Diversifying Across Time

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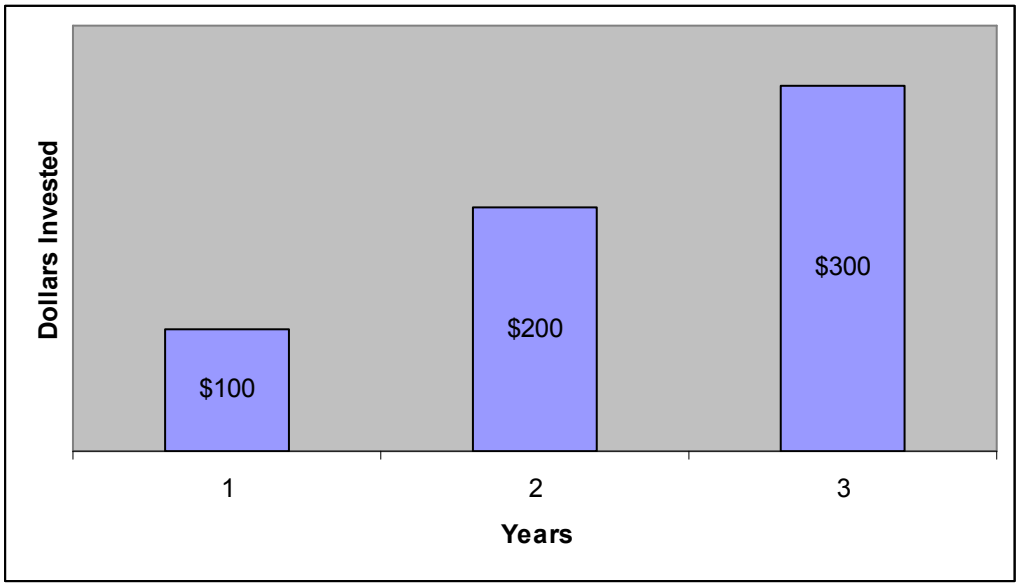
The Plan

- Why it works in theory
- Why it works in practice
- Objections
  - Margin rate
  - Why Stop at 2:1?
  - Human Capital
  - Consume first, save later
  - Psychology
- More Evidence
  - Other countries
  - Monte Carlo

Which is Safer?

Investing

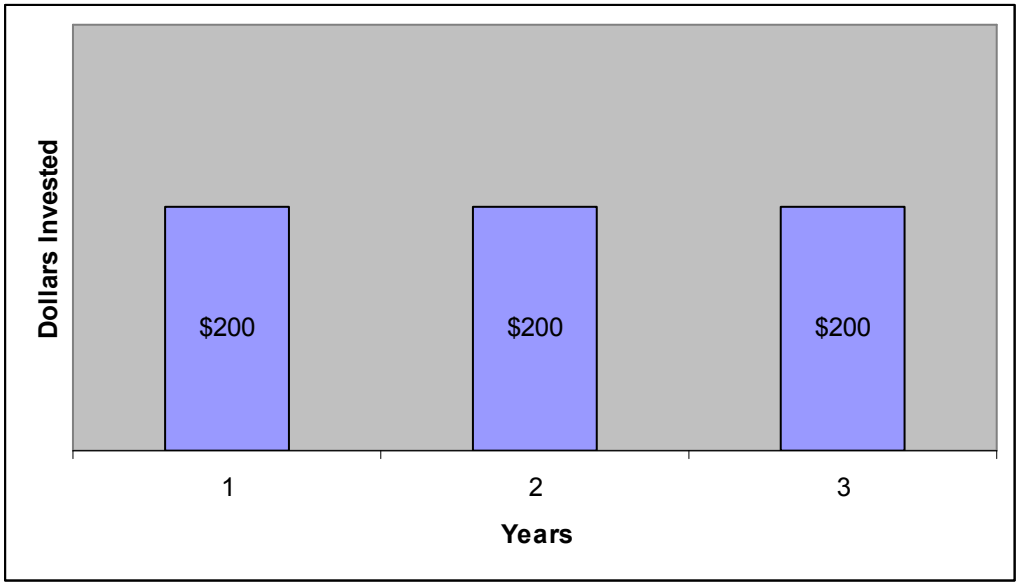
$100, $200, $300



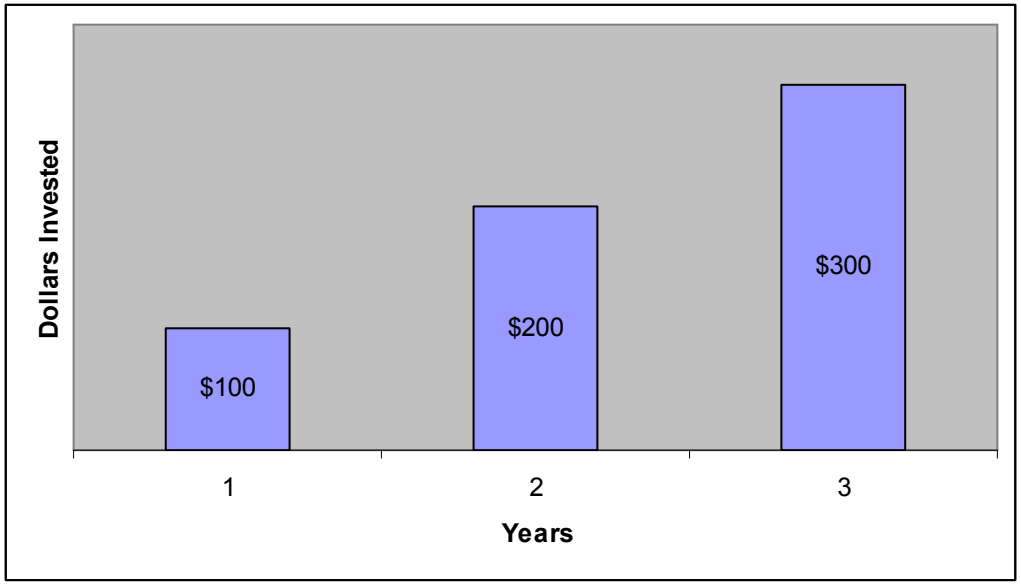
Or …

Same Dollar Years

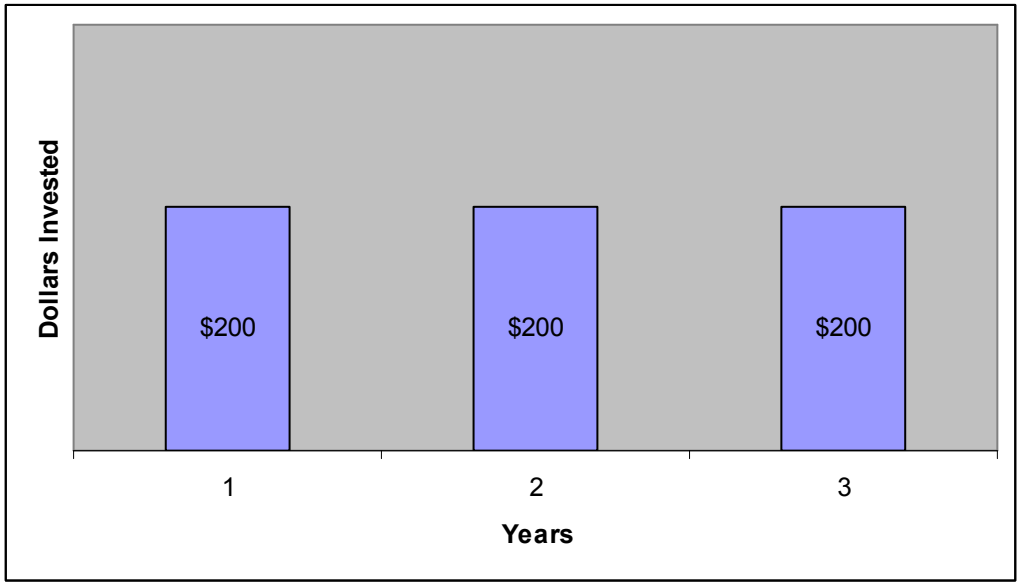
$200, $200, $200



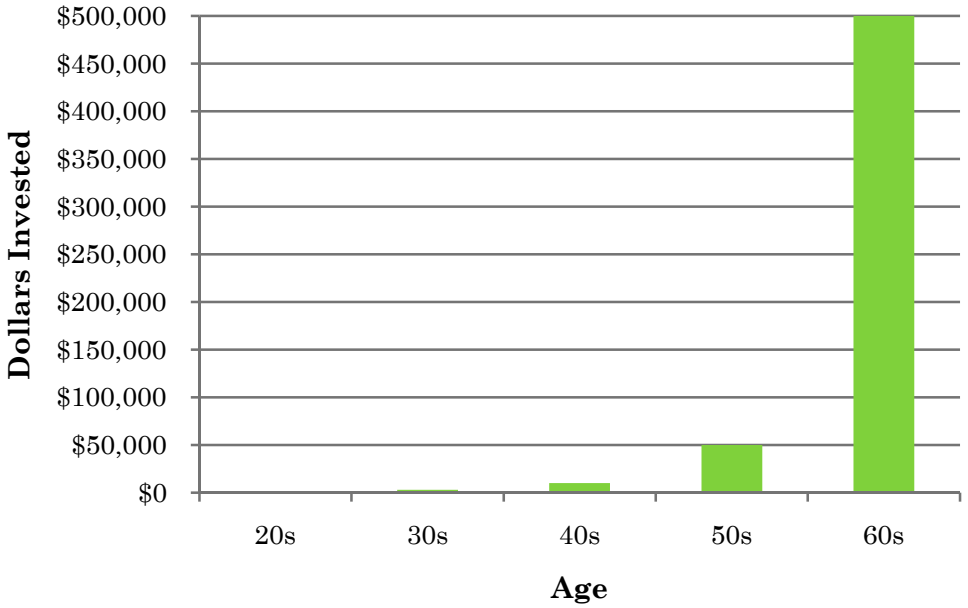
Variance is
$[1^2 + 2^2 + 3^2]\sigma^2$
$= 14\sigma^2$



Or…
Variance is
$[2^2 + 2^2 + 2^2]\sigma^2$
$= 12\sigma^2$

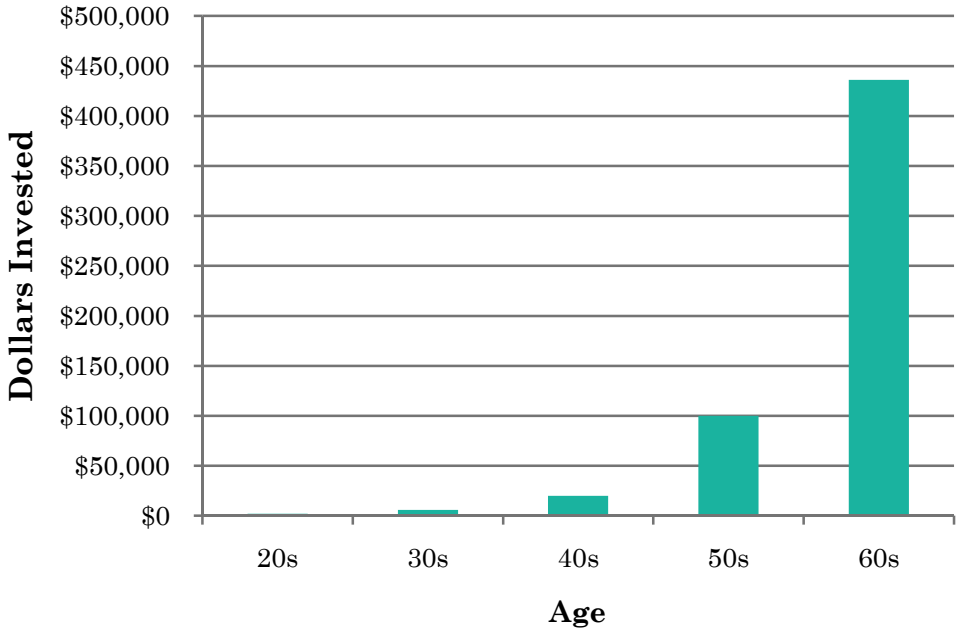


More realisticly
1k, 3k, 10k,
50k, 500k
Variance is $253\sigma^2$



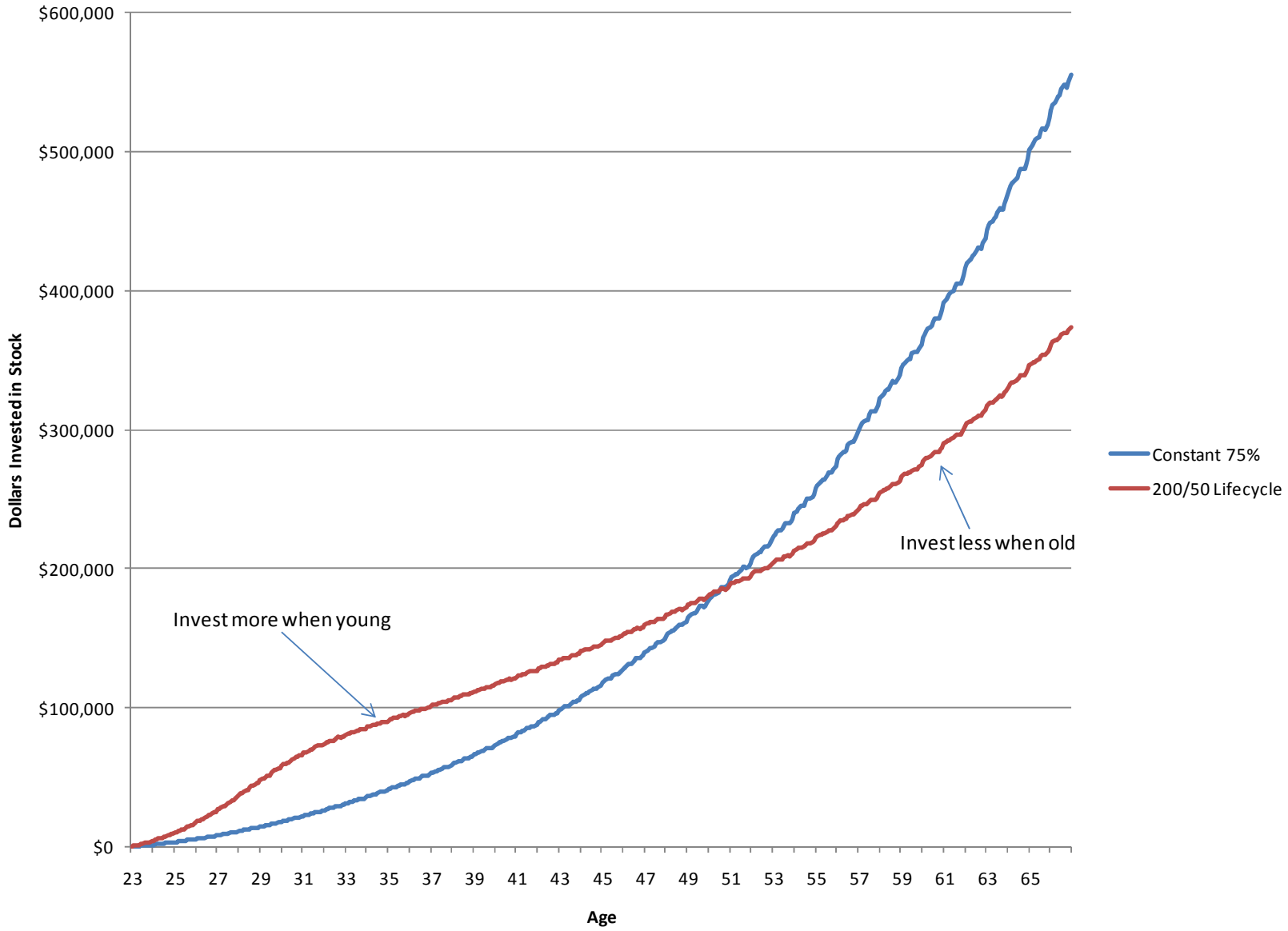
Or…
With leverage
2k, 6k, 20k,
100k, 436k
Variance is $200\sigma^2$

Again, same dollar years



Total Dollar Years

TWO PATHS

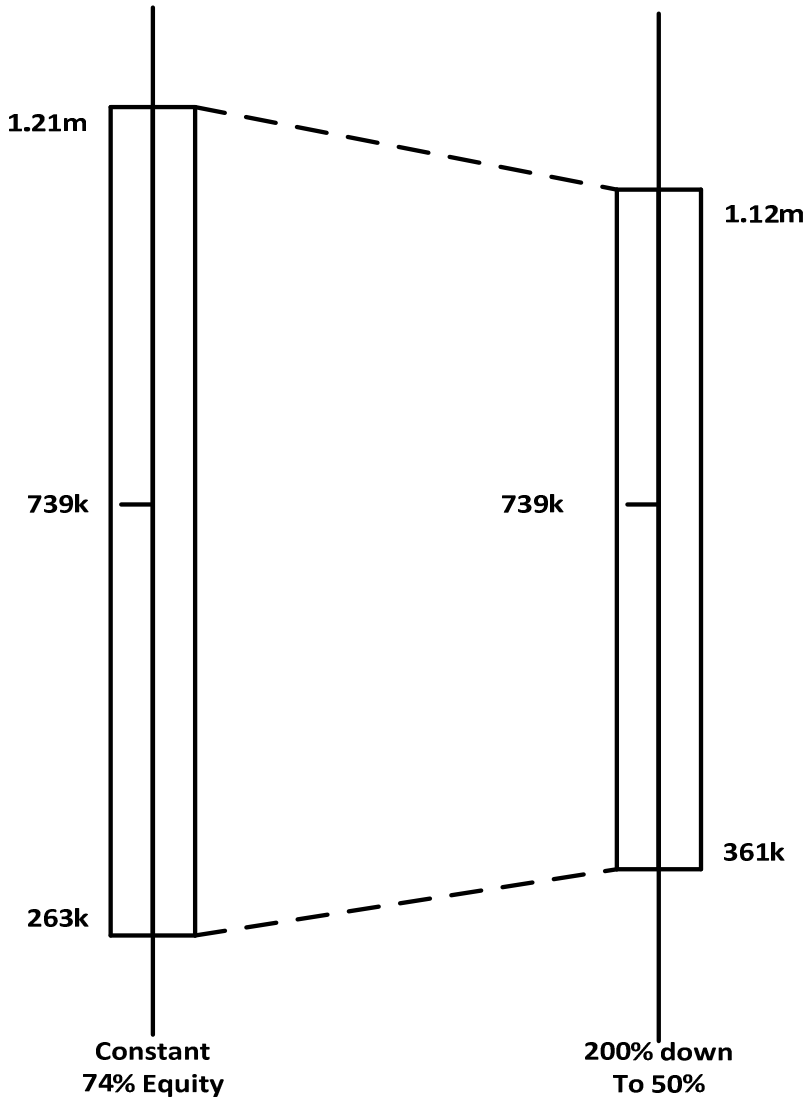

$600,000
$500,000
$400,000
$300,000
$200,000
$100,000
$0
Dollars Invested in Stock
23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65
Age
Invest more when young
Invest less when old
Constant 75%
200/50 Lifecycle

Results from Equally-Aggressive Investing, 1871-2009 (96 simulated investing cohorts)

	200/50 Strategy	74/74 Strategy	Improvement
Max % inv.	200%	74%	
Min % inv.	50.0%	74%	
Mean	$738,684	$738,684	0.0%
St. Dev.	$192,720	$242,833	-20.6%

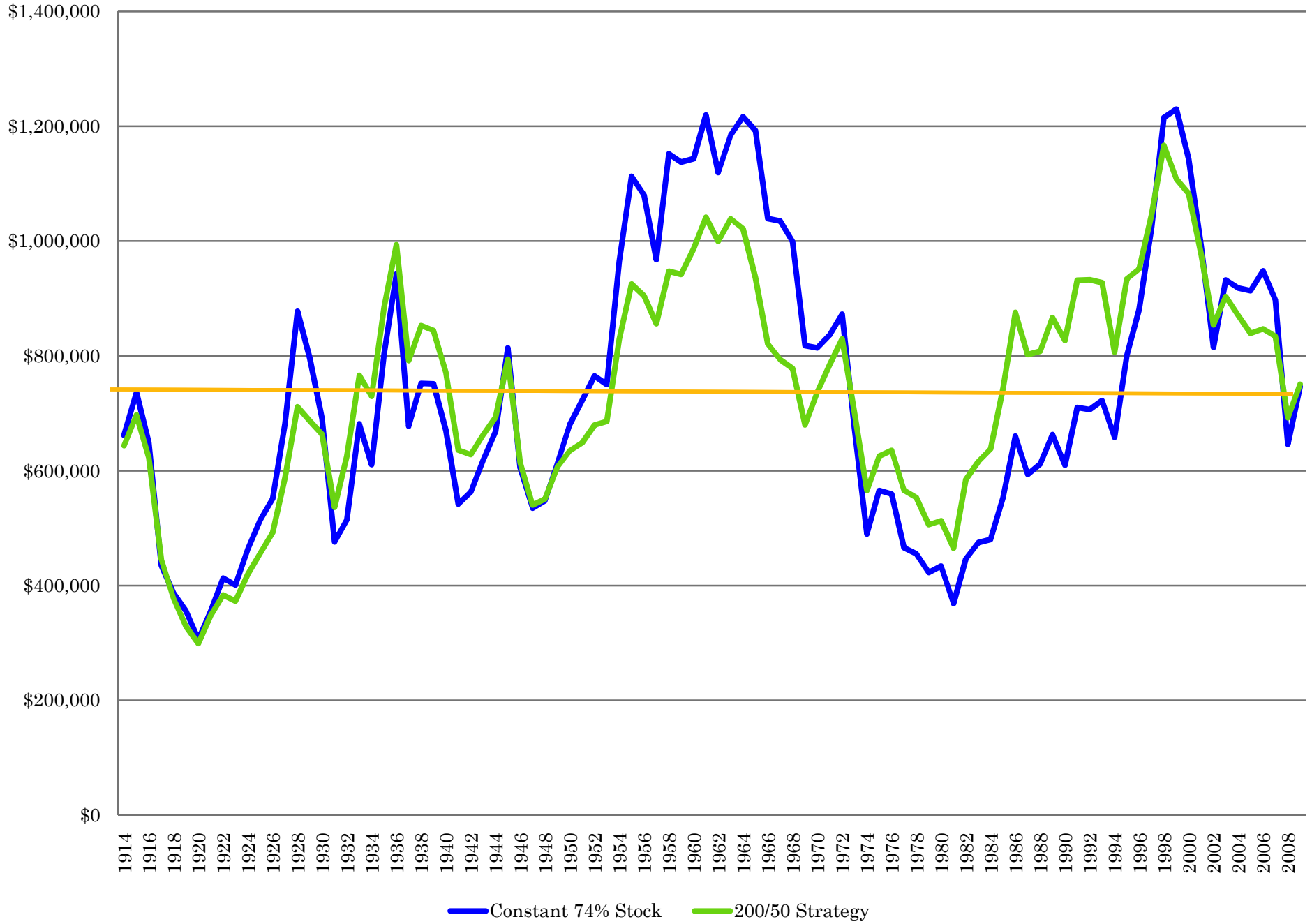
Not the Equity Premium Driving the Results

Same Returns with Lower Risk



Standard Deviation is 21% smaller

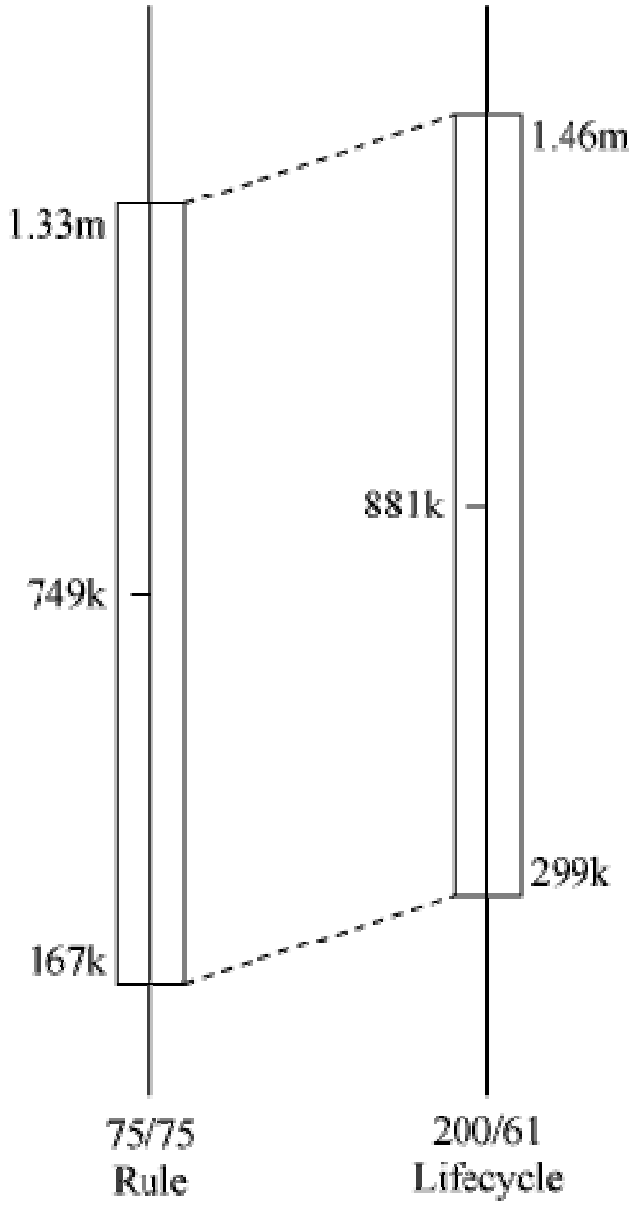
Final Retirement Accumulation by Year



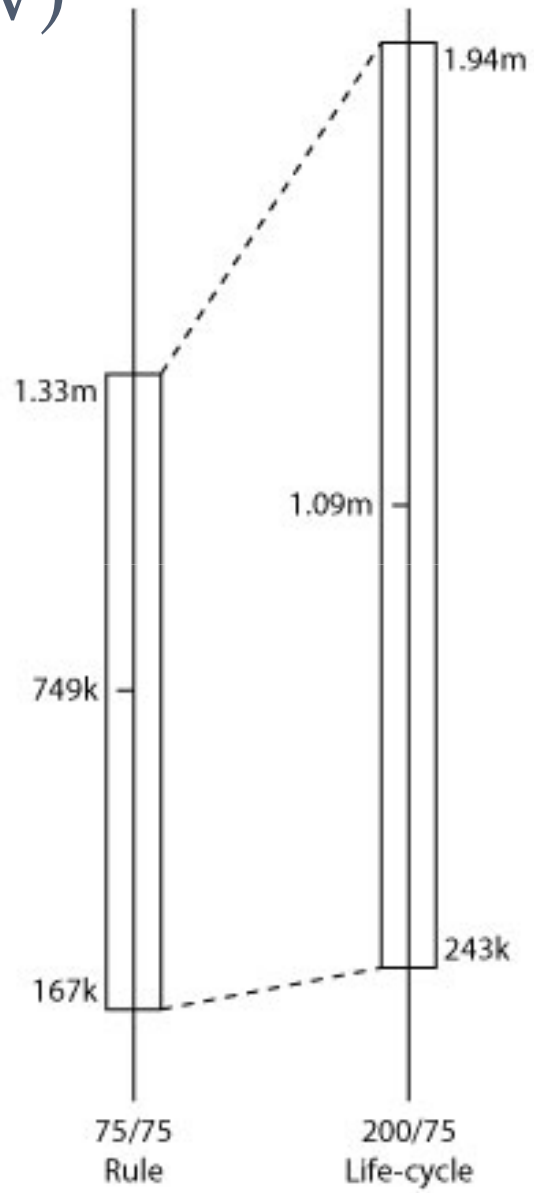
How to Take Advantage of Time Diversification

- 1. Lower risk
- 2. Increase equity exposure – come closer to optimal Merton-Samuelson allocation

Same Absolute Risk with Higher Returns



Same Relative Risk (CoV) with Higher Returns



THE THEORY

- Paul Samuelson and Robert Merton (1969)
- Say you had all your life savings in cash at age 25. How would you invest it in the market?
- Given constant relative risk aversion, pick constant allocation between stocks and bonds, say 50-50, and hold that same allocation over your lifetime.

THE NITTY-GRITTY

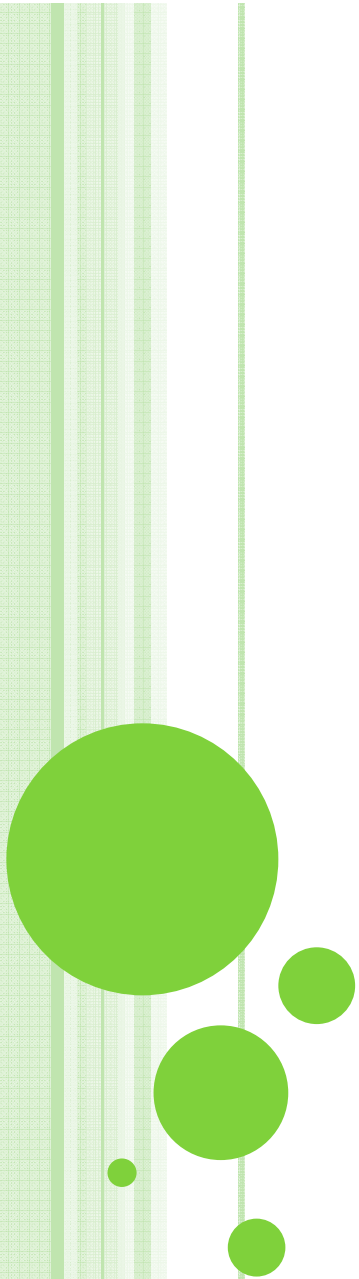
- Merton-Samuelson share depends on three things:
  1) your risk aversion
  2) your beliefs about future stock market returns
  3) your beliefs about future stock market risk

  MS-share = Equity Premium / [RISK^2 * RRA]

- Say Equity Premium = 3.82%, σ=14.29%, RRA =4
- => Target investment is 50%.
  - 50% of your lifetime savings
  - But what if you don't have cash upfront?

APPLYING THIS THEORY TO REALITY

- Problem is young people don't have all their money today
- When young, most of capital is human capital, which gets converted to financial capital.
- Young investors are holding a giant bond in the form of their human capital.



(Dis)count your Chickens Before they Hatch

APPLYING THIS THEORY TO REALITY

- A 40-year old professor with
  - PV future savings = $1 million
  - Current savings = $200k

- Want to allocate 50% of savings to stock
  - Should be ½ of 1.2m, not of 200k
  - Problem is that she doesn't have 600k to invest
  - Use 2:1 leverage to put 400k in the market

  - Even if goal is 30% in stocks
  - 30% of 1.2m, not of 200k
  - Still use 180% leverage to put 360k in the market

Money is fungible

- Inheritance
- Social Security

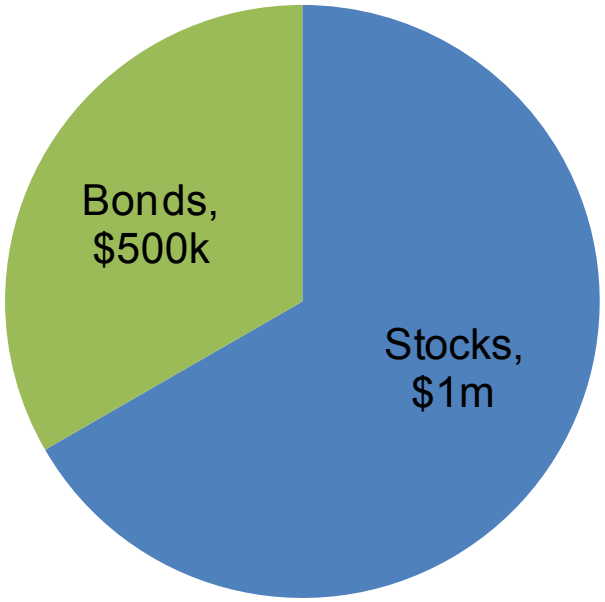
- You can (partially) undo what others do using the part you control

EXAMPLE 1

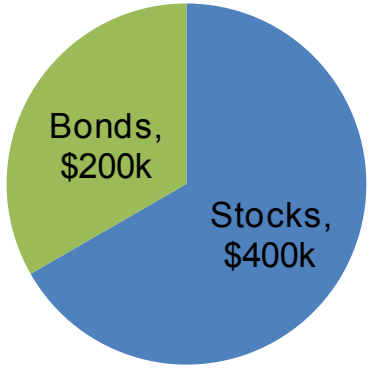
- Your Samuelson share is 67%
  - $1 million in stocks
  - $500k in bonds
- Grandma has $600k in bonds she will be leaving you
- What do you do?

Option 1. Convince Grandma to buy $400k of stocks

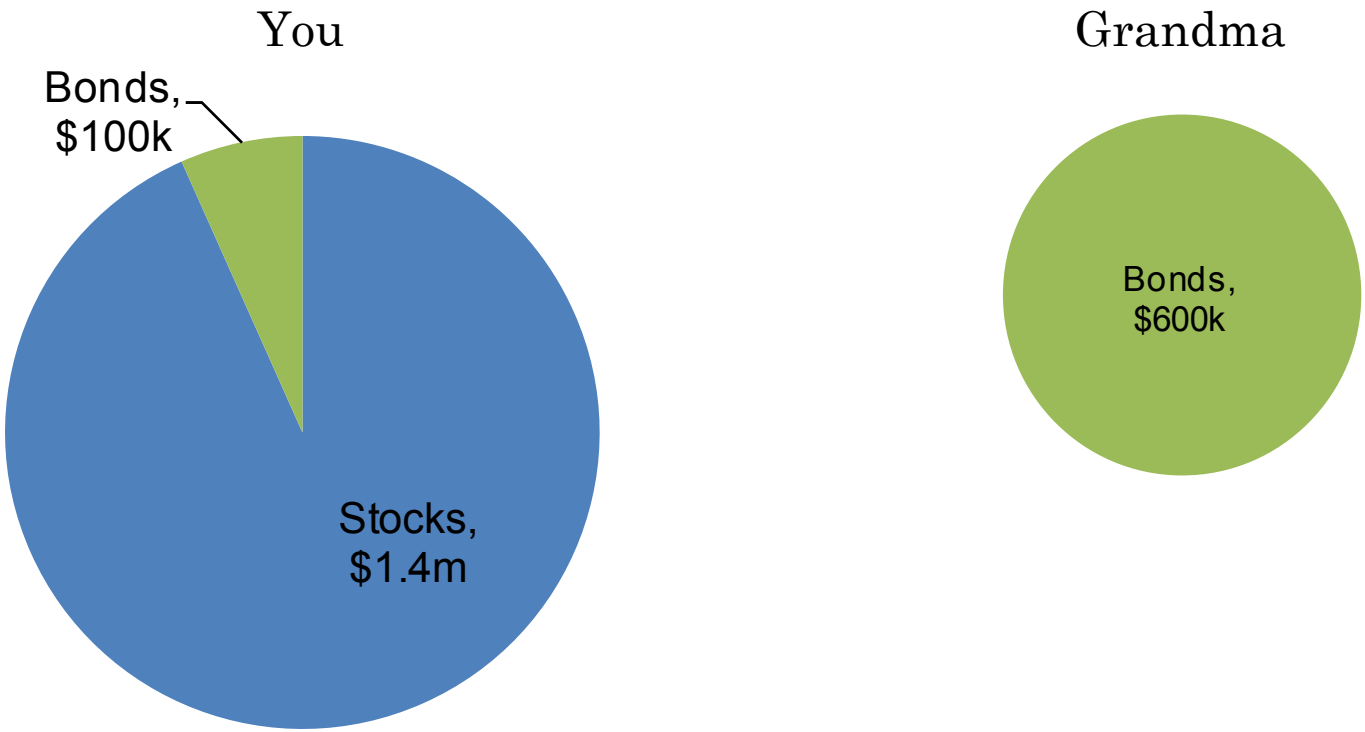
You



Grandma



Option 2. Convert $400k of your bonds to stocks

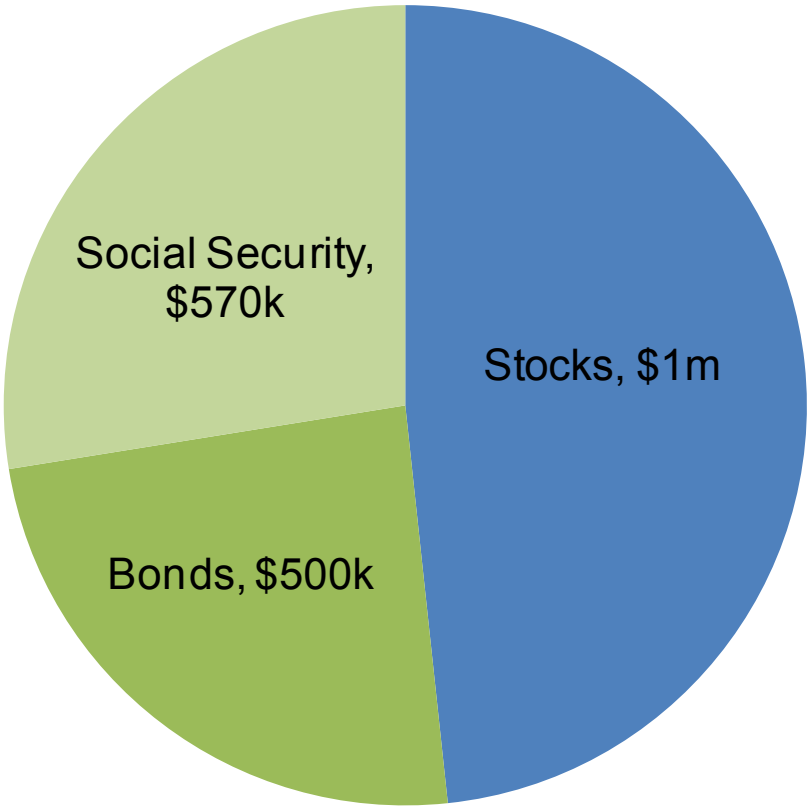


- Now you will have $1.4m in stocks, $100k in gov. bonds, and $600k in granny bonds. Overall, 67% in stocks.

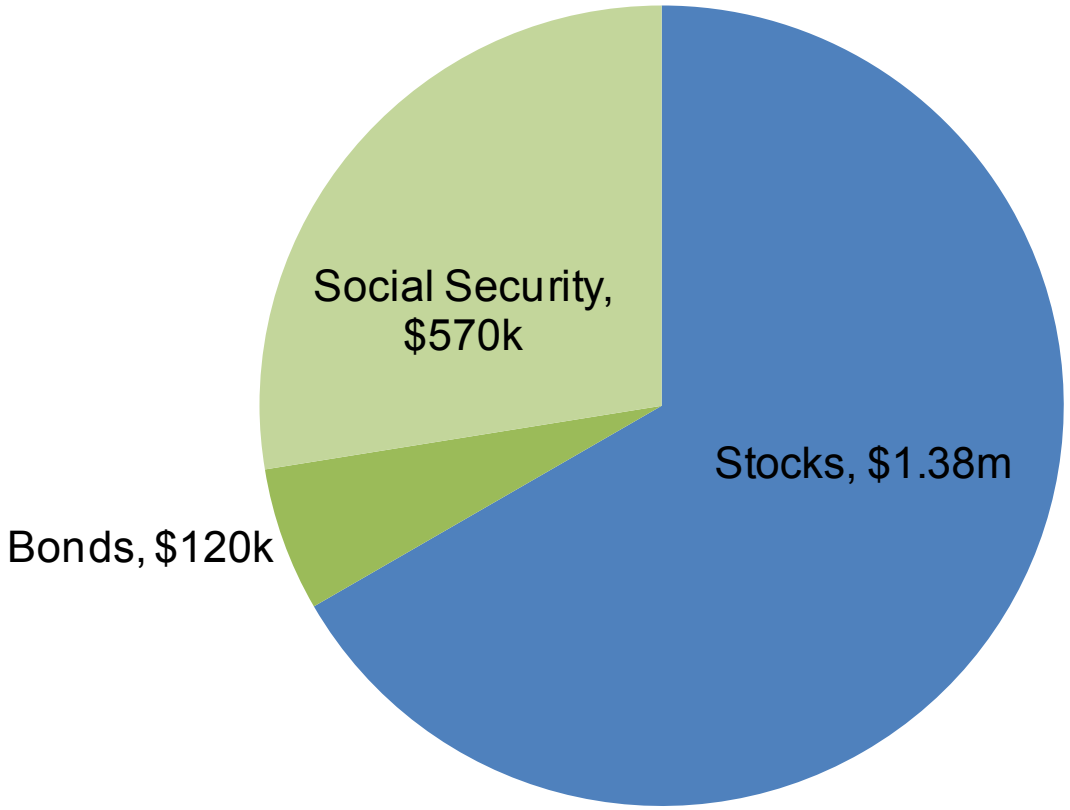
EXAMPLE 2

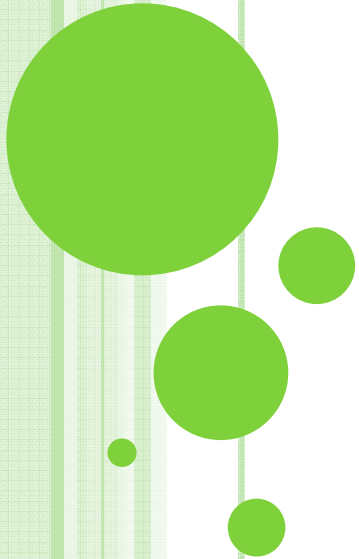
- Your Samuelson share is 67%
  - $1 million in stocks
  - $500k in bonds
- Social Security will provide $30k per year (inflation adjusted)
- An annuity that provides an equivalent inflation-adjusted income would cost $570,000.

Option 1. Ignore social security: really only 1/2.07 = 48% in stocks



Option 2. Convert $380k of your bonds to stocks—Essentially investing 67% of social security annuity.





Historical Simulations

Table IV: Risk Reduction for Leveraged Lifecycle Strategy Relative to Mean-Preserving Constant Strategy for Different Levels of Risk Aversion

	200/λ Strategy	Mean Retirement Wealth	Standard Deviation	**Constant % Stock Strategy**	Mean Retirement Wealth	Standard Deviation	St. Dev. Change from Constant % to 200/λ
CRRA = 2	200/100	$1,496,382	$689,216	128/128	$1,496,382	$946,152	-27.2%
CRRA = 3	200/65	$931,746	$275,432	91/91	$931,746	$377,644	-27.1%
CRRA = 4	200/50	$738,684	$192,720	74/74	$738,684	$242,833	-20.6%
CRRA = 5	200/40	$625,244	$162,116	61/61	$625,244	$185,340	-12.5%

Note: In each row, the 200/λ strategy is derived from the CRRA-contingent Merton-Samuelson share (Equation 1), and the associated λ/λ strategy is the constant strategy that produces the same mean accumulation.

Table V: Utility Gain from Time Diversification for Different Levels of Risk Aversion					
	200/λ Strategy	Cert. Eq. Wealth	**Constant % Stock Strategy**	Cert. Eq. Wealth	Improv. Of 200/λ over Constant %
CRRA = 2	200/100	$1,193,081	128/128	$1,085,252	9.9%
CRRA = 3	200/65	$783,261	91/91	$748,006	4.7%
CRRA = 4	200/50	$609,431	74/74	$589,107	3.4%
CRRA = 5	200/40	$501,269	61/61	$484,051	3.6%
Note: Table adjusts relative risk aversion and presets improvement in real return of target strategy over constant percentage strategy.					

SAME MEAN. ALL GAINS ARE FROM BETTER DIVERSIFICATION

Table VI: Comparison of Certainty Equivalent Wealth for CRRA-Optimal Leveraged Lifecycle Relative to Myopic Merton-Samuelson and Target-Date Strategies for Different Levels of Risk Aversion

	200/λ Strategy	Certainty Equivalent Wealth	**Constant % Stock Strategy**	Certainty Equivalent Wealth	**Traditional Lifecycle Strategy**	Certainty Equivalent Wealth	Improvement of 200/λ over Constant %	Improvement of 200/λ over 90/50
CRRA = 2	200/100	$1,193,081	100/100	$873,018	90/50	$598,733	36.7%	99.3%
CRRA = 3	200/65	$783,261	65/65	$562,765	90/50	$572,631	39.2%	36.8%
CRRA = 4	200/50	$609,431	50/50	$444,226	90/50	$546,634	37.2%	11.5%
CRRA = 5	200/40	$501,269	40/40	$371,378	90/50	$521,853	35.0%	-3.9%

Note: In each row, the 200/λ strategy is derived from the CRRA-contingent Merton-Samuelson share (Equation 1), and the associated λ/λ strategy assumes investors myopically invest the CRRA-contingent Merton-Samuelson share through their working lives.

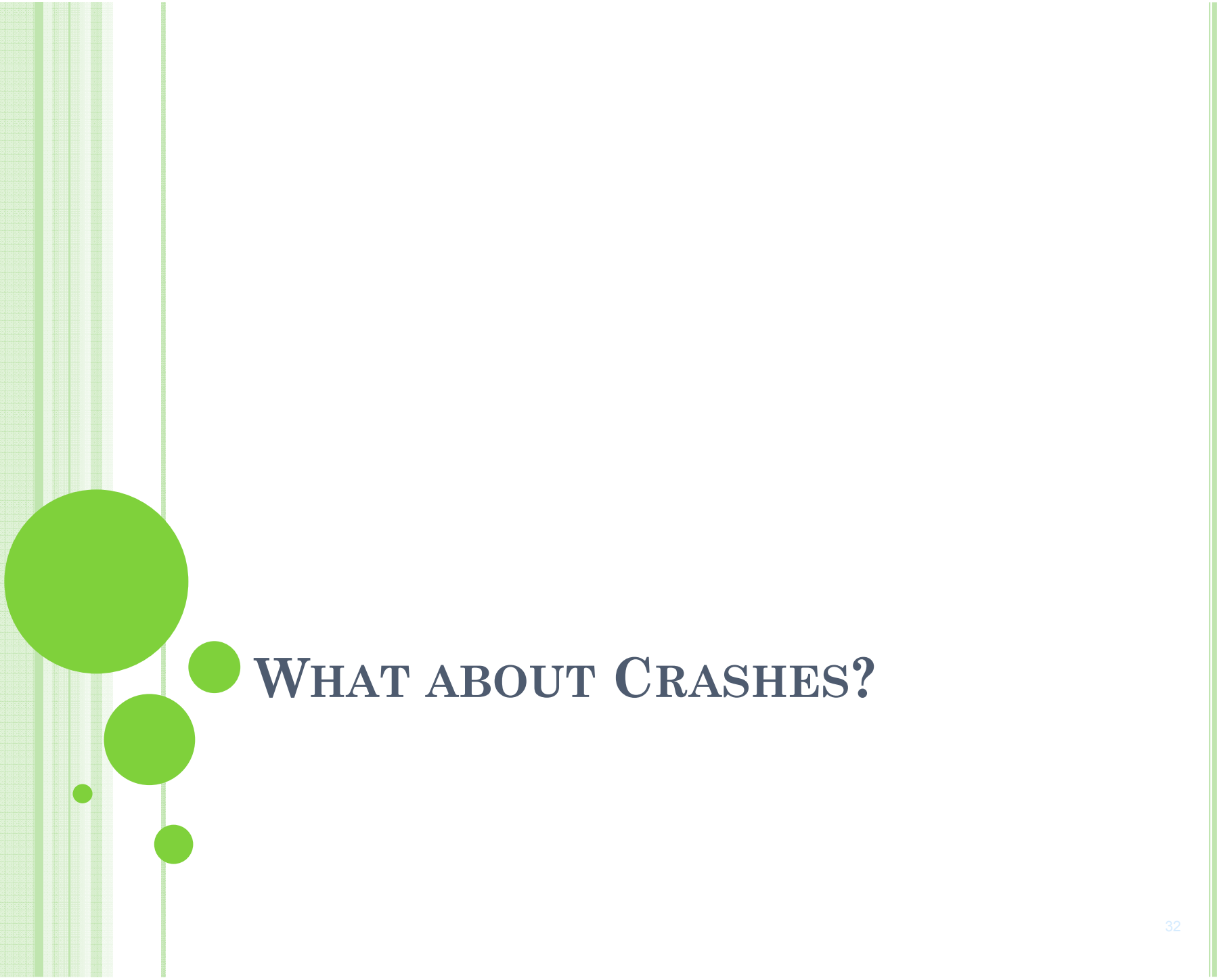
COMPARISON TO LITERATURE

Poterba, Rauh, Venti, and Wise show target date beats equal mean constant share by 3.2%--6% (depending on profession). We show $200/\lambda$ beats target date by 3.5%.

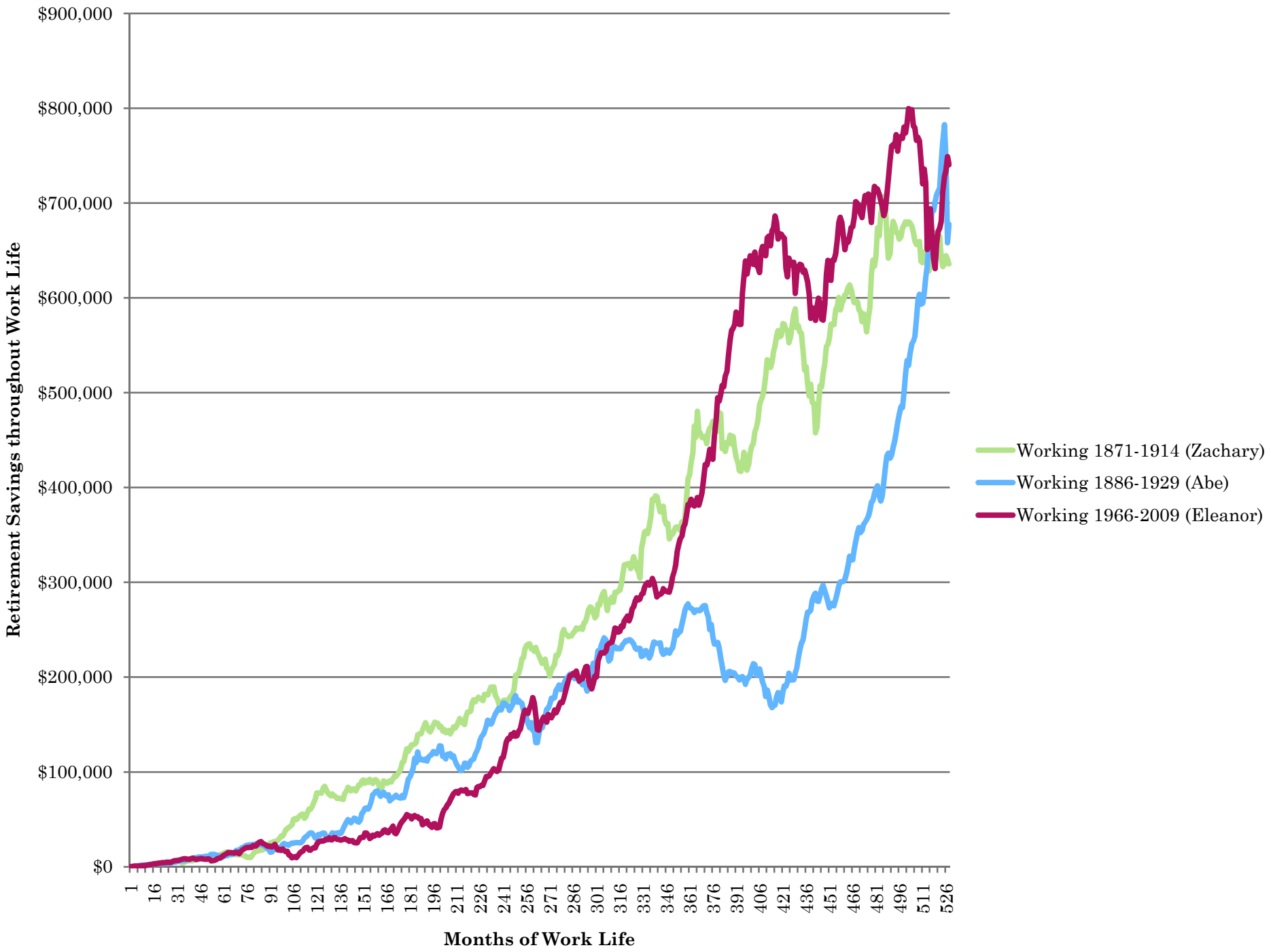
Willen and Kubler find a 1.2% gain in CE relative to the case w/o leverage. Discounted to today and don't start investing till early 50's

Objections

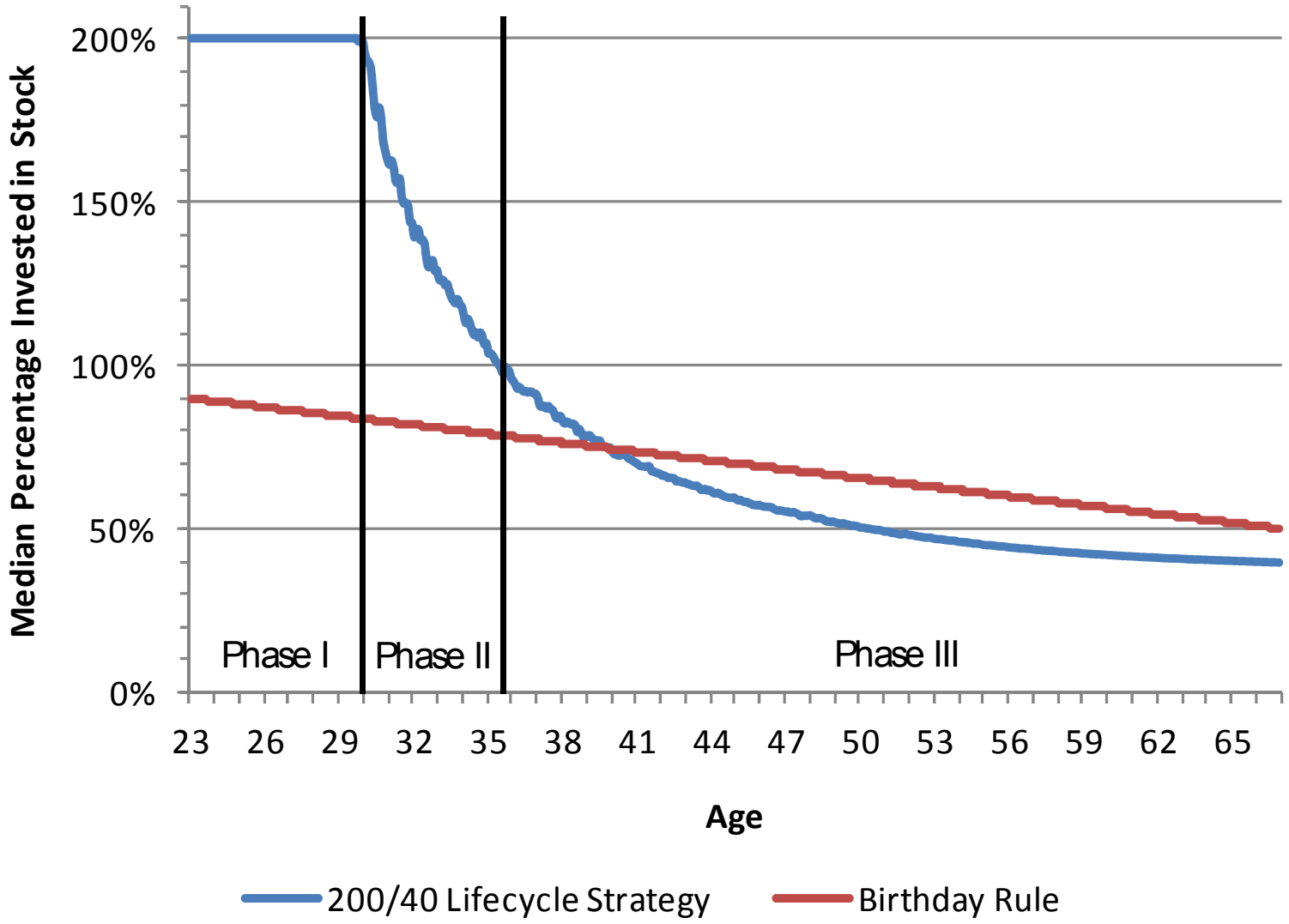
1. What about crashes?
2. Aren't margin loans expensive?
3. Why stop at 2:1?
4. Human capital may be like a stock
5. Consume when young
6. Psychology
7. Last century was special



What about Crashes?


$900,000
$800,000
$700,000
$600,000
$500,000
$400,000
$300,000
$200,000
$100,000
$0
Retirement Savings throughout Work Life
1
16
31
46
61
76
91
106
121
136
151
166
181
196
211
226
241
256
271
286
301
316
331
346
361
376
391
406
421
436
451
466
481
496
511
526
Months of Work Life
Working 1871-1914 (Zachary)
Working 1886-1929 (Abe)
Working 1966-2009 (Eleanor)

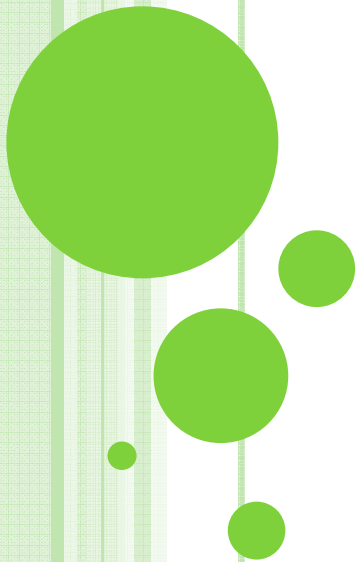
THREE PHASES: 2:1 IS LIMITED



THE COST OF BORROWING?

Competitive Margin Rate Is (Appropriately) Low

- Vanguard and Fidelity charge 7% or 8% or more
- Go Elsewhere!
- Interactive Brokers charges 0.71% to 1.7%.
- Margin lender has 2:1 and can call loan
- In last 138 years, the wholesale loan rate to brokers was 34 basis points above T-bill rate.
- In January 2009, implicit prepaid interest on a LEAP was 0.8%



Why Stop at 2:1?

WHY STOP AT 2:1

Cost of incremental leverage is too expensive.

Table I : Implied Interest Rates for 1 Yr. S&P Calls 1996--2009					
Range of "Leverage" Ratios	Contracts Observed	Average Implied Interest Rate	Mean Spread Over Call-Money Rate	Mean Spread over 1-Year Treasury Note	Marginal Interest Rate at Mean Spread
1.5-2.5	2,202	4.64%	0.26%	1.89%	
2.5-3.5	3,347	5.99%	1.10%	2.79%	8.01%
3.5-4.5	3,071	7.54%	2.47%	4.19%	11.40%
4.5-5.5	2,559	8.65%	3.47%	5.20%	12.53%
5.5-6.5	2,236	9.87%	4.67%	6.42%	15.38%
Option close prices from CRSP used for LEAPS on the S&P 500, between 11 and 12 months to maturity. Implied leverage ratios and implied interest rates were calculated for each contract and then grouped by leverage ratio.					

Human Capital is Correlated to Market

Are you a Stock or a Bond? – Milevsky
Strategy varies by profession

Benzoni, Collin-Dufresne, and Goldstein: A risk-averse (CRRA=5) young worker may actually want to *short* equities

Heaton and Lucas:
Based on 1979–1990 panel of tax returns

33% have negative correlation between wages and the market

33% have correlation between –0.25 and 0.25.

10% have correlation above 0.50.

Don't buy stocks: Consume more when young

- employer matching and tax incentives.
- How to invest what people save.

Psychology: Once burnt twice shy

- needs to be social norm
- new type of target date fund

Not right for everyone!

LAST CENTURY WAS SPECIAL

- Look at other countries
- Monte Carlo simulations
  - Comparison to traditional target date
  - Look at meaner mean
  - Greater volatility

Table VII: Comparison of 200/50 Strategy to 3 Traditional Strategies Invested in English and Japanese Equities							
	11 Nikkei 225 Cohorts (1956 - 2009) (in 100s Yen)						
	200%/50% Lifecycle strategy	90%/50% Strategy (Birthday Rule)	Constant 74%/74% Strategy	Constant 50%/50% Strategy	200/50 Improvement Over 90/50	200/50 Improvement Over 74/74	200/50 Improvement Over 50/50
Max % inv.	200%	90%	74%	50%			
Min % inv.	50.0%	50%	74%	50%			
Mean	¥66,154,903	¥49,890,018	¥45,584,137	¥40,113,979	32.60%	45.13%	64.92%
St. Dev.	¥33,794,530	¥21,208,742	¥21,305,413	¥12,278,965	59.34%	58.62%	175.22%
Min	¥35,008,794	¥27,056,403	¥22,216,969	¥25,130,798	29.39%	57.58%	39.31%
10th pct	¥38,165,700	¥30,803,412	¥25,385,949	¥27,726,040	23.90%	50.34%	37.65%
25th pct	¥43,281,734	¥34,317,473	¥29,711,951	¥30,718,952	26.12%	45.67%	40.90%
Median	¥51,807,633	¥39,831,052	¥38,051,375	¥36,174,717	30.07%	36.15%	43.22%
75th pct	¥74,911,987	¥56,957,418	¥52,248,423	¥45,143,533	31.52%	43.38%	65.94%
90th pct	¥111,268,034	¥82,258,110	¥79,340,443	¥59,548,896	35.27%	40.24%	86.85%
Max	¥154,153,476	¥95,644,253	¥88,764,744	¥62,800,897	61.17%	73.67%	145.46%
Certainty Equivalent (CRRA = 4)	¥49,538,693	¥39,205,097	¥33,768,991	¥34,731,055	26.36%	46.70%	42.64%
	30 FTSE All-Shares Cohorts (1937 - 2009)						
	200%/50% Lifecycle strategy	90%/50% Strategy (Birthday Rule)	Constant 74%/74% Strategy	Constant 50%/50% Strategy	200/50 Improvement Over 90/50	200/50 Improvement Over 74/74	200/50 Improvement Over 50/50
Max % inv.	200%	90%	74%	50%			
Min % inv.	50.0%	50%	74%	50%			
Mean	£591,223	£483,905	£543,146	£404,052	22.18%	8.85%	46.32%
St. Dev.	£128,481	£121,513	£149,940	£103,227	5.73%	-14.31%	24.46%
Min	£331,588	£251,046	£260,363	£205,032	32.08%	27.36%	61.72%
10th pct	£405,028	£303,145	£330,573	£249,951	33.61%	22.52%	62.04%
25th pct	£533,824	£407,786	£466,132	£341,575	30.91%	14.52%	56.28%
Median	£592,006	£499,923	£548,509	£414,580	18.42%	7.93%	42.80%
75th pct	£677,071	£582,586	£652,502	£488,605	16.22%	3.77%	38.57%
90th pct	£750,829	£617,616	£732,984	£518,894	21.57%	2.43%	44.70%
Max	£831,811	£688,049	£847,159	£579,721	20.89%	-1.81%	43.48%
Certainty Equivalent (CRRA = 4)	£522,410	£408,447	£444,194	£337,928	27.90%	17.61%	54.59%

Table VIII: Monte Carlo Comparison of Target-Date (90/50) Strategy with Mean-Preserving Constant and Leveraged Lifecycle Strategies

	90/50 Strategy	65/65 Strategy	200/43 Strategy	Improvement of 200/43 over 90/50	Improvement of 200/43 over 65/65
Mean	$700,857	$700,857	$700,857	0.00%	0.00%
St. Dev.	$305,570	$309,800	$270,414	-11.51%	-12.71%
Certainty Equivalent (CRRA = 4)	$520,299	$510,528	$538,790	3.55%	5.54%

Note: Based on 10,000 Monte Carlo draws from a log-normal distribution with mean 6.61% and st.dev of 14.29%

Table IX: Monte Carlo Comparison of Certainty Equivalent Wealth Across Varying Mean Real Returns

Real Stock Return	Certainty Equivalent of 90/50	Certainty Equivalent of 74/74	Certainty Equivalent of 200/50	Improvement of 200/50 over 90/50	Improvement of 200/50 over 74/74
6.61%	$520,299	$513,816	$547,717	5.3%	6.60%
6.11%	$479,362	$467,563	$496,697	3.6%	6.23%
5.61%	$442,208	$426,025	$450,506	1.9%	5.75%
5.11%	$408,459	$388,691	$408,723	0.1%	5.15%
4.61%	$377,777	$355,104	$370,918	-1.8%	4.45%

Note: Based on 10,000 Monte Carlo draws from a log-normal distribution with st.dev of 14.29% and varying means. Constant strategy set such that mean final accumulation is equal to that of the 200/50 leveraged-lifecycle strategy for 6.61% mean stock return.

Table X: Monte Carlo Comparison of Certainty Equivalent Wealth Across Varying Volatility of Real Returns					
Stock Standard Deviation	Certainty Equivalent of 90/50	Certainty Equivalent of 74/74	Certainty Equivalent of 200/50	Improvement of 200/50 over 90/50	Improvement of 200/50 over 74/74
14.29%	$520,299	$513,816	$547,717	5.3%	6.60%
19.29%	$424,462	$386,385	$418,198	-1.5%	8.23%
24.29%	$338,624	$281,938	$300,647	-11.2%	6.64%
29.29%	$268,228	$204,238	$211,690	-21.1%	3.65%
Note: Based on 10,000 Monte Carlo draws from a log-normal distribution with mean of 6.61 and varying st. devs. Constant strategy set such that mean final accumulation is equal to that of the 200/50 leveraged-lifecycle strategy for 14.29% standard deviation of stock return.					

σ=19.29%, the optimized 200/27 rule has CE of $453,406, which is 6.8% above the 90/50 rule.

σ=24.29%, the optimized 200/17 rule outperforms the target-date fund by 22.4% and

σ=29.29%, the optimized 200/12 rule outperforms the target-date fund by 47.2%

Good news is that VIX is public. Thus easier to rebalance according to change in VIX than mean.

What if Everyone Did It?

If Everyone did it

(We should be so lucky)

- History of mortgages
- History of mutual funds
- Legal issues

Legal Reform

- ERISA Safe harbor
  - All have to be prudent
- Suitability Requirements/Retirement Advisor
  - Suitability and prudent investor duties
- 401k limitations

Conclusions

- Two Ways to Diversify
  - Over Assets and Over Time

- Diversifying across Time can:
  - Reduce Risk by 21% or
  - Increase Expected Return by 45%

  - Raise Certainty Equivalent by 6.6% over equal mean λ/λ
  - Raise Certainty Equivalent by 3.6% over equal mean 90/50
  - Raise Certainty Equivalent by 5.3% to 11.5% over 90/50
    More if RRA is below 4
  - Raise Certainty Equivalent by 35% over λ/λ.

- At least, invest 100% in stocks when young