Dynamics of Leveraged and Inverse ETFs

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Investment comparisons are for illustrative purposes only and not meant to be all-inclusive. To better understand the similarities and differences between investments, including investment objectives, risks, fees, and expenses, it is important to read the products' prospectuses.

Investing involves risk, including possible loss of principal.

Objectives

- Leveraged and inverse funds are not well understood, even by industry professionals
- Goal here is to provide a unified framework to better understand
  - the underlying dynamics of leveraged and inverse ETFs
  - impact on market volatility and liquidity
  - unusual features of their product design, and
  - questions of investor suitability

Agenda

- Basics of leveraged and inverse ETFs
- Return dynamics
- Rebalancing activity and market microstructure
- Buy-and-hold investors and value destruction
- Policy implications

Basics: What are leveraged ETFs?

- Leveraged and inverse Exchange-Traded Funds (ETFs) provide leveraged long or short exposure to the daily return of various indexes, sectors, and asset classes
  - Rapid grown in AUM and volumes
  - Broad coverage: Equity, FX, commodities, fixed income, etc.
  - Offers $2\times$ and $3\times$ long exposure or $-3\times$, $-2\times$, $-1\times$ short exposure to the underlying daily index return (some products cover other return periods)
  - Leveraged mutual funds work in a similar fashion
- These funds have leverage explicitly embedded as part of their product design, i.e., “pre-packaged” margin products
  - Technically should be termed Exchange-Traded Products (ETPs)
  - Creates *intended* and *unintended* characteristics not seen in traditional ETFs

Hot product

- Popular with short-term traders and hedge funds
  - Permits expression of directional views regarding a wide variety of assets
- Used also by individual investors:
  - Levered bet without use of derivatives, swaps, options, futures, or margin
  - Portfolio hedging
  - Limited liability is attractive

Global leveraged ETFs

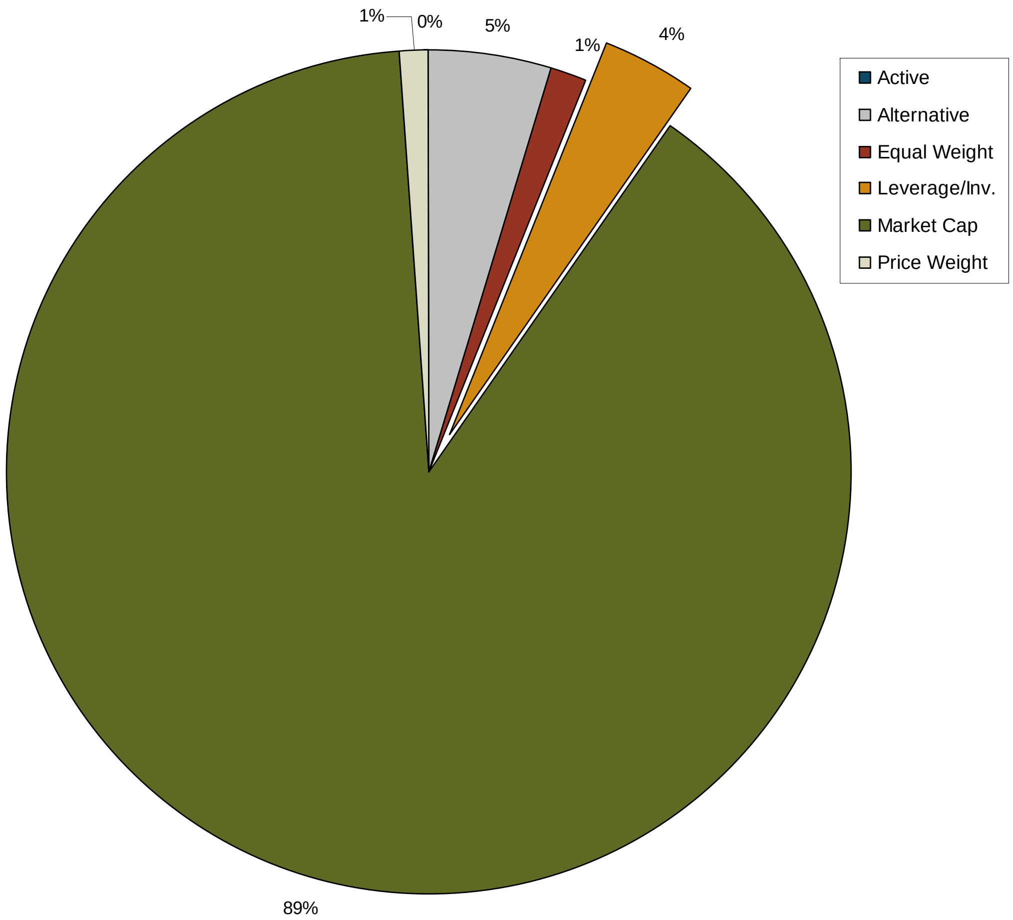
- Horizons BetaPro S&P/TSX Global Gold Bear Plus ETF (HGD-TSX)
- Horizons BetaPro S&P/TSX Global Gold Bull Plus ETF (HGU-TSX)
- Horizons BetaPro S&P/TSX Global Mining Bear Plus ETF (HMD-TSX)
- Horizons BetaPro S&P/TSX Global Mining Bull Plus ETF (HMU-TSX)
- ETFS Leveraged Agriculture ETF (LAGR-LSE)
- ETFS Leveraged All Commodities ETF (LALL-LSE)
- ETFS Leveraged Aluminium ETF (LALU-LSE)
- ETFS Leveraged Cocoa ETF (LCOC-LSE)
- ETFS Leveraged Coffee ETF (LCFE-LSE)
- ETFS Leveraged Copper ETF (LCOP-LSE)
- ETFS Leveraged Corn ETF (LCOR-LSE)
- ETFS Leveraged Cotton ETF (LCTO-LSE)
- ETFS Leveraged Crude Oil ETF (LOIL-LSE)
- ETFS Leveraged Energy ETF (LNRG-LSE)
- ETFS Leveraged Ex-Energy ETF (LNEY-LSE)

- Wide range of global leveraged ETFs
- Include ETFs on commodities (e.g., copper, corn, cotton, etc.)
- Traded in major markets including LSE and TSX

Source: http://etf.stock-encyclopedia.com/category/leveraged-etfs.html

Leveraged and inverse ETFs

AUM by weighting scheme for US equity ETPs, August 2009



Source: BGI and authors' estimates

Mechanics and institutional details

- Leveraged ETFs seek to provide a multiple of the stated return in a given interval, usually a day
  - They gain exposure through total return swaps with a broker dealer versus trading in physicals on margin
  - Futures contracts can also be used in addition to, or instead of, total return swaps. However, given their exchange-imposed standardized specification (to facilitate exchange-based trading and clearing), futures are not as customizable as total return swaps and are more limited in terms of index representation Creations/redemptions for leveraged products are cash versus “in kind” transfers used by traditional ETFs
- Some leveraged and inverse ETFs are managed against customized index benchmarks
  - Explicitly incorporate the financing cost (for leveraged ETFs) or accrued interest (for inverse and leveraged inverse ETFs) in index construction so there is no apparent deviation against the funds' index benchmark

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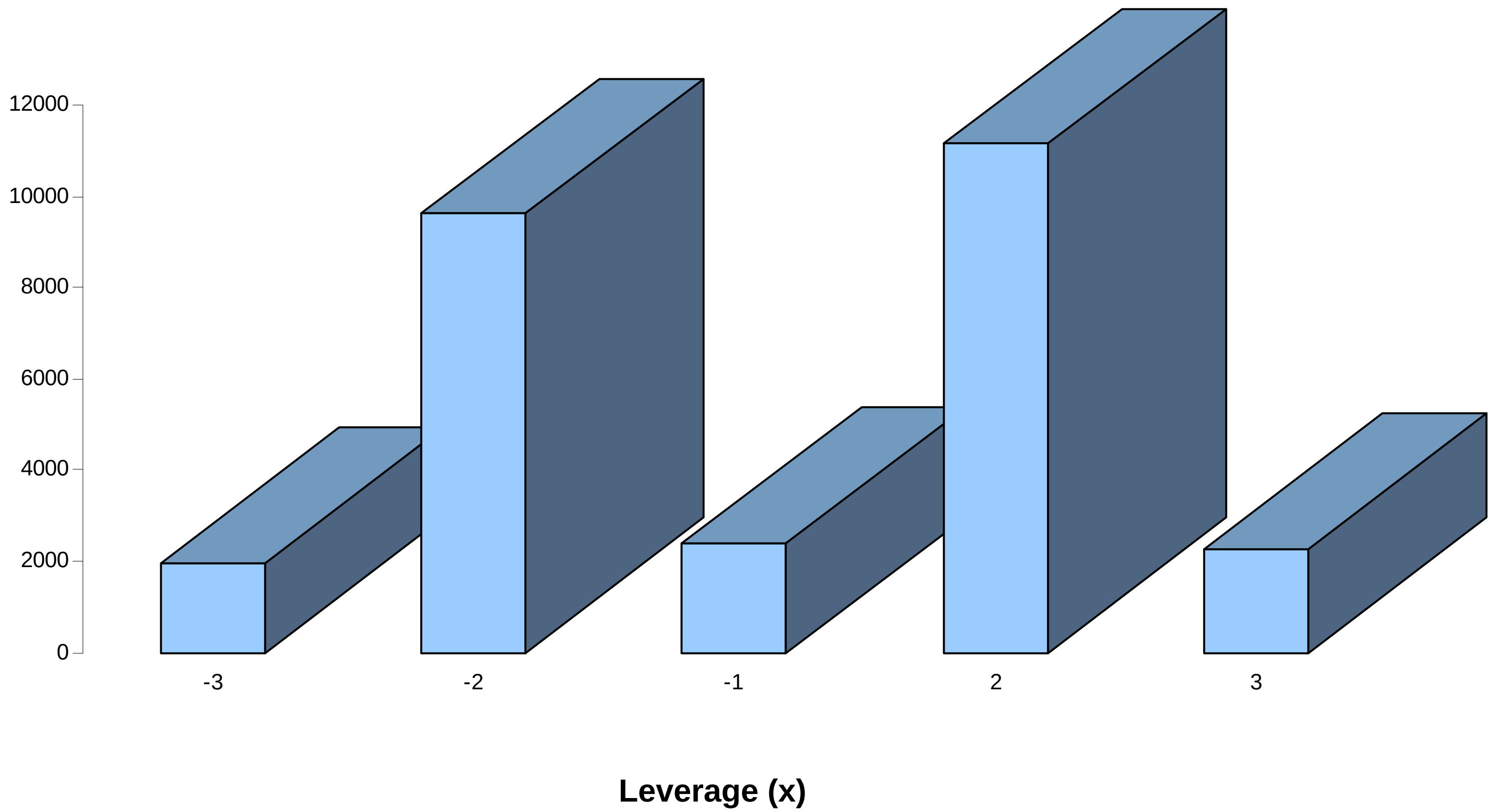
AUM for all inverse and leveraged ETPs
(in millions of dollars as of August 2009)



Source: Authors' estimates based on Bloomberg data and reported leverage for 139 funds

Media attention

- Return dynamics can be confusing to some investors
  - See Wall Street Journal, 2/28/09, " How Managing Risk with ETFs Can Backfire": *Still, many financial advisers believe these funds are a good long-term hedge against falling markets. At a recent conference, roughly 50 financial advisers besieged Matthew Hougan, editor of IndexUniverse.com, a financial Web site, asking him to explain how leveraged ETFs work. Some "are starting to understand," says Mr. Hougan, "but there is still a huge contingent out there who don't."*
- Many sell-side firms offer intraday tools to monitor these flows giving rise to concerns about trading ahead of predictable flows
  - See Wall Street Journal, 12/15/08, "Are ETFs Driving Late-Day Turns?"
  - Note: front-running typically refers to illegal trading by a broker ahead of a customer order that is likely to move prices, whereas rebalancing flows can be computed given public information about AUM, leverage, and index returns, and end of day market volatility

Return dynamics

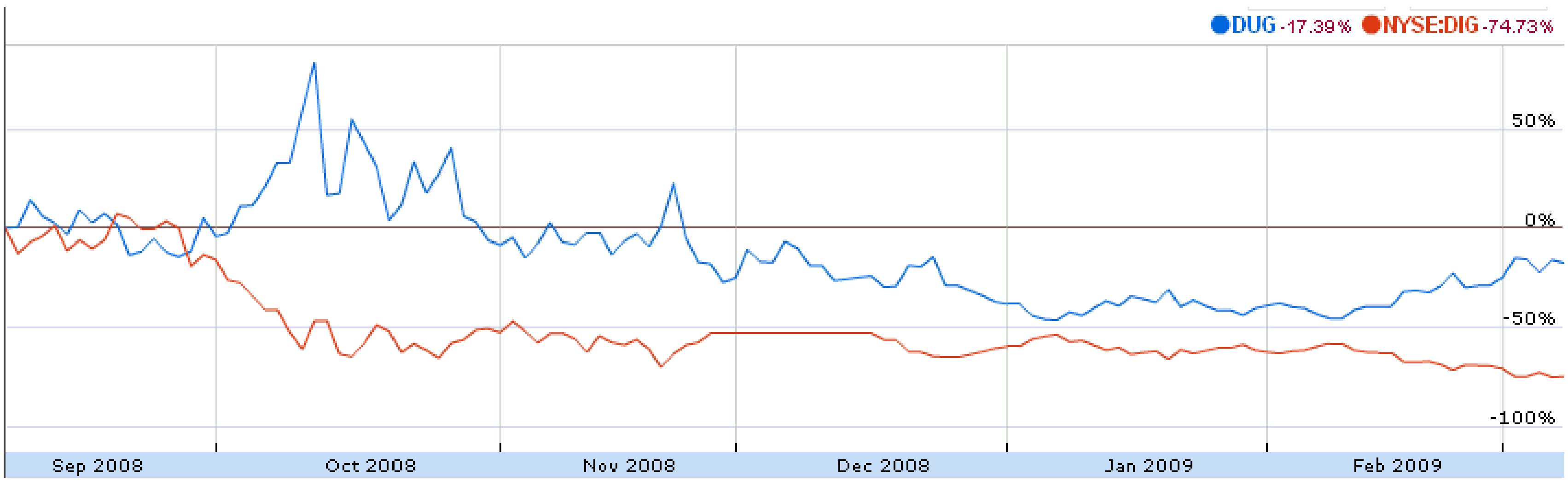
- Leveraged and inverse ETFs are designed to provide leveraged long or short exposure to the *daily* return of various indexes, sectors, and asset classes
- At the daily and intraday level, they track well…

Intraday price movements: ProShares UltraShort Oil & Gas ETF (**DUG**) versus ProShares Ultra ETF (**DIG**): March 2-6, 2009

The performance quoted represents past performance and does not guarantee future results. Investment return and principal value of an investment will fluctuate so that an investor's shares, when sold or redeemed, may be worth more or less than the original cost. Current performance may be lower or higher than the performance quoted.

Mirror Image? Not over longer horizons

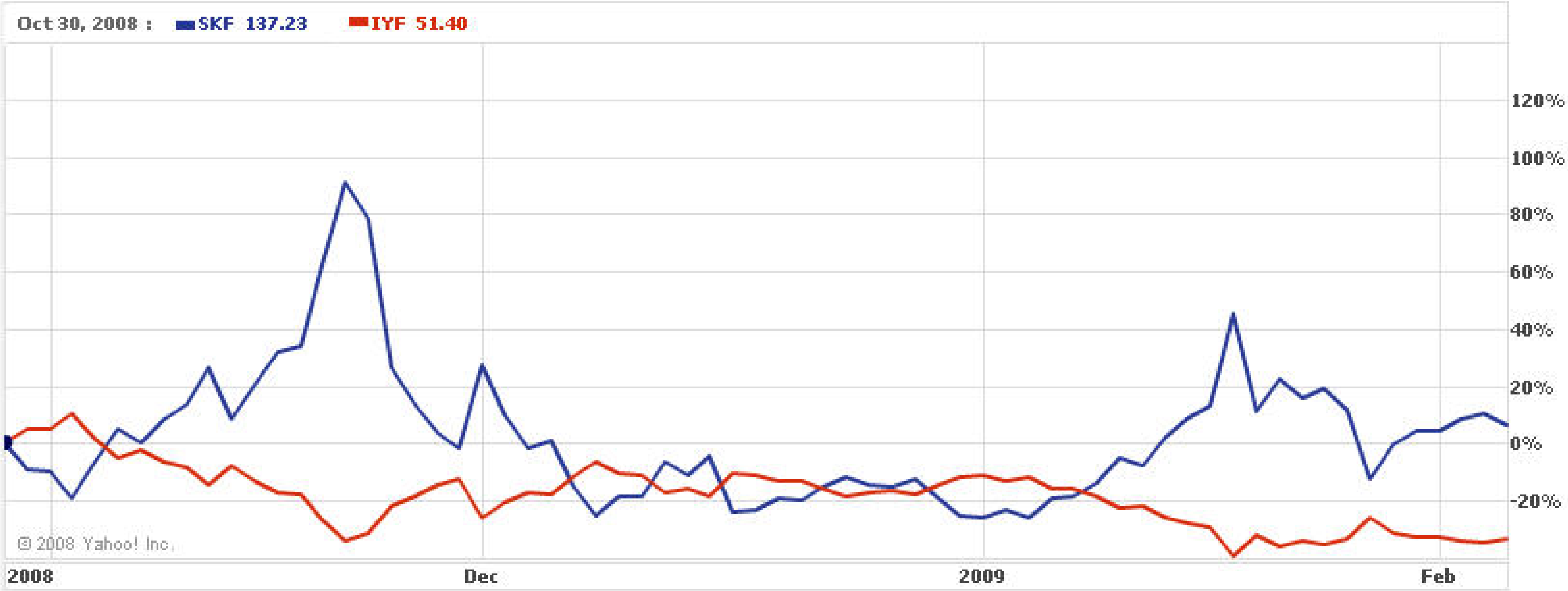
- ProShares UltraShort Oil & Gas ETF (DUG) versus ProShares Ultra ETF (DIG): September 1, 2008- March 6, 2009
- Six Month Returns: **DUG = -17.3%** **DIG = -74.73%**



Source: Google Finance. The performance quoted represents past performance and does not guarantee future results. Investment return and principal value of an investment will fluctuate so that an investor's shares, when sold or redeemed, may be worth more or less than the original cost. Current performance may be lower or higher than the performance quoted.

Divergence over longer horizons

ProShares UltraShort Financials (SKF) versus iShares Dow Jones US Financial Sector (IYF)



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Return divergence

- Individual investors often do not understand that beyond a day, leveraged and inverse ETFs returns might diverge from index returns
- Example: Consider a double-leveraged ETF (x = 2) with an initial NAV of $100
  - Index starts at 100, falls 10% one day and then goes up 10% the subsequent day.
  - Over the two-day period, the index declines by -1% (down to 90, and then climbing to 99).
- While an investor might expect the leveraged fund to decline by twice as much, or -2%, over the two-day period, it actually declines further, by -4% to $96
  - Doubling the index's 10% fall on the first day pushes the fund's NAV to $80.
  - The next day, the fund's NAV climbs to $96 upon doubling the index's 10% gain.

Dynamics of a 2× long ETF

- Day 0
  - Assume AUM = \$100; index value = 100
  - With NAV = \$100, required notional total return swaps is long the index \$200 (or, equivalently, \$100 long plus \$100 swap)
- Day 1
  - Suppose index *falls* 10%; index drops from 100 to 90
  - Swap value = (1-10%)×200 = \$180 for a \$20 loss
  - With AUM now at \$80, the required notional amount of swaps = \$160 (=\$80×2)
  - Swap reset = -\$20 (= \$160 - \$180)
  - Fund reduces its exposure by *selling* \$20

Dynamics of a 2× inverse ETF

- Day 0
  - Assume AUM = \$100; index value = 100
  - With NAV = \$100, swap position is short \$200
- Day 1
  - Suppose index *falls* 10%; index drops from 100 to 90
  - Swap gain is \$20 on short position
  - With AUM now at \$120, the required notional amount of swaps = -\$240 (= -2×\$120)
  - Current swap exposure is -180 = -2×90
  - Required swap reset = -\$60 (= -\$240 - \$180)
  - Fund reduces its exposure by *selling* \$60

There is no pairing off:

*Both long and short levered ETFs rebalance in the **same** direction as the market*

Mechanics of Leveraged Returns

- Notation:
  - S_n = index level on day n = 0, 1,..., N
  - $r_{n-1,n}$ = return on underlying index from date n-1 to n
  - A_n = Leveraged ETF's NAV on day n
  - L_n = Notional amount of total return swaps required before n+1
  - x = Leverage factor (x = -3, -2, -1, 2, 3)
  - E_{n+1} = Exposure of total return swaps on day n+1
- Derivation

$$L_n = xA_n$$

Notional amount of swaps

$$E_{n+1} = L_n(1+r_{n,n+1}) = xA_n(1+r_{n,n+1})$$

Exposure increases with returns

$$A_{n+1} = A_n\ (1 + x\ r_{n,n+1})$$

AUM increases with returns

$$L_{n+1} = xA_n\ (1 + x\ r_{n,n+1})$$

$$\Delta_{n+1} = L_{n+1} - E_{n+1} = A_n\ (x^2 - x)\ r_{n,n+1}$$

Hedge is change in exposure needed

End of day rebalancing flows

$$\Delta_{n+1} = A_n \,(x^2 - x)\, r_{n,n+1}$$

- Hedging term is non-linear and asymmetric
  - Always positive (except when x = 1 for linear, traditional funds)
    - Reset or re-balance flows are always in the same direction as the underlying index's performance, for both long and short
  - Inverse ETF rebalance trades do not offset the corresponding long rebalance trade
    - For example, it takes the value 6 for triple-leveraged (x = 3) and double-inverse (x = -2) ETFs
  - Need for daily re-hedging is unique to leveraged and inverse ETFs
    - Traditional ETFs that are not leveraged or inverse, whether they are holding physicals, total return swaps or other derivatives, have no need to re-balance daily

Example: March 10, 2009

- S&P 500 up 6.37%; Nasdaq up 7.07%
  - All US equity sectors up
  - E-mail from broker at 1:35 EST: "Levered Funds Should Help Power Mkt Into Close"
  - Broker estimates MOC flows by sector. In energy and financials, broker estimates MOC flows to buy of $150 and $468 million, respectively, for that day
  - Actual MOC *net* imbalances on March 10, 2009 at 3:52 PM EST were $176 and $804 million, respectively

Microstructure implications

- Hedging demands have a magnified impact on volatility
  - Increase the market impact coefficient for *all* flows, irrespective of their source.
  - Intuitively, hedging demand provides additional momentum to same day returns that increases the price pressure effect of any signed order imbalance regardless of source
- The price impact is greater with higher volatility, lower market liquidity, higher same day effects, increased AUM, and higher leverage ratios
  - The presence of the lagged hedge induces serial correlation in returns because the previous period's hedge is linearly related to the previous return

$$S_{n+1} - S_n = \frac{\lambda(q_{n,n+1} + (1-\phi)\Delta_{n-1,n}) + w_{n,n+1}}{1 - \lambda\,\phi\,a_n(x^2 - x)}$$

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Aggregate impact rebalancing trades

- Using public information on AUM, leverage, and fund category, can compute the total value of end of day rebalancing activity for a given return in the underlying market index
- Δ(r): The dollar notional rebalancing flow for a given return movement in the underlying index
  - For example if Δ(r) = $150 million and the index return was −2%, then we would predict $300 million of selling rebalancing activity by the close
  - Relationship is linear

$$\Delta_{t_{n-1}}(r_{t_{n-1},t_n}) = \left(\sum_{i=1}^{K} A_{i,t_{n-1}} (x_i^2 - x_i)\, r_{t_{n-1},t_n}\right)$$

Aggregate hedging demand for all ETPs on a particular index

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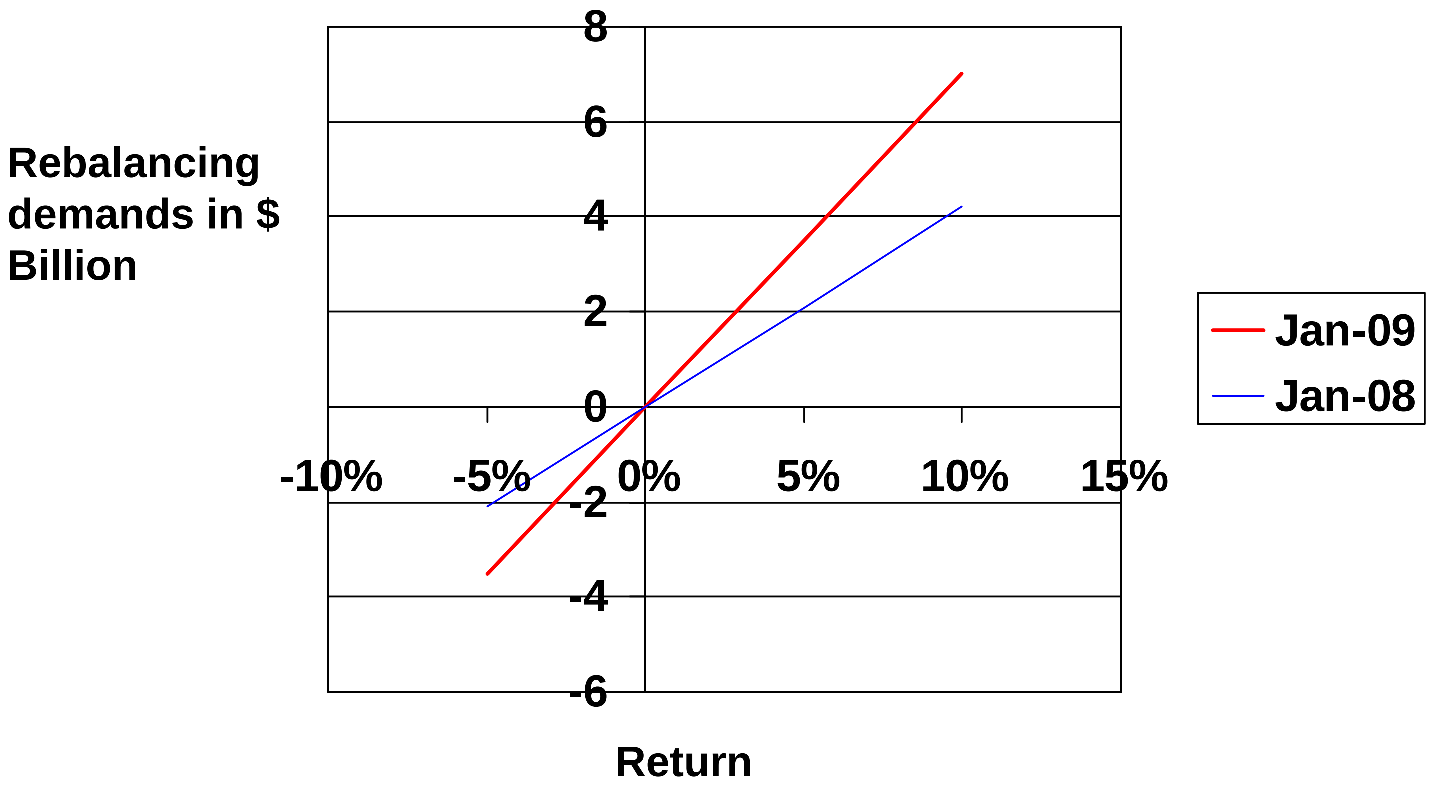
Aggregate rebalancing trades

- compute the total value of end of day rebalancing activity for a broad (equal) movement in US equity market indexes
  - Uses public information on AUM, leverage, and fund category



Breakdown by underlying index

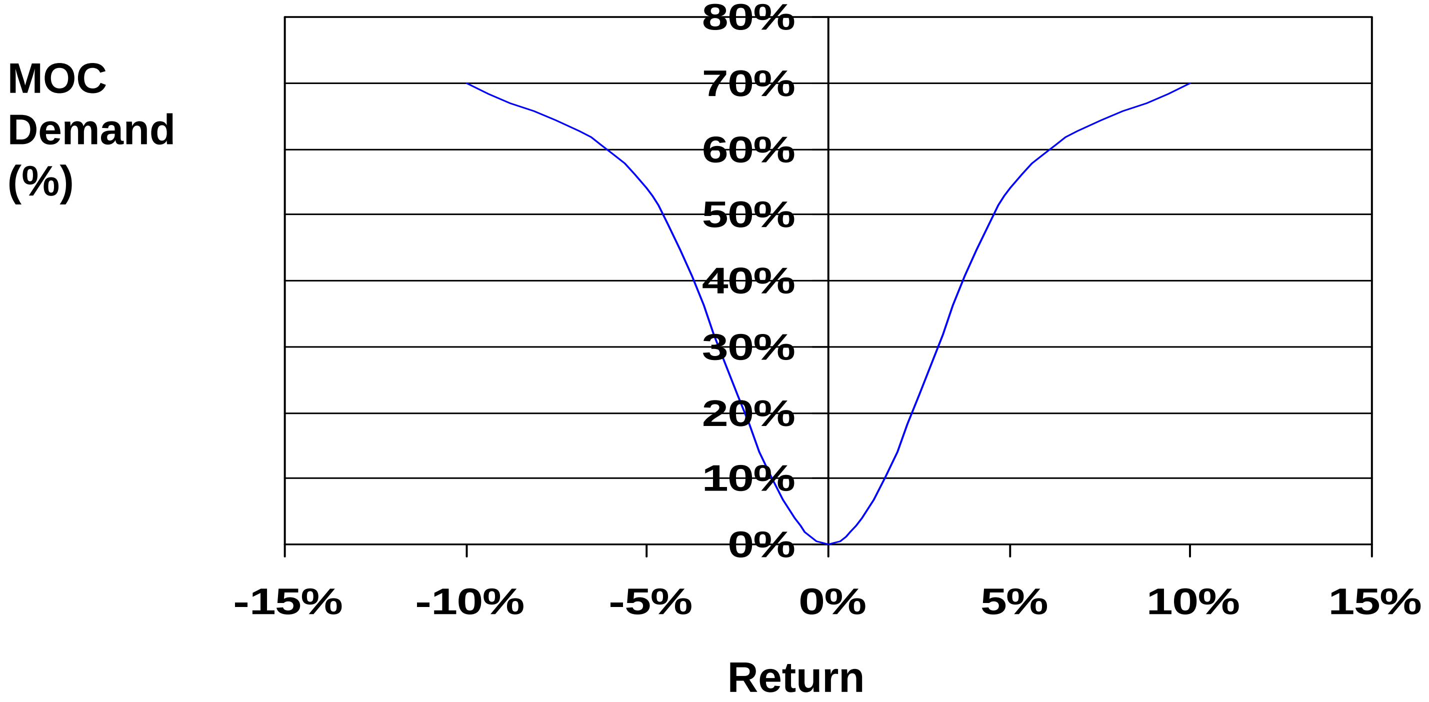
Index	Flows (5%)
S&P 500	$1,570,999,981
Russell 1000 Financials	$945,193,859
DJ US Financials	$509,589,675
Nasdaq 100	$465,252,300
Russell 2000	$437,030,789
DJ US Real Estate	$307,920,825
Russell 1000	$273,226,901
DJ Industrial Average	$267,898,650
DJ US Oil & Gas	$177,467,700
Russell 1000 Energy	$97,372,319
DJ US Basic Materials	$86,387,700
Russell 1000 Technology	$58,012,800
S&P Midcap 400	$41,765,175
DJ US Consumer Services	$26,274,150
DJ US Technology	$24,028,125
DJ US Semiconductors	$22,838,850
S&P Smallcap 600	$19,608,000
Russell Midcap	$16,336,508
DJ US Industrials	$13,957,500
DJ US Consumer Goods	$9,718,650
DJ US Health Care	$8,151,000
Russell 2000 Growth	$6,748,650
Russell 1000 Growth	$6,605,100
Russell 2000 Value	$6,401,025
DJ US Utilities	$6,284,925
Russell 1000 Value	$5,058,750
Russell Midcap Growth	$4,291,575
S&P Financial Select	$4,274,364
Russell Midcap Value	$3,776,250
Russell 3000	$2,193,600
DJ US Telecommunications	$1,757,850
S&P Energy Select	$1,464,617
S&P Technology Select	$1,359,340
S&P Health Care Select	$921,703
	$5,430,169,206

- Estimates of $\Delta(r)$ for $r = 5\%$ based on major US listed ETFs and ETNs on broad equity indexes, excluding commodity and FX funds
- Total rebalancing flows are $5.43 billion as of August 2009 for a broad 5% market move
- Substantial concentration in some sectors including financials
- Note: Rebalancing activity need not necessarily occur at the closing auction itself, but will typically occur before the close

Source: Authors' estimates based on Bloomberg data

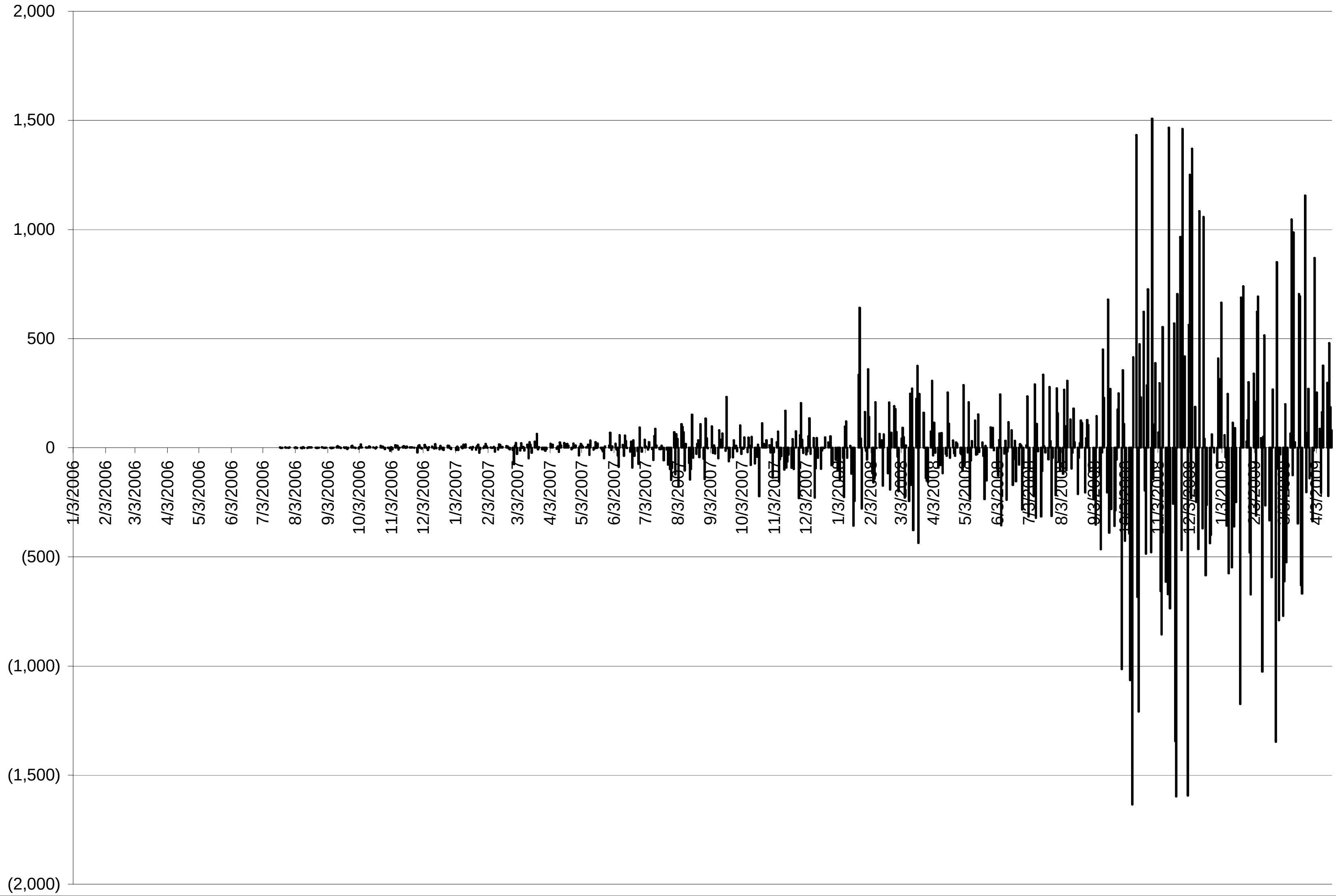
Rebalancing trades relative to closing volume

- Current context
  - Non-rebalancing MOC volumes ~ 5-6% of total volume
  - Volume from 3:00-close ~ 28%; from 3:30-close ~ 20%;
  - Graph shows potential size of rebalancing if concentrated at the close: Non-linear percentage of volume and convexity of cost function



Source: Authors' estimates based on Bloomberg data

Large cap rebalancing flows (millions)



Source: Authors' estimates based on Bloomberg data

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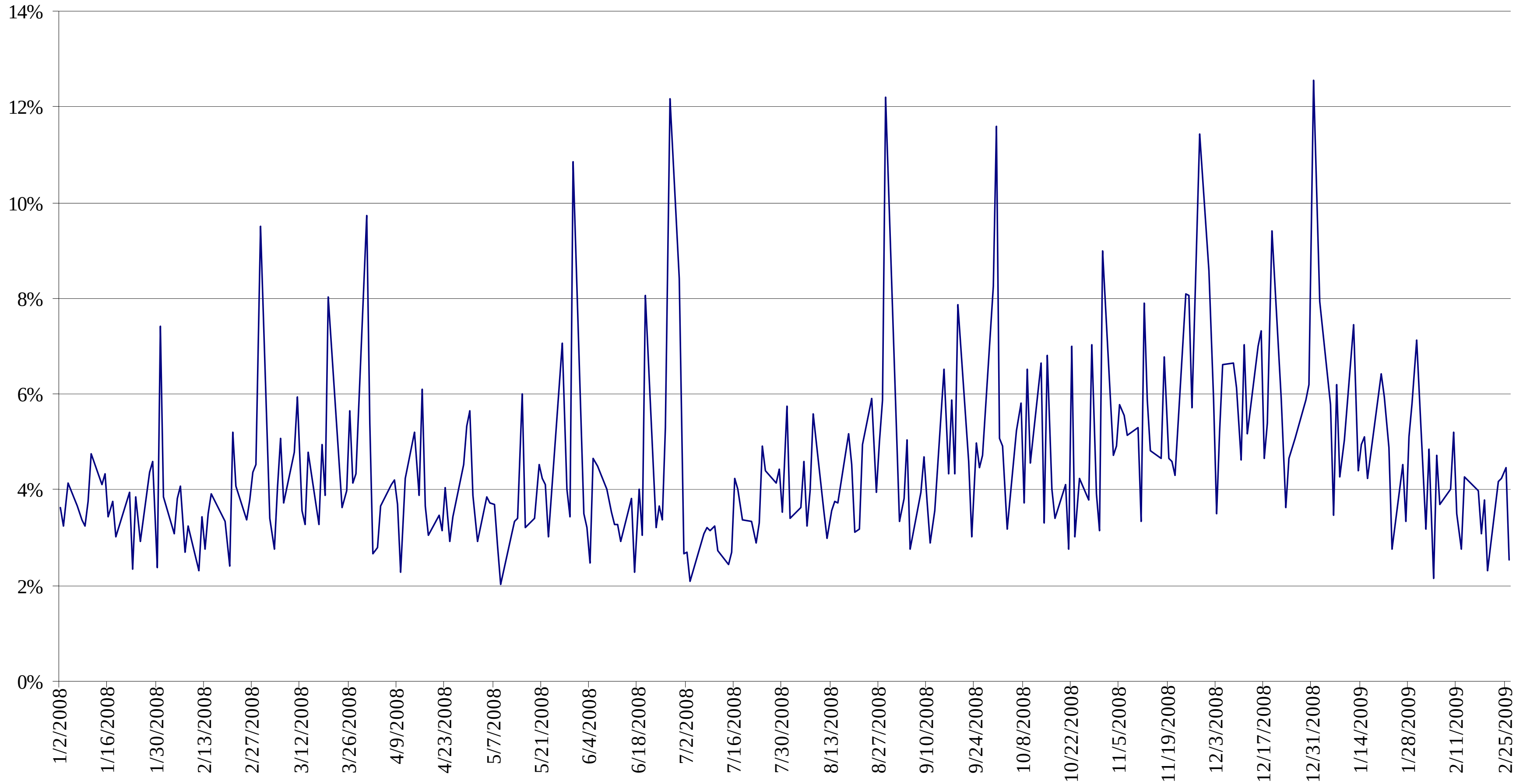
MOC Activity: S&P 500

- MOC volume as a percentage of underlying volume
- Pronounced seasonalities (end of month) imply we should use median volumes to assess closing liquidity



Regression models

- Is there evidence that re-balancing flows contribute to greater return movements towards the close?
- Simple test: Look at 2x and 3x long and short large cap leveraged ETFs (SSO, SDS, BGU, BGZ) re-balancing flows from 1/1/2006-4/15/2009
  - Estimate model of closing returns (3pm EST to close) for SPX on the day's return and re-balancing flows

$$r_{t_n^-, t_n} = \beta_0 + \beta_1 r_{t_n - 1;\, t_n} + \beta_2 \Delta_{t_n}$$

  - With 825 degrees of freedom, $R^2 = 0.35$

$$r_{t_n^-, t_n} = \underset{}{0.01} + \underset{(5.806)}{0.249}\, r_{t_n - 1;\, t_n} + \underset{(2.503)}{0.541}\, \Delta_{t_n}$$

Source: Authors' estimates based on Bloomberg data

Regression models

- Also examine volatility in terms of second moment (i.e., absolute returns) on VIX and the magnitude of daily re-balance flows in large cap levered funds
  - Look at 2x and 3x long and short large cap leveraged ETFs (SSO, SDS, BGU, BGZ) re-balancing flows from 1/1/2006-4/15/2009
  - Likely conservative as other levered funds and non-exchange traded levered funds will also induce flows in large cap names

$$|r_{t_n^-,t_n}| = \beta_0 + \beta_1 VIX_{t_n} + \beta_2 |\Delta_{t_n}|$$

  - With 825 degrees of freedom, $R^2 = 0.46$

$$|r_{t_n^-,t_n}| = -0.18 + \underset{(12.186)}{0.023} VIX_{t_n} + \underset{(6.422)}{0.724} |\Delta_{t_n}|$$

Source: Authors' estimates based on Bloomberg data

Returns

- Turn now to analysis of long-run returns to a leveraged ETF
    - Average ETF holding periods vary considerably across funds
    - Also considerable variation across investors within a fund
- Although most products explicitly target a multiple of a single day's return, some investors hold these products much longer
    - Concerns have prompted guidance from the SEC and FINRA ("Leveraged and Inverse ETFs: Specialized Products with Extra Risks for Buy-and-Hold Investors," August 19, 2009)
    - However, some argue that actual investors have generally tracked the index well over longer holding periods (See Joanne Hill and George Foster, "Understanding Returns Of Leveraged And Inverse Funds," IndexUniverse.com, August 25, 2009)
    - We want to analyze how investors fare over arbitrary intervals of time given underlying return dynamics

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Return dynamics

- Model in continuous time to allow for any rebalancing interval

$$A_{n+1} = A_n \, (1 + x \, r_{n,n+1})$$

$$\frac{A_{n+1} - A_n}{A_n} = x \left(\frac{S_{n+1} - S_n}{S_n} \right) \quad \Rightarrow \quad \frac{dA_t}{A_t} = x \frac{dS_t}{S_t}$$

- Index level follows geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad \Rightarrow \quad dA_t = x\mu A_t dt + x\sigma A_t dW_t$$

$$d \ln(S_t) = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dW_t \quad \Rightarrow \quad d \ln(A_t) = \left(x\mu - \frac{x^2 \sigma^2}{2} \right) dt + x\sigma dW_t$$

Long-term value

- Solve this differential equation for the total value
- Express the leveraged ETF's gross return as follows:

$$\left(\frac{A_n}{A_0}\right) = \left(\frac{S_n}{S_0}\right)^x e^{\frac{(x-x^2)\sigma^2 \; t_N}{2}}$$

Returns reflect the sample path over the time horizon (random variables)

Deterministic scalar that tends to zero over time, and is an increasing function of volatility over the period

Lognormal distribution

- Basic properties: Suppose *Z* is distributed normally with mean *m* and standard deviation *s*
  - *Moment generating function* $E\,[e^{kZ}] = e^{km+k^2s^2/2}$
  - *Median* $M\,[e^{Z}] = e^{m}$
- Lognormal distribution is positively skewed; mean reflects a few extreme sample paths
- Median might be a better representation of the average investor's experience
- Example: Suppose index returns are equally likely to be 4% or -2%
  - Over a successive two-day period, the four possible cumulative returns are 8.16%, 1.92%, 1.92%, and -3.96%
  - The average two-day cumulative return is thus 2.01%, but only one sample path of the four exceeds this

Longer term returns

- Mean return (excluding costs) is just *x* times index return
- Substituting for the mean return for a leveraged ETF, we get three conditions under which the *median* return is negative

$$m = (x\,\mu - x^2\,\sigma^2/2)\,t_N$$

1. *x* < 0 and μ > 0 (e.g., inverse and leveraged inverse ETFs and positive index drift);
2. x > 0 and μ < 0 (e.g., leveraged long ETFs with negative index return drift); and
3. x > 0 (leveraged long) and high volatility relative to drift

$$0 < \mu < \frac{x\sigma^2}{2}$$

- A special case of 3 is when the median index return is positive but the corresponding median leveraged ETF return is negative

$$\frac{\sigma^2}{2} < \mu < x\sigma^2$$

Frictions

- Dealer costs
  - Proportional (with factor θ) to notional exposure desired
  - Previous analysis goes through substituting x' = (1- θ)x for x

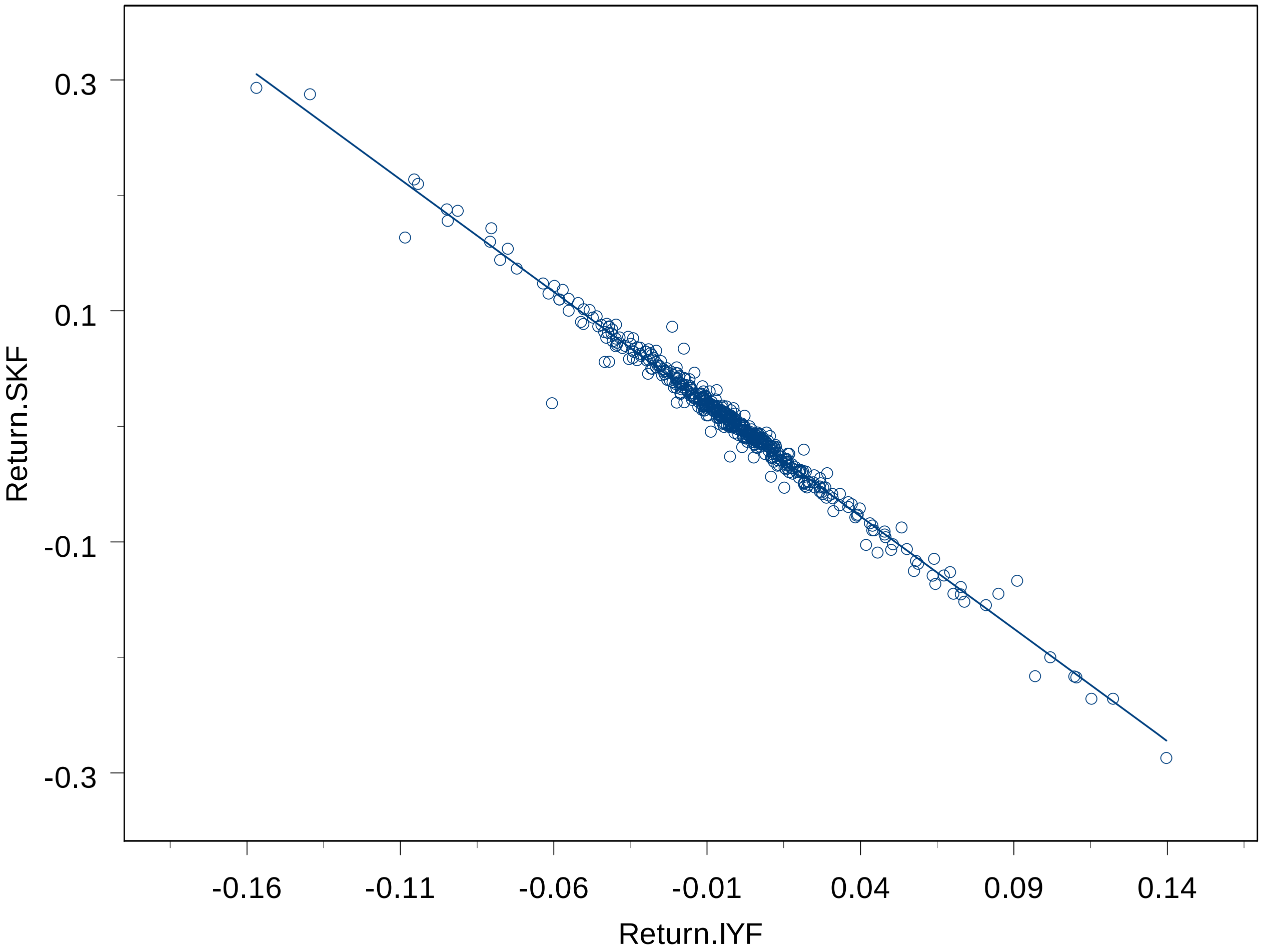
$$\frac{dA_t}{A_t} = (1\text{-}\theta)x\frac{dS_t}{S_t}$$

- Market impact costs
  - Expected costs proportional to expected (absolute) rebalancing volume
  - Costs increase with square root of rebalancing interval and volatility
  - Doubling the time between rebalances does not double cost; rather cost increase 44%

$$\lambda(x^2 - x)\sigma\sqrt{\tau}$$

Regression Analysis of daily returns

ProShares Ultra Short Financials (SKF) vs. Dow Jones US Financial Services (IYF)



February 1, 2007 to February 1, 2009

– **Slope x' = -1.94 (versus theoretical x = -2)**

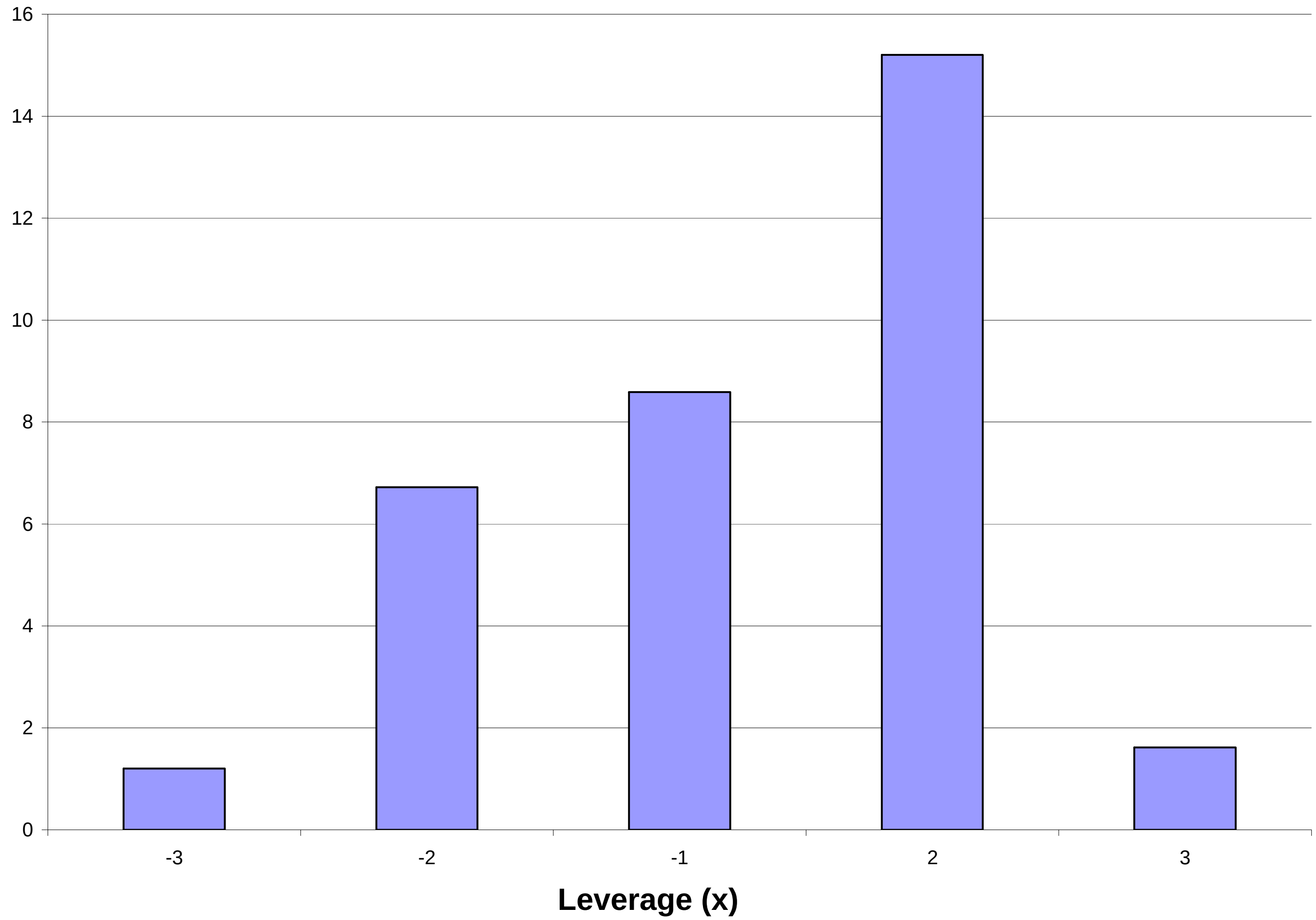
– **Fit R^2 = 0.98 (daily observations)**

– **Implied transaction cost θ = 3%**

Other considerations

- For longer term investors, the drag on returns is reinforced by two factors:
  - Transaction costs
    - Explicit: Management fees range from 75-95 basis points, higher than the average ETF
    - Implicit: Bid-ask spreads are reasonable, but keep in mind the holding period, and there are implicit costs from daily rebalancing
  - Taxes: Leveraged and inverse ETFs are not designed with respect to tax efficiency
    - Large capital gains distributions can yield an after-tax performance for an investor that is much less than a strategy using leverage with less turnover (The impact of taxes, however, will vary depending on each individual investor's particular circumstances)

US Equity Leveraged and Inverse ETFs
Average holding period (days)



Source: Authors' estimates based on Bloomberg data and reported leverage for 139 funds

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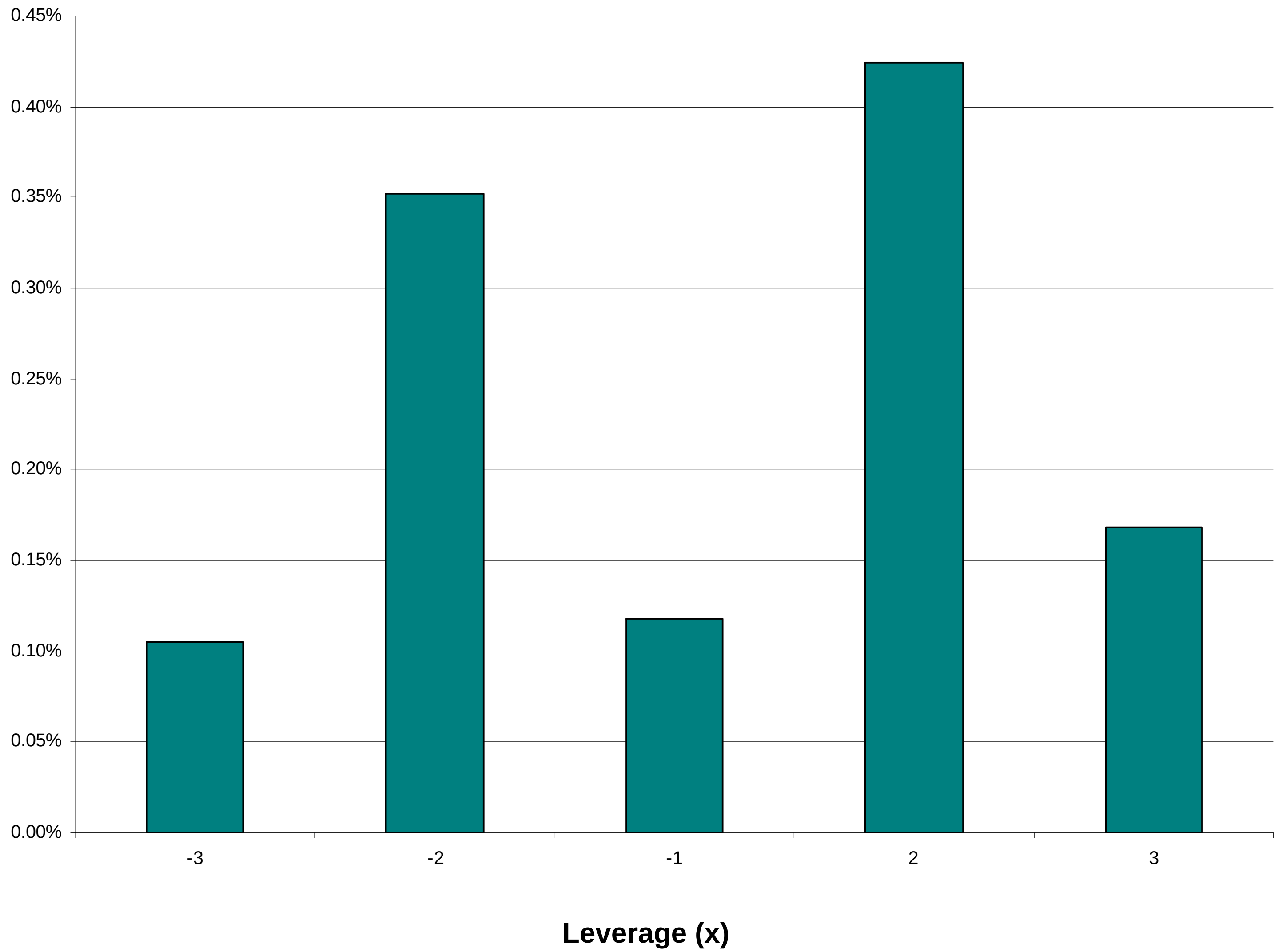
US Equity Leveraged and Inverse ETFs

Average Bid-Ask Spread (%)



Source: Authors' estimates based on Bloomberg data and reported leverage for 139 funds

Policy implications: Protecting investors

- Leveraged funds are not well understood both by investors and industry professionals
  - Track returns well at the daily level, but not necessarily over longer periods
  - Regulators have allowed more leveraged funds; the genie is out of the bottle
  - Do investors understand the risks despite warnings? Buying leveraged funds on margin?
- The gross return of a leveraged ETF is path-dependent
  - under certain conditions can lead to value erosion for a buy-and-hold investor
  - Specifically, erosion occurs with high volatility in underlying index and with inverse ETFs over the longer horizon given positive drift
  - The erosion in value for longer term investors is reinforced by the drag on returns from high transaction costs and tax considerations
- Solutions: Mandate better disclosure? Qualifications? Margin requirements?

Policy implications: Protecting markets

- Regulation is needed not only to protect investors but also others from larger consequences
  - Example: Motorcycle helmet laws exist not only to protect riders but also to mitigate societal costs of injuries
- The daily re-leveraging of leveraged and inverse funds requires the manager to short gamma
  - Trading in the same direction of the market amplifies impact of volume shocks (analogy to portfolio insurance in 1987)
  - Can exacerbate volatility towards the close
  - Counter-party risk considerations
- Predictable flows give rise to concerns
  - No way for regulators to monitor EOD demands
  - Effects greatest for highly levered products on narrow indexes
- Solutions: Better monitoring of hedging demands; more scrutiny of applications on thinly traded indexes

Summary

- Inverse and leveraged ETFs illustrate the importance of careful product development
- Characterized by issues that are often cited as contributors to the financial crisis
  - Sensibility: Does it make sense to run a fund aimed at investors with a one-day horizon?
  - Lack of transparency: How well do investors understand these products? Fund companies?
  - Leverage: Should margin be available to all investors?
  - Systemic risk: Are the broader microstructure implications really understood?