, employees are not allowed to borrow to fund leveraged investments for their future.

We recognize that legal constraints are not the primary reason that people fail to buy enough stock when they are young. Despite compelling theory and empiricism, many people have a strong psychological aversion to mortgaging their retirement savings. While families are encouraged to buy a house on margin, they are discouraged (and with regard to their 401(k) accounts prohibited) from buying equities on margin. We are taught to think of leverage investments as having the goal of short-term speculation instead of long-term diversification. As a result, most people have too little diversification across time and too little exposure to the market when young. Based on theory and historical data, the cost of these mistakes is substantial.

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[23] Vigdor (2006), however, finds increased housing prices as a result of higher leverage, and Lamont and Stein (1999) estimate increased sensitivity of housing prices in markets with levered ownership.

VII.   References

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