Managing the Risk of Variable Annuities: a Decomposition Methodology

Presentation to the Q Group

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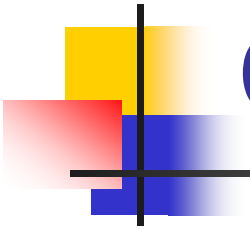
Introduction: Purpose

- Variable annuities: new products for household life-cycle consumption
- Decomposition method: a process to commercialization
- Significant potential market for asset management



Outline

- Describe variable annuities and their guarantees
- Valuation of a guarantee and a variable annuity
- Describe the decomposition methodology
- Optimal hedging results
- Implications to investment management

Description of the Variable Annuities

- $1 trillion market
- Separate account
- Guarantees
  - Guaranteed minimum account value
  - Guaranteed minimum death benefits
  - Guaranteed minimum withdrawal benefits
  - Guaranteed minimum income benefits

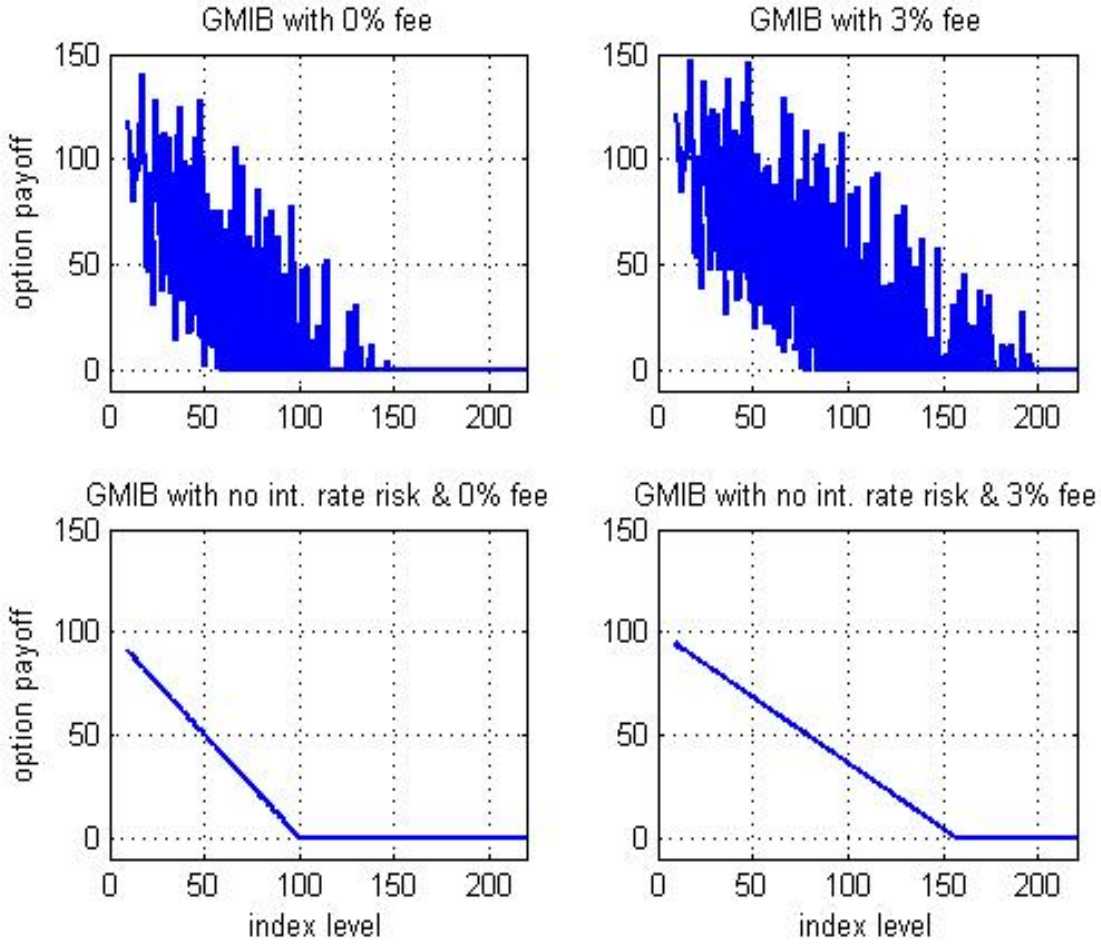
Salient Economic Features

- Fees structure
- Withdrawal penalty
- Customer behavior on lapsation
- Embedded options

Describe Guaranteed Minimum Income Benefits: A model

- No mortality risk, lapsation,
- Interest rate risk and equity risk
- Single premium deposit
- ‘Annuity’ is a zero coupon bond 10 year bond
- 15 year accumulation period

Describe GMIB: Impact of interest rate risk



Payoffs Profile

- GMIB is an equity put option with stochastic strike price
- Fees are positively directly related to the strike of the option
- GMIB as an exchange option
- Importance of the specification of the interest rate model

Valuation of GMIB: Interest rate and equity risk

- The 2-Factor Ho-Lee (2004)
  - Arbitrage-free 2-factor interest rate model
  - No negative interest rates in valuing annuity
  - No explosive interest rates in modeling equity return
  - Recombining lattice
  - Decouples stock return and bond rates
  - Specify the interest rate distribution consistent with the market prices

Two Factor Generalized Ho-Lee Model

$P_{i,j}^{n}(T)$ is the price of a T year bond at time n, at state (i, j)

$$P_{i,j}^{n}(T) = \frac{P\ (n+T)}{P\ (n)} \prod_{k=1}^{n} \frac{(1+\delta_{0,1}^{k-1}(n-k))}{(1+\delta_{0,1}^{k-1}(n-k+T))} \frac{(1+\delta_{0,2}^{k-1}(n-k))}{(1+\delta_{0,2}^{k-1}(n-k+T))} \prod_{k=0}^{i-1} \delta_{k,1}^{n-1}(T) \prod_{k=0}^{j-1} \delta_{k,2}^{n-1}(T)$$

where

$$\delta_{i,1}^{n}(T) = \delta_{i,1}^{n}\delta_{i,1}^{n+1}(T-1)\left(\frac{1+\delta_{i+1,1}^{n+1}(T-1)}{1+\delta_{i,1}^{n+1}(T-1)}\right)$$

and

$$\delta_{i,2}^{n}(T) = \delta_{i,2}^{n}\delta_{i,2}^{n+1}(T-1)\left(\frac{1+\delta_{i+1,2}^{n+1}(T-1)}{1+\delta_{i,2}^{n+1}(T-1)}\right)$$

The one period forward volatilities are defined by

$$\delta_{i,1}^{m} = \exp\left(-2 \cdot \sigma_1(m) \min\left(R_{i,1}^{m}, R\right) \Delta t^{3/2}\right)$$

and

$$\delta_{i,2}^{m}(1) = \delta_{i,2}^{m} = \exp\left(-2 \cdot \sigma_2(m) \min\left(R_{i,2}^{m}, R\right) \Delta t^{3/2}\right)$$

where

$$R_{i,1}^{m} \Delta t = -\log\left(\frac{P(n+1)}{P(n)}\right) + \sum_{k=0}^{n-1} \log\left(\frac{(1+\delta_{0,1}^{k-1}(n-k))}{(1+\delta_{0,1}^{k-1}(n-k+T))}\right) + \sum_{j=0}^{i-1} \delta_{k,1}^{n-1}(T)$$

and

$$R_{i,2}^{m} \Delta t = -\log\left(\frac{P(n+1)}{P(n)}\right) + \sum_{k=0}^{n-1} \log\left(\frac{(1+\delta_{0,2}^{k-1}(n-k))}{(1+\delta_{0,2}^{k-1}(n-k+T))}\right) + \sum_{j=0}^{i-1} \delta_{k,2}^{n-1}(T)$$

Equity Risk

- Equity risk
  - Lognormal
  - Arbitrage-free relative to the short rate
  - Correlations with interest rate risks

$$S(t+1) = S(t)\exp\left(r(t) - 0.5\sigma^2 + \sigma Z\right)$$

Valuation of GMIB: Market Data

- Equity instantaneous volatility: 20%
- USD zero swap curve (July 31, 2002)
- Swaptions Black Vol

Swap Tenor (years)

Option Term (years)	1yr	2yr	3yr	4yr	5yr	6yr	7yr	8yr	9yr	10yr
1yr	44.40	36.50	32.90	29.70	27.60	25.70	25.40	24.00	23.40	23.30
2yr	31.20	28.80	27.10	25.30	24.20	22.90	23.00	21.80	21.30	21.10
3yr	27.00	25.30	24.30	23.10	22.20	21.30	21.10	20.40	20.00	19.90
4yr	24.00	22.80	21.90	21.20	20.70	19.90	19.70	19.10	18.70	18.60
5yr	22.30	21.30	20.80	19.90	19.40	18.70	18.50	18.00	17.70	17.40
7yr	19.80	19.10	18.50	18.10	17.60	16.90	16.70	16.30	16.00	15.80
10yr	17.40	16.40	15.90	15.60	15.10	14.50	14.50	14.10	13.90	13.70

Valuation of GMIB: Scenarios

- 5000 randomly generated interest rate scenarios
- Equity scenarios constructed from each interest rate scenario
- The cost of the guarantee

$$\max(B(T) - V(T), 0)$$

Valuation of VA: Model

- f is the fee (eg 300 bpt)
- P is the single premium
- e is the expenses (eg 60 bpt)
- PV(fees)=(1-(1-f)^n)P
- VA= ((f-e)/f)PV(fees)-GMIB

Valuation of VA: Results

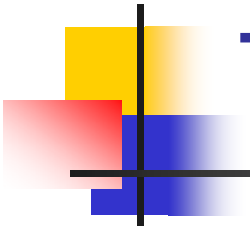
Fees	0%	1%	2%	3%
PV of fees	0.00	5.83	19.06	30.54
GMIB	4.47	5.47	6.57	7.78
VA	-4.47	0.36	12.49	22.76

Decomposition Method

- The need to hedge the guarantees
- Re-insurance
- Dynamic hedging
- Static hedging
- Quasi-static hedging: the decomposition enables us to build the investment benchmarks

Decomposition Method-Description

- Determine the pathwise values
- The Arrow-Debreu state-time securities identify the structured product
- Replicating the pathwise values by benchmark securities
- Risk neutral probabilities and risk-adjusted pathwise value distribution

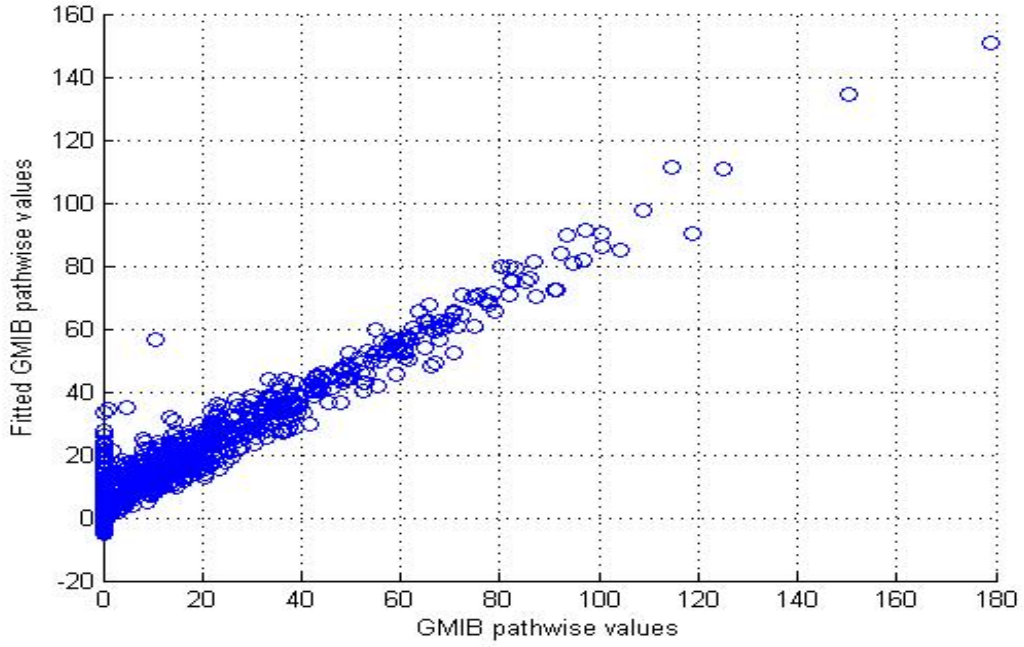


The Procedure

- Determine the pathwise values of the structured product
- Specify the hedging instruments
- Determine the pathwise values of the benchmark securities
- Determine the optimal hedging (replicating) portfolio

GMIB Decomposition Results

- Hedging instruments: 25 puts and calls
- Stepwise regression



Decomposition of GMIB: No interest rate risk

A	B	C	D	E
Hedging Instrument	Strike	Dollar Value	% Position	t-stat
Cash		0	0.00	-5.36
Equity Put	100	3.89	100.00	infinity

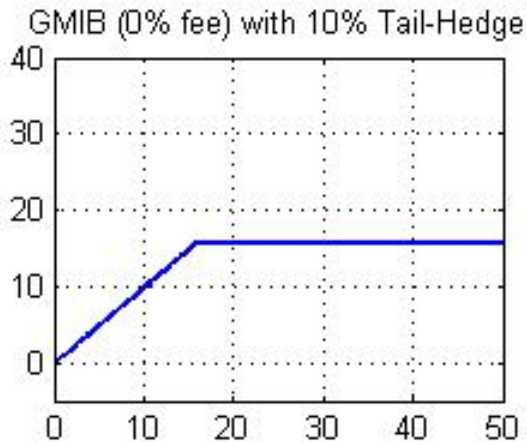
Decomposition of GMIB

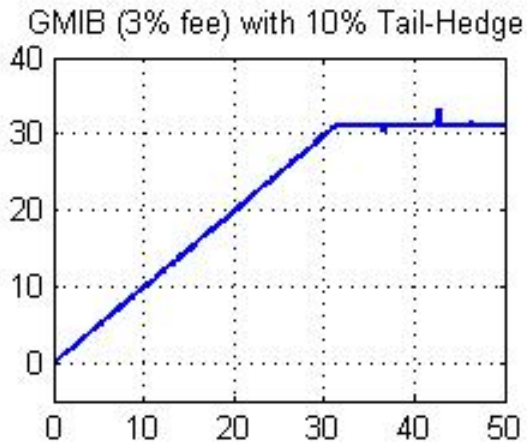
A	B	C	D	E
Hedging Instrument	Strike	Dollar Value	Percentage Position	t-statistic
Cash		2.3	29.56	9.36
Equity Put	80	0.38	4.88	6.06
Equity Put	130	1.96	25.19	4.03
Equity Put	150	1.37	17.61	1.81
Equity Put	185	3.65	46.92	9.14
Bond Call	35	-42.3	-543.70	-16.01
Bond Call	45	37.87	486.76	14.26
Bond Call	75	1.28	16.45	3.06
Bond Call	95	1.03	13.24	6.58
Bond Call	125	0.24	3.08	8.37

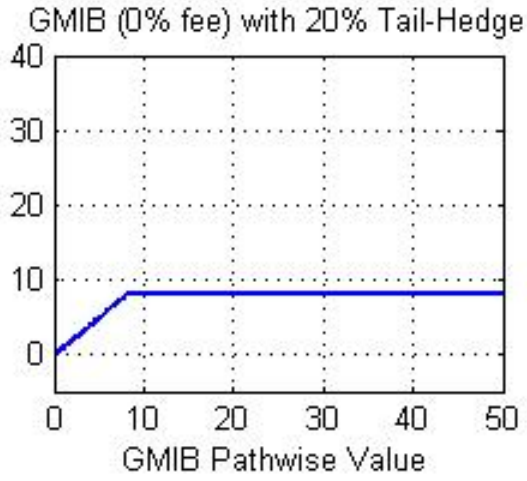
Hedging the GMIB 10% Tail Risk

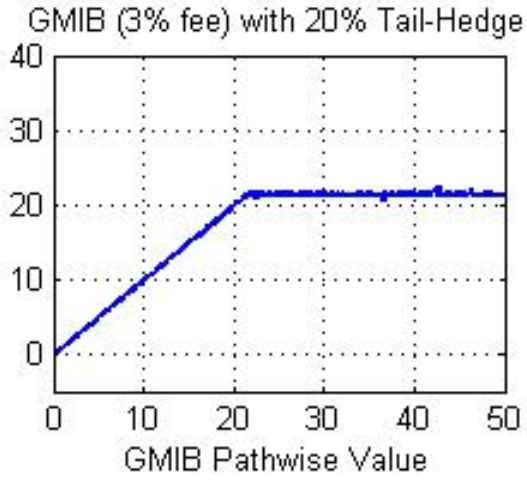
A	B	C	D	E
Hedging Instrument	Strike	Dollar Value	Percentage Position	t-statistic
Cash		1.80	49.72	8.8
Equity Put	110	1.38	38.12	13.14
Equity Put	130	0.70	19.34	25.7
Bond Call	100	1.22	33.70	13.52
Bond Call	70	3.79	104.70	20.31
Bond Call	55	-5.27	-145.58	-15.67

Hedging the GMIB Tail Risk

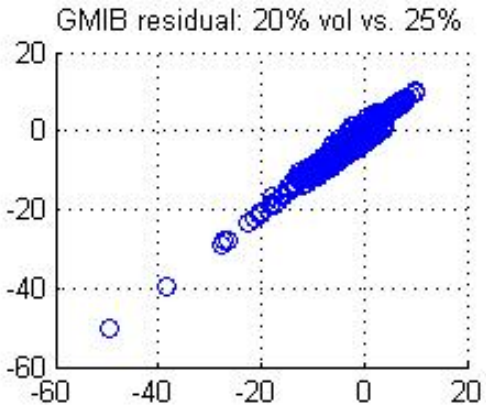


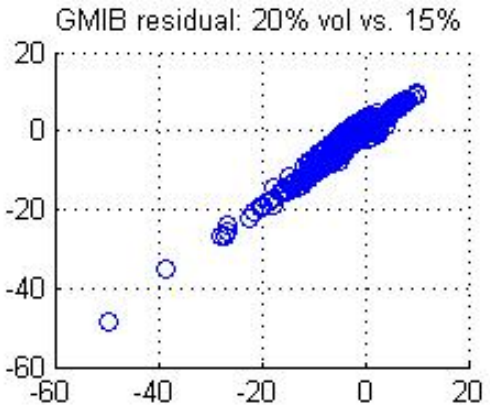


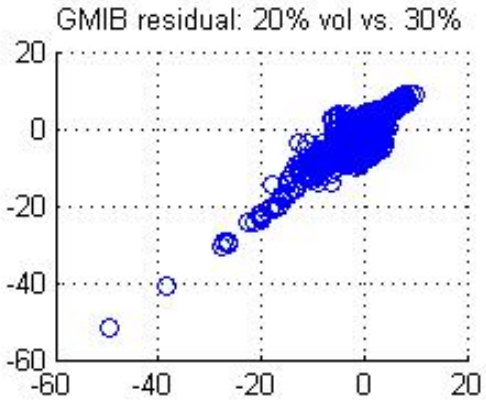


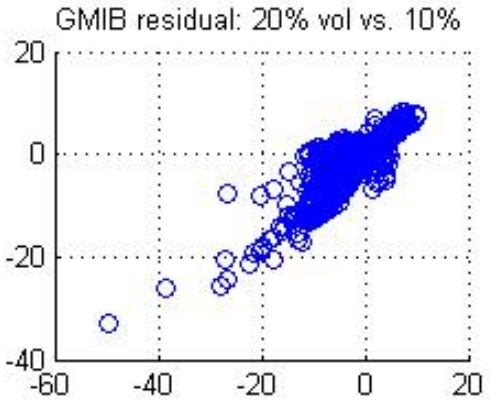


Hedge Stability With Respect to Volatility









Applications of the Decomposition Method to Fixed Income

- Decomposition of a Companion bond
- FHLMC 1747Q = -1.258 cash + TSY – 1.458 C(95-18,8.17%) – 8.325 F(95-08,6.45%) + 6.461 F (95-08, 6.87%)
- Tested on IO, PO, and other CMOs
- Prepayment risk and the extension risk measured by the caps and floors

Applying to the General Account

- Single premium deferred annuities
- Embedded option: lapsation related to the rise of interest rates
- Wallace (1997): implementation of the liability benchmark at Transamerica.

Advantages of the Decomposition

- Clear specification of risks
- Use as benchmarks for the investment process
- Managing the tail risk
- Identify the more complex risks sources: path dependency, inflation

Investment Process

- Investment objectives: Risk and return analysis
  - Determining the tail risk
- Benchmark construction
  - Portfolio strategy: Determining benchmark
- Hedging portfolio revision
  - Investment management: Manage against the benchmark
- Feedback effect
  - Return attribution
- The role of asset management: evaluate the business process of supplying VA

Conclusions: Commercialization

- Innovations and new product designs as riders to variable annuities
- Importance of identifying the investment process
- Price discovery process and completing the market
- Using the economic indices (eg. Inflation, housing value) as hedging instruments
- Significant potential market