Market Equilibrium in a NonCAPM World

Harry M. Markowitz

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1. **Market Efficiency: A Theoretical Distinction and So What?**

For the most part this talk will present material from Markowitz (2005), copy attached. As that paper notes: with the aid of some simplifying assumptions, the Capital Asset Pricing Model comes to dramatic conclusions about practical matters such as how to choose an investment portfolio, how to estimate expected returns and how to value financial assets. The paper illustrates that when one clearly unrealistic CAPM assumption is replaced by a more real-world version, some of the dramatic, practical conclusions of CAPM no longer follow. Specifically, the market portfolio is no longer a mean-variance efficient portfolio, expected returns are no longer linearly related to betas, and there is no representative investor. This has implications for financial practice, research and pedagogy.

2. **JLMSim in its CME (Capital Market Equilibrium) mode of operation.**

The talk will also describe how the CME mode of operation for the JLMSim stock market simulator allows its use to compute market equilibrium expected returns for markets too complex to be solved analytically. Jacobs, Levy and Markowitz 2004 (copy attached) describes JLMSim's original DA (Dynamic Analysis) mode of operation. As such it is an asynchronous, discrete-event, stock market simulator which allows users to specify:

how many securities are in the simulated word,

how many different types of investor's there are,

how many investors of each type,

how often investors of a given type reoptimize,

what is their risk aversion,

whether they can sell short (subject to a Reg-T type constraint), and other constraints on their portfolio choice,

which security analyst do investors of the type use for their mean, variance and covariance estimates,

which type of trader do they use for executing the orders they generate.

Concerning the various security analysts, the JLMSim user specifies rules they use in making estimates. Concerning traders, the user specifies rules they use to set bid and offer prices, and to revise these if orders remain incomplete.

Prices and transactions are endogenous, occurring when buy orders generated by one investor match sell orders generated by another.

The CME (Capital Market Equilibrium) mode of operation was suggested by the Black-Litterman (BL) procedure for estimating expected returns. The BL procedure determines expected returns based on a given covariance matrix, security weights in the market portfolio, and user "views" about some or all of the securities (or asset classes) in the analysis. If the user has no views then the BL procedure returns the CAPM equilibrium expected returns consistent with the specified covariance matrix and market weights.

But if investor constraints in fact are not those of CAPM, the BL procedure gives the wrong answer; i.e., the BL expected returns will typically *not* imply the specified market holdings. Specifically, if borrowing is constrained the BL expected returns will typically imply a greater weight for high-risk securities in the market portfolio than the user specified. The JLMSim CME mode of operation was developed to determine expected returns that would imply specified market weights for a nonCAPM population of investors.

In its CME mode of operation, as in its DA mode, JLMSim allows the user to specify the number of securities in the market, the number of different types of investors, the number of investors of each type, the distribution of their starting wealth, their risk aversion coefficients and constraints on their choice of portfolio. Also, in both modes the user in effect specifies the amount of cash in the system net of any borrowing. In the CME mode the user specifies current interest rates for borrowing and lending. In the DA mode the user has the option of having a simulated "Federal Reserve" change interest rates.

Unlike the DA mode, with the CME mode the user inputs a target market portfolio and an estimated covariance matrix shared by all investors. The objective of the analysis is to find expected return estimates, shared by all investors, which lead to this market portfolio, given the covariance estimates and the specified population of investors.

Unlike the DA mode, which may have many security analysts each with its own method for estimating expected returns and covariances, in the CME mode covariances are an input and there is only one security analyst producing expected return estimates for all investors. This analyst increases the estimated expected return for a security if its share of the market portfolio is below the target weight; decreases it if it is below the target weight. Here "target weight" includes fraction invested in cash (net of borrowing). Thus, if cash is below (above) target in the market portfolio, i.e., the holding of securities in general are above (below) target, then expected returns of securities will tend to be raised (lowered) generally.

As in the DA mode, in the CME mode of operation investors have efficient frontiers computed on the basis of their current estimates of means and covariances
(both shared by all investors in the CME mode); they pick an ideal portfolio depending on their various risk aversion coefficients; decide how far they will attempt to move from their current portfolio towards their ideal; and place the resulting buy and sell orders with traders. An investor's trader checks to see if there are matching orders on each security's buy and sell books, and, if not, places uncompleted orders on the books. This trading process moves securities from investors with one risk aversion to those with others, as appropriate given current expected return estimates. As part of the process, prices are set (where orders cross) and this in turn implies market portfolio weights. As noted, expected return estimates are altered to drive market weights towards target weights. Thus, in the CME mode the entire estimation/optimization/trading process in effect constitutes an algorithm for determining expected returns that imply prescribed market portfolio weights consistent with a given covariance matrix and population of investors. We find that this "algorithm" converges quite rapidly.

One major difference between the CME mode and the DA mode is in the handling of deposits and withdrawals. In its DA mode of operation JLMSim can be run with no deposits and withdrawals or these can be generated randomly. In the CME mode, on the other hand, at the end of each day the gains of each investor is withdrawn, or any losses replaced by a deposit, bringing the investor's wealth back to its original level. Thus the equilibrium prices reached by the "algorithm" are consistent with the specified population of investors, including their respective wealth levels, as well as the specified covariance matrix and market weights.

References:

Jacobs, Bruce I., Kenneth N. Levy, and Harry M. Markowitz. (2004) *Financial Market Simulation* Journal of Portfolio Management (30th Anniversary): pp. 142-152

Markowitz, Harry M. (2005) *Market Efficiency: A Theoretical Distinction and So What?* Financial Analysts Journal Vol. 61, (5), pp.17-30, September/October