Measuring Default Risk Premia from Default Swap Rates and EDFs*

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Abstract

This paper estimates the time-series behavior of default risk premia for U.S. corporate debt over 2000-2004, based on the relationship between default probabilities, as estimated by Moody's KMV EDFs, and default swap (CDS) market rates. The default-swap data, obtained through CIBC from 39 banks and specialty dealers, allow us to establish a strong link between actual and risk-neutral default probabilities for the 93 firms in the three sectors that we analyze: broadcasting and entertainment, healthcare, and oil and gas. We find dramatic variation over time in risk premia, from peaks in the third quarter of 2002, dropping by roughly 50% to late 2003.

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1 Introduction

This paper estimates the time-series behavior of default risk premia for U.S. corporate debt, based on a close relationship between default probabilities, as estimated by the Moody's KMV EDF measure, and default swap (CDS) market rates. The default-swap data, obtained by CIBC from 39 banks and specialty dealers, allow us to establish a strong link between actual and risk-neutral default probabilities for the 93 firms in the three sectors that we analyzed: broadcasting and entertainment, healthcare, and oil and gas.

Based on over 180,000 CDS rate quotes, we find that 5-year EDFs explain over 74% of the cross-sectional variation in 5-year CDS rates, after controlling for sectoral and temporal effects. We find that the marginal impact of default probability on credit spreads is proportionately much greater for high-credit-quality firms than for low-credit-quality firms. For a given default probability, we find substantial variation over time in credit spreads. For example, after peaking in the third quarter of 2002, credit risk premia declined steadily and dramatically through late 2003, when, for a given default probability, credit spreads were on average roughly 50% lower than at their peak, after controlling for sectoral effects. A potential explanation is that major default losses in prior months had restricted the availability of capital in the corporate debt sector, driving risk-premia to high levels by mid-2002, and that fresh capital flowed into the risk sector over the subsequent months in order to take advantage of the high premiums offered, eventually (but not immediately) driving risk premia down. This is similar to the explanation offered by **?**) for dramatic increases in catastrophic risk insurance premiums after major losses of capital, with subsequent slow declines in premia over time as new capital is attracted into the sector.

Our study is based on price data from an extensive database of credit default swap (CDS) rates from CIBC, and from MoodysKMV estimated default frequency (EDF) data. Panel regression and time-series models are used to estimate default risk premia, which we measure as the ratio of risk-neutral to actual default probabilities. This ratio may be viewed as the proportional premium for bearing default risk. For example, if this ratio is 2.0 (for a particular firm, date, and maturity date), then market-based insurance that pays one dollar in the event of default would be priced at twice the probability of default by that date, ignoring the time value of money.

While Fisher (1959) took a simple regression approach to explaining yield spreads on corporate debt in terms of various credit-quality and liquidity related variables, Fons (1987) gave the earliest empirical analysis, to our knowledge, of the relationship between actual and risk-neutral default probabilities. Driessen (2005) recently estimated the relationship between actual and risk-neutral default probabilities, using U.S. corporate bond price data (rather than CDS data), and using average long-horizon default frequencies by credit rating (rather than contemporaneous firm-by-firm EDFs). Driessen reported an average risk premium across his data of 1.89, after accounting for tax and liquidity effects, that is roughly in line with the estimates that we provide here. While the conceptual foundations of Driessen's study are similar to ours, there are substantial differences in our respective data sources and methodology. First, the time periods covered are different. Second, the corporate bonds underlying Driessen's study are less homogeneous with respect to their sectors, and have significant heterogeneity with respect to maturity, coupon, and time period. Each of our CDS rate observations, on the other hand, is effectively a new 5-year par-coupon credit spread on the underlying firm that is not as corrupted, we believe, by tax and liquidity effects, as are corporate bond spreads. Most importantly, we do not rely on historical average default rate by credit rating as a proxy for current conditional default intensity.

Because the corporate bonds in Driessen's study involve taxable coupon income, Driessen was forced to estimate the portion of the bond yield spread that is associated with taxes. As for the estimated actual default probabilities, Driessen's reliance on average frequency of default for bonds of the same rating rules out conditioning on current market conditions, which Kavvathas (2001) and others have shown to be significant. Reliance on default frequency by rating also rules out consideration of distinctions in default risk among bonds of the same rating. Moody's KMV EDF measures of default probability provide significantly more power to discriminate among the default probabilities of firms (Kealhofer (2003), Kurbat and Korbalev (2002)). Blanco, Brennan, and Marsh (2003) show that CDS rates represent somewhat fresher price information than do bond yield spreads. Indeed, our enquiries of market participants have led us to the view that default swaps, because they are "un-funded exposures," in the language of dealers, have rates that are less sensitive to liquidity effects than are bond yield spreads.

Bohn (2000), Delianedis and Geske (1998), G. Delianedis Geske and Corzo (1998), and Huang and Huang (2003) use structural approaches to estimating the relationship between actual and risk-neutral default probabilities, generally assuming that the Black-Scholes-Merton model applies to the asset value process, and assuming constant volatility. **?**) have found that these structural models tend to fit the data rather poorly, and typically underestimate credit spreads, especially for shorter maturity bonds.

The potential applications of our study are numerous, and include: *(i)* the relationship between risk and expected return for the credit component of corporate debt, and *(ii)* analysis of the extent to which the default risk premia of different firms have common factors, as well as the dynamics and macroeconomics of these common factors. These applications can, in turn, be further applied to a range of pricing and portfolio investment decisions involving corporate credit risk.

A weakness of our study is the lack of data bearing on risk-neutral mean loss given

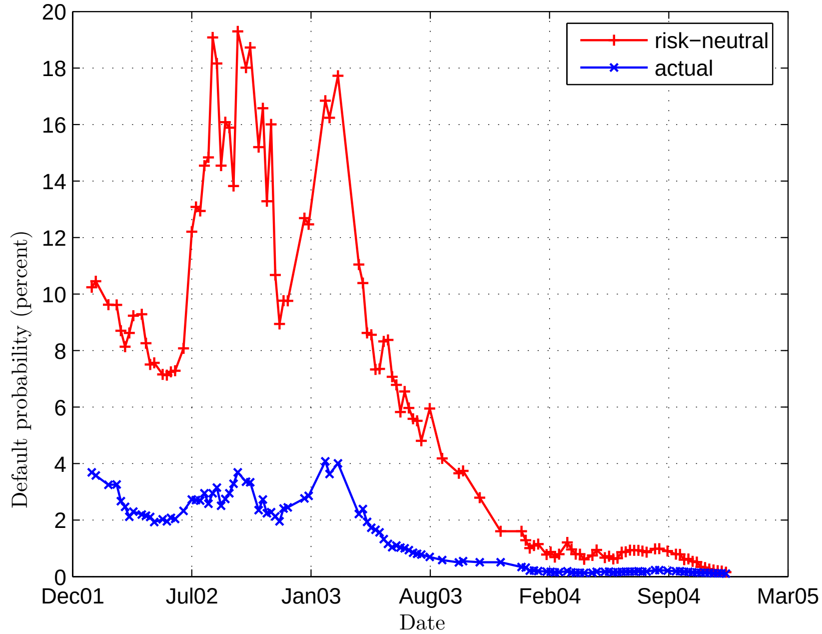

Figure 1: Estimated actual and risk-neutral 1-year default probabilities for Royal Caribbean Cruises.

default (LGD). The highest annual cross-sectional sample mean of loss given default during our sample period was reported by Altman, Brady, Resti, and Sironi (2003) to be approximately 75%. Using 75% as a rough estimate for risk-neutral mean loss given default, our measured relationship between CDS and EDF implies that short-term risk-neutral default probabilities are roughly double of their actual-probability counterparts, on average, although this premium is much higher for high quality firms, and lower for low quality firms, is much higher for firms in the broadcasting-entertainment sector than for firms in oil-and-gas or healthcare. In particular, this ratio was dramatically across sectors and firms in mid-2002 than in late 2003. If the risk-neutral mean LGD were constant over time, then our results on relative changes over time in default risk premia would be relatively unaffected. The results of Altman, Brady, Resti, and Sironi (2003), however, indicate that average realized LGDs tend to be positively correlated with aggregate default rates. As a robustness check, we provide some indication of the potential impact of such correlation on estimated CDS rates.

As an illustrative example, Figure 2, which shows estimated actual and risk-neutral 1-year default probabilities for Royal Caribbean Cruises, and is consistent with the typical pattern in our sample of high default risk premia in the third quarter of 2002.

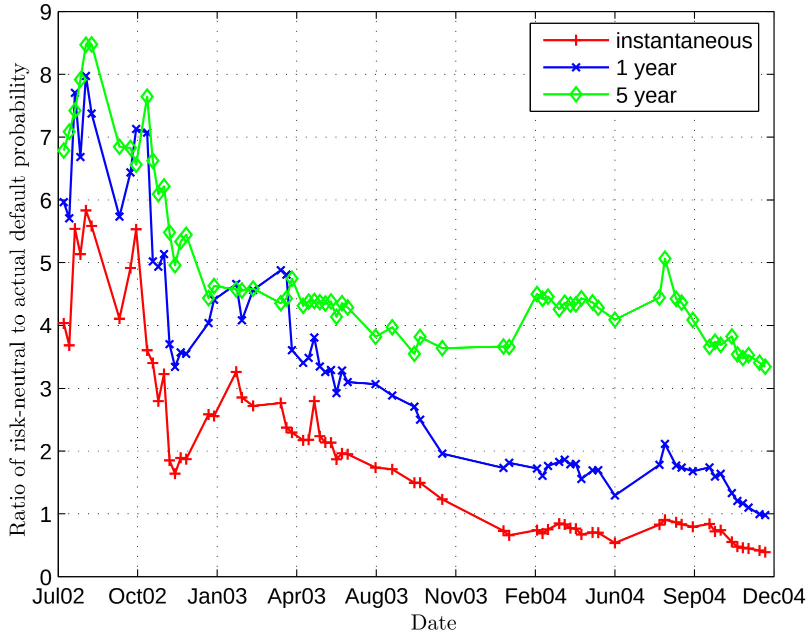

Figure 2: Estimated proportional risk premia for the broadcasting-and-entertainment sector, at various maturities.

The remainder of the paper is structured as follows. Section 2 describes our data, including a brief introduction to default swaps and to the construction of the Moody's KMV EDF measure of default probability. Section 3 presents panel-regression evidence of a strong relationship between CDS rates and EDFs across several sectors, with higher risk premia for high-quality firms, and dramatically declining risk premia from mid-2002 to late 2003. Section 4 introduces a simple time-series model of actual default intensities, and a maximum-likelihood approach to parameter estimation. Section 4 also contains parameter estimates for each firm, based on 12 years of monthly observations of 1-year EDFs for each firm. Section 5 provides a reduced-form pricing model for default swaps, based on time-series models of actual and risk-neutral default intensities. Section 5.2 introduces our parameterization of the time-series model for risk-neutral default intensities, using both EDFs and CDS rates. Section 5.3 provides estimates of the parameters for each of the three sectors. Section 6 discusses the results, and then concludes.

2 The EDF and CDS Data

This section discusses our data sources for conditional default probabilities and for default swap rates.

2.1 The EDF Data

Moody's KMV provides its customers with, among other data, current firm-by-firm estimates of conditional probabilities of default over time horizons that include the benchmark horizons of 1 and 5 years. For a given firm and time horizon, this "EDF" estimate of default probability is fitted non-parametrically from the historical default frequency of other firms that had the same estimated "distance to default" as the target firm. The distance to default of a given firm is, roughly speaking, the number of standard deviations of annual asset growth by which its current assets exceed a measure of book liabilities. The liability measure is, in the current implementation of the EDF model, equal to the firm's short-term book liabilities plus one half of its long-term book liabilities. Estimates of current assets and the current standard deviation of asset growth ("volatility") are calibrated from historical observations of the firm's equity-market capitalization and of the liability measure. The calibration is based on the model of Black and Scholes (1973) and Merton (1974), by which the price of a firm's equity may be viewed as the price of an option on assets struck at the level of liabilities. Crosbie and Bohn (2002) and Kealhofer (2003) provide more details on the KMV model and the fitting procedures for distance to default and EDF. While one could criticize the EDF measure as an estimator of the "true" conditional default probability, it has a number of important merits for business practice and for our study, relative to other available approaches to estimating conditional default probabilities. First, it is readily available for essentially all public U.S. companies, and for a large fraction of foreign public firms. (There is a private-firm EDF model, which we do not rely on, since our CDS data are for public firms.) Second, while the EDF model is based on a single covariate, distance-to-default, for default prediction, and one might wish to exploit additional covariates (Duffie, Saita, and Wang (2005), Shumway (2001)), the distance-to-default (DD) covariate has a strong underlying theoretical basis in the Black-Scholes-Merton model, within which DD is a sufficient statistic for conditional default probabilities.

Third, the EDF is fitted non-parametrically to the distance-to-default, and is therefore not especially sensitive, at least on average, to model mis-specification. While the measured distance-to-default is itself based on a theoretical option-pricing model, the function that maps DD to EDF is consistently estimated in a stationary setting. That is, conditional on only the distance to default, the measured EDF is equal to the "true" DD-conditional default probability as the number of observations goes to infinity, under typical mixing and other technical conditions for non-parametric qualitative-response estimation.

An alternative industry measure of default likelihood is the average historical default frequency of firms with the same credit rating as the target firm. This measure is often used, for example, in implementations of the Credit Metrics approach (www.creditmetrics.com), and is convenient given the usual practice by financial-services firms of tracking credit quality by internal credit ratings based on the approach of the major recognized rating agencies such as Moody's and Standard and Poors. The ratings agencies, however, do not claim that their ratings are intended to be a measure of default probability, and they acknowledge a tendency to adjust ratings only gradually to new information, a tendency strongly apparent in the empirical analysis of Behar and Nagpal (1999), Lando and Skødeberg (2002), Kavvathas (2001), Nickell, Perraudin, and Varotto (2000), among others.

The Moody's KMV EDF measure is also extensively used in the financial services industry. For example, from information provided to us by Moody's KMV, 40 of the world's 50 largest financial institutions are subscribers. Indeed, it is the only widely used name-specific major source of conditional default probability estimates of which we are aware, covering over 26,000 publicly traded firms.

Our basic analysis in Section 3 directly relates daily observations of 5-year CDS rates to the associated daily 5-year EDF observations. In order to develop a time-series model of default intensities, however, we turn in Section 4 to monthly observations of 1-year EDFs. By sampling monthly rather than daily, we mitigate equity market microstructure noise, including intra-week seasonality in equity prices, and we also avoid the intra-month seasonality in EDFs caused by monthly uploads of firm-level accounting liability data. By using 1-year EDFs rather than 5-year EDFs, our intensity estimates are less sensitive to model mis-specification, as the 1-year EDF is theoretically much closer to the intensity than is the 5-year EDF.

2.2 Default Swaps and the CDS Database

A default swap, often called, with inexplicable redundancy, a "credit default swap" (CDS), is an over-the-counter derivative security designed to transfer credit risk. With minor exceptions, a default swap is economically equivalent to a bond insurance contract. The buyer of protection pays periodic (usually quarterly) insurance premiums, until the expiration of the contract or until a contractually defined credit event, whichever is earlier. For our data, the stipulated credit event is default by the named firm. If the credit event occurs before the expiration of the default swap, the buyer of protection receives from the seller of protection the difference between the face value and the market value of the underlying debt, less the default-swap premium that has accrued since the last default-swap payment date. The buyer of protection normally has the option to substitute other types of debt of the underlying named obligor. The most popular settlement mechanism at default is for the buyer of protection to submit to the seller of protection debt instruments of the named firm, of the total notional amount specified in the default-swap contract, and to receive in return a cash payment equal to that notional amount, less the fraction of the default-swap premium that has accrued (on a time-proportional basis) since the last regular premium payment date.

The CDS rate is the annualized premium rate, as a fraction of notional. Using an actual-360 day-count convention, the CDS rate is thus four times the quarterly premium. Our observations are at-market, meaning that they are bids or offers of the default-swap rates at which a buyer or seller of protection is proposing to enter into new default swap contracts, without an up-front payment. Because there is no initial exchange of cash flows on a standard default swap, the at-market CDS rate is, in theory, that for which the net market value of the contract is zero. In practice, there are implicit dealer margins that we treat by assuming that the average of the bid and ask CDS rates is the rate at which the market value of the default swap is indeed zero.

For the purpose of settlement of default swaps, the contractual definition of default normally allows for bankruptcy, a material failure by the obligor to make payments on its debt, or a restructuring of its debt that is materially adverse to the interests of creditors. The inclusion, or not, of restructuring as a covered default event has been a question of debate among the community of buyers and sellers of protection. ISDA, the industry coordinator of standardized OTC contracts (www.isda.org), has arranged a consensus for a standardized contractual definition of default that, we believe, is likely to be reflected in most of our data. This consensus definition of default has been adjusted over time, and to the extent that these adjustments during our observation period are material, or to the degree of heterogeneity in our data over the definition of default that is applied, our results could be affected. The contractual definition of default can affect the estimated risk-neutral implied default probabilities, since of course a wider definition of default implies a higher risk-neutral default probability.

If restructuring is included as a contractually covered credit event, then there is the potential for significant heterogeneity at default in the market values of the various debt instruments of the obligor, as fractions of their respective principals, especially when there is significant heterogeneity with respect to maturity. The resulting cheapest-to-deliver option can therefore increase the loss to the seller of protection in the event of default. Without, at this stage, data bearing on the heterogeneity of market value of the pool of deliverable obligations for each default swap, we are in effect treating the cheapest-to-deliver option value as a constant that is absorbed into the estimated risk-neutral fractional loss L^* to the seller of protection in the event of default. While we vary L^* as a parameter, we generally assume that L^* is constant across the sample. To the extent that L^* varies over time or across issuers, our implied risk-neutral default probabilities would be corrupted. This is not crucial, as we shall show, when modeling the CDS rates implied by a given EDF. This robustness also applies to the mark-to-market pricing of old default swaps, which is an increasingly important activity, given that the notional amount of debt covered by default swaps is almost doubling each year, and is expected to reach 4 trillion U.S. dollars in 2004, according to the British Bankers Association (www.bba.org).

For a given level of seniority (our data are based on senior unsecured debt instruments), there is less recovery-value heterogeneity if the event of default is bankruptcy or failure to pay, for these events normally trigger cross-acceleration covenants that cause debt of equal seniority to convert to immediate obligations that are *pari passu*, that is, of equal priority. In any case, the option held by the buyer of protection to deliver from a list of debt instruments will cause the effective fractional loss given default to the seller of protection to be the maximum fractional loss given default of the underlying list of debt instruments. If restructuring is included as a covered default event, the impact of this cheapest-to-deliver option is, within the current "modified" ISDA standard contract, mitigated by a contractual restriction on the types of deliverable debt instruments, especially with respect to maturity.

Ignoring the cheapest-to-deliver effect, the CDS rate is, in frictionless markets, extremely close to the par-coupon credit spread of the same maturity as the default swap, as shown by Duffie (1999). Our results thus speak to the relationship between EDFs and corporate credit spreads. We are told by market participants that asset swaps, synthetic approximations of par-coupon bonds, as explained in Duffie (1999), trade at "par spreads" that are, on average, becoming closer to CDS rates as the CDS market matures and grows in volume, liquidity, and transparency. This is confirmed to some extent in empirical studies by Longstaff, Mithal, and Neis (2003) and Blanco, Brennan, and Marsh (2003), provided one measures bond spreads relative to interest-rate swap yields.

Our CIBC data set consists of over 180,000 intra-day CDS rate quotes on 93 firms from three Moody's industry groups. The sources of these quotes include 27 investment banks and 12 default-swap brokers. The cross-sectional concentration of the number of quotes by source is shown in Figure 3. A breakdown of the number of quotes by banks and by default-swap brokers is given in Table 1.

We selected three representative Moody's North American industry groups: Broadcasting and Entertainment, Oil and Gas, and Healthcare. The CDS quotes are for 1-year, 3-year, and 5-year, quarterly premium, senior unsecured, US-Dollar-denominated, at-the-money default swaps. The 5-year quotes are the most liquid, and are the basis for most of our results. A company from any of these three sectors is included in our study if and only if at least 1,000 historical pairs of CDS bid and ask quotes for that firm were available during the sample period. The range of credit qualities of

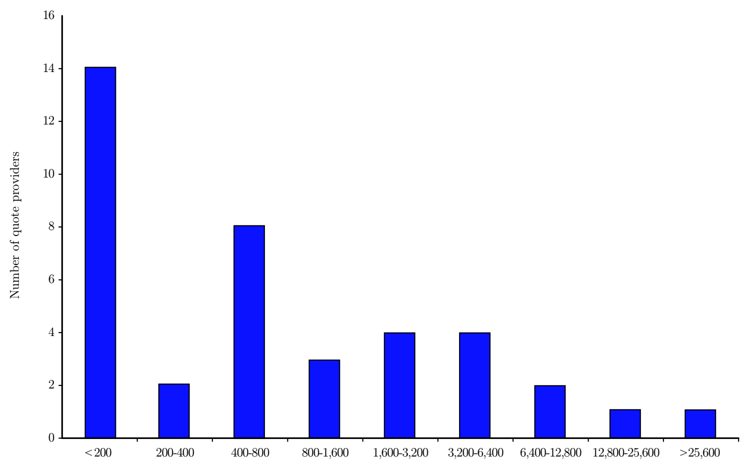

Figure 3: Distribution of CDS quote providers by number of quotes provided. Data source: CIBC.

Table 1: Breakdown of number of CDS quotes by type of source

	Total	Median	Average	Sources	Min	Max
Banks	57,023	485	2,112	27	4	14,843
Brokers	124,837	624	10,403	12	31	108,838
All	181,860	562	4,663	39	4	108,838

the included firms may be judged from Figure 4, which shows, for each credit rating, the number of firms in our study of that median Moody's rating during the sample period. Figure 4 indicates a concentration of Baa-rated firms. Daily CDS mid-point rate quotes were estimated from intra-day bid and ask quotes.[1]

Figure 5 shows a histogram of the ratio of quotes to the daily median quote for the same name, after removing the points associated with the median quote itself (of which there are approximately 38,500). The plot shows substantial intraday variation in CDS quotes of a given name.

The firms that we studied from the broadcasting-and-entertainment industry are listed in Table 2, along with their median 1-year EDF and median Moody's credit rating during the sample period from June 2000 to December 2004, and the number of CDS quotes available for each. The same information covering firms from the healthcare and oil-and-gas industries is provided in Appendix C.

[1]We used the following algorithm: (a) If a bid and an ask were present, we record the bid-ask spread. (b) If the bid is missing, we subtract the average bid-ask spread to estimate the ask. (c) If the ask is missing, we add the average bid-ask spread to estimate the bid. (d) From the resulting bid and ask, we calculate the mid-quote as the average of the bid and ask quotes.

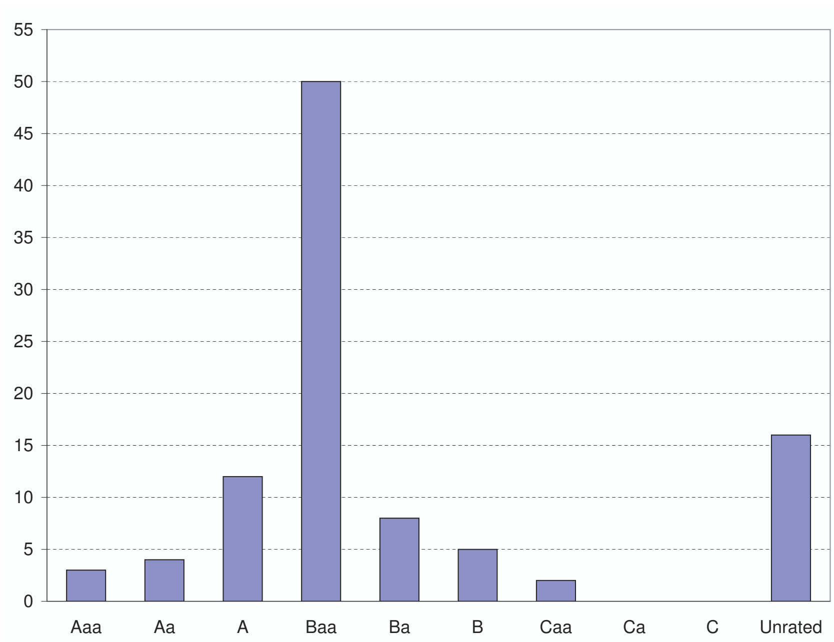

Figure 4: Distribution of firms by median credit rating during the sample period. Sources: CIBC and Moody's.

3 Panel Regression Analysis

A simple preliminary linear model of the relationship between a firm's 5-year CDS (Y_i) and the 5-year EDF (X_i) measured in basis points on the same day is

$$Y_i = \underset{(0.879)}{33.050} + \underset{(0.005)}{1.615} X_i + e_i, \qquad (1)$$

where X_i is the observed 5-year EDF of a given firm on a given day, Y_i is an observed CDS rate of the same firm on the same day, e_i is a random disturbance. Standard errors are shown parenthetically. The ordinary-least-squares (OLS) coefficient estimates and standard errors are based on 33,912 paired EDF-CDS observations from December 2000 to December 2004, with most observations during 2002 through 2004. The associated coefficient of determination, R^2, is 0.728. Figure 6 illustrates the fit of (1), for all firms in our study, and all time periods. The 5-year CDS rate is estimated to increase by approximately 16 basis points for each 10 basis point increase in the 5-year EDF. If one were to take the risk-neutral expected loss given default to be, say, 75% and the default intensities (actual and risk-neutral) to be constant, this would imply an average ratio of risk-neutral to actual default intensity φ of approximately $(16/0.75)/10$, or 2.0.

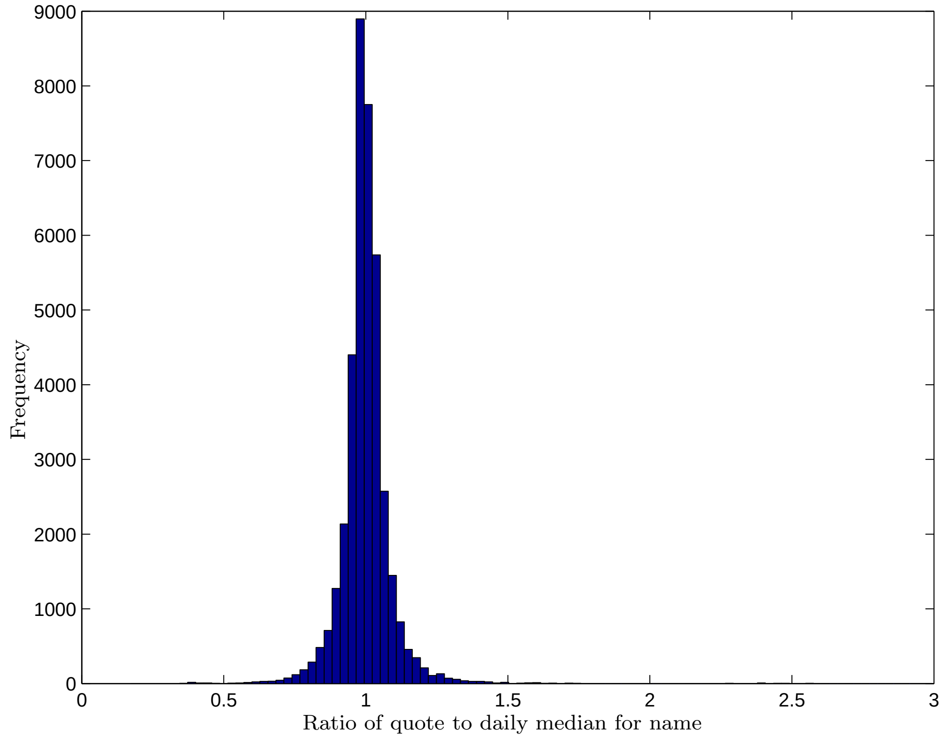

Figure 5: Intraday distribution of ratio of five-year CDS bids to median bid, after removing the median bids. Source: CIBC.

Linearity of the CDS-EDF relationship, however, is placed in doubt by the sizable intercept estimate of roughly 30 basis points, more than 30 times its standard error. Absent an unexpectedly large liquidity impact on CDS rates, the fitted default swap rate should be closer to zero at low levels of EDF. While there may be mis-specification due to the assumed homogeneity of the relationship over time and across firms, we have verified with sector and quarterly regressions that the associated intercept estimates are unreasonably large in magnitude. We also noted that scatter plots of the CDS-EDF relationship indicated a pronounced concavity at low levels of EDF. That is, the sensitivity of credit spreads to a firm's estimated default probability seems to decline at larger levels of default risk. There is also apparent heteroskedasticity, with dramatically greater variance for higher EDFs. The slope of the fit illustrated in Figure 6 is thus heavily influenced by the CDS-to-EDF relationship for lower-quality firms.

Table 2: Broadcasting and Entertainment Firms

Name of Firm	Median EDF (basis points)	Median Rating	No. Quotes
Adelphia Communications Corp	378	N/A	228
Belo Corp	6	Baa3	1,168
Brunswick Corp	9	Baa2	1,390
Charter Communications Inc	600	N/A	456
Clear Channel CommunicationsInc	41	Baa3	3,330
Comcast Cable Communications	–	Baa3	1,182
Comcast Corp	40	Baa3	2,723
COX Communications Inc	17	Baa3	4,956
Cox Enterprises Inc	–	Baa3	1,058
Historic TW Inc	–	Baa1	1,462
Interpublic Group of Cos Inc	229	Baa3	1,095
Knight-Ridder Inc	3	A2	1,290
LibertyMediaCorp	48.5	Baa3	2,244
Mediacom Communications Corp	857	Caa1	168
News America Holdings	–	N/A	1,165
News America Inc	–	Baa3	1,679
OmnicomGroup	38	Baa1	2,539
Primedia Inc	939.5	B3	332
Royal Caribbean Cruises Ltd	107	Ba2	1,043
Sabre Holdings Corp	64.5	Baa2	1,467
Time Warner Inc	135	Baa1	5,549
Viacom Inc	18	A3	3,997
Walt Disney Co	23	Baa1	4,459

We next considered the log-log specification[2]

$$\log Y_i = \alpha + \beta \log X_i + z_i, \tag{2}$$

for coefficients α and β, and a residual z_i. The fit, illustrated in Figure 7, shows much less heteroskedasticity. (One notes granularity associated with the very low log-EDFs of extremely high-quality firms.)

We have taken CDS rate observations (Y_i) by two approaches: *(i)* the daily median CDS for each given name, and *(ii)* all CDS observations for that day. The second approach, which has substantially more CDS observations per EDF observation, has by construction a lower coefficient of determination (R^2), and is likely to have more precise estimates of the intercept and slope coefficients, α and β. (This is necessarily

[2]We also examined the fit, by non-linear least squares, of the model, $Y_i = \alpha X_i^{\beta} + u_i$, which differs from (2) by having a residual that is additive in levels, rather than additive in logs. An informal comparison shows that the non-linear least-squares model is somewhat preferred for lower-quality firms.

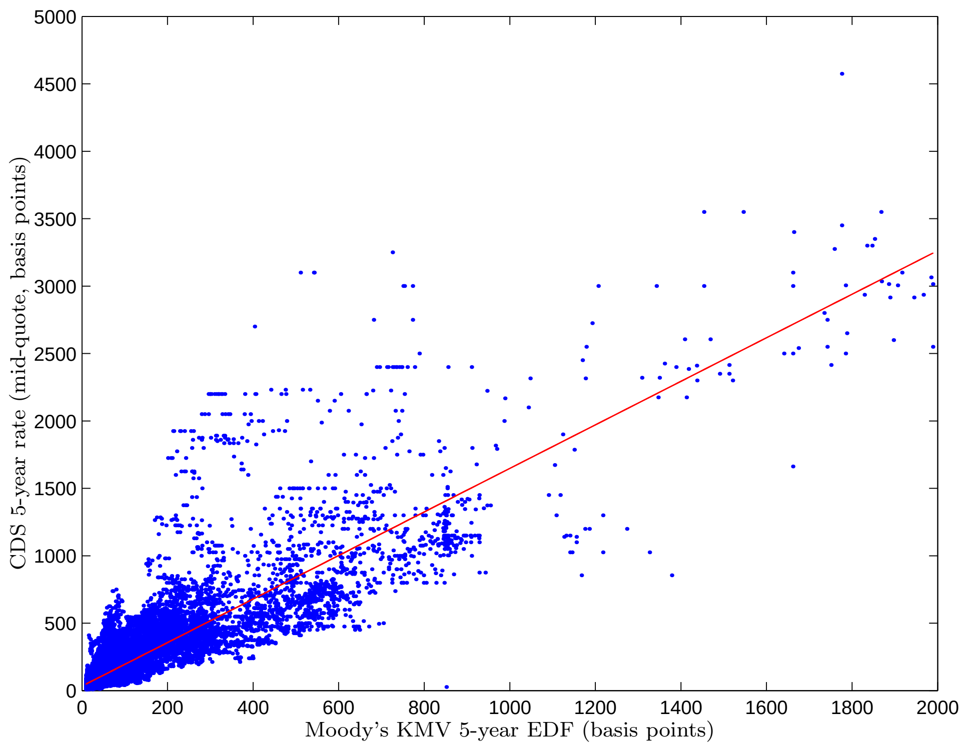

Figure 6: Scatter plot of EDF and CDS observations and OLS fitted relationship. Source: CIBC (CDS) and Moody's KMV (EDF).

so if the model is correctly specified.) It is from this model with more observations that we would thus anticipate getting a more precise notion of how CDS rates are related to EDFs.[3]

One might have considered a model in which the CDS rate is fit to both 5-year and 1-year EDF observations, given the potential for additional influences of near-term default risk on CDS rates. We have found, however, that the 1-year and 5-year EDFs are extremely highly correlated. As might be expected, adding 1-year EDFs to the regression has no major impact on the quality of fitted CDS rates, and involves substantial noise in the slope coefficients. We do not report the results for the multiple regressions. In any case, the 5-year EDF captures the average effect of default risk over the 5-year period, as does the CDS. This is not to suggest, however, that default risk premia implicit in the CDS rates necessarily have the same term structure. We have little information about this term structure to report at this time. (We plan to

[3]Technically, the two cases (daily median CDS observations, and all CDS observations) would not both be consistent with equation (2), since the median is an order statistic that depends on sampling noise in a non-linear fashion. We prefer, in any case, the median to the average daily CDS observation as we believe it to be more robust to outliers induced by observation noise.

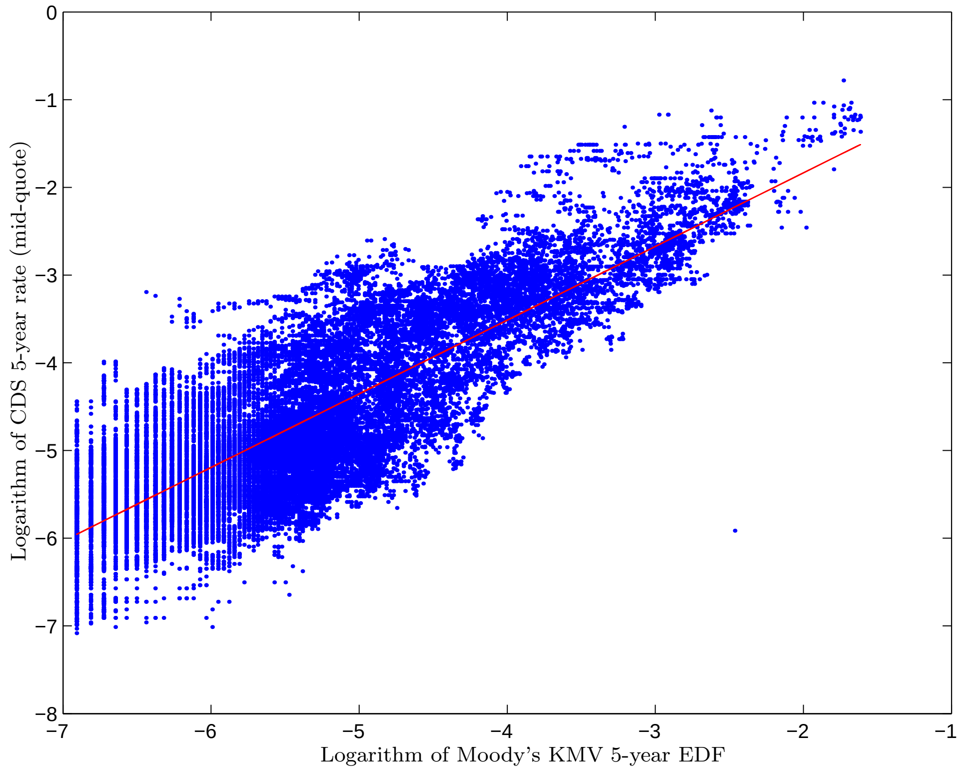

Figure 7: Scatter plot of EDF and CDS observations, logarithmic, and OLS fitted relationship. Source: CIBC (CDS) and Moody's KMV (EDF).

later analyze short-maturity CDS data.)

We also control for changes in the CDS-to-EDF relationship across time and across sectors. Table 11, found in the appendix, presents the results of a regression of the logarithm of the daily median CDS rate on the logarithm of the associated daily 5-year EDF observation, including dummy variables for sectors and months. For example, extracting from Table 11 the fit implied for the oil-and-gas sector, we have

$$\log \mathrm{CDS}_i \;=\; \underset{(0.027)}{1.186} \;+\; \underset{(0.004)}{0.830} \log \mathrm{EDF}_i + \sum \hat{\beta}_j D_{\mathrm{month}\ j}(i) + z_i, \qquad (3)$$

where $\hat{\beta}_j$ denotes the estimate for the dummy multiplier for month j, with j running from December 2000 through April 2004, and z_i denotes the residual. We obtain an R^2 of about 75.5%. From the one-standard-deviation confidence band implied by normality of the residuals for the logarithmic fit, the associated confidence band for a given CDS rate places it between 59% and 169% of the fitted rate.

From the dummy coefficient estimate for the healthcare sector, the CDS rate for a healthcare firm is estimated to be 20% higher than that of an oil-and-gas firm with the

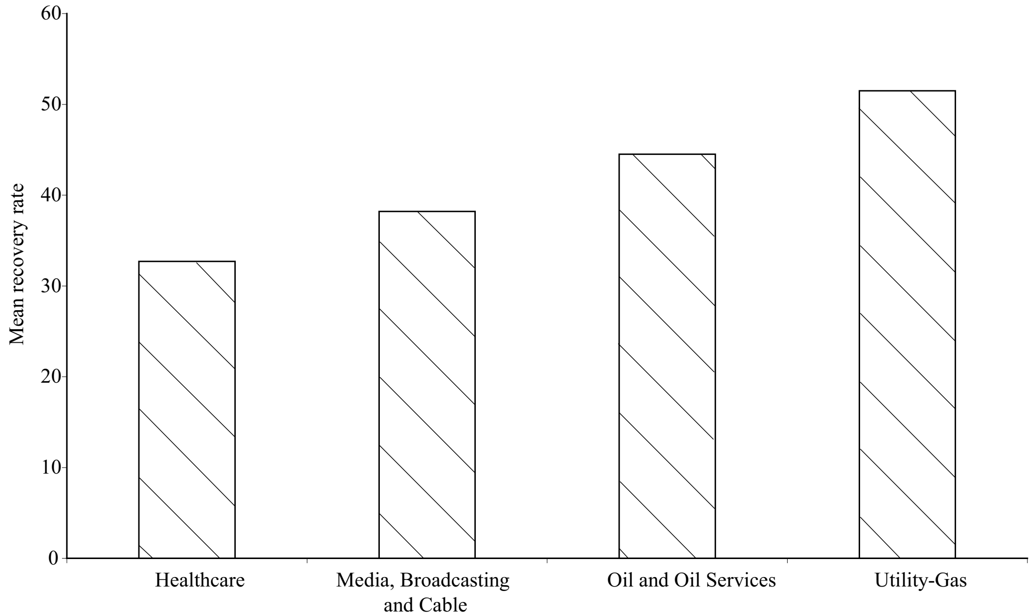

Figure 8: Sectoral differences in average default recovery, 1982-2003. Source: Moody's Investor Services.

same EDF. A broadcasting-and-entertainment firm is estimated to have a 42% higher CDS than an oil-and-gas firm with the same reported EDF. As one can see from Figure 8, showing selected Moody's average sectoral default recoveries for 1982 to 2003, some of these sectoral spread-to-EDF differences are due to sectoral differences in default recovery. For example, assuming that the ratio of the risk-neutral mean loss given default in the oil-and-gas sector to another sector is the same as the ratio of the empirical average loss given default, then broadcasting-entertainment spreads would be approximately $62\%/52\% - 1 = 19\%$ higher than oil-and-gas sector, for equal risk-neutral default probabilities. Similarly, healthcare spreads would be approximately $67\%/52\% - 1 = 29\%$ higher than oil-and-gas sector, for equal risk-neutral default probabilities.[4]

The fitted model also shows highly significant variation in risk premia across the months of 2002 and 2003, with the highest risk premia during the third quarter of 2002, when, for a given EDF, spreads are estimated to have been roughly double than what they were in December 2003. Figure 9 illustrates this variation over time with a plot of the dummy variables of the regression model (3), indicating the percentage increase in CDS rates at a given EDF associated with each month. The broadcasting

[4]From the Moody's sectoral data, the average recovery for the oil and gas sector is estimated from the simple average of the of the Moody's "Oil and Oil Services" and the "Utility-Gas" sectors, at 48%. Broadcasting and Entertainment recoveries are estimated at the 'Media Broadcasting and Cable' average of 38%, and Healthcare at 32.7%.

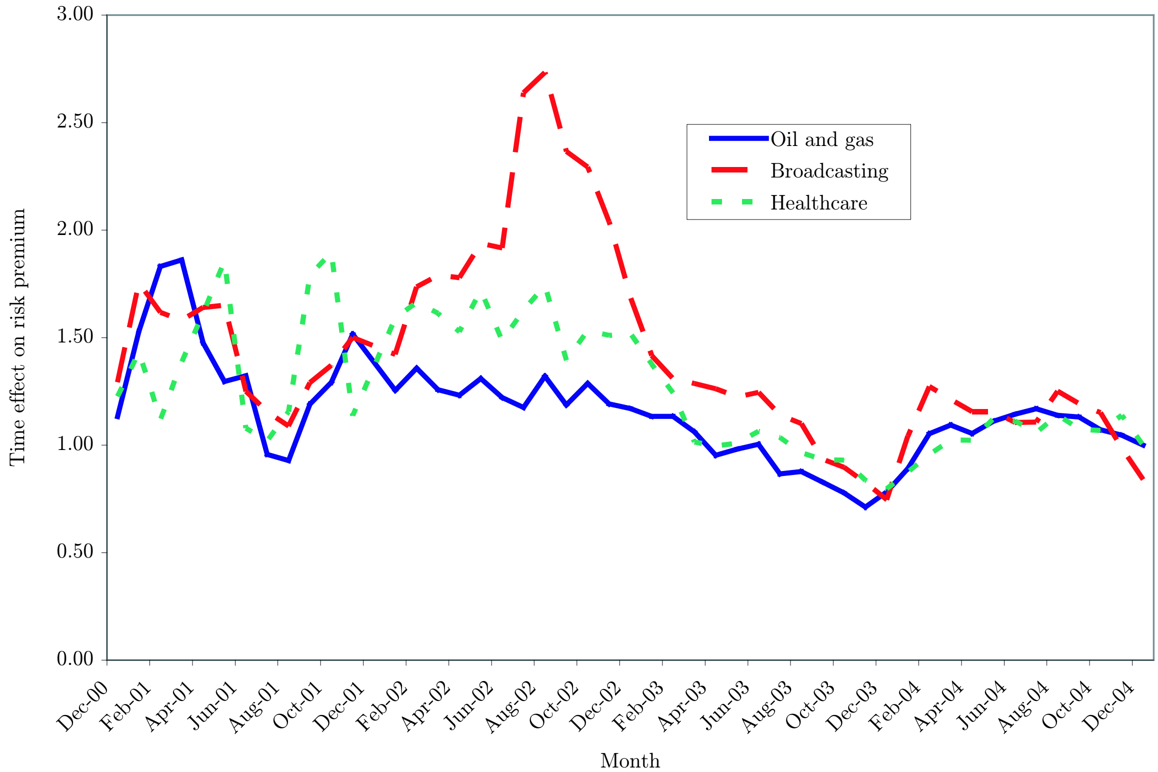

Figure 9: Monthly dummy multipliers in CDS-to-EDF fit.

and entertainment sector, in particular, shows dramatic reductions in risk premia from mid 2002 (around the times of default of Adelphia and Worldcom) to late 2003.

In Table 13 in Appendix C we report the results of the regression of the logarithm of the daily median CDS rate on the logarithm of the associated daily 5-year EDF observation when including dummy variables for each sector-month pair:

$$\log \mathrm{CDS}_i \;=\; \underset{(0.047)}{1.448} \;+\; \underset{(0.015)}{0.760}\,\log \mathrm{EDF}_i + \sum_{\text{sector } s, \text{month } j} \hat{\beta}_{s,j}\, D_{s,j}(i) + z_i. \qquad (4)$$

Here, $\hat{\beta}_{s,j}$ denotes the estimate for the dummy multiplier for sector s and month j, with j running from December 2000 through December 2004 ($33,912$ observations in all), and z_i denotes the residual. We obtain an R^2 of about 74.4%. Figure 15 shows that the index of default risk premium for the broadcasting-and-entertainment sector peaks during July and August of 2002, and that, in July 2002, it was at a 9-month and 6-month high for the healthcare and the oil-and-gas industry, respectively.

4 Actual Default Intensity from EDF

The default intensity of an obligor is the instantaneous mean arrival rate of default, conditional on all current information. To be slightly more precise, we suppose that default for a given firm occurs at the first event time of a (non-explosive) counting process N with intensity process λ, relative to a given probability space $(\Omega, \mathcal{F}, P)$ and information filtration $\{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions. In this case, so long as the obligor survives, we say that its default intensity at time t is λ_t. Under mild technical conditions, this means that, conditional on survival to time t and all information available at time t, the probability of default between times t and $t+h$ is approximately $\lambda_t h$ for small h. We also adopt the relatively standard simplifying doubly-stochastic, or Cox-process, assumption, under which the conditional probability at time t, for a currently surviving obligor, that the obligor survives to some later time T, is

$$p(t,T) = E\left(e^{-\int_t^T \lambda(s)\,ds} \,\middle|\, \mathcal{F}_t\right). \tag{5}$$

For our analysis, we ignore mis-specification of the EDF model itself, by assuming that $1 - p(t, t+1)$ is indeed the current 1-year EDF. From the Moody's KMV data, then, we observe $p(t, t+1)$ at successive dates t, $t+h$, $t+2h, \ldots$, where h is one month. From these observations, we estimate a time-series model of the underlying intensity process λ, for each firm. In total, we analyzed 84 firms.

After some preliminary diagnostic analysis of the EDF data set, we opted to specify a model under which the logarithm $X_t = \log \lambda_t$ of the default intensity satisfies the Ornstein-Uhlenbeck equation

$$dX_t = \kappa(\theta - X_t)\,dt + \sigma\,dB_t, \tag{6}$$

where B is a standard Brownian motion, and θ, κ, and σ are constants to be estimated. The behavior for $\lambda = e^X$ is sometimes called a Black-Karasinski model.[5] This leaves us with a vector $\Theta = (\theta, \kappa, \sigma)$ of unknown parameters to estimate from the available monthly EDF observations of a given firm. We have 144 months of 1-year EDF observations for most of the firms in our sample, for the period January, 1993 to December, 2004.

In general, given the log-autoregressive form of the default intensity in (6), there is no closed-form solution available for the 1-year EDF, $1 - p(t, t+1)$ from (5). We therefore rely on numerical lattice-based calculations of $p(t, t+1)$. We have implemented the two-stage procedure for constructing trinomial trees proposed by Hull and White (1994), as well as a more rapid algorithm, explained in the Appendix B,

[5] See Black and Karasinsky (1991).

based on approximation of the solution in terms of a basis of Chebyshev polynomials. (Our current parameter estimates are for the trinomial-tree algorithm.)

The maximum likelihood estimator (MLE) $\hat{\Theta}$ of the parameter vector Θ is then obtained, firm by firm, using a fitting algorithm described in the appendix. That is, for a given firm, $\hat{\Theta}$ solves

$$\sup_{\Theta} \quad \mathcal{L}\left(\{1 - p(t_i, t_i + 1) : 1 \leq i \leq N\}; \Theta\right),$$

where $t_1, t_2, \ldots, t_N$ are the N observation times for the given firm, and $\mathcal{L}$ denotes the likelihood score of observed EDFs given Θ. This is not a routine MLE for a discretely-observed Ornstein-Uhlenbeck model, for several reasons:

1. Evaluation of the likelihood score requires a numerical differentiation of the modeled EDF,

$$G(\lambda(t); \Theta) = 1 - E_{\Theta}\left(e^{-\int_t^{t+1} \lambda(s)\, ds} \;\middle|\; \lambda(t)\right),$$

   where E_{Θ} denotes expectation associated with the parameter vector Θ.

2. As indicated by Kurbat and Korbalev (2002), Moody's KMV caps its 1-year EDF estimate at 20%. Since this truncation, if untreated, would bias our estimator, we explicitly account for this censoring effect on the associated conditional likelihood, as explained in Appendix A.

3. Moody's KMV also truncates the EDF below at 2 basis points. Moreover, there is a significant amount of integer-based granularity in EDF data below approximately 10 basis points, as indicated in Figure 7. We therefore remove from the sample any firm whose sample-mean EDF is below 10 basis points.

4. There were occasional missing data points. These gaps were also treated exactly, assuming the event of censoring is independent of the underlying missing observation.

5. For a small number of firms, an exceptional 1-month fluctuation in the 1-year EDF generated an obviously unrealistic estimate of the mean-reversion parameter κ for that company. We ignored Enron's data point for December 2002, the month it defaulted. Similarly, Magellan Health Services filed for protection under Chapter 11 in March 2003 (we used the EDFs through February 2003), and Adelphia Communications petitioned for reorganization under Chapter 11 in June 2002 (we used the EDFs through May 2002). For Forest Oil, we ignored the outlier months of January and February 1993. Finally, we removed Dynergy from our data set as its 1-year EDF is capped at 20% for most of 2002 and 2003.

Table 3 lists the firms for which we have EDF data, showing the number of monthly observations for each as well as the number of EDF observations that were truncated at 20%. Frequency plots of the estimated volatility and mean-reversion coefficients, σ and κ, are shown in Figures 11 and 10, respectively. The estimated parameter vector for each firm is provided in Table 14, found in Appendix C.

One notes significant dispersion across firms in the estimated parameters. Monte-Carlo analysis revealed substantial small-sample bias in the MLE estimators, especially for mean reversion (see Table 15 in Appendix C). We therefore obtain sector-by-sector estimates for κ and σ, while allowing for a firm-specific long-run mean parameter θ. Towards this end, we introduce a joint distribution of EDFs across firms in a given industry sector by imposing joint normality of the Brownian motions driving each firm's EDFs, with a flat cross-firm correlation structure. In particular, we generalize Equation (6) by assuming that the logarithm $X_t^i = \log \lambda_t^i$ of the default intensity of firm i satisfies the Ornstein-Uhlenbeck equation

$$dX_t^i \;=\; \kappa \left(\theta^i - X_t^i\right) dt + \sigma \left(\sqrt{\rho}\, dB_t^c + \sqrt{1-\rho}\, dB_t^i\right), \tag{7}$$

where B^c and B^i are independent standard Brownian motions, independent of $\{B^j\}_{j\neq i}$, and the constant pairwise correlation coefficient ρ is an additional parameter to be estimated. The sector-by-sector estimates of the extended parameter vector

$$\Theta = (\{\theta^i\}, \kappa, \sigma, \rho)$$

are shown in Table 4 and in Table 16 in Appendix C.[6]

5 Risk-Neutral Intensity from CDS and EDF

This section explains our methodology for extracting risk-neutral default intensities, and probabilities, from CDS and EDF data.

5.1 Default Swap Pricing

We begin with a simple reduced-form arbitrage-free pricing model for default swaps. Under the absence of arbitrage and market frictions, and under mild technical conditions, there exists a "risk-neutral" probability measure, also known as an "equivalent martingale" measure, as shown by Harrison and Kreps (1979) and Delbaen and Schachermayer (1999). In our setting, markets should not be assumed to be complete, so the martingale measure is not unique. This pricing approach nevertheless allows us, under its conditions, to express the price at time t of a security paying

[6] The intensity λ is measured in basis points.

Table 3: Number of observations of 1-year EDFs. Data: Moody's KMV.

Ticker	sector†	not censored	capped at 0.02%	capped at 20%	total	Ticker	sector	not censored	capped at 0.02%	capped at 20%	total
253647Q	B&E	55	18	0	73	IPG	B&E	119	1	0	120
ABC	H	113	0	0	113	JNJ	H	26	118	0	144
ABT	H	82	62	0	144	KMG	O&G	123	21	0	144
ADELQ	B&E	97	0	16	113	KMI	O&G	142	2	0	144
AGN	H	126	18	0	144	KMP	O&G	139	3	0	142
AHC	O&G	137	7	0	144	KRI	B&E	70	50	0	120
AMGN	H	36	108	0	144	L	B&E	81	21	0	102
APA	O&G	144	0	0	144	LH	H	120	0	0	120
APC	O&G	144	0	0	144	LLY	H	101	43	0	144
BAX	H	144	0	0	144	MCCC	B&E	56	0	0	56
BC	B&E	120	0	0	120	MDT	H	39	105	0	144
BEV	H	142	0	2	144	MGLH	H	103	0	19	122
BHI	O&G	144	0	0	144	MMM	H	42	78	0	120
BJS	O&G	144	0	0	144	MRK	H	50	70	0	120
BLC	B&E	115	5	0	120	MRO	O&G	144	0	0	144
BMY	H	54	90	0	144	NBR	O&G	144	0	0	144
BR	O&G	129	15	0	144	NEV	O&G	137	0	0	137
BSX	H	123	21	0	144	NOI	O&G	97	0	0	97
CAH	H	144	0	0	144	OCR	H	131	13	0	144
CAM	O&G	113	0	0	113	OEI	O&G	124	0	0	124
CCU	B&E	142	2	0	144	OMC	B&E	120	0	0	120
CHIR	H	144	0	0	144	OXY	O&G	132	12	0	144
CHK	O&G	130	0	13	143	PDE	O&G	144	0	0	144
CHTR	B&E	50	0	11	61	PFE	H	36	84	0	120
CMCSA	B&E	144	0	0	144	PHA	H	77	23	0	100
CNG	U	72	13	0	85	PKD	O&G	144	0	0	144
COC	O&G	45	0	0	45	PRM	B&E	107	0	3	110
COP	O&G	133	11	0	144	PXD	O&G	144	0	0	144
COX	B&E	116	0	0	116	RCL	B&E	141	0	0	141
CVX	O&G	39	105	0	144	RIG	O&G	136	4	0	140
CYH	H	97	0	0	97	SBGI	B&E	113	0	0	113
DCX	A	68	6	0	74	SGP	H	61	59	0	120
DGX	H	93	0	0	93	SLB	O&G	85	35	0	120
DIS	B&E	103	41	0	144	SUN	O&G	120	0	0	120
DO	O&G	97	10	0	107	THC	H	144	0	0	144
DVN	O&G	135	9	0	144	TLM	O&G	126	18	0	144
DYN	U	121	0	13	134	TRI	H	67	0	0	67
EEP	O&G	114	6	0	120	TSG	B&E	94	3	0	97
ENRNQ	O&G	105	1	1	107	TSO	O&G	135	0	0	135
EP	O&G	143	0	1	144	TWX	B&E	144	0	0	144
EPD	O&G	76	0	0	76	UCL	O&G	117	3	0	120
F	A	143	1	0	144	UHS	H	120	0	0	120
FST	O&G	142	0	0	142	UNH	H	113	7	0	120
GDT	H	115	2	0	117	VIA	B&E	139	5	0	144
GENZ	H	144	0	0	144	VLO	O&G	144	0	0	144
GLM	O&G	120	0	0	120	VPI	O&G	144	0	0	144
GM	A	144	0	0	144	WFT	O&G	143	1	0	144
HAL	O&G	144	0	0	144	WLP	H	137	7	0	144
HCA	H	140	4	0	144	WMB	U	135	1	8	144
HCR	H	120	0	0	120	WYE	H	86	58	0	144
HMA	H	95	25	0	120	XOM	O&G	0	120	0	120
HRC	H	131	0	4	135	XTO	O&G	120	0	0	120
HUM	H	142	0	0	142	YBTVA	B&E	120	0	0	120
ICCI	T	65	0	0	65						

† A: Automobile; B&E: Broadcasting and Entertainment; H: Healthcare; O&G: Oil and Gas; R: Retail; T: Transportation; U: Utilities.

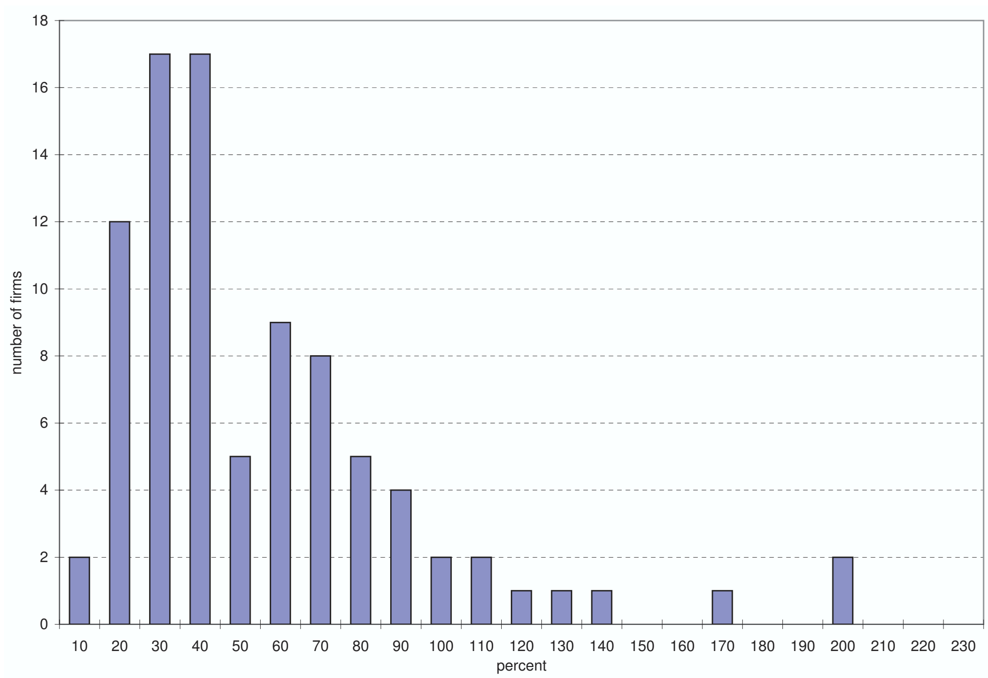

Figure 10: Distribution of estimated default intensity mean-reversion parameters (κ).

some amount, say W, at some stopping time $\tau > t$, of

$$S_t = E^Q \left(e^{-\int_t^\tau r(u)\, du} W \;\middle|\; \mathcal{F}_t \right), \tag{8}$$

where r is the short-term interest-rate process[7] and E^Q denotes expectation with respect to an equivalent martingale measure Q, that we fix. One may view (8) as the definition of such a measure Q. The idea is that the actual measure P and the risk-neutral measure Q differ by an adjustment for risk premia.

Under our earlier assumption of default timing according to a default intensity process λ (under the actual probability measure P that generates our data), Artzner and Delbaen (1992) show that there also exists a default intensity process λ^* under Q. Even though we have assumed the double-stochastic property under P, this need not imply the same convenient double-stochastic property under Q as well. Indeed, Kusuoka (1999) gave a counterexample. We will nevertheless assume the double-stochastic property under Q. (Sufficient conditions are given in Duffie (2001),

[7]Here, r is a progressively measurable process with $\int_0^t |r(s)|\, ds < \infty$ for all t, such that there exists a "money-market" trading strategy, allowing investment at any time t of one unit of account, with continual re-investment until any future time T with a final value of $e^{\int_t^T r(s)\, ds}$.

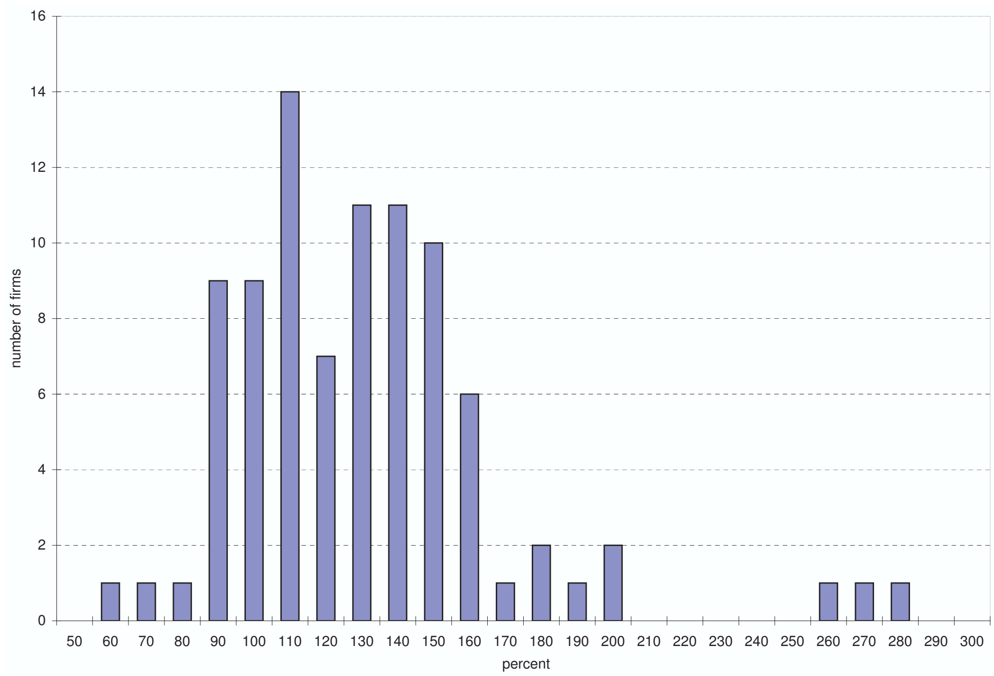

Figure 11: Distribution of estimated default intensity volatility parameters (σ).

Appendix N.) Thus, we have

$$Q(\tau > T \mid \mathcal{F}_t) = p^*(t,T) = E^Q\left(e^{-\int_t^T \lambda^*(u)\,du} \;\middle|\; \mathcal{F}_t\right), \tag{9}$$

provided the firm in question has survived to t.

For convenience, we assume independence, under Q, between interest rates on the one hand, and on the other the default time τ and loss given default. We have verified that, except for levels of volatility of r and λ^* far in excess of those for our sample, the role of risk-neutral correlation between interest rates and default risk is in any case negligible for our parameters. This is not to suggest that the magnitude of the correlation itself is negligible. (See, for example, Duffee (1998).) It follows from (8) and this independence assumption that the price of a zero-coupon defaultable bond with maturity T and zero recovery at default is given by

$$d(t,T) = \delta(t,T)p^*(t,T), \tag{10}$$

where $\delta(t,T) = E_t^Q\left(e^{-\int_t^T r(s)\,ds}\right)$ is the default-free market discount and $p^*(t,T)$ is the risk-neutral conditional survival probability of (9).

Table 4: Sector EDF-implied default intensity parameters.

	$\hat{\kappa}$	$\hat{\sigma}$	$\hat{\rho}$	no. firms
Oil and Gas	0.470	1.223	0.243	40
Healthcare	0.421	1.231	0.124	25
Broadcasting and Entertainment	0.427	1.232	0.232	19

Extensions to the case of correlated interest rates and default times are treated, for example, in Lando (1998).

A default swap stipulates quarterly payments by the buyer of protection of premiums at an annual rate of c, as a fraction of notional, until the default-swap maturity or default, whichever is first. From (10), the market value of the payments by the buyer of protection at the origination date of a default swap of unit notional size is thus $cg(t)$, where

$$g(t) = \frac{1}{4} \sum_{i=1}^{n} \delta(t, t(i)) p^*(t, t(i)), \tag{11}$$

for premium payment dates $t(1), \ldots, t(n)$. The market value of the potential payment by the seller of protection on this default swap is

$$h(t, c) = E^Q \left(\delta(t, \tau) W_\tau^c 1_{\tau \leq t(n)} \;\middle|\; \mathcal{F}_t \right), \tag{12}$$

for the payment at default, if it occurs at time t, of

$$W_t^c = L_t^* - c \left(t - \frac{\lfloor 4t \rfloor}{4} \right), \tag{13}$$

where $\lfloor x \rfloor$ denotes the largest integer less than x, and where L_t^* denotes the risk-neutral expected fractional loss of notional at time t, assuming immediate default.[8] The second term in (13) is a deduction for accrued premium.

The current CDS rate is that choice $C(t)$ for the premium rate c at which the market values of the payments by the buyer and seller of protection are equal. That is, $C(t)$ solves

$$C(t) g(t) = h(t, C(t)). \tag{14}$$

Noting that $h(t, c)$ is linear with respect to c, this is a linear equation to solve for

[8] A more precise definition of L_t^* is given on page 130 of Duffie and Singleton (2003).

$C(t)$.

We turn to the calculation of $h(t,c)$. By the doubly-stochastic property (see, for example, Duffie (2001), Chapter 11), we first condition on (λ^*, L^*), and then use the conditional risk-neutral density $e^{-\int_t^s \lambda^*(u)\,du}\lambda^*(s)$ of τ at time s to get

$$h(t,c) = \int_t^{t(n)} \delta(t,s) E^Q \left(e^{-\int_t^s \lambda^*(u)\,du} \lambda^*(s) W_s^c \,\middle|\, \mathcal{F}_t \right) ds. \tag{15}$$

We take L^* to be constant and use, as a numerical approximation of the integral in (15),

$$h(t,c) \simeq \sum_{i=1}^{n} \delta\left(t, \frac{t(i)+t(i-1)}{2}\right) \left[p^*(t,t(i-1)) - p^*(t,t(i))\right] \left(L^* - \frac{c}{8}\right), \tag{16}$$

which involves a time discretization of the integral in (15) that, in effect, approximates, between quarter ends, with a linear discount function and risk-neutral survival function. Then $C(t)$ is calculated from (14) using this approximation.

5.2 Model Specification

For a parametric specification of the risk-neutral default intensity process $\lambda^{*,i}$ of firm i, motivated by our regression results, we suppose that

$$\log \lambda_t^{*,i} = \beta_0 + \beta_1 \log \lambda_t^i + \beta_2 \log v_t + u_t^i, \tag{17}$$

where β_0, β_1 and β_2 are constants, $X^i = \log \lambda^i$ is as specified earlier by (7), and v is the geometric average over a subset J of default intensities $\{\lambda^i\}_{i\in J}$ of firms in the industry group,[9]

$$\log v_t = \frac{1}{|J|} \sum_{i\in J} X_t^i. \tag{18}$$

u^i satisfies

$$du_t^i = -\kappa_u(\theta_u^i - u_t^i)\,dt + \sigma_u\sqrt{\rho_u}\,d\xi_t^c + \sigma_u\sqrt{1-\rho_u}\,d\xi_t^i, \tag{19}$$

for a constant correlation parameter ρ_u and where, under the actual probability measure P, we take ξ^c and ξ^i to be independent standard Brownian motions, independent of the Brownian motions B^c and $\{B^j\}$ of (7).

[9] J usually includes all non-defaulted firms i in the industry group, and v is interpreted as an indicator of sector-wide default risk.

The risk-neutral distribution of $(\lambda^{*,i}, \lambda^i)$ is specified by assuming that

$$\sqrt{\rho}\, dB_t^c + \sqrt{1-\rho}\, dB_t^i = -\frac{\kappa}{\sigma}\gamma\, dt + \sqrt{\rho}\, d\tilde{B}_t^c + \sqrt{1-\rho}\, d\tilde{B}_t^i \tag{20}$$

and

$$\begin{aligned} \sqrt{\rho_u}\, d\xi_t^c + \sqrt{1-\rho_u}\, d\xi_t^i &= -\frac{\tilde{\kappa}_u}{\sigma}\theta_u^i\, dt - \frac{\tilde{\kappa}_u - \kappa_u}{\sigma} u_t^i\, dt \\ &\quad + \sqrt{\rho_u}\, d\tilde{\xi}_t^c + \sqrt{1-\rho_u}\, d\tilde{\xi}_t^i, \end{aligned} \tag{21}$$

where $\tilde{B}^c$, $\tilde{B}^i$, $\tilde{\xi}^c$, and $\tilde{\xi}^i$ are independent standard Brownians motion under the risk-neutral measure Q, independent of $\{\tilde{B}^j\}_{j\neq i}$ and $\{\tilde{\xi}^j\}_{j\neq i}$, and where γ and $\tilde{\kappa}_u$ are constants. In addition to the parameter vector Θ, the model for λ^* requires an estimator of the parameter vector

$$\Theta^* = (\beta_0, \beta_1, \beta_2, \gamma, \{\theta_u^i\}, \kappa_u, \sigma_u, \rho_u, \tilde{\kappa}_u).$$

5.3 Estimation Strategy and Results

For any given firm, we estimate the parameters (Θ, Θ^*) for the joint model of actual and risk-neutral intensity processes in a two-step procedure. First, we estimate the parameter vector Θ of the actual intensity model λ following the procedure described in Section 4. In a second step, fixing the estimate of Θ, and treating this estimate as though in fact equal to the true parameter vector, we estimate the parameter vector Θ^* governing the risk-neutral intensity process λ^* on a sector-by-sector basis. For this second step, our data consists of weekly observations of both 1-year and 5-year default swap rates and 1-year EDFs, over a time period from June 2000 through December 2004. As with the actual default intensity model, this is not a routine MLE procedure since the evaluation of the likelihood function requires a numerical differentiation of the modeled CDS rate $C(t)$ determined by (14), which we approximate using (16). In the current implementation, we only use pairs of CDS-EDF observations where neither the CDS or the EDF data is missing, and for which the EDF is not censored at 20%. In addition, we remove from the sample any firm whose sample-mean EDF is below 10 basis points.

In this current implementation, we impose several over-identifying restrictions in the form of moment conditions on the parameter vector Θ^*, not only to limit the computational burden and to facilitate the estimation procedure for Θ^* in hours rather than days, but also in order to obtain more robust parameter estimates. Preliminary investigations have shown that, by restricting Θ^* so that the parameter-implied stationary means of $\exp(u)$ and u are equal, respectively, to 1 and to the model-implied sample mean across all firms in a given sector, one obtains considerable improvement

Table 5: Sector CDS-implied risk-neutral default intensity parameter estimates

parameter estimates	Oil and Gas	Healthcare	Broadcasting and Entertainment
$\hat{\beta}_0$	0.960	0.576	-2.685
$\hat{\beta}_1$	0.483	0.522	0.400
$\hat{\beta}_2$	0.630	0.629	1.594
$\hat{\gamma}$	0.030	-0.175	-0.421
$\hat{\theta}_u$	-0.858	-0.269	-0.302
$\hat{\kappa}_u$	0.229	0.189	0.234
$\hat{\sigma}_u$	0.887	0.452	0.531
$\hat{\rho}_u$	0.304	0.213	0.529
$\hat{\tilde{\kappa}}_u$	0.137	-0.200	-0.230
sector log-likelihood	-1.040	-1.195	-1.323
no. firms	33	16	13

in the interpretability of the parameter estimates, facilitating the comparison of the implied values for λ and λ^*. On the same grounds, and because the data were unable to pin down the market-price-of-credit-risk factor γ exactly, we choose β_0, β_1, β_2, and γ such that the sum of squared errors between the logarithm of the observed 1-year and 5-year CDS rates and the logarithm of their implied counterparts when using λ^* as in (17) but ignoring the process u, is minimized. Finally, we choose σ_u and $\tilde{\kappa}_u$ so that the implied sample mean and sample standard deviation of the standardized innovations $\epsilon^i_{t+h}, \epsilon^i_{t+2h}, \ldots$ of u^i_t, defined by

$$u^i_{t+h} \quad = \quad \theta^i_u + e^{-\kappa_u h}(u^i_t - \theta_u) + \sigma_u \sqrt{\frac{1 - e^{-2\kappa_u h}}{2\kappa_u}}\, \epsilon^i_{t+h}. \tag{22}$$

are close to 0 and 1, respectively, for each firm i in the sector.[10]

Sector-by-sector parameter estimates for the Broadcasting and Entertainment, Healthcare, and Oil and Gas industry are summarized in Table 5,[11] and sector-by-sector sample moments of the estimated risk premia, that is, the ratio of estimated risk-neutral to estimated actual default intensities, are provided in Table 6. Estimates and summary statistics by firm are listed in Table 17, Appendix C. Figure 12 shows the implied sample paths of λ^* and λ for Royal Caribbean Cruises, and Figure 13 displays the time series of Royal Caribbean's estimated default risk premia.

For example, extracting from Table 5 the fit implied for the healthcare sector, we

[10]Closeness is measured in terms of sum of squared differences.

[11]Both λ and λ^* are measured in basis points.

Table 6: Sample moments for estimated risk premia

	mean	median	min	max	1^{st} quartile	3^{rd} quartile
Oil and Gas	2.166	1.613	0.121	15.135	0.776	2.925
Healthcare	2.141	1.829	0.602	8.720	1.318	2.614
Broadcasting and E.	1.991	1.464	0.117	12.070	0.724	2.720

have

$$\log \lambda_t^* = 0.576 + 0.522 \log \lambda_t + 0.629 \log v_t + u_t,$$

or equivalently,

$$\lambda_t^* = 1.779\, \lambda_t^{0.522}\, v_t^{0.629}\, e^{u_t},$$

where λ_t and λ_t^* are measured in basis points. So, for an actual default intensity of 100 basis points, a geometric average of all default intensities in the sector of 100 basis points, and $u_t = 0$, we get a risk-neutral default intensity of roughly 357 basis points. If the default intensity of firm i increases by 1%, then, everything else being equal, the risk-neutral default intensity $\lambda^{*,i}$ increases by roughly β_1%, and similarly, if the default intensities for each firm in the sector increase by 1%, $\lambda^{*,i}$ increases by roughly $(\beta_1 + \beta_2)$%. The risk-neutral distribution of λ and u are estimated as

$$\begin{aligned} d \log \lambda_t &= 0.470((\hat{\theta}^i + 0.175) - \log \lambda_t)\, dt + 1.223\, d\tilde{B}_t, \\ d \log u_t &= -0.200 u_t\, dt + 0.452\, d\tilde{\xi}_t, \end{aligned}$$

where $\hat{\theta}^i$ is reported in Table 14, Appendix C. The sample averages of the estimated risk premia are 2.17, 2.14, and 2.70 for the oil-and-gas, healthcare, and broadcasting-and-entertainment sector, respectively. Additional sector-by-sector sample statistics of the estimated risk premia are provided in Table 6.

As a diagnostic check, we examine the behavior of the standardized innovations $\epsilon_{t+h}, \epsilon_{t+2h}, \ldots$ of u_t, defined in (22). Under the specified model, and under the actual probability measure P, these innovations are standard normals. Table 7 lists the sample mean and the sample standard deviation (SD) of the fitted versions of these standardized innovations, for each of the three sectors. Finally, Figure 14 shows the associated histogram of fitted ϵ_t, merging across all firms, plotted along with the standard normal density curve.

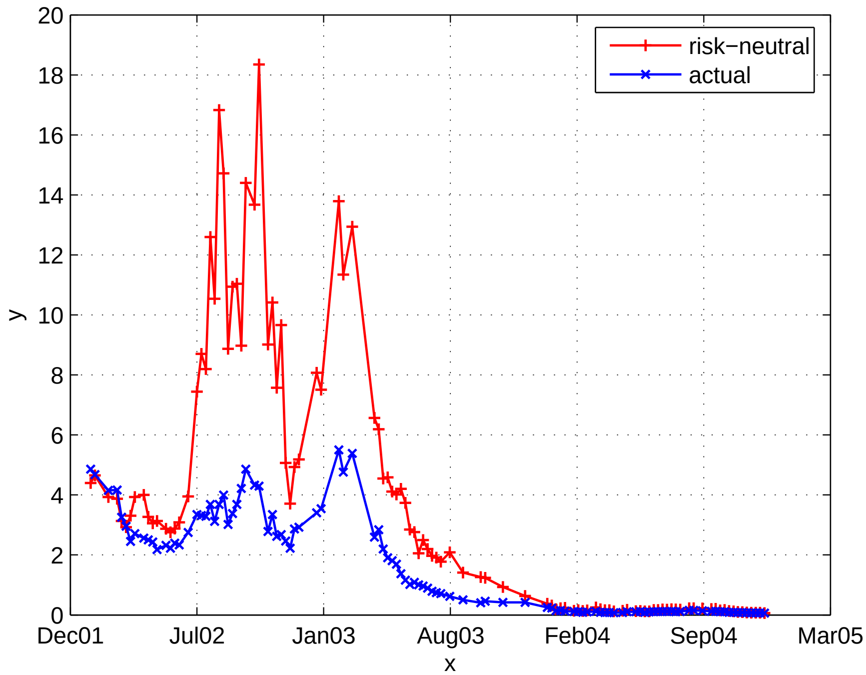

Figure 12: Implied default intensities for Royal Caribbean Cruises. Sources: Moody's KMV and CIBC.

6 Discussion and Conclusion

We compare our results on default risk premia to those available in the literature. Using the structural model of Leland and Toft (1996), Huang and Huang (2003) calibrated parameters for the model determining actual and risk-neutral default probabilities, by credit rating, that are implied from equity-market risk premia, recoveries, initial leverage ratios, and average default frequencies. All underlying parameters were obtained from averages reported by the credit rating agencies, Moody's and Standard and Poors, except for the equity-market risk premia, which were obtained by rating from estimates by Bhandari (1999). At the five-year maturity point, the estimated ratios of annualized risk-neutral to actual five-year default probabilities are reported in Table 8. In magnitude, the results are roughly consistent with those of Driessen (2005). One notes that the risk premium typically declines as default probability increases, as suggested by the results of our basic log-log regression model, and as captured by our time-series formulation.

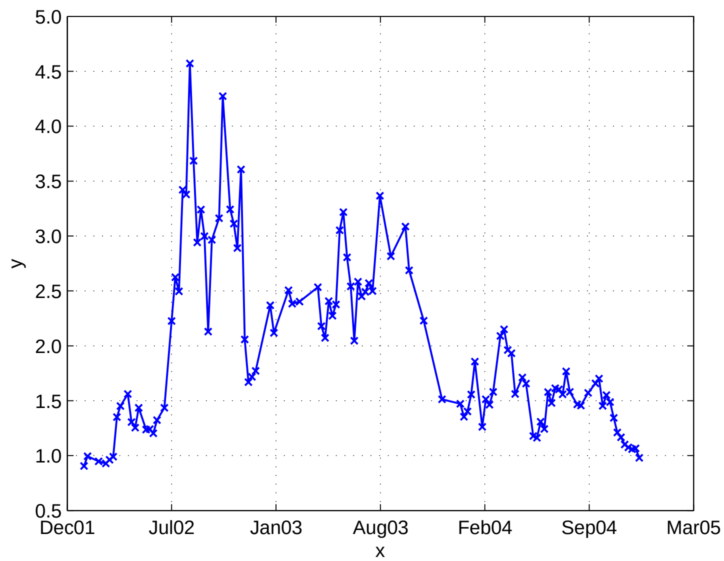

Figure 13: Estimated default risk premia, λ^*/λ, for Royal Caribbean Cruises.

Table 7: Sample moments for standardized innovations

	Mean	SD
Oil and Gas	-0.049	0.980
Healthcare	-0.046	0.938
Broadcasting and Entertainment	-0.033	0.930
All	-0.045	0.959

Table 8: Five-year default risk premium implied by structural-model results of Huang and Huang (2003)

Initial Rating	Premium (ratio)	$Q(\tau < 5)$ (percent)	$P(\tau < 5)$ (percent)
Aaa	1.7497	0.04	0.02
Aa	1.7947	0.09	0.05
A	1.7322	0.25	0.15
Baa	1.4418	1.22	0.84
Ba	1.1658	9.11	7.85
B	1.1058	25.61	23.41

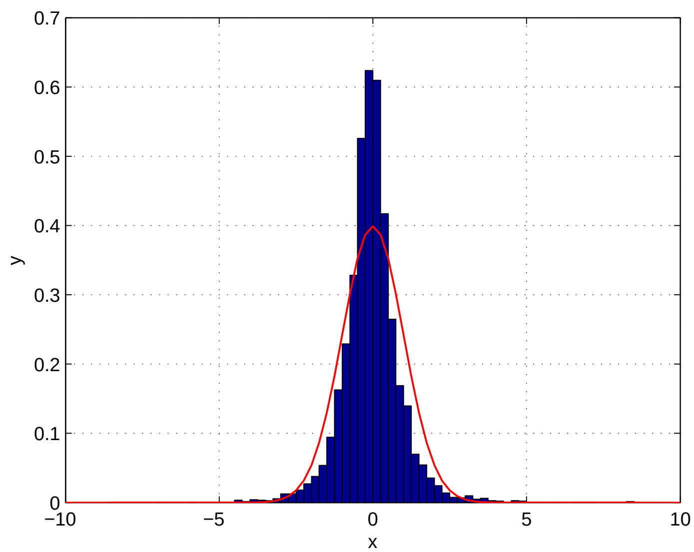

Figure 14: Estimated innovations ϵ across all sectors, and the standard normal density.

A MLE for Intensity from EDFs

This appendix shows our methodology for MLE estimation of the parameters of the default intensity, including the effects of missing EDF data as well as censoring of EDFs by truncation above 20%. Our data is the monthly observed EDF level Y_i at month i, for each of N month-end times $t_0, t_1, \ldots, t_N$.

From (6), for any time t and time step h (which is 1/12 in our application), the discretely sampled log-intensity process X satisfies

$$X_{t+h} = b_0 + b_1 X_t + \epsilon_{t+h}, \tag{A.1}$$

where $b_1 = e^{-\kappa h}$, $b_0 = a/\kappa(1 - b_1)$, and $\epsilon_{t+h}, \epsilon_{t+2h}, \ldots$ are *iid* normal with mean zero and variance $\sigma_\epsilon = \sigma^2(1 - e^{-2\kappa h})/(2\kappa)$.

For a given firm, we initialize the search for the parameter vector $\Theta = (a, \kappa, \sigma)$ as follows. First, we regress $\log(1 - Y_i)$ on $\log(1 - Y_{i-1})$, using only months at which both the current and the lagged EDF are observed and not truncated at 20%. The associated regression coefficient estimates, denoted by $\hat{b}_0$ and $\hat{b}_1$, are considered to be starting estimates of b_0 and b_1, respectively. The sample standard deviation of the fitted residuals, $\hat{\sigma}_\epsilon$, is our starting estimate for σ_ϵ. We then start the search for $\Theta = (a, \kappa, \sigma)$ at

$$\begin{aligned}
\kappa_0 &= -\frac{\log(\hat{b}_1)}{h}, \\
a_0 &= \frac{\hat{b}_0}{1 - \hat{b}_1}\kappa_0, \\
\sigma_0 &= \hat{\sigma}_\epsilon \sqrt{\frac{2\kappa_0}{1 - \exp(-2\kappa_0 h)}}.
\end{aligned}$$

If Θ is the true parameter vector, then $Y_i = G(\lambda(t_i); \Theta)$, where G is defined via (7).

Suppose, to pick an example of a censoring outcome from which the general case can easily be deduced, that for months $k+1$ through $\bar{k} \geq k+1$, inclusive, the EDFs are truncated at $\zeta = 20\%$, meaning that the censored and observed EDF is 20%, implying that the actual EDF was larger than or equal to 20%, and moreover that the EDF data from month $l + 1 > \bar{k} + 1$ to month $\bar{l}$ are missing. Let $\mathcal{I} = \{i : k+1 \leq i \leq \bar{k}\} \cup \{i : l+1 \leq i \leq \bar{l}\}$ denote the censored and missing month numbers. Then the likelihood of the observed non-censored EDFs $Y = \{Y_i : i \notin \mathcal{I}\}$ evaluated at outcomes $y = \{y_i : i \notin \mathcal{I}\}$, using the usual abuse of notation for measures, is defined by

$$\begin{aligned}\mathcal{L}(Y,\mathcal{I};\Theta)\,dy &= \prod_{n=0}^{k-1} P(Y_{n+1}\in dy_{n+1}; Y_n = y_n, \Theta) \\ &\quad\times\ P(Y_{k+1}\geq\zeta,\ldots,Y_{\bar{k}}\geq\zeta, Y_{\bar{k}+1}\in dy_{\bar{k}+1}; Y_k = y_k,\Theta) \\ &\quad\times \prod_{n=\bar{k}+1}^{l-1} P(Y_{n+1}\in dy_{n+1}; Y_n = y_n,\Theta) \\ &\quad\times\ P(Y_{\bar{l}+1}\in dy_{\bar{l}+1}; Y_l = y_l,\Theta) \\ &\quad\times \prod_{n=\bar{l}+1}^{N-1} P(Y_{n+1}\in dy_{n+1}; Y_n = y_n,\Theta),\end{aligned}$$

where $P(\,\cdot\,; Y_n = y_n;\Theta)$ denotes the distribution of $\{Y_{n+1}, Y_{n+2},\ldots\}$ associated with initial condition y_n for Y_n, and associated with parameter vector Θ. A maximum likelihood estimator (MLE) $\hat{\Theta}$ for Θ solves

$$\sup_{\Theta}\mathcal{L}(Y,\mathcal{I};\Theta). \tag{A.2}$$

For $z\in\mathbb{R}$, we let $g(z;\Theta) = G(e^z;\Theta)$, and let $Z_i^\Theta = g^{-1}(Y_i;\Theta)$ denote the logarithm of the default intensity at time t_i that would be implied by a non-censored EDF observation Y_i, assuming the true parameter vector is Θ. Letting $Dg(\,\cdot\,;\Theta)$ denote the partial derivative of $g(\,\cdot\,;\Theta)$ with respect to its first argument, and using standard change-of-measure arguments, we can rewrite the likelihood as

$$\begin{aligned}\mathcal{L}(Y,\mathcal{I};\Theta) &= \prod_{n=0}^{k-1} P(Z_{n+1}^\Theta; Z_n^\Theta,\Theta)\left(Dg(Z_{n+1}^\Theta;\Theta)\right)^{-1} \\ &\quad\times\ P(Y_{k+1}\geq\zeta,\ldots,Y_{\bar{k}}\geq\zeta; Y_k = y_k, Y_{\bar{k}+1} = y_{\bar{k}+1},\Theta) \\ &\quad\times\ P(Z_{\bar{k}+1}^\Theta; Z_k^\Theta,\Theta)\left(Dg(Z_{\bar{k}+1}^\Theta;\Theta)\right)^{-1} \\ &\quad\times \prod_{n=\bar{k}+1}^{l-1} P(Z_{n+1}^\Theta; Z_n^\Theta,\Theta)\left(Dg(Z_{n+1}^\Theta;\Theta)\right)^{-1} \\ &\quad\times\ P(Z_{\bar{l}+1}^\Theta; Z_l^\Theta,\Theta)\left(Dg(Z_{\bar{l}+1}^\Theta;\Theta)\right)^{-1} \\ &\quad\times \prod_{n=\bar{l}+1}^{N-1} P(Z_{n+1}^\Theta; Z_n^\Theta,\Theta)\left(Dg(Z_{n+1}^\Theta;\Theta)\right)^{-1}.\end{aligned} \tag{A.3}$$

The second term on the right-hand side of (A.3) is equal to

$$\begin{aligned} q(Y;\Theta) &= P(Z^{\Theta}_{k+1} \geq g^{-1}(\zeta;\Theta), \ldots, Z^{\Theta}_{\bar{k}} \geq g^{-1}(\zeta;\Theta)\,; \\ &\qquad Z^{\Theta}_{k} = g^{-1}(y_k;\Theta), Z^{\Theta}_{\bar{k}+1} = g^{-1}(y_{\bar{k}+1};\Theta), \Theta). \end{aligned}$$

In the remainder of this appendix, we describe how to compute $q(Y;\Theta)$ by Monte Carlo integration, and hence $P(Y_{k+1} \geq \zeta, \ldots, Y_{\bar{k}} \geq \zeta; Y_k = y_k, Y_{\bar{k}+1} = y_{\bar{k}+1}, \Theta)$. In order to simplify notation we suppress Θ in what follows. We observe that for any time t between times s and u, the conditional distribution of $X(t)$ given $X(s)$ and $X(u)$ is a normal distribution with mean $M(t \,|\, s,u)$ and variance $V(t \,|\, s,u)$ given by

$$\begin{aligned} M(t \,|\, s,u) &= \frac{1 - e^{-2\kappa(u-t)}}{1 - e^{-2\kappa(u-s)}} M(t \,|\, s) + \frac{e^{-2\kappa(u-t)} - e^{-2\kappa(u-s)}}{1 - e^{-2\kappa(u-s)}} M(t \,|\, u), \\ V(t \,|\, s,u) &= \frac{V(t \,|\, s) V(u \,|\, t)}{V(u \,|\, s)}, \end{aligned}$$

where, for times t before u, we let

$$\begin{aligned} M(u \,|\, t) &= \theta + e^{-\kappa(u-t)}(X(t) - \theta) \\ V(u \,|\, t) &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(u-t)}) \\ M(t \,|\, u) &= e^{\kappa(u-t)}(X(u) - \theta(1 - e^{-\kappa(u-t)})) \end{aligned}$$

denote the conditional expectation and variance, respectively, of $X(u)$ given $X(t)$, and the conditional expectation of $X(t)$ given $X(u)$. As a consequence, letting $Z_k = X(t_k)$, we can easily simulate from the joint conditional distribution of $(Z_{k+1}, \ldots, Z_{\bar{k}})$ given Z_k and $Z_{\bar{k}+1}$ which is given by

$$\begin{aligned} P(Z_{k+1}, \ldots, Z_{\bar{k}} \,|\, Z_k, Z_{\bar{k}+1}) &= P(Z_{k+1} \,|\, Z_k, Z_{\bar{k}+1}) \\ &\quad \prod_{j=1}^{\bar{k}-(k+1)} P(Z_{k+j+1} \,|\, Z_{k+j}, Z_{\bar{k}+1}). \end{aligned}$$

We are now in a position to estimate the quantity in (A.4) by generating some "large" integer number J of independent sample paths $\{(Z^j_{k+1}, \ldots, Z^j_{\bar{k}}); 1 \leq j \leq J\}$ from the joint conditional distribution of $(Z_{k+1}, \ldots, Z_{\bar{k}})$ given Z_k and $Z_{\bar{k}+1}$, and by computing the fraction of those paths for which $Z^j_i \geq g^{-1}(\zeta)$ for all i in $\{k+1, \ldots, \bar{k}\}$.

B Solution of Log-Normal Intensity Model

This appendix provides an algorithm, prepared for this project by Gustavo Manso, for computing the survival probability of (5), and related expectations of the form

$E(e^{-\int_0^t \lambda(s)\,ds} F(\lambda_t))$, for a well-behaved function $F : [0, \infty) \to \mathbb{R}$. The algorithm allows for a generalization of the log-normal intensity model to a model that is, in logarithms, autoregressive with a mixture-of-normals innovation, allowing for fat tails and skewness. Matlab code is downloadable at the web site www.stanford.edu/∼/manso/numerical/.

INPUTS: Parameters $(k, m_1, v_1, p, m_2, v_2, m)$ and initial log-intensity $x \in [a, b]$.

OUTPUT: Let $y(j) = \lambda(t_j)$, for equally spaced times $t_0, t_1, \ldots, t_m$. The output is

$$S(0, x) = E\left[\exp\left(-\sum_{j=1}^{m} y(j)\right) F(y(m))\right],$$

where

$$\begin{aligned} \log y(j) &= -k \log y(j-1) + W(j) + Z(j), \\ \log y(0) &= x, \end{aligned}$$

and $W(j)$ is normal, mean m_1, variance v_1, $Z(j)$ is, with probability p, equal to 0 (no jump) and with probability $1 - p$, normal with mean m_2, variance v_2. All $W(j)$ and $Z(j)$ are independent.

Step 1 Compute $K \geq N + 1$ Chebyshev interpolation nodes on $[-1, 1]$:

$$z_k = -\cos\left(\frac{2k-1}{2K}\pi\right),\ k = 1, \ldots, K.$$

Step 2 Adjust the nodes to the $[a, b]$ interval:

$$x_k = (z_k + 1)\left(\frac{b-a}{2}\right) + a,\ k = 1, \ldots, K.$$

Step 3 Evaluate Chebyshev polynomials:

$$T_n(z_k) = \cos(n \cos^{-1} z_k),\ k = 1, \ldots, K \ \text{ and } \ n = 1, \ldots, N.$$

Step 4 Recursive Integration:

- Boundary condition: $S(m, x) = F(\exp(x))$, for $x \in [a, b]$.

- For $j = m : -1 : 0$,

  1. Numerical Integration:

$$S(j, x_k) = \pi^{-\frac{1}{2}} \sum_{i=1}^{I} \omega_i \left[pq(j+1, u_a(i, x_k)) + (1-p)q(j+1, u_b(i, x_k)) \right],$$

  where

$$\begin{aligned} q(j, u) &= S(j+1, u) \exp(-\exp(u)), \\ u_a(i, x) &= \sqrt{2v_1}\phi_i + (m_1 - kx), \\ u_b(i, x) &= \sqrt{2(v_1 + v_2)}\phi_i + (m_1 + m_2 - kx), \end{aligned}$$

  and (ω_i, ϕ_i), $i = 1, \ldots, I$, are I-point Gauss-Hermite quadrature weights and nodes.[12]

  2. Compute the Chebyshev coefficients:

$$c_n = \frac{\sum_{k=1}^{K} S(j, x_k) T_n(z_k)}{\sum_{k=1}^{K} T_n(z_k)^2} \text{ for } n = 0, \ldots, N,$$

  to arrive at the approximation for $S(j, x)$, $x \in [a, b]$:

$$\widehat{S}(j, x) = \sum_{n=0}^{N} c_n T_n \left(2\frac{x-a}{b-a} - 1 \right).$$

C Additional Background Statistics

This appendix contains additional background statistics regarding the firms studied. Section 2 contains the data regarding firms from the broadcasting-and-entertainment industry. This appendix includes information regarding the firms studied from the healthcare and the oil-and-gas industries.

[12] See Judd (1998), page 262, for a table with (ω_i, ϕ_i).

Table 9: Healthcare firms

Firm Name	Median EDF	Median Rating	No. Quotes
Abbott Laboratories	4.0	A1	1,845
Allergan Inc	3.0	A3	2,137
Amerisource Bergen Corp	83.5	Ba3	437
Amgen Inc	2.0	A2	2,159
Baxter International Inc	32.0	Baa1	2,252
Beverly Enterprises Inc	1,086.0	B1	285
Boston Scientific Corp	5.0	Baa1	1,813
Bristol-Myers Squibb Co	22.0	A1	2,063
Cardinal Health Inc	15.0	Baa3	1,753
Chiron Corp	12.0	Baa2	1,920
Community Health Systems Inc	98.0	N/A	307
Eli Lilly & Co	3.0	Aa3	1,942
Genzyme Corp	24.0	N/A	1,657
Guidant Corp	5.0	Baa1	1,407
HCA Inc	23.0	Ba2	891
Health Management Associates Inc	10.0	N/A	2,222
Healthsouth Corp	–	N/A	318
Humana Inc	40.0	Baa3	1,925
Johnson & Johnson	2.0	Aaa	1,654
Laboratory Corp Of America Holdings	12.0	Baa3	1,635
Manor Care Inc	21.0	Baa3	1,168
Medtronic Inc	2.0	N/A	2,093
Merck & Co Inc	5.0	Aa2	1,516
Minnesota Mining & Manufacturing Co (3M)	2.0	Aa1	1,655
Pfizer Inc	2.0	Aaa	1,504
Pharmacia Corporation	9.0	Aaa	1,116
Quest Diagnostics	10.0	Baa2	1,230
Schering-Plough Corporation	25.0	Baa1	1,658
Tenet Healthcare Corporation	67.0	B3	–
Triad Hospitals Inc	148.0	B2	519
United Health Group Inc	2.0	A3	1,442
Universal Health Services Inc	33.5	Baa3	1,237
Wellpoint Health Networks	–	Baa1	1,580
Wyeth	17.0	Baa1	2,150

Table 10: Oil and gas firms

Firm Name	Median EDF	Median Rating	No. Quotes
Amerada Hess Corp	20.0	Ba1	1,284
Anadarko Petroleum Corp	43.0	Baa1	2,696
Apache Corp	11.0	A3	2,217
Baker Hughes Inc	15.0	A2	2,207
BJ Services Co	17.0	Baa2	1,588
BurlingtonResourcesInc	10.0	Baa1	2,056
Chesapeake Energy Corp	177.0	Ba3	1,152
Chevron Texaco Corp	3.0	N/A	1,897
Conoco Phillips Holding Co	15.5	A3	1,677
Cooper Cameron Corp	29.0	Baa1	1,518
XTO Energy Inc	6.0	Baa3	1,225
Diamond Offshore Drilling	25.0	Baa2	2,331
EL Paso Corp	1,000.0	Caa1	2,294
Exxon Mobil Corp	2.0	N/A	1,329
Forest Oil Corp	107.0	Ba3	367
Global Marine Inc	12.0	Baa1	1,401
Halliburton Co	86.0	Baa2	2,139
Kerr-Mc Gee Corp	38.0	Baa3	2,170
Kinder Morgan Energy Partners LP	12.0	Baa1	2,263
Kinder Morgan Inc	7.0	Baa2	2,003
National-Oilwell Inc	31.0	Baa2	1,201
Occidental Petroleum Corp	8.0	Baa1	2,581
Parker Drilling Co	446.5	B2	449
Conoco Phillips	6.0	A3	2,929
Pioneer Natural Resources Co	44.0	Baa3	1,001
Pride International Inc	113.5	Ba2	1,228
Shell Oil Co	–	Aa2	1,373
Sunoco Inc	7.5	Baa2	1,536
Talisman Energy Inc	5.0	N/A	1,425
Transocean Inc	71.5	Baa2	2,487
Unocal Corp	6.0	Baa2	1,441
Marathon Oil Corp	10.0	Baa1	2,024
Valero Energy Corp	35.5	Baa3	2,637
Vintage Petroleum Inc	229.0	Ba3	556
Weatherford International Ltd	35.0	Baa1	2,874
Enron Corp	51.5	WR	361
Devon Energy Corporation	19.0	Baa2	2,878
Enterprise Products Partners LP	5.0	N/A	1,439
Enbridge Energy Partners LP	6.0	N/A	1,192
Nabors Industries Ltd	41.0	N/A	2,594
Schlumberger Ltd	8.0	N/A	1,673
Schlumberger Technology Corp	–	A2	1,021
Duke Energy Field Services Llc	–	Baa2	1,004

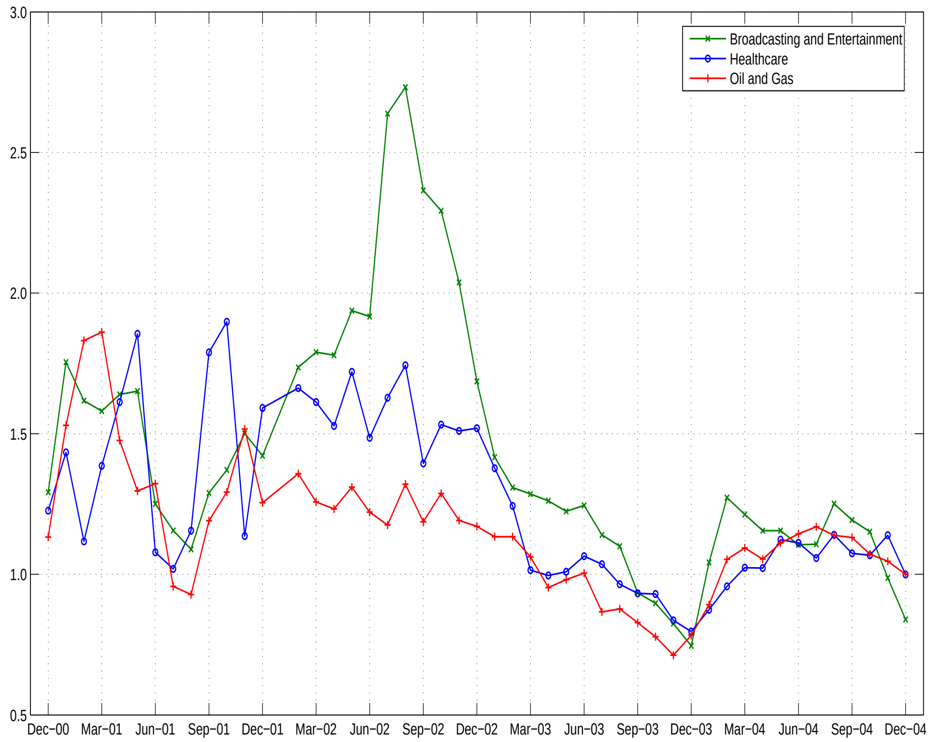

Figure 15: Monthly dummy multipliers by industry sector in CDS-to-EDF fit.

Table 11: CDS-EDF regression results

Results for Daily Median CDS Data

	Levels Model†	Standard Error	Log-Log Model†	Standard Error	Log-Log Model	Standard Error
Number of CDS Samples	18,259		18,259		22,501	
Intercept	-28.877	7.846	0.912	0.029	1.186	0.027
Slope	1.541	0.007	0.828	0.004	0.830	0.004
Broadcasting Dummy	104.985	3.187	0.389	0.010	0.348	0.009
Healthcare Dummy	21.439	3.053	0.196	0.010	0.181	0.009
Dec-00 Dummy	2.605	29.525	0.545	0.094	0.077	0.040
Jan-01 Dummy	31.516	22.927	0.554	0.073	0.296	0.071
Feb-01 Dummy	42.600	21.568	0.594	0.069	0.330	0.067
Mar-01 Dummy	63.467	21.050	0.704	0.067	0.439	0.065
Apr-01 Dummy	45.407	18.248	0.556	0.058	0.296	0.056
May-01 Dummy	35.145	20.701	0.416	0.066	0.152	0.061
Jun-01 Dummy	22.945	21.569	0.215	0.069	-0.050	0.068
Jul-01 Dummy	18.772	19.937	0.117	0.064	-0.113	0.063
Aug-01 Dummy	-8.447	15.164	0.049	0.048	-0.219	0.047
Sep-01 Dummy	2.341	23.433	0.180	0.075	-0.084	0.073
Oct-01 Dummy	20.670	17.013	0.264	0.054	0.018	0.053
Nov-01 Dummy	20.341	22.436	0.322	0.072	0.129	0.069
Dec-01 Dummy	-8.964	69.162	0.194	0.221	-0.106	0.235
Jan-02 Dummy	33.728	12.343	0.385	0.039	0.122	0.038
Feb-02 Dummy	51.169	11.568	0.504	0.037	0.245	0.032
Mar-02 Dummy	48.130	9.778	0.460	0.031	0.209	0.030
Apr-02 Dummy	43.675	9.485	0.459	0.030	0.203	0.029
May-02 Dummy	63.273	9.417	0.544	0.030	0.289	0.029
Jun-02 Dummy	55.498	9.631	0.489	0.031	0.222	0.029
Jul-02 Dummy	130.020	9.265	0.571	0.030	0.285	0.028
Aug-02 Dummy	152.160	9.375	0.635	0.030	0.362	0.029
Sep-02 Dummy	101.648	9.617	0.488	0.031	0.219	0.029
Oct-02 Dummy	124.762	9.230	0.544	0.030	0.275	0.028
Nov-02 Dummy	105.308	9.413	0.471	0.030	0.202	0.029
Dec-02 Dummy	74.419	9.884	0.428	0.032	0.152	0.030
Jan-03 Dummy	52.690	9.594	0.351	0.031	0.057	0.029
Feb-03 Dummy	43.064	9.892	0.313	0.032	0.017	0.030
Mar-03 Dummy	19.895	9.626	0.239	0.031	-0.062	0.029
Apr-03 Dummy	15.600	9.618	0.187	0.031	-0.102	0.029
May-03 Dummy	23.330	9.671	0.183	0.031	-0.086	0.030
Jun-03 Dummy	27.770	9.605	0.219	0.031	-0.051	0.029
Jul-03 Dummy	11.421	9.646	0.113	0.031	-0.157	0.029
Aug-03 Dummy	4.165	10.808	0.085	0.034	-0.184	0.033
Sep-03 Dummy					-0.266	0.031
Oct-03 Dummy					-0.302	0.031
Nov-03 Dummy					-0.363	0.034
Dec-03 Dummy					-0.390	0.043
Jan-04 Dummy					-0.153	0.032
Feb-04 Dummy					-0.006	0.032
Mar-04 Dummy					0.058	0.030
Sum of Squared Residuals	517,328,076		5,270		6,154	
Total Sum of Squares	2,064,101,382		20,300		25,127	
R^2	0.749		0.740		0.755	

† Regressions are based on CDS data for the period December 2000 through September 2003.

Table 12: CDS-EDF regression results

Results for Intraday CDS Data

	Levels Model†	Standard Error	Log-Log Model†	Standard Error
Number of CDS Samples	40,844		40,844	
Intercept	-35.010	5.167	1.026	0.023
Slope	1.583	0.005	0.785	0.003
Broadcasting Dummy	58.204	1.552	0.322	0.006
Healthcare Dummy	30.340	1.928	0.229	0.007
Dec-00 Dummy	38.421	23.172	0.618	0.088
Jan-01 Dummy	62.357	17.417	0.655	0.066
Feb-01 Dummy	57.496	16.120	0.633	0.061
Mar-01 Dummy	79.335	15.178	0.789	0.058
Apr-01 Dummy	68.074	12.643	0.631	0.048
May-01 Dummy	51.590	14.461	0.443	0.055
Jun-01 Dummy	37.881	15.176	0.263	0.058
Jul-01 Dummy	25.792	13.116	0.194	0.050
Aug-01 Dummy	5.740	10.373	0.125	0.040
Sep-01 Dummy	5.060	16.124	0.207	0.061
Oct-01 Dummy	35.723	11.398	0.313	0.043
Nov-01 Dummy	36.374	15.471	0.391	0.059
Dec-01 Dummy	18.565	57.107	0.278	0.218
Jan-02 Dummy	45.780	7.755	0.484	0.030
Feb-02 Dummy	59.139	7.147	0.592	0.027
Mar-02 Dummy	49.396	6.082	0.531	0.023
Apr-02 Dummy	33.833	5.833	0.456	0.022
May-02 Dummy	61.333	5.873	0.575	0.022
Jun-02 Dummy	43.272	5.883	0.509	0.022
Jul-02 Dummy	112.068	5.708	0.622	0.022
Aug-02 Dummy	128.001	5.781	0.674	0.022
Sep-02 Dummy	91.658	5.854	0.534	0.022
Oct-02 Dummy	101.796	5.674	0.558	0.022
Nov-02 Dummy	89.819	5.775	0.499	0.022
Dec-02 Dummy	63.924	6.096	0.442	0.023
Jan-03 Dummy	42.351	6.127	0.355	0.023
Feb-03 Dummy	36.320	6.168	0.341	0.024
Mar-03 Dummy	21.240	6.078	0.276	0.023
Apr-03 Dummy	20.482	6.109	0.250	0.023
May-03 Dummy	21.402	6.093	0.221	0.023
Jun-03 Dummy	25.479	5.989	0.239	0.023
Jul-03 Dummy	9.514	6.196	0.117	0.024
Aug-03 Dummy	8.619	6.823	0.103	0.026
Sum of Squared Residuals	792797345		11500	
Total Sum of Squares	2,952,660,064		35,702	
R^2	0.731		0.678	

† Regressions are based on CDS data for the period December 2000 through September 2003.

Table 13: CDS-EDF regression results

	Results for Daily Median CDS Data					
	Estimate†	Std. Error	Estimate†	Std. Error	Estimate†	Std. Error
Number of CDS Samples	33,912					
Intercept	1.448	0.047				
Slope	0.760	0.015				
Sector-month dummy	Oil and Gas		Broadcasting and E.		Healthcare	
Dec-00	0.124	0.006	0.256	0.007	0.204	0.011
Jan-01	0.425	0.011	0.562	0.009	0.360	0.008
Feb-01	0.605	0.004	0.481	0.010	0.111	0.007
Mar-01	0.621	0.005	0.458	0.010	0.326	0.009
Apr-01	0.389	0.009	0.495	0.013	0.478	0.009
May-01	0.260	0.012	0.502	0.019	0.618	0.009
Jun-01	0.280	0.014	0.223	0.014	0.075	0.008
Jul-01	-0.044	0.015	0.145	0.016	0.019	0.010
Aug-01	-0.075	0.021	0.085	0.014	0.144	0.005
Sep-01	0.174	0.032	0.254	0.020	0.582	0.005
Oct-01	0.256	0.019	0.316	0.019	0.641	0.008
Nov-01	0.416	0.015	0.406	0.014	0.128	0.008
Dec-01 and Jan-02	0.227	0.017	0.352	0.020	0.465	0.011
Feb-02	0.306	0.020	0.551	0.023	0.508	0.021
Mar-02	0.229	0.018	0.582	0.021	0.478	0.021
Apr-02	0.209	0.016	0.576	0.024	0.424	0.015
May-02	0.270	0.015	0.661	0.024	0.542	0.014
Jun-02	0.200	0.016	0.651	0.024	0.396	0.009
Jul-02	0.162	0.018	0.970	0.030	0.487	0.012
Aug-02	0.278	0.017	1.005	0.031	0.556	0.012
Sep-02	0.171	0.018	0.861	0.029	0.332	0.011
Oct-02	0.253	0.019	0.830	0.031	0.427	0.012
Nov-02	0.175	0.018	0.712	0.029	0.412	0.013
Dec-02	0.157	0.016	0.522	0.028	0.418	0.014
Jan-03	0.125	0.016	0.348	0.027	0.320	0.015
Feb-03	0.126	0.015	0.269	0.026	0.217	0.014
Mar-03	0.060	0.014	0.251	0.024	0.015	0.010
Apr-03	-0.048	0.014	0.232	0.021	-0.004	0.009
May-03	-0.019	0.014	0.202	0.020	0.009	0.006
Jun-03	0.004	0.012	0.219	0.018	0.062	0.006
Jul-03	-0.143	0.012	0.131	0.015	0.035	0.007
Aug-03	-0.132	0.010	0.095	0.013	-0.036	0.005
Sep-03	-0.188	0.010	-0.068	0.013	-0.070	0.004
Oct-03	-0.251	0.011	-0.109	0.013	-0.073	0.004
Nov-03	-0.340	0.010	-0.193	0.009	-0.179	0.004
Dec-03	-0.245	0.007	-0.293	0.007	-0.228	0.003
Jan-04	-0.115	0.006	0.041	0.006	-0.135	0.001
Feb-04	0.052	0.004	0.241	0.006	-0.044	0.000
Mar-04	0.089	0.003	0.193	0.006	0.023	0.001
Apr-04	0.053	0.003	0.144	0.005	0.022	0.001
May-04	0.106	0.002	0.144	0.005	0.116	0.001
Jun-04	0.134	0.002	0.100	0.006	0.106	0.001
Jul-04	0.156	0.002	0.102	0.006	0.056	0.002
Aug-04	0.130	0.001	0.224	0.006	0.131	0.002
Sep-04	0.123	0.001	0.176	0.006	0.072	0.002
Oct-04	0.069	0.002	0.141	0.005	0.066	0.003
Nov-04	0.045	0.001	-0.013	0.004	0.130	0.002
Dec-04	reference		-0.175	0.004	-0.001	0.002
Sum of Squared Residuals	8,742.760					
Total Sum of Squares	34,098.286					
R^2	0.744					

† Regressions are based on CDS data for the period December 2000 through December 2004.

Table 14: Fitted parameters of default intensity models

Ticker	$\hat{\theta}$	$\hat{\kappa}$	$\hat{\sigma}$	Ticker	$\hat{\theta}$	$\hat{\kappa}$	$\hat{\sigma}$
253647Q	†	–	–	IPG	4.547	0.169	1.007
ABC	4.061	1.980	2.521	JNJ	†	–	–
ABT	†	–	–	KMG	2.231	0.239	0.910
ADELQ	5.869	0.539	1.541	KMI	1.593	1.670	2.753
AGN	1.284	0.366	0.902	KMP	2.537	0.383	1.041
AHC	2.215	0.574	1.116	KRI	†	–	–
AMGN	†	–	–	L	3.225	0.269	1.086
APA	2.211	0.298	0.909	LH	2.216	0.169	1.307
APC	2.033	0.238	0.855	LLY	†	–	–
BAX	2.360	0.649	1.148	MCCC	6.124	1.034	1.522
BC	2.761	0.377	1.082	MDT	†	–	–
BEV	4.740	0.586	1.424	MGLH	6.636	0.125	1.059
BHI	1.850	0.202	0.793	MMM	†	–	–
BJS	2.897	0.730	1.310	MRK	†	–	–
BLC	2.316	0.248	1.073	MRO	2.232	0.359	0.897
BMY	†	–	–	NBR	2.936	1.080	1.559
BR	1.897	0.401	0.994	NEV	4.562	0.274	0.955
BSX	1.813	0.701	1.822	NOI	3.571	1.196	1.900
CAH	2.183	0.595	1.293	OCR	2.445	0.245	1.311
CAM	3.167	0.592	1.101	OEI	4.192	0.331	1.227
CCU	2.445	0.390	1.483	OMC	2.755	1.209	1.769
CHIR	2.575	0.617	1.222	OXY	-0.378	0.078	0.696
CHK	3.261	0.167	1.265	PDE	4.234	0.811	1.533
CHTR	11.718	0.116	1.062	PFE	†	–	–
CMCSA	3.317	0.528	0.972	PHA	†	–	–
CNG	†	–	–	PKD	5.741	0.141	1.201
COC	2.702	1.959	1.965	PRM	6.693	0.054	1.142
COP	†	–	–	PXD	3.777	0.437	1.311
COX	2.345	0.647	1.387	RCL	2.661	0.382	1.210
CVX	†	–	–	RIG	2.216	0.313	1.377
CYH	4.285	0.957	1.478	SBGI	5.183	0.677	1.500
DCX	3.502	0.680	1.353	SGP	2.861	0.149	0.571
DGX	0.408	0.154	0.856	SLB	1.846	0.289	0.871
DIS	1.773	0.360	0.879	SUN	2.452	0.307	0.933
DO	1.882	0.298	1.437	THC	3.526	0.393	1.002
DVN	2.274	0.335	1.392	TLM	1.886	0.278	1.200
DYN	‖	–	–	TRI	4.189	0.656	0.842
EEP	1.657	0.194	0.867	TSG	3.678	0.241	1.255
ENRNQ	‡	–	–	TSO	4.097	0.544	1.299
EP	5.014	0.264	1.040	TWX	3.253	0.296	1.097
EPD	1.731	1.377	2.666	UCL	1.506	0.188	0.833
F	2.568	0.401	1.127	UHS	3.039	0.880	1.168
FST	4.478	0.825	1.345	UNH	1.786	0.302	1.235
GDT	1.618	0.562	1.083	VIA	2.133	0.657	1.452
GENZ	2.309	0.817	1.486	VLO	2.682	0.281	1.038
GLM	2.283	0.307	1.025	VPI	4.047	0.751	1.547
GM	3.008	0.974	1.358	WFT	2.132	0.189	1.102
HAL	2.967	0.407	1.457	WLP	2.646	0.740	1.469
HCA	2.004	0.427	1.740	WMB	3.699	0.211	1.258
HCR	2.679	0.361	1.016	WYE	1.812	0.536	0.875
HMA	2.115	0.259	0.961	XOM	†	†	†
HRC	4.033	0.398	1.668	XTO	2.299	0.155	0.978
HUM	3.936	0.370	1.390	YBTVA	5.440	0.793	1.535
ICCI	6.164	0.610	1.402				

† No estimates provided; the sample mean of the 1-year EDF is less than 10 basis points.

‡ No estimates within admissible parameter region; the estimate for the mean-reversion parameter κ is negative.

‖ Firm removed from data set.

Table 15: MC distribution of default intensity parameter estimates

	$\hat{\theta}$	$\hat{\kappa}$	$\hat{\sigma}$
"true parameters"	4.00	0.50	1.00
	10 years of data		
mean	3.95	0.87	1.21
std. dev.	(0.50)	(0.40)	(0.26)
	50 years of data		
mean	3.98	0.58	1.04
std. dev.	(0.21)	(0.15)	(0.09)

Table 16: Fitted parameters of default intensity models

Oil and Gas		Healthcare		Broadcasting and E.	
Ticker	$\hat{\theta}$	Ticker	$\hat{\theta}$	Ticker	$\hat{\theta}$
AHC	2.093	ABC	4.544	ADELQ	5.862
APA	2.864	AGN	1.778	BC	3.063
APC	2.411	BAX	2.292	BLC	2.248
BHI	2.675	BEV	4.706	CCU	2.485
BJS	3.230	BSX	1.978	CHTR	6.359
BR	1.708	CAH	2.322	CMCSA	3.436
CAM	3.430	CHIR	2.767	COX	2.479
CHK	5.378	DGX	3.930	DIS	1.503
COC	2.861	GDT	2.191	IPG	3.338
DO	2.632	GENZ	2.548	L	2.175
DVN	2.588	HCA	2.781	MCCC	5.943
EEP	2.095	HCR	2.809	OMC	2.883
ENRNQ	1.915	HMA	2.063	PRM	4.417
EP	3.647	HRC	4.032	RCL	3.219
EPD	2.530	HUM	3.670	SBGI	5.021
FST	4.863	LH	3.922	TSG	2.915
GLM	3.174	MGLH	5.796	TWX	3.278
HAL	3.261	OCR	2.965	VIA	2.289
KMG	2.199	SGP	1.404	YBTVA	5.324
KMI	2.329	THC	3.462		
KMP	2.460	TRI	4.711		
MRO	2.728	UHS	2.091		
NBR	3.169	UNH	2.932		
NEV	4.450	WLP	3.621		
NOI	3.942	WYE	3.305		
OEI	3.890				
OXY	2.490				
PDE	4.514				
PKD	4.364				
PXD	3.891				
RIG	2.897				
SLB	1.805				
SUN	2.743				
TLM	2.191				
TSO	4.377				
UCL	3.288				
VLO	2.301				
VPI	3.021				
WFT	4.137				
XTO	1.461				

Table 17: Fitted parameters and summary statistics for risk-neutral default intensity models

Oil and Gas

Ticker	in J ?	s	mean(λ)	median(λ)
AHC	1	0.880	3.238	3.265
APA	1	0.880	1.090	1.050
APC	1	0.880	0.990	0.862
BHI	1	0.880	0.755	0.568
BJS	1	0.880	1.201	0.585
BR	1	0.880	2.374	2.367
CAM	1	0.880	0.349	0.318
CHK	1	0.880	1.803	1.870
COC	0	0.859	2.841	2.855
DO	1	0.880	1.215	1.125
DVN	1	0.880	2.731	2.088
EEP	0	0.859	4.447	4.547
EP	0	0.859	1.837	1.272
EPD	0	0.859	10.469	10.694
GLM	0	0.859	0.677	0.633
HAL	1	0.880	2.233	1.335
KMG	1	0.880	1.827	1.700
KMI	1	0.880	5.456	5.046
KMP	0	0.859	2.330	2.220
MRO	0	0.859	1.494	1.491
NBR	1	0.880	0.666	0.499
NOI	0	0.859	0.335	0.280
OXY	0	0.859	3.418	3.285
PDE	1	0.880	3.385	3.220
PXD	1	0.880	5.223	5.682
RIG	0	0.859	0.716	0.668
SLB	0	0.859	0.971	0.918
SUN	0	0.859	2.086	1.938
TLM	1	0.880	3.813	3.509
UCL	0	0.859	1.612	1.587
VLO	1	0.880	3.232	2.603
WFT	1	0.880	0.534	0.510
XTO	0	0.859	4.514	4.474

Healthcare

Ticker	in J ?	s	mean(λ)	median(λ)
AGN	1	0.900	3.880	3.796
BAX	1	0.900	1.252	0.878
BSX	1	0.900	2.752	2.738
CAH	1	0.900	1.821	1.815
CHIR	1	0.900	1.782	1.749
DGX	0	0.828	1.645	1.773
GDT	0	0.828	2.614	2.439
GENZ	0	0.828	3.315	2.030
HCR	0	0.828	1.626	1.559
HMA	0	0.828	2.442	2.449
HUM	1	0.900	1.082	1.044
LH	0	0.828	1.727	1.729
SGP	0	0.828	1.203	1.045
UHS	0	0.828	1.235	1.215
UNH	0	0.828	4.092	3.822
WYE	1	0.900	1.913	1.810

Broadcasting and Entertainment

Ticker	in J ?	s	mean(λ)	median(λ)
BC	0	1.477	1.052	1.061
BLC	0	1.477	2.433	2.343
CCU	1	1.575	2.156	1.632
CMCSA	1	1.575	3.068	1.521
COX	0	1.477	3.482	3.174
DIS	1	1.575	2.156	1.910
IPG	0	1.477	0.860	0.812
L	0	1.477	1.998	1.419
OMC	0	1.477	1.429	1.412
RCL	0	1.477	1.952	1.663
TSG	0	1.477	0.480	0.285
TWX	1	1.575	0.909	0.617
VIA	1	1.575	2.432	2.022

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