Exotic Beta Revisited

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NOT FOR QUOTATION[2]

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ABSTRACT

While traditional portfolios are defined by their asset class allocations—especially to equities—we propose portfolios comprised of simple and intuitive risk premia that are uncorrelated with equities, which we call "exotic betas." Dynamically adjusting exposures across these exotic betas to reflect observable variation in risk premia over time is akin to value investing and can improve both risk and return in traditional portfolios. These exotic betas perform well over a variety of market conditions, are low cost, and can be delivered in transparent and relatively liquid form.

Introduction

The financial crisis of the past few years has many lessons, one of which is that the investment landscape is continually evolving and finding appropriate sources of return is a constant challenge. Finding alpha on a consistent basis is especially hard in today's competitive environment. Hedge funds, proprietary traders, and even high frequency algorithms scour the markets continuously hoping to find inefficiencies before others can do the same.

Market beta is a consistent source of return in the long run, but it has also disappointed over the last decade, particularly relative to its risk which is as great as ever. Further, many investors expect the equity risk premium to be lower in the future than it has been historically.

How should investors respond to these realities? One way institutional investors have appropriately responded is by focusing even more carefully than ever before on the drivers of risk and return in their portfolios, not simply in terms of stocks versus bonds or alpha versus beta, but in terms of developing a better and more complete understanding of the underlying risk factors and their liquidity, leverage, return per unit of risk, expected correlation in stress scenarios, capacity, sustainability, and so on.

Five years ago, well before the crisis, we began discussing a concept we called *exotic beta*.[3] We continue to believe this concept is useful to investors, but in this article we reexamine the idea and update it in the context of the events of the past few years.

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[2] This research paper is intended to educate readers about the concept of exotic beta. It is not an offering document for any investment vehicle, a recommendation to buy or sell any particular assets or fund, or a proxy for any portfolios that are currently managed, or may in the future be managed, by Kepos Capital. In addition, this paper (i) does not constitute investment advice and (ii) does not take into account any individual personal circumstances or other factors that may be important in making investment decisions; no advisory or fiduciary relationship is created by the distribution hereof.

I. DEFINING EXOTIC BETA

Initially, we defined exotic beta as an exposure to a risk factor that is both uncorrelated with global equity markets and has a positive expected return. We described it then as existing on a continuum—a "spectrum"—between alpha and beta. Like alpha, it is a source of uncorrelated return, but unlike alpha, exotic beta is not an opportunity created by short-term inefficiencies. Exotic betas are risk premia created by long-term exposures to compensated risk factors. They are transparent, relatively well known, and intuitive. What differentiates exotic betas from the market beta is simply the source of the premium.

There is little doubt that there is one principal risk in the global financial markets, the market beta or even more simply put, equity risk. We saw it in action during the financial crisis. And as we have seen over the past few years, when that dominant risk factor has a large negative shock, no matter where the shock originates many other risk factors become highly correlated. Particularly when markets experience a financial crisis and liquidity dries up, many seemingly unrelated risks, which in normal times are uncorrelated with the market, experience large negative shocks, if only because investors need to reduce risk and to find liquidity. But as we have also seen in this crisis, that is not true for all priced risks; for example, those who insured against hurricanes and earthquakes were not materially affected by the financial crisis.

Exotic beta is not a new idea to portfolio theory. Modern finance has long recognized that there may be many different priced risks in the investment landscape. The idea behind including exotic beta in an investor's portfolio emerges from the traditional concept that combining market beta with alpha from active management and relatively passive exposures to uncorrelated risk factors can result in a more efficient portfolio. This is a key concept in what has come to be called the "endowment" model of portfolio construction.

In the context of traditional portfolio theory the return from an exotic beta is alpha and, therefore, exotic betas should be considered as a potential source of alpha in a diversified portfolio. But there are many dimensions that distinguish exotic beta from we have termed "active alpha." These include time horizon, capacity, liquidity, and perhaps most importantly a judgment about whether there is a real or perceived risk that justifies a premium. For all these reasons, the approaches that institutional investors take to source returns from exotic beta should be very different from those required to source active alpha. And because exotic betas are more transparent, more liquid, and have more capacity than sources of active alpha, the fees should be lower as well.

In an excellent recent review of modern financial theory and its implications for the Norwegian sovereign wealth fund, Ang, Goetzmann and Schaefer (2009) [4], focus considerable attention on their view that, "opportunities for the fund lie in the potential to seek return through both fixed and time-

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[3] See http://www2.goldmansachs.com/gsam/pdfs/USI/education/aa_beyond_alpha.pdf. We credit the term "exotic beta" to George Main at DGAM.

[4] See http://www.regjeringen.no/upload/FIN/Statens%20pensjonsfond/rapporter/AGS%20Report.pdf.

varying exposure to factor risk."[5] We agree wholeheartedly, and we have been referring to exactly those types of opportunities as exotic beta.

Why "exotic" beta?

It is a perhaps unfortunate market convention to call exposure to any asset class or risk factor, not just the market portfolio, a "beta." Investors often refer, for example, to their "beta to commodities" or their "beta to credit risk." Following in this spirit, and to distinguish these factor risks from the market beta, we refer to such risk exposures that have a positive risk premium, but are uncorrelated with the global market risk as "exotic" betas.

But we have learned several important lessons during the recent financial crisis about managing exposures to exotic betas. First of all, though often slow moving, the exposures to exotic betas should not be expected to be constant over time. This lesson was underscored by the size of the market moves: even those portfolios designed to weather all storms were forced to safe harbor. But more generally the volatilities, the correlations, and in particular the risk premiums associated with different risk factors do change over time and so should the exposures to them. A second lesson was that investors need to be very cautious about crowding, excessive leverage, and exposures to liquidity needs. And finally, we learned that many risk exposures have predictable reactions to market stress and while not all risk can be eliminated that careful attention to managing the time varying risk exposures can significantly improve the expected performance of a portfolio of exotic beta exposures.

Investors should be aware that the premia from exotic beta may not persist in equilibrium and consequently the excess returns to any particular factor may decline over time. If enough investors try to take advantage of a particular risk factor, as for example we have seen in recent years with respect to commodities, then the prices of the financial instruments reflecting that factor may be driven up to the point where the premium will disappear or, as was the case with commodities, the prices may be driven up even beyond that point. But because the financial markets are constantly evolving and experiencing shocks, we expect new opportunities to arise over time (just as we expect existing ones to decay). Thus, investors must continually monitor the risk premia associated with each risk factor in order to decide when they are attractive. In this sense investing in exotic betas may be viewed as a form of value investing, only as applied to risk factors rather than individual assets.

The source of exotic beta risk premia

In order to understand why—and to recognize when—the premium embedded in an exotic beta has gone away, we need to understand why it exists in the first place. One possible explanation is behavioral: most investors do not behave in the perfectly rational manner of our theoretical models. Rational investors—according to these models such as the Sharpe (1964) Capital Asset Pricing Model—should care about the risk of their portfolio of assets or their total wealth, and thus the volatility of individual assets doesn't matter, only "covariance" matters. In equilibrium, a risk premium should not be paid for volatility, but rather a risk premium should only be paid for a particular type of covariance.

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[5] See the section "Factor Risk Opportunities," pages 119 through 141.

A well-known result from the Capital Asset Pricing Model is that the particular covariance that deserves a risk premium is the covariance with the market portfolio. The intuition behind this result is that cash flows which tend to disappear in bad states of nature are less desirable. This is why a portfolio that is long equities deserves a risk premium, even in equilibrium. The cash flows from equities are available primarily in good times. In bad times their values are severely diminished. This covariance is the source of the significant equity risk premium. And while a short equity index position is just as volatile as a long position, such a position does not earn a premium. In fact, because it pays off in bad states of nature, a short position acts as a form of insurance and therefore its expected return is significantly negative. With commodities, there is no obvious covariance with good or bad times. Because this covariance is generally close to zero, and often negative, even though commodities are quite volatile, no significant risk premium deserves to be paid.

"Covariance matters" is the theory, but in financial markets it doesn't always work that way. In fact there is a common misperception that because individual asset 'volatility' measures the probability of losing money, volatility per se requires compensation. If most investors exposed to a risk factor demand compensation for volatility, then a premium will exist. We believe this undeserved risk premium for volatility exposure is a primary source of the return in many exotic betas.

Another source of return in exotic betas is demand for liquidity. Some have argued, for example, that much of the historical return from exposure to commodities, and particularly commodity futures, comes from providing liquidity to hedging by commodity producers. We agree. But, if liquidity supply is the source of return, then it is important to monitor relative supply and demand. In commodity markets, the slope of the futures price curve—whether a given commodity is *backwardated* (downward sloping), or in *contango* (upward sloping)—is a clear indicator of the relative supply and demand for hedging in that market. Futures markets that are backwardated tend to reflect excess demand for long hedges (i.e., the short futures positions of commodities producers push the futures curve down relative to the spot market). In recent years, as investors have moved in to supply liquidity, the expected returns, as indicated by the typical contango shape of the futures curves, have in most cases been much less positive or, indeed, negative.

A third reason for the existence of returns from exotic beta is that global market returns have "fat" tails on the downside, and investors are getting paid a premium to accept the risk of a market crash, which as we have seen, can be concentrated in certain exotic betas. Exposure to default risk is an example of where this was clearly the case in the recent financial crisis, because the left-tailed distribution of returns is undesirable to many.

Still another explanation for the premium may be the aversion that many investors have to assets that have gone down in value, and conversely the attraction that many investors have to assets that have gone up. It takes a strong conviction on either side to become a contrarian—to sell what has gone up significantly or over a long period of time—and buy assets that have disappointed. In our opinion, this behavioral bias accounts for the persistence in most asset classes of a value risk premium over time.

Some have argued that leverage creates the value in exotic betas, rather than the expected returns inconsistent with equilibrium. This argument is misleading. Leverage is a useful tool, but does not create value in and of itself. Leverage is helpful if it relaxes a constraint that otherwise prevents a portfolio from achieving a desired set of risk allocations, but leverage doesn't create the opportunity. Certainly Modern Portfolio Theory never assumed these constraints, so there is nothing "post-modern" about these concepts. These opportunities exist only if the positive expected excess returns are inconsistent with equilibrium. In these cases, leverage can be used to increase the overall risk and return for a given level of capital, but the opportunity remains the same, whether leveraged or not.[6] Moreover, excessive leverage can lead to liquidity needs at the worst possible time and thus leverage exposure needs to be managed extremely carefully.

There are many types of metrics that can be used as measures of valuation for exotic betas. Some metrics, such as deviations from long-term trend returns, can be applied across many types of risk factors. Other metrics, such as yield spreads in fixed income, backwardation versus contango in commodities, or deviations from purchasing power parity in currencies, are specific to a particular asset class. In most cases, a particular exotic beta will have several metrics that can be used to measure whether the expected risk premium is higher or lower than average.

Where do exotic betas belong?

How should investors think about exotic betas in the context of asset allocation and strategic benchmarks? We support the use of strategic benchmarks as targets for long term asset allocation; such benchmarks embed an agreed-upon risk tolerance as well as a portfolio of agreed-upon asset classes with their associated long-term risks and expected returns. They can also embed the particular structure of an investment objective, for example the interest rate exposures of long-lived pension obligations. Benchmarks play a useful, and indeed critical, role as a stable, agreed-upon long-term reference against which to measure shorter-term risks embedded in portfolios.

Ang, Goetzman and Shaefer (2009) argue that exotic betas (which they call "factor exposures") should also be embedded in benchmarks. They support a value bias, for example, in the equity benchmarks. This may be desirable for some very stable long-term risk premia, but our feeling is that the risk premia for exotic betas vary over time and investors need to understand, monitor and manage their exposures on a shorter time frame than is practical in the context of a strategic benchmark. We certainly agree that it may make sense to engineer a value tilt into equity portfolios. The value tilt is a great example of a well-known, intuitive, long-term risk exposure that has paid off extremely well over time but with substantial variation in return. In this sense it is a perfect example of an exotic beta. It is, nonetheless, a risk that needs to be sized appropriately alongside all other risks. And, as we have seen in recent years—and as had been seen in the past—value tends to underperform during periods of market distress. **For most institutional investors we think it makes sense to manage the portfolio of exotic beta risks more dynamically outside the context of the strategic benchmark.**

[6] In the CAPM, the set of optimal portfolios with borrowing and lending at the risk free rate is known as the Capital Market Line.

II. ANALYZING EXOTIC BETA

Value investing in equities is a good example of an exotic beta. However, most investors think of value investing as a style, or a skill sourced from individual managers, not a risk factor. Indeed, we can and should separate the exposure to a risk factor which is created by a passive tilt toward a value equities benchmark, from the skill needed to create the alpha that would outperform such a benchmark. Moreover, the risk of an equity portfolio with a value tilt is primarily exposure to equities, not to value.

Using futures contracts, however, we can hedge the overall equity exposure from such a portfolio, thus creating a pure exotic beta risk and, finally, we can easily create a very liquid pure value tilt across stocks using individual equities, equity index futures contracts, or ETFs. By separating the basic risk exposure of a value tilt from the active management skill provided by a value manager we can achieve several benefits: First, we can size the risk exposure based on risk-return trade-offs rather than manager capacity. Second, we can separate the overall equity exposure decision from the value tilt decision. Finally, we can easily, and at very low cost, adjust the size of the value exposure without adjusting individual manager allocations.

There are several examples of exotic betas in the fixed income markets. For example, a simple portfolio with passive exposures to global bonds hedged to equities creates a very nice example of an exotic beta. Over the past three decades, as global bond market yields have declined significantly, the returns from exposure to higher duration bonds has performed extremely well on a risk-return basis, and has been negatively correlated with global equity markets. Note, however, that from a valuation perspective fixed income markets have become much less attractive in recent years and this fact highlights the need to monitor forward-looking measures of expected risk premia and to adjust exposures accordingly. Moreover, if inflation becomes a significant risk in the future, then the correlations of bonds and equities should be expected to increase since an inflation shock hurts both bonds and equities.

There is no natural premium from long exposure in currencies,[7] but an exotic beta can be created in currencies by purchasing cheap currencies on a purchasing power basis and selling those that are expensive. Another example of an exotic beta exposure in currencies is a portfolio that buys currencies with high short rates and sells currencies with low short rates, a simple generalization of the idea of a currency carry trade. However, while a portfolio with currency carry trades embedded in it has performed very well historically, there is a well-known danger in such a portfolio, which is that it does very poorly when investors become risk averse. This is a particularly good example of an exotic beta for which it is important to monitor market conditions closely and to adjust exposures accordingly.

As mentioned above, there are also opportunities to create exotic betas in commodities, based on the shapes of forward curves, and there are additional examples in credit, in volatility, in insurance markets, in real estate and in other assets.

[7] Even without a risk premium, Black (1989) shows that it is optimal to hold some currency risk in your portfolio.

For the purposes of this paper, we define 10 liquid exotic betas. In all cases, the market risk of the defined risk premium is hedged against global equity market risk so that the exotic betas are designed to be uncorrelated with global equity markets.

- ***Equity Value:*** long exposure to countries with the highest fundamental value relative to price hedged by short exposure to the opposite.
- ***Bond Yields:*** GDP-weighted combination of sovereign bonds.
- ***Bond Yields Value:*** long the sovereign bonds in countries with the highest yields and short those in countries with the lowest yields.
- ***Term Structure:*** long the sovereign term structure in countries with the steepest curves hedged by short the opposite.
- ***Commodities:*** volume-weighted combination of commodity futures subject to constraints of 50% in the energy sector, tilted towards commodities with the greatest backwardation in prices.
- ***Real Assets:*** market cap weighted combination of REIT securities.
- ***Currency Value:*** long exposure to high interest rate and weak purchasing power parity currencies hedged by short exposure to low interest rate and strong PPP currencies.
- ***Volatility:*** short equity index volatility in most market conditions except long equity index volatility when there are signs of significant financial stresses ahead.
- ***Credit:*** equal-weight combination of the 6 major credit sectors of US investment grade, US high yield, European investment grade, European high yield, mortgages and asset-backed securities.
- ***Catastrophe Bonds:*** market cap weighted combination of bonds linked to catastrophic reinsurance risk.

This is not an exhaustive list, but we believe these factors capture a broad spectrum of uncorrelated risk premia available in global markets and which can readily be implemented in an investment portfolio with liquid securities and derivatives. Further, these premia are related in different ways to the rationales discussed in Section I.

Table 1 displays some relevant summary statistics on each of the exotic betas and Figure 1 charts their cumulative returns over time. Because of data availability, we have different start dates for each series, but all end at December 2010. We standardize the realized volatility of each exotic beta to 10% so that the average annual excess return (over 1-month T-bills) and maximum drawdown may be directly compared across the 10 risk premia. The distributions of the exotic betas are quite attractive when compared to a passive exposure in market risk, even when considered individually.

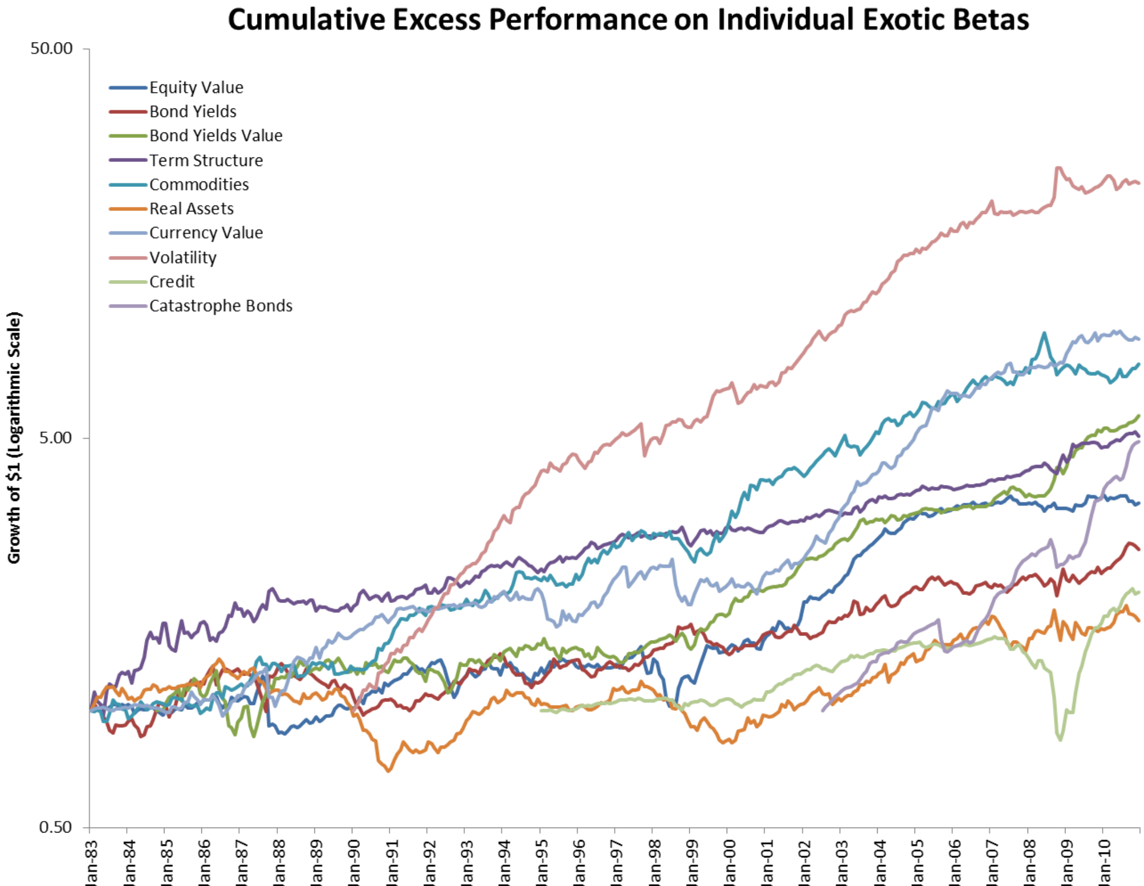
Table 1

Exotic beta strategy	Start Date	Volatility	Average Annual Excess Return	Sharpe Ratio	Maximum Drawdown	Correlation with ACWI
Equity Value	Jan-83	10.0%	4.9%	0.49	-30%	0.15
Bond Yields	Feb-83	10.0%	3.9%	0.39	-26%	(0.05)
Bond Yields Value	Nov-84	10.0%	7.2%	0.72	-32%	(0.00)
Term Structure	Jan-83	10.0%	6.3%	0.63	-15%	(0.05)
Commodities	Dec-82	10.0%	7.9%	0.79	-25%	(0.01)
Real Assets	Dec-82	10.0%	2.4%	0.24	-48%	0.09
Currency Value	Dec-82	10.0%	8.4%	0.84	-24%	0.12
Volatility	Feb-90	10.0%	15.5%	1.55	-17%	(0.15)
Credit	Feb-95	10.0%	4.9%	0.49	-46%	0.45
Catastrophe Bonds	Aug-02	10.0%	19.6%	1.96	-17%	0.25
MSCI ACWI	Dec-82	14.8%	5.7%	0.39	-52%	1.00
MSCI at 10% Vol	Dec-82	10.0%	3.9%	0.39	-35%	1.00

Once again, the Sharpe ratios reported above are after hedging out expected beta. The magnitude of the Sharpe ratios in many cases is quite high, and we would say higher than one should expect in the future. After standardizing the MSCI ACWI return to the same 10% volatility for comparison to the exotic betas, the maximum equivalent drawdown in the MSCI ACWI is 35%, which is not that different from the exotic betas themselves.[8]

[8] Note there is more correlation in Credit and Cat Bonds with ACWI than we should expect. This is because our simulated hedge never looks forward in the data—only backwards—and thus does not expect the degree of correlation between this exotic beta and global equities during the credit crunch. Because the negative returns were so large in that period, they significantly influence the full-sample correlations.

Figure 1



While we have removed most of the correlation to global equities in our exotic betas, this does not guarantee that the correlations among them are low. In fact, the cross-correlations vary by risk premium, but the average cross correlation across the 10 exotic betas is -0.01. The complete table of correlations is shown in Table 2.

Table 2

	Equity Value	Bond Yields	Bond Yields Value	Term Structure	Commodities	Real Assets	Currency Value	Volatility	Credit	Cat Bonds
Equity Value	1.00	(0.22)	0.17	(0.13)	0.09	0.12	0.28	0.18	0.14	(0.21)
Bond Yields	(0.22)	1.00	(0.07)	0.17	(0.06)	0.06	(0.21)	(0.28)	(0.01)	0.38
Bond Yields Value	0.17	(0.07)	1.00	(0.38)	(0.05)	(0.00)	0.06	0.09	(0.04)	(0.00)
Term Structure	(0.13)	0.17	(0.38)	1.00	(0.01)	(0.03)	(0.11)	(0.15)	(0.15)	(0.14)
Commodities	0.09	(0.06)	(0.05)	(0.01)	1.00	(0.02)	0.10	0.04	0.11	(0.04)
Real Assets	0.12	0.06	(0.00)	(0.03)	(0.02)	1.00	0.04	(0.06)	0.11	(0.07)
Currency Value	0.28	(0.21)	0.06	(0.11)	0.10	0.04	1.00	0.21	0.17	(0.20)
Volatility	0.18	(0.28)	0.09	(0.15)	0.04	(0.06)	0.21	1.00	(0.24)	(0.19)
Credit	0.14	(0.01)	(0.04)	(0.15)	0.11	0.11	0.17	(0.24)	1.00	0.32
Catastrophe Bonds	(0.21)	0.38	(0.00)	(0.14)	(0.04)	(0.07)	(0.20)	(0.19)	0.32	1.00

III. CONTRASTING EXOTIC BETA TO RISK PARITY AND HEDGE FUND REPLICATION

There is a related portfolio concept in circulation today known as "risk parity." In its basic form, risk parity simply normalizes various asset classes to a constant risk, then equal-weights them into a portfolio. For example, consider the following 10 asset classes:

Table 3

ASSET CLASS	Start Date	Native Annualized Volatility	Sharpe Ratio	Maximum Drawdown (at 10% Vol)	Correlation with ACWI
Inflation-Linked Bonds	Jul 1981	6.5%	0.04	-57.2%	0.13
US Bonds	Feb 1976	5.7%	0.35	-57.1%	0.10
Developed Int'l Bonds	Feb 1990	2.9%	0.89	-29.4%	0.09
Emerging Market Bonds	Feb 1994	14.0%	0.50	-24.5%	0.57
High Yield Bonds	Sep 1983	8.9%	0.51	-40.9%	0.56
Real Estate	Dec 1982	26.5%	0.30	-46.6%	0.57
Commodities	Jan 1970	20.0%	0.29	-41.9%	0.17
US Equities	Feb 1975	15.3%	0.42	-39.3%	0.84
Developed Int'l Equities	Feb 1973	13.7%	0.43	-38.0%	0.93
Emerging Market Equities	Feb 1988	19.4%	0.56	-29.3%	0.65
Risk Parity Portfolio	Jan 1985	N/A	0.74	-37.3%	0.81

The correlations between asset classes are displayed in Table 4. As casual empiricism would suggest, the average cross correlation is fairly high at 0.30. Notable exceptions to this average are the very small correlations between equities and bonds, and the approximately zero correlations between commodities and everything else.

The Risk Parity Portfolio has better historical risk and return characteristics than a prototypical traditional institutional allocation, primarily due to the relatively higher allocations to smaller asset classes with higher historical returns as well as the exceptionally high Sharpe ratio for global bonds as yields have continuously dropped over the past 25 years. This type of portfolio is also sometimes referred to as an "alternative beta" portfolio, and this form of investing is gaining traction among institutional investors. Therefore, we will consider the Risk Parity Portfolio as a benchmark for comparison to our notion of exotic beta. Note, however, that parity is not built around risk premia (it is, rather, built around asset classes), nor are the asset classes orthogonalized to market risk.

Table 4

	Inflation-Linked Bonds	US Bonds	Developed Int'l Bonds	Emerging Market Bonds	High Yield Bonds	Real Estate	Commodities	US Equities	Developed Int'l Equities	Emerging Market Equities
Inflation-Linked Bonds	1.00	0.37	0.62	0.37	0.24	0.21	0.09	0.16	0.12	0.10
US Bonds	0.37	1.00	0.68	0.35	0.31	0.22	(0.02)	0.25	0.05	(0.02)
Developed Int'l Bonds	0.62	0.68	1.00	0.27	0.12	0.16	(0.15)	0.08	0.09	0.04
Emerging Market Bonds	0.37	0.35	0.27	1.00	0.52	0.39	0.17	0.53	0.54	0.62
High Yield Bonds	0.24	0.31	0.12	0.52	1.00	0.58	0.11	0.57	0.50	0.46
Real Estate	0.21	0.22	0.16	0.39	0.58	1.00	0.01	0.61	0.43	0.36
Commodities	0.09	(0.02)	(0.15)	0.17	0.11	0.01	1.00	0.12	0.12	0.21
US Equities	0.16	0.25	0.08	0.53	0.57	0.61	0.12	1.00	0.67	0.56
Developed Int'l Equities	0.12	0.05	0.09	0.54	0.50	0.43	0.12	0.67	1.00	0.64
Emerging Market Equities	0.10	(0.02)	0.04	0.62	0.46	0.36	0.21	0.56	0.64	1.00

There are many ways to access risk premia. Certainly many existing portfolios would already include exposures to some degree to the exotic beta risk factors we have described. The exposures come naturally with many forms of active management, from investments in hedge funds, as well as from investments in asset classes such as commodities. More generally they also arise from tilts of the strategic benchmark away from market capitalization weights by, for example, overweighting emerging markets.

There is no right or wrong way to access exotic betas. What we would stress is simply that investors should be aware of their exposures, should understand them, and should manage them purposefully. In addition, investors should not pay excessive fees for exotic beta or for market beta. Some exotic betas are available in liquid markets and are relatively easy to create by investors directly through asset purchases and sales, while others can be accessed and managed easily through the use of derivatives. In other cases the exposures are more difficult to obtain, either because they exist in less liquid markets or because creating exposures requires active management of hedges, and in these cases the exposures are probably best sourced through professional asset managers.

As noted in the introduction, exotic betas exist along a spectrum running from pure active alpha to passive market beta. For this reason, the optimal sourcing of exotic betas also spans the spectrum from passive portfolios such as ETFs and index funds on one side to actively managed accounts and vehicles (e.g., hedge funds, commodity pools) on the other. The key issue in any portfolio is to understand what risks are being sourced, and the size of the exposures. The next step is proactively to make judgments about expected returns of the risk factors and from these judgments to make decisions about where there are opportunities to increase allocations and where it would be more appropriate to reduce exposures.

In this context we recommend the use of "implied views."[9] The implied views of a portfolio are the expected excess returns for which the portfolio positions represent an optimal portfolio relative to estimated correlations and volatilities. From a mathematical perspective, the implied views are a simple exercise which basically amounts to running the portfolio optimization exercise in reverse.[10]

To illustrate this concept, consider the Risk Parity Portfolio described above. Using the historical covariance of these asset classes, we can back out the implied expected returns for each asset class that would make that portfolio optimal, shown in Table 5. Because correlations are not zero, the implied expected returns and Sharpe ratios across asset classes are not equal.

Table 5—An Equal Risk Combination of Asset Classes

ASSET CLASS	Implied Risk Premium	Implied Sharpe Ratio
Inflation-Linked Bonds	2.6%	0.40
US Bonds	2.2%	0.39
Developed Int'l Bonds	1.0%	0.36
Emerging Market Bonds	8.2%	0.58
High Yield Bonds	4.8%	0.54
Real Estate	12.8%	0.48
Commodities	4.0%	0.20
US Equities	8.5%	0.56
Developed Int'l Equities	7.0%	0.51
Emerging Market Equities	9.4%	0.49

[9] A term originally coined in Black and Litterman (1992).

[10] It is a simple matrix multiplication, but it does leave open one degree of freedom because if all expected excess returns are multiplied by a constant the optimal portfolio remains unaltered. In Table 5, we normalize the scale of the expected returns to the risk parity portfolio's Sharpe ratio over our full sample, 0.74.

A common misconception is that a risk parity portfolio makes no statement about expected returns, or that it assumes expected returns are equal. As the above example illustrates, this is far from the case. In fact, an equal-risk combination of underlying risk premia is only optimal under widely varying returns across asset classes. The implied Sharpe ratio of 0.20 for Commodities is driven primarily by the fact that it is more lowly correlated with the other risk premia so, even with a relatively lower expected return per unit of risk, it is still optimal to hold an equal weight in this asset class.

Naturally, one can define an equal-risk combination of our exotic betas as well.[11] The implied views from this portfolio—shown in Table 6—demonstrate once again how varied the expected returns may be. Term structure is considerably lower than any other risk premium because of its unusually low correlation with the other exotic betas. In contrast, equity value, currency value and credit all imply relatively high Sharpe ratios due to their relatively high correlations with the other premia. In our opinion, the implied Sharpe ratios from an equal-risk portfolio of exotic betas are no more sensible than the implied Sharpe ratios from the risk parity portfolio of asset classes.

Table 6

Equal Risk Combination of Exotic Betas

RISK PREMIUM	Implied Risk Premium	Implied Sharpe Ratio
Equity Value	6.9%	0.69
Bond Yields	3.8%	0.38
Bond Yields Value	3.8%	0.38
Term Structure	0.4%	0.04
Commodities	5.7%	0.57
Real Assets	5.6%	0.56
Currency Value	6.6%	0.66
Volatility	3.0%	0.30
Credit	6.9%	0.69
Catastrophe Bonds	4.1%	0.41

Exotic Beta versus Hedge Fund Replication

Many have noted in recent years that much of the returns from hedge funds can be attributed to time-varying exposures to well known risk factors, many of which are the same risks that we refer to as exotic betas.[12] We demonstrate that with our data by regressing the monthly returns of the HFRI hedge fund index on the monthly returns of the MSCI ACWI and all ten of our exotic betas. Over the period August 2002 to December 2010—the longest period for which we have all the exotic beta histories—the

[11] Here we normalize the scale of the expected returns to the Sharpe ratio on an equal-risk combination of exotic betas over our full sample, 1.51.
[12] See, for example, Hseih and Fung (2004).

average monthly excess return on the hedge fund index above 1-month T-bills is 0.46%. As displayed in Table 7, over this same period, the global equity index plus exotic beta model explains 86.4% of the variation in monthly performance.[13] Consistent with Asness, Krail and Liew (2001), equity beta is the most important variable. However, commodities, credit and cat bonds also are statistically significant factors in hedge fund performance. While the others are not statistically significant at the usual levels, the fact that nearly all the coefficients are positive indicates a strong association between hedge fund returns and our exotic betas. As the intercept shows, net of all these factors, hedge fund excess returns are actually -0.12% per month. Over this same period, the regression on MSCI ACWI alone delivers an R-square of 75.5% and an intercept of 0.28% per month, indicating that our exotic betas account for an additional 40 basis points in average monthly hedge fund returns, approximately 85% of their total.

Table 7—Linear Regression of HFR Hedge Fund Index on Global Equities and Exotic Beta

EXPLANATORY VARIABLES	Regression Coefficient	Standard Error	t-Stat
Intercept	-0.12%	0.12%	(0.99)
MSCI ACWI	0.32	0.02	15.35
Equity Value	0.07	0.05	1.36
Bond Yields	-0.07	0.04	(1.96)
Bond Yields Value	0.06	0.05	1.10
Term Structure	0.09	0.06	1.60
Commodities	0.14	0.03	5.33
Real Assets	-0.01	0.02	(0.60)
Currency Value	0.05	0.04	1.49
Volatility	0.03	0.03	0.81
Credit	0.08	0.03	2.59
Catastrophe Bonds	0.08	0.03	2.50
R-squared	86.4%		

While this demonstrates that the index of hedge funds does deliver many or most of our exotic betas, it is probably suboptimal to source exotic betas this way. First, the primary risk factor exposure in actively managed strategies is simply the market beta. Although this is well known, and was clearly, and not surprisingly, illustrated once again in the recent financial crisis, it bears repeating. There is no good reason for investors to source a significant amount of market beta from these strategies, especially given performance fees associated with such long-term equity exposure.

Second, many of the risk factors which hedge funds provide exposure to, including especially market beta, are easily sourced more cheaply directly in the markets or through the use of derivative instruments. This ease of sourcing is the basis for a recent addition to the financial marketplace, the

[13] If credit and catastrophe bonds are dropped from the analysis, it can be extended back to February 1990. The results are substantially similar with an R-square of 64.6% and the same sign and similar t-stats on each of the explanatory variables. Over the longer period, the intercept is positive 0.40% per month.

hedge fund replicating portfolio. The idea behind the replicating portfolio is that by monitoring the returns of hedge funds one can reasonably accurately measure their risk exposures over time and then provide directly a much cheaper package of derivatives that approximates the returns from those exposures. While a hedge fund replicating portfolio may be a relatively inexpensive way to approximate many of the risk factors embedded in actively managed portfolios, these exposures are imprecisely estimated on lagged returns and therefore cannot capture the dynamics of factors in real time. It remains to be seen whether the time variation of the risks embedded in such funds consistently adds value.

We think it is fair to characterize the hedge fund portfolio as providing considerable exposure both to market risk and to exotic betas, along with its exposure to active alpha. Exposures to exotic beta within actively managed portfolios of managers do not pose a problem in and of themselves, but investors are advised to measure and monitor these factor exposures. To better manage exotic beta exposures, the hurdles above suggest that there are better ways to access exotic betas than through the creation of a hedge fund portfolio or a hedge fund replicating portfolio.

Over time, more managers will offer direct exotic beta strategies that focus on creating risk premia from transparent, well-known, intuitive risk factors that are uncorrelated or lowly correlated with market beta. Such products have lower fees and more transparency than do hedge funds, and we think they make more sense when an investor is looking to source exotic betas. As described above, alternative beta and risk parity products are closely related cousins to these concepts, as well.

IV. IMPLEMENTING AN EXOTIC BETA STRATEGY

For many investors, the broad exposure to equity markets—the market beta—is the dominant risk in the portfolio. For this reason, the degree of market exposure in the strategic benchmark is almost always a decision made by a governing board. Depending on how much detail is included, exposures to some of the exotic beta risk factors may also be incorporated in strategic benchmarks, at least in part, through the allocations to different asset classes. Many of the exposures to exotic betas, however, will typically not be determined by the benchmark. Most benchmarks today, for example, include substantial allocations to hedge funds, but not to the many risk factors that are pervasive in that space. As most investors readily understand, looking only at the credit component of the fixed-income allocation in a portfolio that also includes a separate and significant allocation to hedge funds would be incomplete.

For this reason, we advocate building a risk budget that includes the capability to monitor the risk factor exposures across all the components of an investor’s portfolio. Readily available, third-party risk monitoring software will generally provide that information. We also recommend creating a risk budget for the exotic beta exposures. This is true whether the exposures are created directly by the investor, through external managers, or through some combination. The risk budget allows investors to understand and manage the amount of risk to their overall portfolios from the risk factor exposures. It also allows investors to have a context for understanding the sources of active risk relative to the benchmark, and to have a context for anticipating the level of excess returns from the exotic beta program. In some cases, investors may want a dedicated allocation to this category of return that is separate and aside from their other strategic allocations. Based on our research and historical returns, such an allocation is most efficiently carved from the strategic equity allocation.

Sample risk budget

Let us build a sample risk budget across five types of risks in client portfolios: equity, fixed income, hedge funds, the risk parity portfolio, and exotic beta. We proxy equity and fixed income with the MSCI ACWI and the Barclay's Global Aggregate, while for hedge funds we use the asset-weighted HFR composite index of hedge fund returns (HFRI). For the risk parity and exotic beta portfolios, we use the risk-parity combinations of asset classes and exotic betas described on pages 10-14, normalized to 10% volatility. In the next section, we explore how to improve the exotic beta strategy through more purposeful and dynamic allocations, but the equal-risk combination is sufficient to illustrate our results.

The assets we consider in this example are summarized in Table 8. We believe the historical Sharpe ratios for global equities are roughly in line with current long-term expectations. On the other hand, our risk-parity portfolio and equal-risk combination of exotic betas may overstate the expected return due to the fact they are simulations created with the benefit of hindsight. The prospective returns for global fixed income is also reduced because during this time period global yields have declined significantly. In addition, the HFR composite index may overstate performance due to survivor and backfill bias, to name several of many possible biases.[14] In order to reduce estimation bias in the data, we use Black-Litterman (1991) to "shrink" the expected returns towards the Black (1989) global CAPM equilibrium. The shrinkage coefficients we choose for fixed income and risk parity are equivalent to placing 75% weight on the historical average and 25% on the CAPM expected return. For exotic beta and hedge funds, we downweight the history more substantially by placing 50% weight on the empirical average. Table 8 also displays the historical returns, Sharpe ratios, correlations and Black-Litterman Sharpe ratios for the five risk types over the period 1985 to 2010.

[14] Survivor bias occurs when dead funds are purged from the database. In early periods of their existence, HFRI appears to have removed some dead funds. Self-reporting and backfill bias occur when managers only start reporting their performance after they have a strong track record, and then backfill their returns since inception. Barry (2003) documents backfill bias in the HFR database, with approximately 50% of new managers backfilling some or all of their prior history.

Table 8

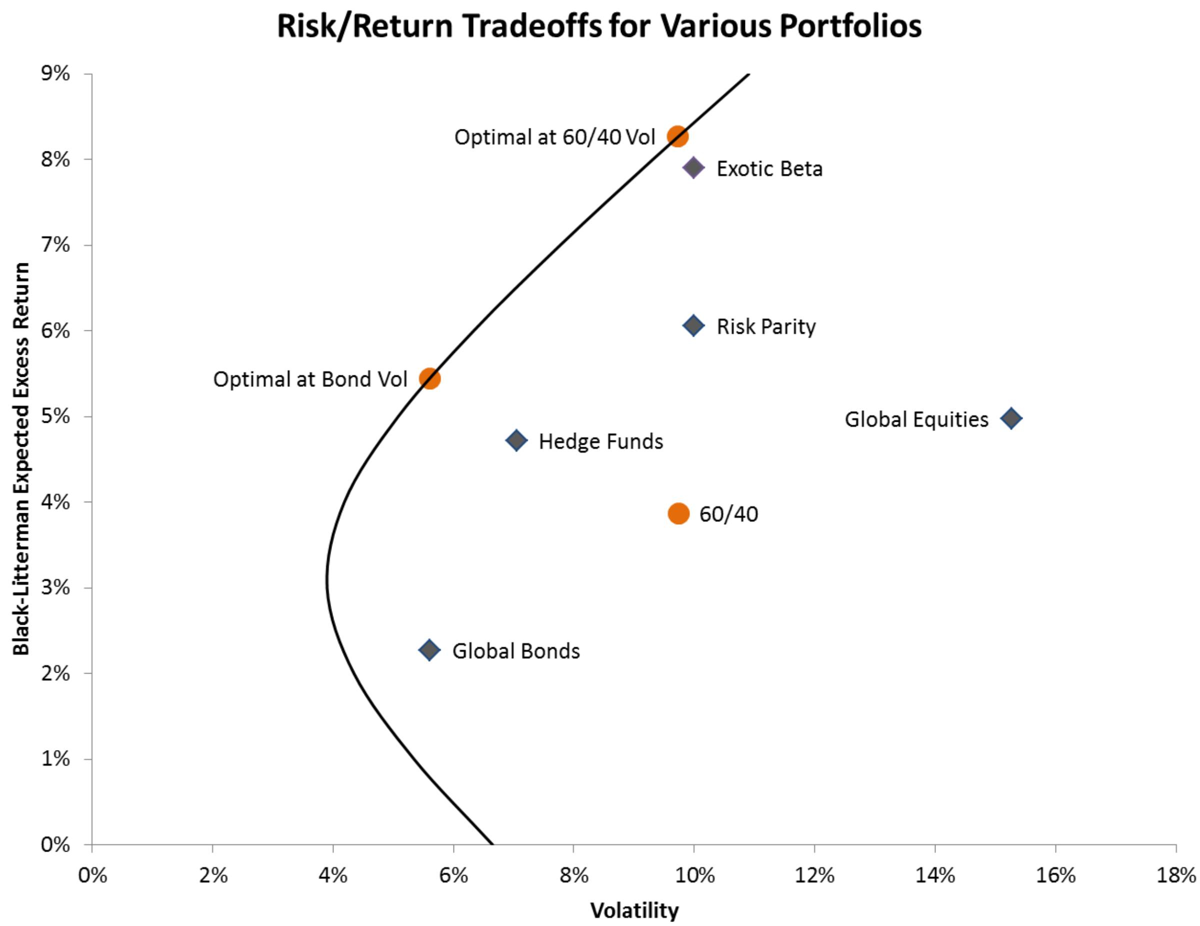
ASSET CLASS/ STRATEGY	Historical Average Excess Return	Historical Sharpe Ratio	Black-Litterman Sharpe Ratio
Global Equities	5.0%	0.33	0.33
Global Bonds	2.9%	0.52	0.40
Risk Parity	7.2%	0.72	0.61
Exotic Beta	15.1%	1.51	0.79
Hedge Funds	7.7%	1.09	0.67

	Global Equities	Global Bonds	Risk Parity	Exotic Beta	Hedge Funds
Global Equities	1.00	0.14	0.81	0.23	0.78
Global Bonds	0.14	1.00	0.40	0.12	0.12
Risk Parity	0.81	0.40	1.00	0.51	0.77
Exotic Beta	0.23	0.12	0.51	1.00	0.35
Hedge Funds	0.78	0.12	0.77	0.35	1.00

Notice that the correlations are in general quite low across these risk groupings with the exception of the correlation between hedge funds, global equities, and the risk parity portfolio which are all about 0.80. Since equities are a substantial element in the risk parity portfolio, this correlation is not unexpected, and the degree of equity risk in hedge funds is also widely documented. See, for example, Asness, Krail and Liew (2001).

The risk/return tradeoffs are quite different across the risk categories. And when we begin to consider portfolios of these risks, the resulting portfolios can significantly differ from traditional allocations. In Figure 2 we display the Black-Litterman expected excess return and volatility for each of the five risks after normalizing the Risk Parity and Exotic Beta portfolios to 10% volatility, as well as the efficient frontier. We also show three combinations of these 5 assets: (1) a traditional 60% equities, 40% fixed income portfolio, (2) a portfolio optimized to the same volatility level as the 60/40 portfolio, and (3) a portfolio optimized to the same volatility level as global bonds. We require the asset weights sum to 100% (i.e., no additional leverage) and no shorting is permitted.

Figure 2



The 60/40 portfolio is a good approximation of the risk and return for a traditional institutionally invested portfolio. However, the optimal combination of assets with the same volatility as 60/40 has 0% in global equities, 0% in global bonds, 6% in risk parity, 94% in exotic beta and 0% in hedge funds. What's driving this, of course, is the high return per unit of risk in the exotic beta portfolio, as well as the no leverage constraint. When targeting a lower volatility level—equivalent to global bonds—the optimal combination is slightly more sensible at 0% in global equities, 27% in global bonds, 0% in risk parity, 37% in exotic beta and 35% in hedge funds.

These results suggest that exotic beta should play a meaningful role in a client's portfolio alongside hedge funds, global equities and global bonds. We discuss what this means in the next section. In most cases an investor managing its own exposures will have a different set of issues to deal with, but this should give some idea of how one might approach the problem if there are no constraints or existing positions to worry about.

Managing a portfolio of exotic betas

Given an appropriate allocation to exotic beta in a portfolio, how should a portfolio of the underlying exotic betas be managed? We believe there are three elements to this process. The first is identifying the exotic betas as discussed above, and importantly, these are justified risk premia for a real or perceived risk by investors. The second is choosing a strategic allocation to each exotic beta. The final element is a slow-moving dynamic allocation that chooses both an overall risk level and the conditional optimal tactical allocation to each exotic beta. As discussed above, we believe this dynamic process should be thought of as value investing in risk premia: when a given risk premia is relatively higher (and volatility relatively lower), we should in general increase our exposure to that factor.

As referenced above, we believe optimizing a portfolio of exotic betas is best handled through the Black-Litterman (1990) asset allocation model. The intuition behind Black-Litterman is that one can define a set of expected returns that is consistent with a risk model as well as some notion of equilibrium, and then average those expected returns with tactical views expressed through proprietary models. The so-called "equilibrium" expected returns are those implied by a given allocation, which in the usual global asset allocation application of Black-Litterman is the global capitalization-weighted portfolio motivated by Black's global version of the Capital Asset Pricing Model (CAPM). In the case of our exotic betas, we have already hedged out the CAPM beta, so in equilibrium according to the CAPM these exotic betas should have risk premia of approximately zero. Since the basic notion of exotic beta is that there are excess returns to equilibrium, we need to choose a method to forecast the risk premia on the underlying risk factors.

We choose a straightforward Bayes-Stein approach to forecasting the risk premia as in Jorion (1986). Bayes-Stein is motivated by the fact that the true expected returns are never observable and are therefore estimated in data with error. Because the relatively higher realized average returns are more likely to have a positive average error, and the relatively lower realized average returns are more likely to have a negative average error, by averaging or "shrinking" the realized average returns towards a meaningful center point the resulting forecasts have smaller errors and better approximate reality.[15] In our case, we define the strategic risk premia on each exotic beta by shrinking its Sharpe ratio towards the grand mean Sharpe ratio, which is the average Sharpe ratio across all the exotic betas.[16]

We compare the return and risk characteristics for three strategic allocation approaches in Table 9.[17] The first portfolio is the equal-risk combination of underlying exotic betas introduced above. To review, this portfolio is constructed using only the backward-looking estimates of risk to mitigate look-ahead bias, and always targets 10% volatility. The realized volatility of the equal risk exotic beta portfolio is not quite 10%, indicating that the expanding window estimates of risk are slightly upward biased. Not

[15] Technically, the original Stein approach shows that the mean squared error of a forecast can be improved by introducing a bias through shrinkage. A seminal paper illustrating this shrinkage approach in asset allocation earlier than Black-Litterman (1990) is Jorion (1985).

[16] There is an optimal amount of shrinkage under various assumptions. In our case, we have chosen to fix the shrinkage to 75% weight on the grand mean and 25% on the individual exotic beta mean.

[17] Marginal contribution to risk is the decomposition of total variance into the components that are specific to each asset in a portfolio.

surprisingly given our earlier discussion of implied views, the marginal contribution to risk is not the same for all risk premia. In fact, the dispersion in risk is fairly extreme, with 26% coming from equity value, almost 20% from each of commodity and currency risk, and 0% coming from term structure risk. While this portfolio might have parity in terms of standalone volatility from each risk premium, the risk allocation is far from equal.

Table 9

EXOTIC BETA PORTFOLIOS	Equal Risk	Equal Sharpe	Black-Litterman
Average Excess Return	12.6%	15.3%	16.1%
Annualized Volatility	8.3%	9.3%	9.3%
Sharpe Ratio	1.51	1.64	1.73
Marginal Contribution to Risk:			
Equity Value	26%	10%	9%
Bond Yields	4%	11%	11%
Bond Yields Value	6%	11%	11%
Term Structure	0%	5%	6%
Commodities	17%	20%	21%
Real Assets	9%	9%	7%
Currency Value	19%	13%	14%
Volatility	6%	11%	12%
Credit	11%	5%	5%
Catastrophe Bonds	3%	5%	5%
Total	100%	100%	100%

The second portfolio we consider assumes that all exotic betas have the same Sharpe ratio. Formally, we set the Sharpe ratio for each exotic beta in every month equal to the grand mean Sharpe ratio across exotic betas, using only backward-looking data. The portfolio is then optimized with a fixed risk-aversion parameter using an expanding window estimate of volatility and correlation. The risk-aversion parameter is chosen to achieve approximately 10% annualized volatility. In this portfolio, the range of risk allocated to each premia is 5% to 20%, somewhat more diversified than the equal-risk portfolio. This portfolio also has a slightly higher Sharpe Ratio in sample, which derives from the more balanced allocation of risk across premia.

A third approach to this problem is to optimize using Black-Litterman (with a fixed risk aversion) where the expected returns are a weighted average of the grand mean Sharpe ratio and the empirical Sharpe ratio estimated at each point in the sample, once again using only data up to that point in time. For simplicity, the shrinkage parameter is chosen to place 75% weight on the grand mean and 25% on the historical return. This application of Black-Litterman to asset allocation across the exotic betas has the most diversified combination of all the three portfolio approaches. Of course, it leans towards those risk premia with relatively higher Sharpe ratios and lower cross correlations—particularly commodities, currency and volatility—but it does this only using the historical mean that would have been estimated at each point in time. The resulting portfolio realizes a Sharpe ratio that is 15% above the equal-risk portfolio.

While the differences between these three approaches are not massive, the Black-Litterman approach probably benefits from the fact that it places some weight on historical volatilities, correlations and mean returns, whereas the equal risk approach only considers historical volatilities.

Dynamic exposures to exotic betas

The previous section examined three approaches to create a strategic allocation across the underlying exotic betas. However, we do not believe the expected risk premia are constant over time, or even that they vary together. For example, after a series of defaults, all credit spreads widen significantly. Some of this might reflect an increased probability of subsequent defaults, but much of this is simply driven by an increased demand for insurance against default. The same process occurs in catastrophic insurance risk after a season of storms or a large earthquake. In many cases, the change in the risk premium (or insurance premium if you view it from the underlying buyer of the insurance) is a multiple of the steady-state risk premium just before an event. It would be nearly impossible to forecast these events, but with the benefit of all the history up to that point we can assess the attractiveness of the risk premia relative to its past, both on an expected return and a risk basis.

In building forecasts of risk premia, we incorporate valuation-type metrics as their key ingredients. These might include the level of spread relative to history in credit markets, for example, or the level of implied volatility relative to history in equity index volatility. Other forecasting models might consider the underlying components of supply and demand for hedging in each of these markets. There is a large universe of factors motivated by the idea of "value", and in all cases they are motivated by the idea that risk premia revert to their means over the long term.

Table 10

STRATEGY COMPARISON	Volatility	Sharpe Ratio	Maximum Drawdown (at 10% vol)	Correlation with ACWI
Exotic Beta Static B-L	8.3%	1.73	-18%	0.16
Exotic Beta Dynamic B-L	9.8%	1.86	-19%	0.16
Global Equities	15.3%	0.34	-35%	1.00

Table 10 compares the return and risk of the static Black-Litterman combination of the exotic betas with a dynamic Black-Litterman implementation that incorporates a simple timing model with a tracking error of 2.5%. The timing model incorporates measures of valuation in each exotic beta relative to its history up to that point, as well as measures of each exotic beta's historical return. The Sharpe ratio on the timing strategy considered independently is 0.83 in backtests, and we have chosen a risk level for timing that results in only moderate swings in allocation over time. Table 11 displays the range of tilting that occurs in our backtests since 1986. For example, the risk level of equity value ranges from 0.38 to 2.01 times its full-sample average risk level. Broadly, the timing model results in exposure shifts that are between 50% and 150% of the static weights, with the largest range being 15% to 200% of the static weight.

Table 11

EXOTIC BETA	Range of Tilts (x Average)	
	Max	Min
Equity Value	2.01	0.38
Bond Yields	1.42	0.78
Bond Yields Value	1.24	0.83
Term Structure	1.42	0.80
Commodities	1.26	0.72
Real Assets	1.27	0.65
Currency Value	1.46	0.33
Volatility	1.52	0.81
Credit	1.93	0.14
Catastrophe Bonds	1.96	0.12

The improvement in Sharpe ratio over the static implementation is a modest 8%, although the drawdowns and correlation to global equities are similar between the two. For comparison, we report the same statistics for global equities. The equity portfolio has a Sharpe ratio only 18% as large as that of our proposed strategy, and its drawdown is approximately twice as large. Of course, the equity portfolio is not necessarily a substitute for exotic beta, but it serves as a useful benchmark for the level of return and risk that one generally receives from investing in traditional risk premia.

Managing the risk to an exotic beta portfolio is in many ways similar to that in any portfolio. A targeted level of overall risk is agreed upon and the allocations to individual exotic betas are crafted to optimize return while being consistent with the overall risk target. Even abstracting from changes in expected returns, obviously as volatilities and correlations change over time the weights would need to be adjusted.

Conclusions

Many risk factors—which we call exotic betas—are neither equity exposure nor pure skilled active management. In this paper we have updated our thoughts about including exotic betas in investor portfolios in light of the 2007/2008 financial crisis. We continue to recommend that investors create a framework for understanding, appropriately sizing, and managing the multiple risk factors which provide sources of returns in their portfolios.

One of the many investment themes that survived the financial crisis is that the exposures to exotic beta are very attractive in a portfolio context. However, an important caveat which has been learned is that such exposures should not be expected to be completely static over time. In addition to being dynamic, we believe it is optimal for investors to access these risk premia in relatively lower cost, more transparent, and customized implementations. Based on our impression of currently available products, it seems that the industry is already headed this way.

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