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What is Risk Neutral Volatility?

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The Risk Neutral Probability Distribution for the S&P 500 Index

The market price of a risky asset depends on the market's assessment of the probability distribution for its future payoff together with the market's risk aversion. Neither of these can be observed directly but a combination of the two, the "Risk Neutral Probability Density" (RND), can be extracted from market prices for options.

The Risk Neutral Density for the S&P 500 index embodies the market's forecast of the true probability distribution of the index level on option expiration date (i.e., the empirical density under the "P" measure) modified by the market's attitudes about bearing risk (i.e., a transformation from the "P" measure to the "Q" measure).

Prices for options and other derivatives are determined under the "risk neutral" Q measure.

In the Black-Scholes (BS) model, the only difference between the empirical P measure and the Q measure is the mean. The mean return under the empirical returns distribution is equal to the riskless rate plus an appropriate risk premium. Risk neutralization rolls all risk premia into the (modified) probabilities of the Q distribution, whose mean becomes simply the riskless rate.

The Risk Neutral Probability Distribution for the S&P 500 Index

The probability distribution for the index level on option expiration date can be estimated in many different ways. The form of the density itself is unknown; extensive research suggests it has fatter tails and other deviations from a lognormal and its shape changes over time.

The factors that go into the risk adjustment process are also unknown. Extensions of the BS model introduce additional stochastic factors like stochastic volatility and jumps of indeterminate size and frequency.

Risk premia for unspanned (i.e., unhedgeable) factors are often treated simply as unknown constants to be estimated. But empirical evidence suggests that this is a gross oversimplification.

Which leaves a major question: Market prices for derivative securities depend heavily on volatility, but ***"What is Risk Neutral Volatility?"***

Why Should We Care?

Option parameters implied from market prices are felt to be more accurate, or at least more meaningful, than those obtained strictly from statistical analysis of historical data

- option pricing entails forecasting the future, not analyzing historical data
- IV (or VIX) as the measure of expected volatility
- Black-Scholes IV depends on the BS model; RND is not model-dependent
- the RND measures how options are being priced, not how they ought to be priced in theory

BS assumed the returns distribution came from a constant volatility lognormal diffusion. This is too simple. Which differences are most important to investors and market makers?

- tail behavior (extreme values, left tail vs. right tail)
- time variation
- trading range over the option's lifetime, or over a shorter holding period

Why Should We Care? continued

The BS model uses the principle of delta-hedging to price options as redundant securities. Delta-hedging is how market makers manage their risk, although it is far from riskless in practice. How risky delta-hedging actually is may affect option pricing.

- selling volatility versus buying volatility
- jumps vs. diffusions
- puts versus calls

Understanding the multiple risk premia in market option prices is important

- significant components of return
- volatility risk of different types (e.g., volatile returns vs. uncertainty about volatility)
- how to extract the best volatility forecast from implied volatility?
- how much is the market is charging / paying for bearing different kinds of risk?
- how do all of these vary through time?

The Risk Neutral Probability Distribution for the S&P 500 Index

Breeden and Litzenberger (Journal of Business, 1978) showed how the risk neutral probability distribution for S_T, the value of the underlying asset on option expiration day can be extracted from a set of market option prices.

Two major problems in constructing a complete risk neutral density from a set of market option prices:

1. How to smooth and interpolate option prices to limit pricing noise and produce a smooth density

2. How to extend the distribution to the tails beyond the range of traded strike prices.

But one terrific advantage is that unlike implied volatility, the risk neutral density is model-free.

Research Questions

This project develops the Breeden-Litzenberger technology to extract RNDs from daily S&P 500 index option prices and then compute the standard deviations.

I explore whether, and how much, the risk neutral volatility is influenced by a broad set of exogenous factors that are expected to be related to estimation of the empirical density and to the risk neutralization process. The main objective is not to build a formal model of option risk premia, but to establish a set of stylized facts that any such model will need to be consistent with.

Several broad questions to be addressed include:

- What return and volatility-related factors are most important to investors in forming the forecasts of the empirical probability density that are embedded in the RND?
- Is the market primarily forward-looking or backward-looking in gauging volatility? What are the relevant time horizons it focuses on?
- What factors influence the process of risk neutralization?
- Do the answers to these questions differ for long maturity vs. short maturity options, or over different time periods?

Previous Literature

There is a broad literature on extracting risk neutral probabilities, and a great deal more on closely related issues like implied volatility. Previous research has followed several distinct tracks.

An incomplete selection

The RND as a window on market expectations. (particularly market expectations about monetary policy and exchange rates):

Bates(1991,1996) ; Gemmill and Saflekos (2000), Campa, Chang and Reider (1998); Malz(1997); Melick and Thomas (1997)

The RND as a window on market risk preferences.

Ait-Sahalia and Lo (1998,2000); Jackwerth, J.C. (2000); Bliss and Panigirtzoglou (2004);

Developing returns processes that produce a given RND: (especially implied binomial and trinomial trees)

Rubinstein, M. (1994, 1998); Derman and Kani (1993, 1998); Jackwerth and Rubinstein(1996) Jackwerth, J.C. (1999);

Methodology: The best procedure for modeling and extracting an RND from market prices

Shimko(1993); Bahra(1996); Bliss and Panigirtzoglou (2002); Bondarenko (2003); Jackwerth, J.C. (2004); Jiang, G.J. and Y.S. Tian (2005)

Extracting the Risk Neutral Density from Options Prices in Theory

The value of a call option is the expected value under the risk neutral distribution of its payoff on the expiration date T, discounted back to the present.

$$C = e^{-rT} \int_X^{\infty} (S_T - X) f(S_T) dS_T$$

Taking the partial derivative with respect to X,

$$\frac{\partial C}{\partial X} = -e^{-rT} \int_X^{\infty} f(S_T) dS_T = -e^{-rT} \left[1 - F(X)\right]$$

$$\boxed{F(X) = e^{rT} \frac{\partial C}{\partial X} + 1}$$

F(X) is the cumulative probability distribution. Taking the derivative again gives f(X), the risk neutral probability density, RND.

$$\boxed{f(X) = e^{rT} \frac{\partial^2 C}{\partial X^2}}$$

Extracting the Risk Neutral Density, a more intuitive approach

Consider a call option that allows you to buy a share of some underlying stock for a price of 101 one month from now. If the stock price in one month is above 101, you will exercise the option. The market price for this option is 5.00 .

There is a second call option that allows you to buy 1 share of the same stock for a price of 100 in one month. The market price for Option 2 is 5.70.

Stock price in 1 month	**90**	**95**	**100**	**101**	**105**	**110**
Option 1 value	0	0	0	0	4	9
Option 2 value	0	0	0	1	5	10

For every stock price above 101, the second option pays 1 more than the first option.

The market values that extra 1 that option 2 pays if the stock price is above 101 as being worth 5.70 – 5.00 = 0.70. So (roughly speaking) the market is saying the probability the stock price will be above 101 is 70%.

Extracting the Risk Neutral Density from Options Prices in Practice

We use three options with sequential strike prices X_{n-1}, X_n, and X_{n+1} to obtain an approximation to F(X) centered on X_n.

$$F(X_n) \approx e^{rT}\left[\frac{C_{n+1} - C_{n-1}}{X_{n+1} - X_{n-1}}\right] + 1$$

The RND is the second partial derivative, which is approximated by

$$f(X_n) \approx e^{rT}\frac{C_{n+1} - 2C_n + C_{n-1}}{(\Delta X)^2}$$

S&P 500 Index Options Prices Jan. 5, 2005

S&P close 1183.74

Option expiration 3/18/05

	Calls				Puts			
Strike price	**Best bid**	**Best offer**	**Average price**	**Implied volatility**	**Best bid**	**Best offer**	**Average price**	**Implied volatility**
500	-	-	-	-	0.00	0.05	0.025	0.593
550	-	-	-	-	0.00	0.05	0.025	0.530
600	-	-	-	-	0.00	0.05	0.025	0.473
700	-	-	-	-	0.00	0.10	0.050	0.392
750	-	-	-	-	0.00	0.15	0.075	0.356
800	-	-	-	-	0.10	0.20	0.150	0.331
825	-	-	-	-	0.00	0.25	0.125	0.301
850	-	-	-	-	0.00	0.50	0.250	0.300
900	-	-	-	-	0.00	0.50	0.250	0.253
925	-	-	-	-	0.20	0.70	0.450	0.248
950	-	-	-	-	0.50	1.00	0.750	0.241
975	-	-	-	-	0.85	1.35	1.100	0.230
995	-	-	-	-	1.30	1.80	1.550	0.222
1005	-	-	-	-	1.50	2.00	1.750	0.217
1025	-	-	-	-	2.05	2.75	2.400	0.208
1050	134.50	136.50	135.500	0.118	3.00	3.50	3.250	0.193
1075	111.10	113.10	112.100	0.140	4.50	5.30	4.900	0.183
1100	88.60	90.60	89.600	0.143	6.80	7.80	7.300	0.172
1125	67.50	69.50	68.500	0.141	10.10	11.50	10.800	0.161
1150	48.20	50.20	49.200	0.135	15.60	17.20	16.400	0.152
1170	34.80	36.80	35.800	0.131	21.70	23.70	22.700	0.146
1175	31.50	33.50	32.500	0.129	23.50	25.50	24.500	0.144
1180	28.70	30.70	29.700	0.128	25.60	27.60	26.600	0.142
1190	23.30	25.30	24.300	0.126	30.30	32.30	31.300	0.141
1200	18.60	20.20	19.400	0.123	35.60	37.60	36.600	0.139
1205	16.60	18.20	17.400	0.123	38.40	40.40	39.400	0.139
1210	14.50	16.10	15.300	0.121	41.40	43.40	42.400	0.138
1215	12.90	14.50	13.700	0.122	44.60	46.60	45.600	0.138
1220	11.10	12.70	11.900	0.120	47.70	49.70	48.700	0.136
1225	9.90	10.90	10.400	0.119	51.40	53.40	52.400	0.137
1250	4.80	5.30	5.050	0.117	70.70	72.70	71.700	0.139
1275	1.80	2.30	2.050	0.114	92.80	94.80	93.800	0.147
1300	0.75	1.00	0.875	0.115	116.40	118.40	117.400	0.161
1325	0.10	0.60	0.350	0.116	140.80	142.80	141.800	0.179
1350	0.15	0.50	0.325	0.132	165.50	167.50	166.500	0.198
1400	0.00	0.50	0.250	0.157	-	-	-	-
1500	0.00	0.50	0.250	0.213	-	-	-	-

At the money

Extracting the Risk Neutral Density from Options Prices in Practice

Obtaining a well-behaved risk neutral density from market option prices is a nontrivial exercise. Here are the main steps we follow.

1. Use bid and ask quotes, rather than transactions prices. Eliminate options too far in or out of the money.

2. Use out of the money calls, out of the money puts, and a blend of the two at the money

3. Construct a smooth curve in strike-implied volatility space

4. Interpolate the IVs using a 4th degree smoothing spline fitted to the bid-ask spread

5. Convert the interpolated IV curve back to option prices and extract the middle portion of the risk neutral density

6. Append tails to the Risk Neutral Density from a Generalized Pareto Distribution (GPD)

The following pictures give an accelerated summary of what the effect of each of these steps is.

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1400	0.00	0.50	0.250	0.157	-	-	-	-
1500	0.00	0.50	0.250	0.213	-	-	-	-

At the money (between 1180 and 1190)

Not used

Blended

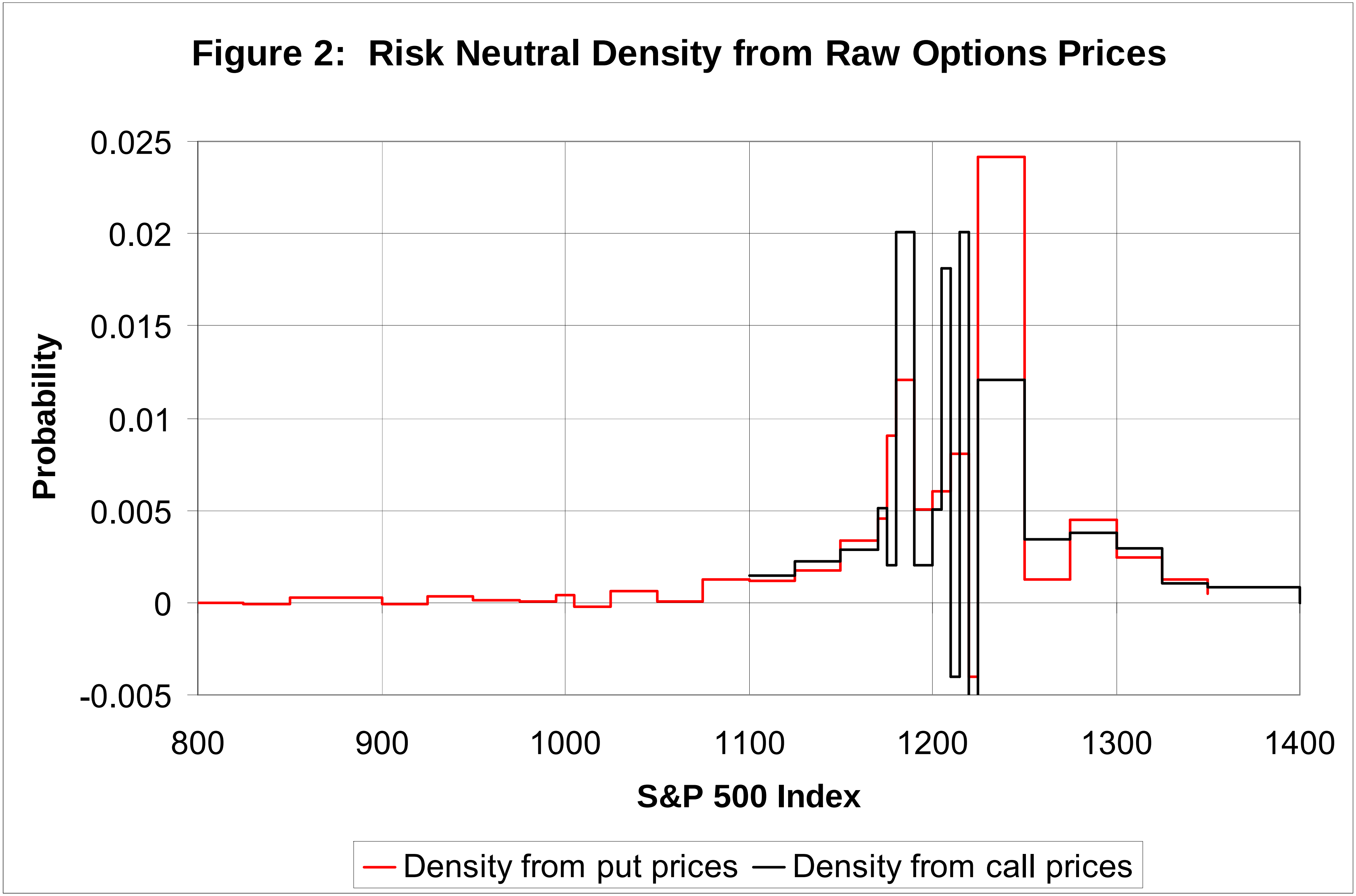


Figure 2: Risk Neutral Density from Raw Options Prices
Probability
0.025
0.02
0.015
0.01
0.005
0
-0.005
800
900
1000
1100
1200
1300
1400
S&P 500 Index
Density from put prices
Density from call prices

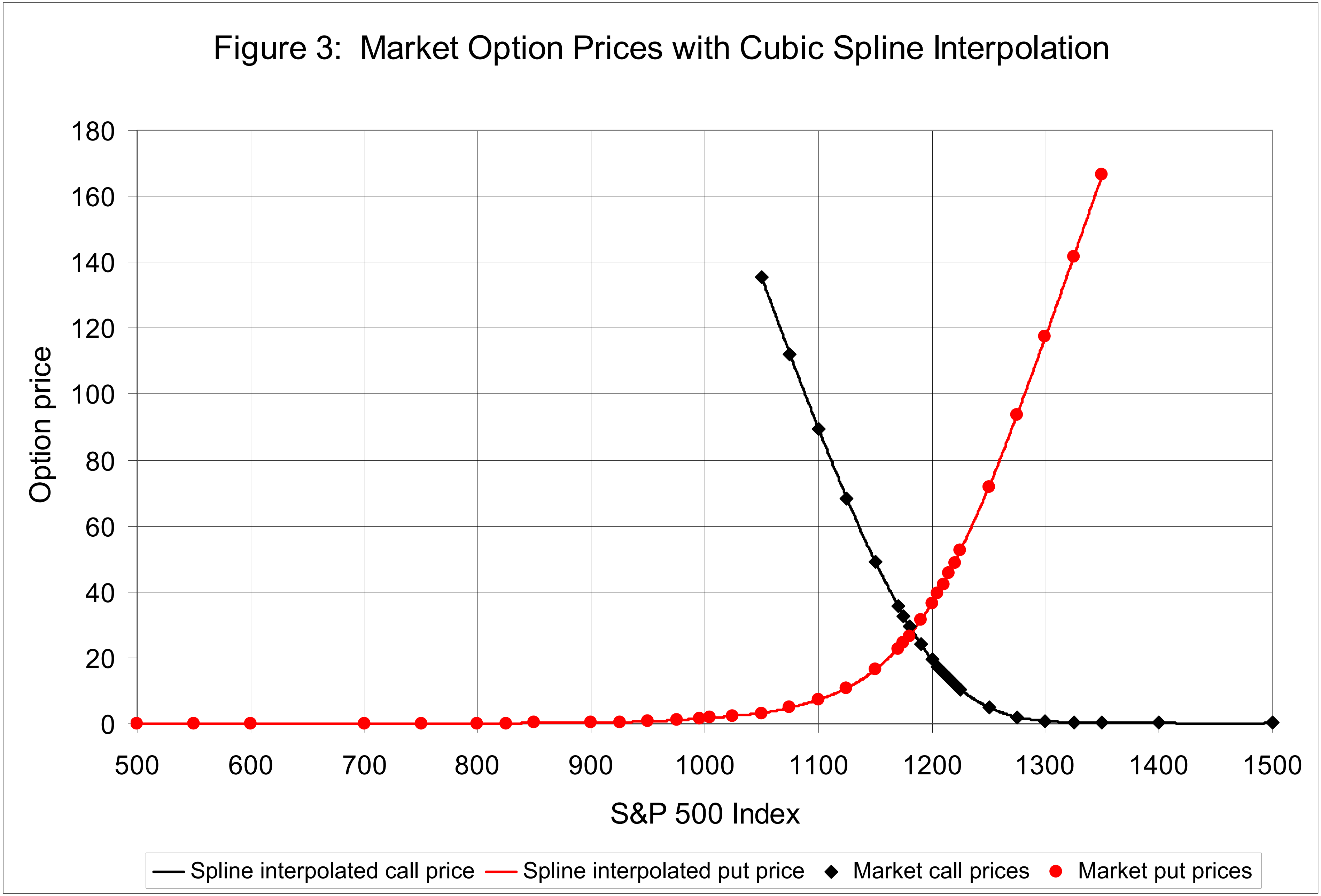
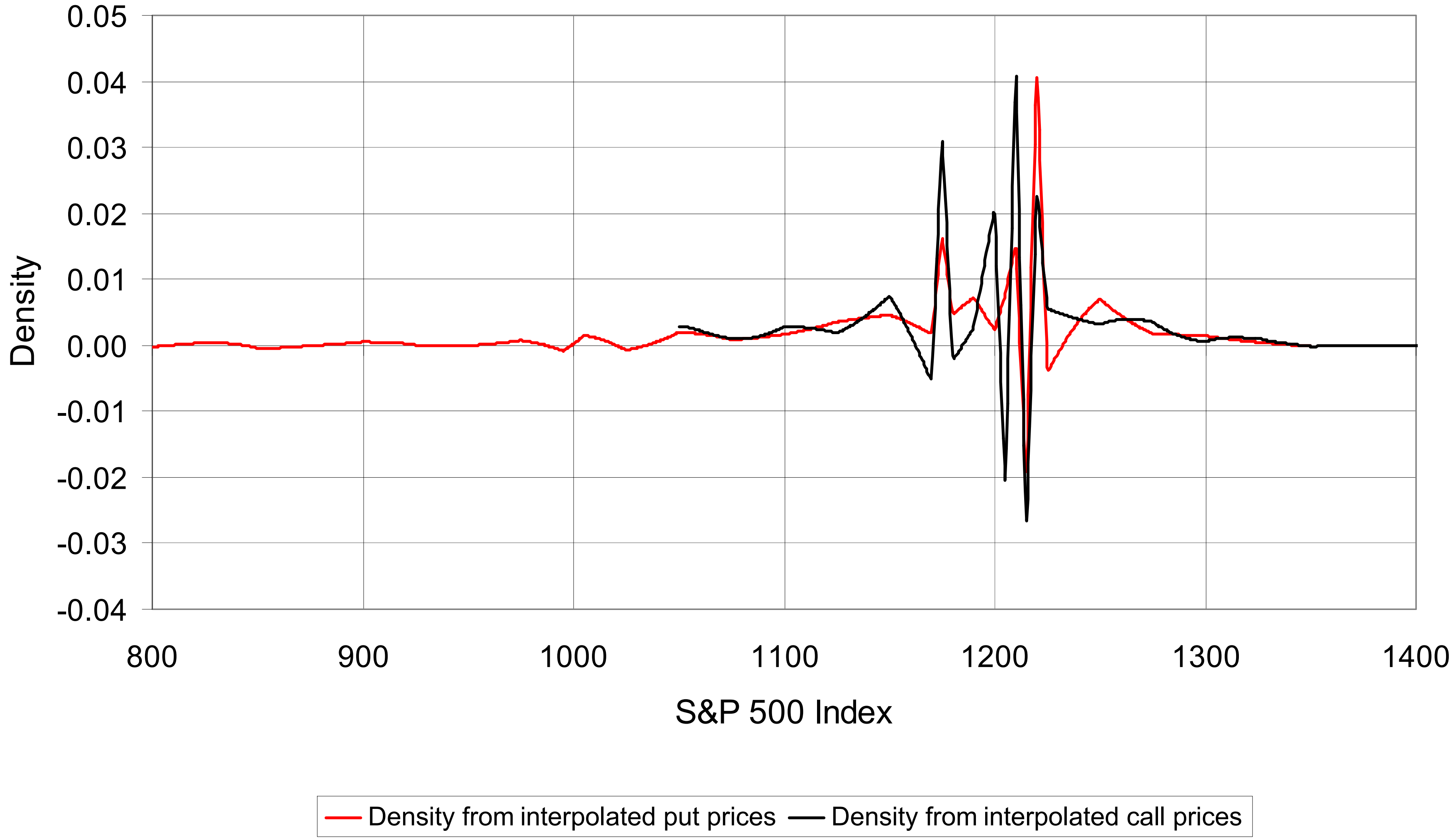


Figure 3: Market Option Prices with Cubic Spline Interpolation
180
160
140
120
100
80
60
40
20
0
Option price
500
600
700
800
900
1000
1100
1200
1300
1400
1500
S&P 500 Index
Spline interpolated call price
Spline interpolated put price
Market call prices
Market put prices

Figure 4: Densities from Option Prices with Cubic Spline Interpolation



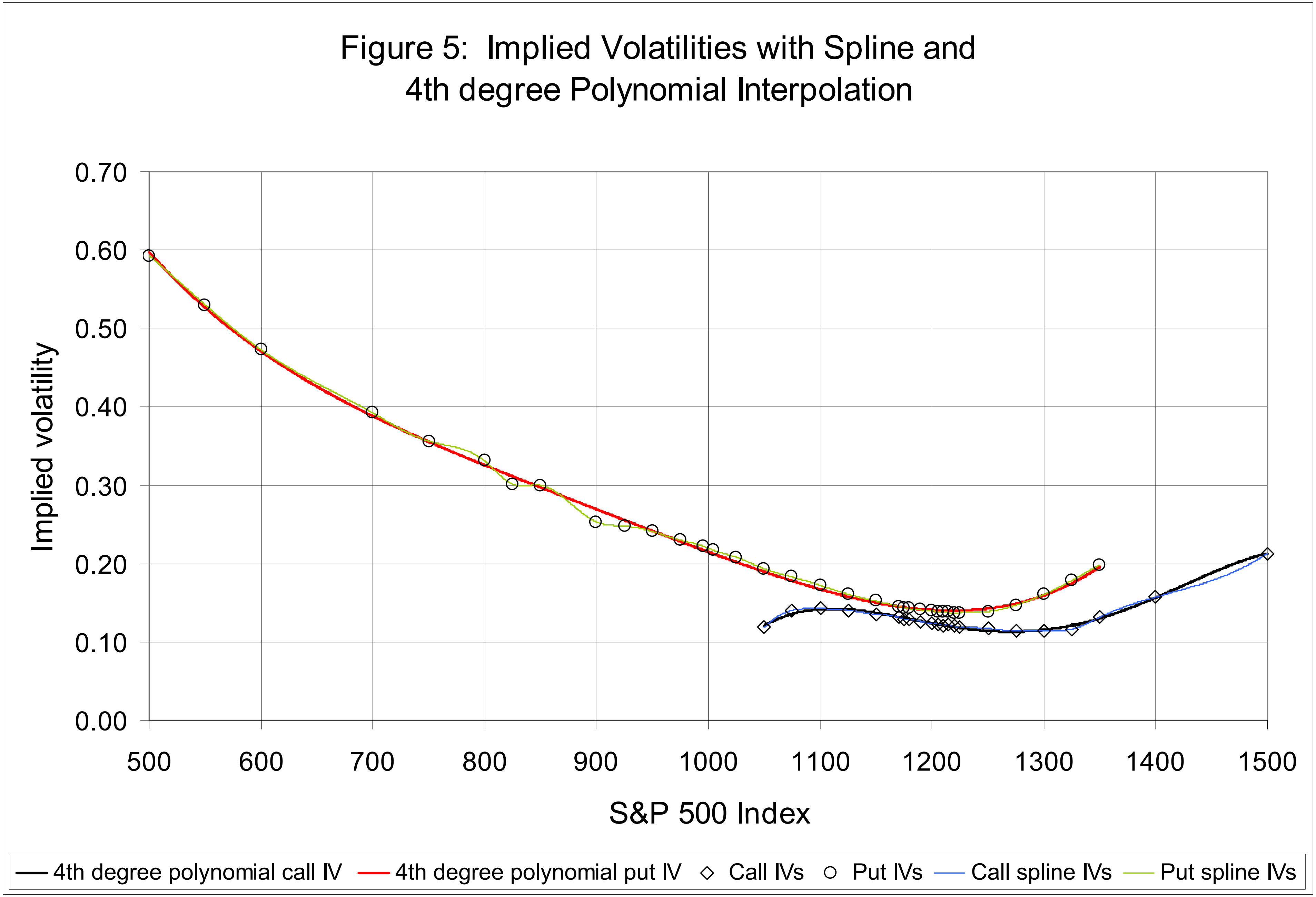


Figure 5: Implied Volatilities with Spline and
4th degree Polynomial Interpolation
Implied volatility
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
500
600
700
800
900
1000
1100
1200
1300
1400
1500
S&P 500 Index
4th degree polynomial call IV
4th degree polynomial put IV
Call IVs
Put IVs
Call spline IVs
Put spline IVs

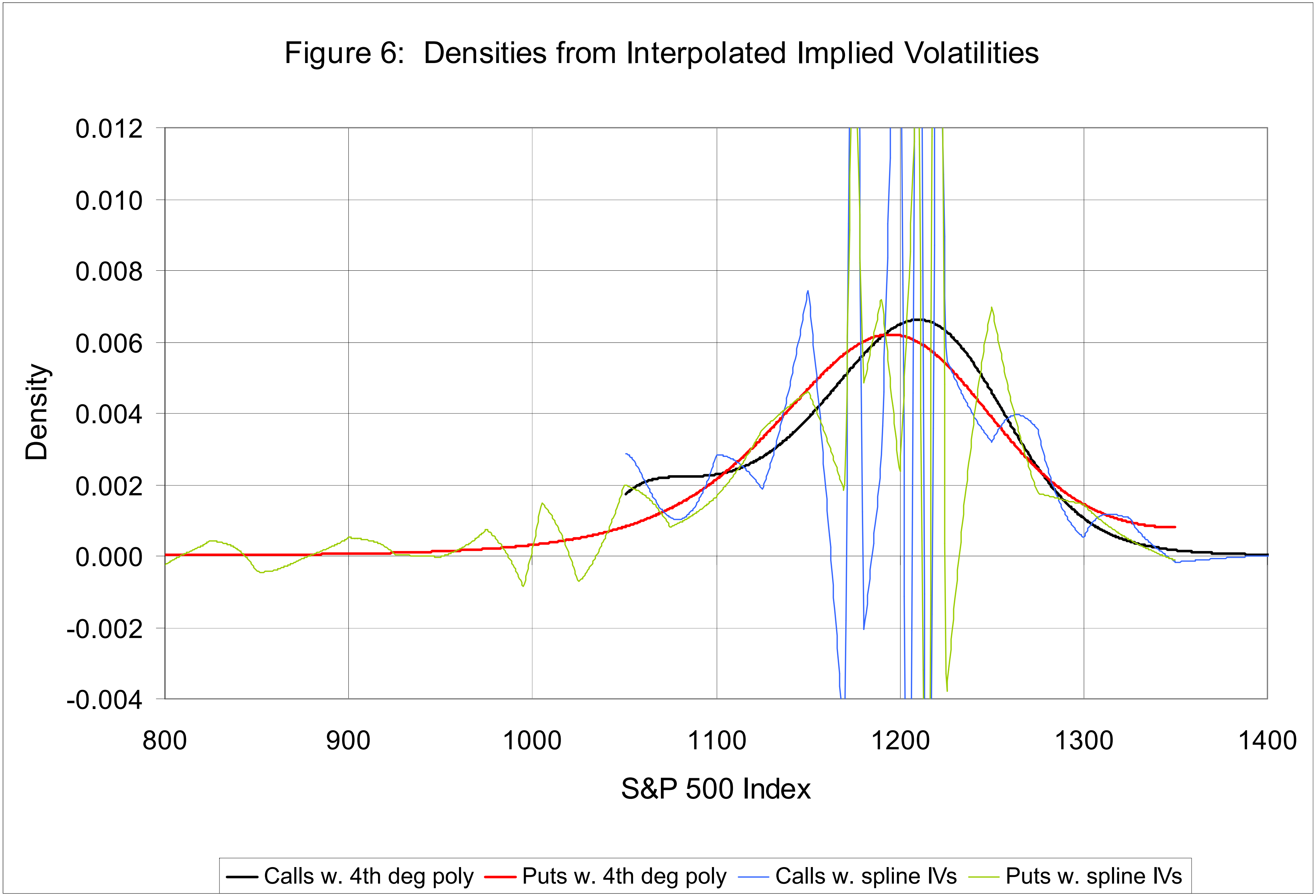
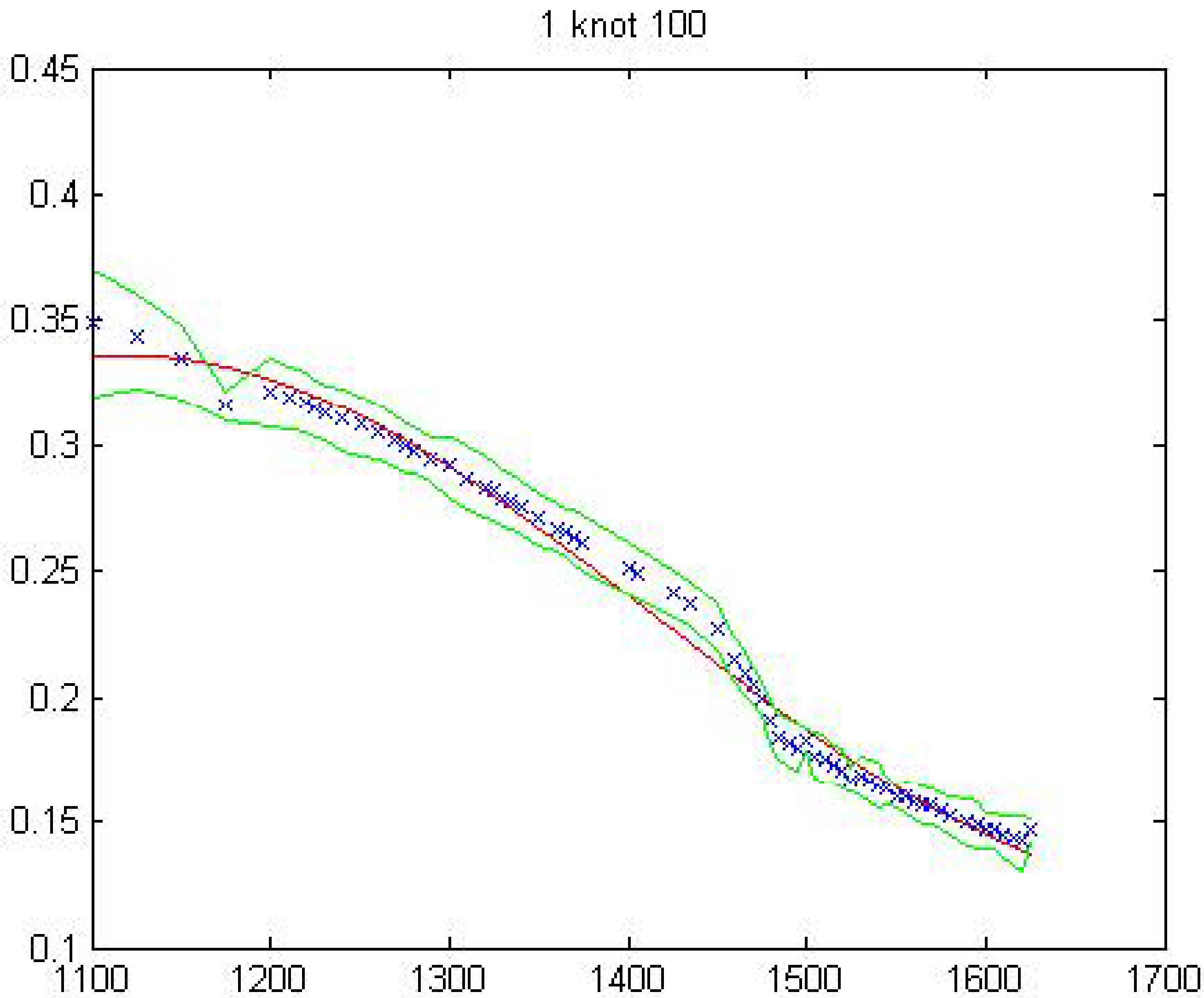
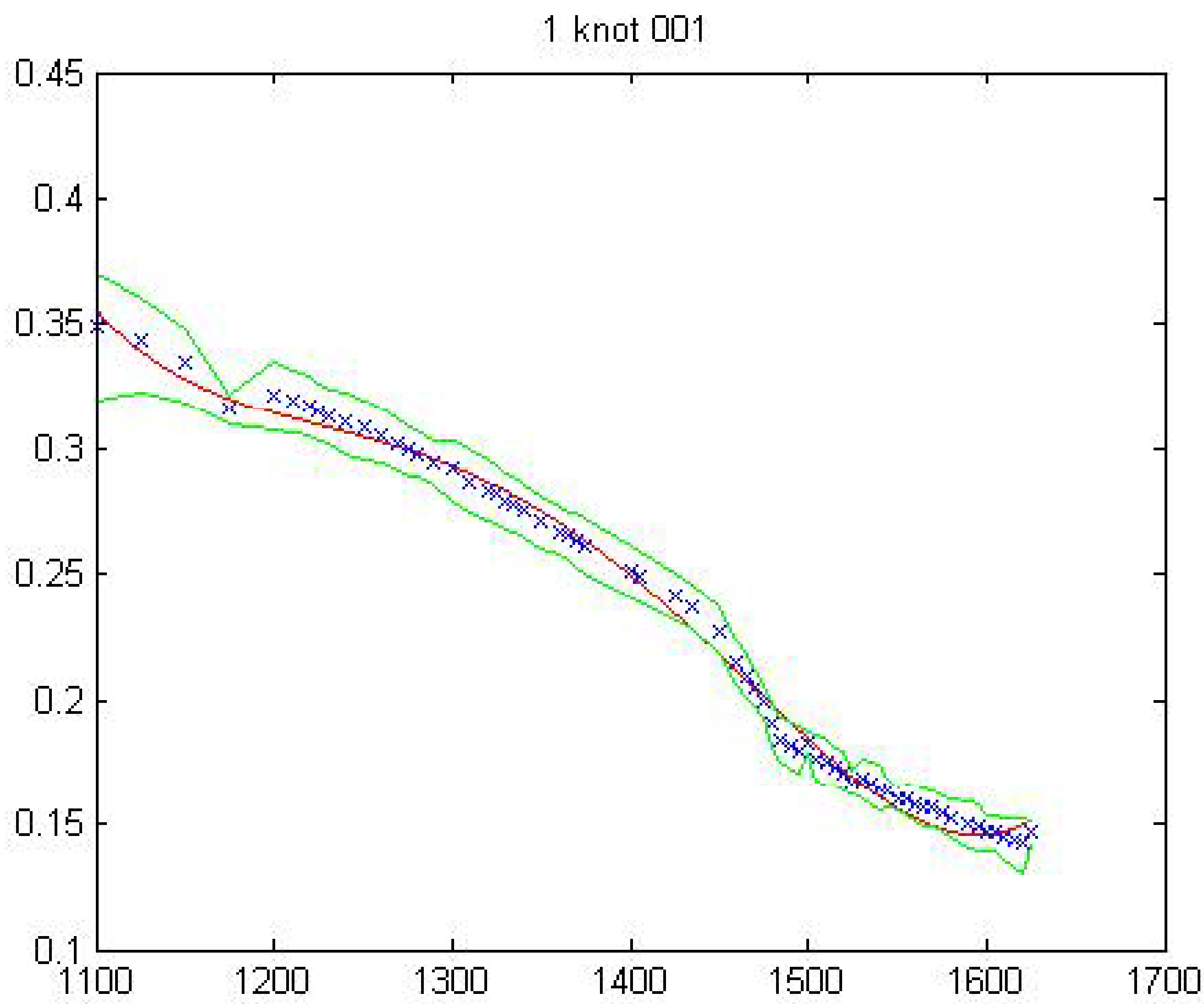


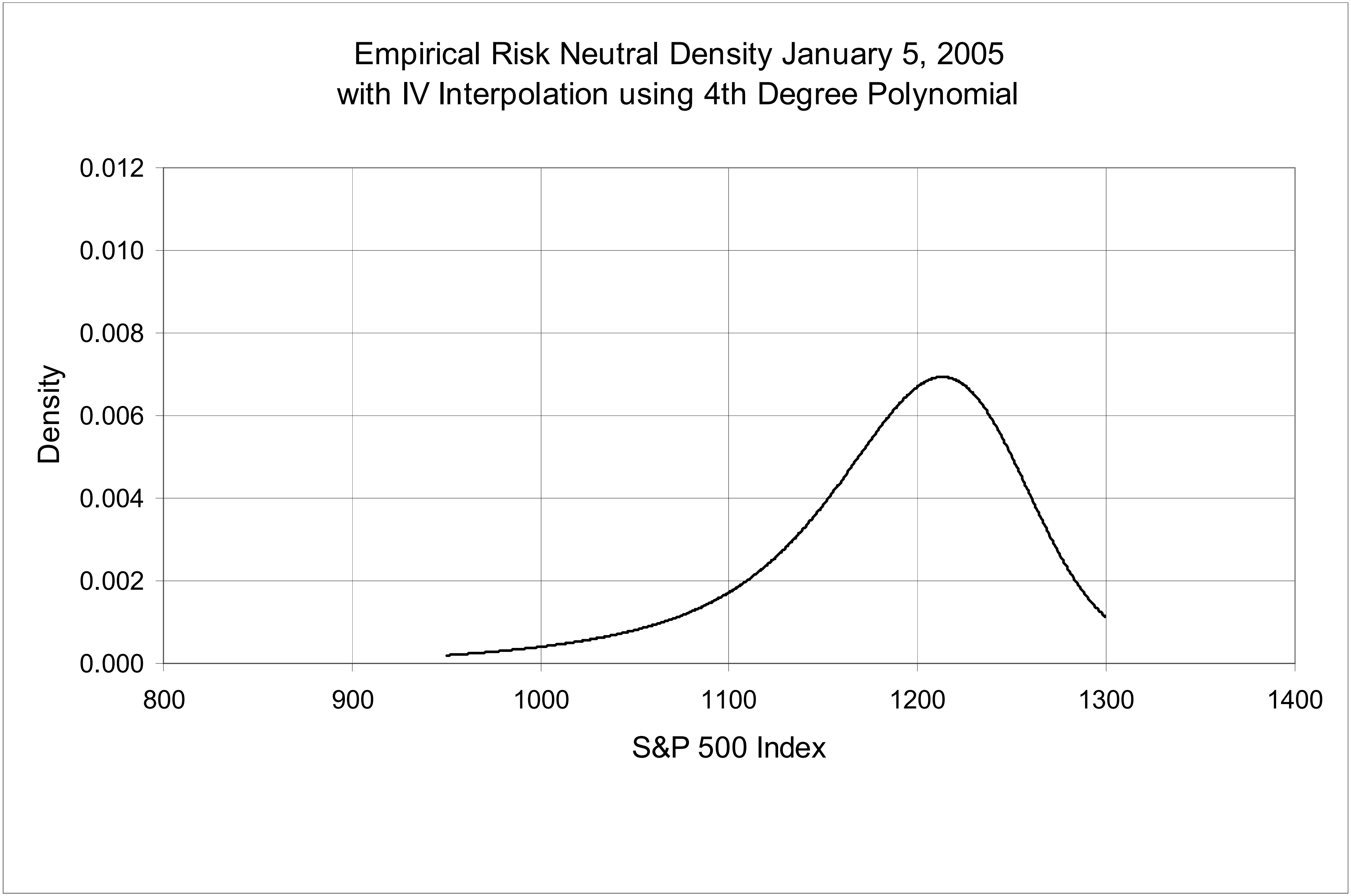
Figure 6: Densities from Interpolated Implied Volatilities
Density
0.012
0.010
0.008
0.006
0.004
0.002
0.000
-0.002
-0.004
800
900
1000
1100
1200
1300
1400
S&P 500 Index
Calls w. 4th deg poly
Puts w. 4th deg poly
Calls w. spline IVs
Puts w. spline IVs

4th Degree Smoothing Splines with Bid-Ask Spread Adjustment



4th Degree Smoothing Spline with Bid-Ask Spread Adjustment




Empirical Risk Neutral Density January 5, 2005
with IV Interpolation using 4th Degree Polynomial
Density
0.012
0.010
0.008
0.006
0.004
0.002
0.000
800
900
1000
1100
1200
1300
1400
S&P 500 Index

Extracting the Risk Neutral Density from Options Prices in Practice

Only the middle portion of the empirical RND spanned by the set of option strikes can be estimated this way.

Some estimation techniques generate tails automatically, like fitting one or a mixture of parametric distributions (e.g., lognormal, Student-t) to option prices.

We take the empirical RND from the data and append tails from a Generalized Pareto (GPD) distribution. The fitted tails contain the correct total probability and are fitted to match the shape of the extreme portions of the empirical RND.

By the Fisher-Tippett Theorem, the tails of any member of a very broad class of distributions in the limit resemble the tail of a Generalized Extreme Value distribution.

The GPD is the distribution of random draws from a density function, that exceed some high threshold value. It is closely related to the GEV, and in particular, it has the same tail shape parameter. Either density works well for our purpose, but the GPD has the advantage of only requiring estimation of two parameters, rather than 3 for the GEV.

The Generalized Pareto (GPD) Distribution

We complete the risk neutral density by adding tails from a GPD density.

The GPD distribution has two parameters, along with the tail cutoff value c that is chosen by the user:

c = lower limit of the tail

σ = scale

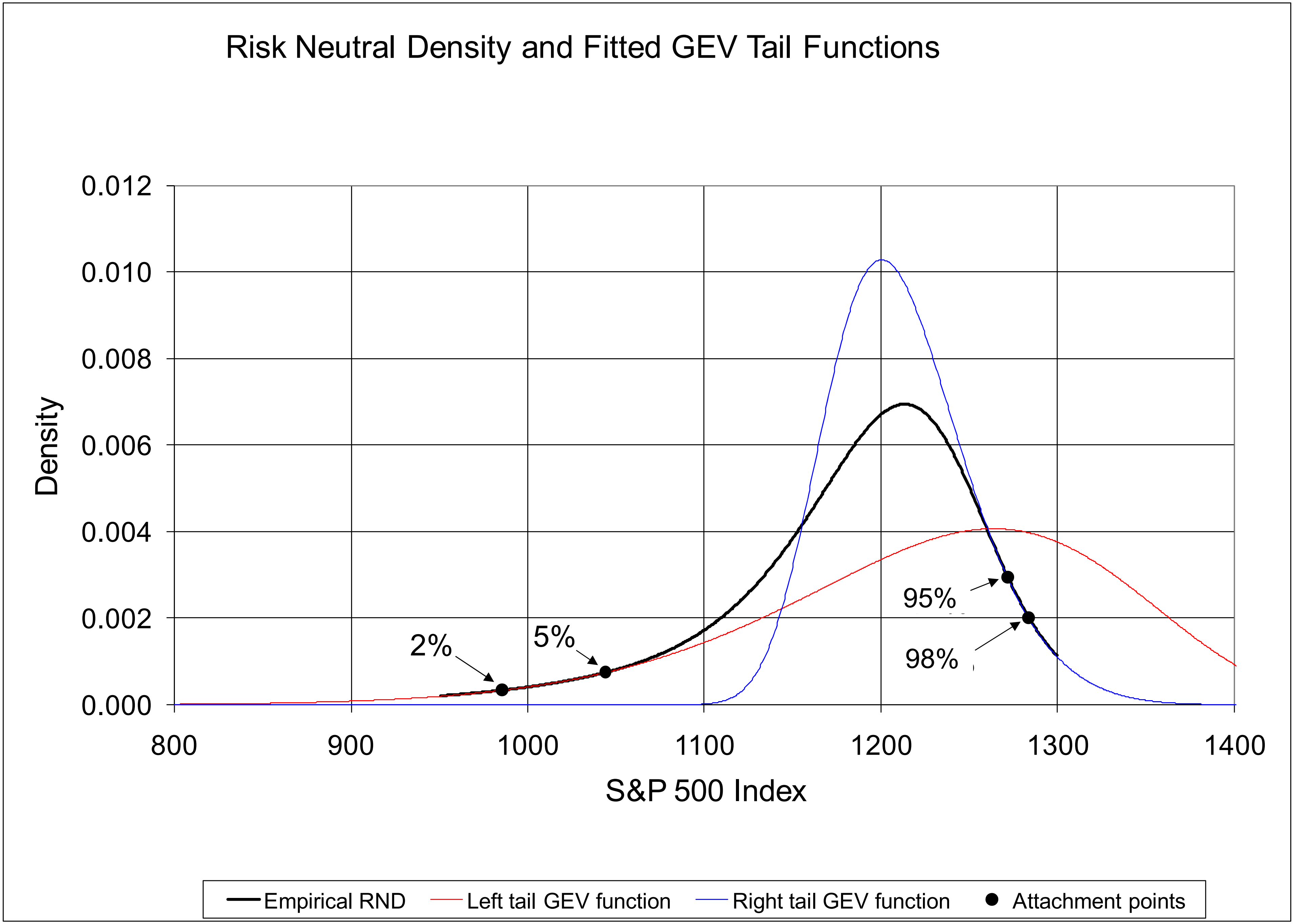
ξ = tail shape

$$F(S_T \mid S_T \geq c) = \begin{cases} 1 - \left(1 + \xi\left(\dfrac{S_T - c}{\sigma}\right)\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\left(\dfrac{S_T - c}{\sigma}\right)\right) & \text{if } \xi = 0 \end{cases}$$

ξ > 0 corresponds to a Fréchet distribution, that has a heavier tail than a Gaussian;

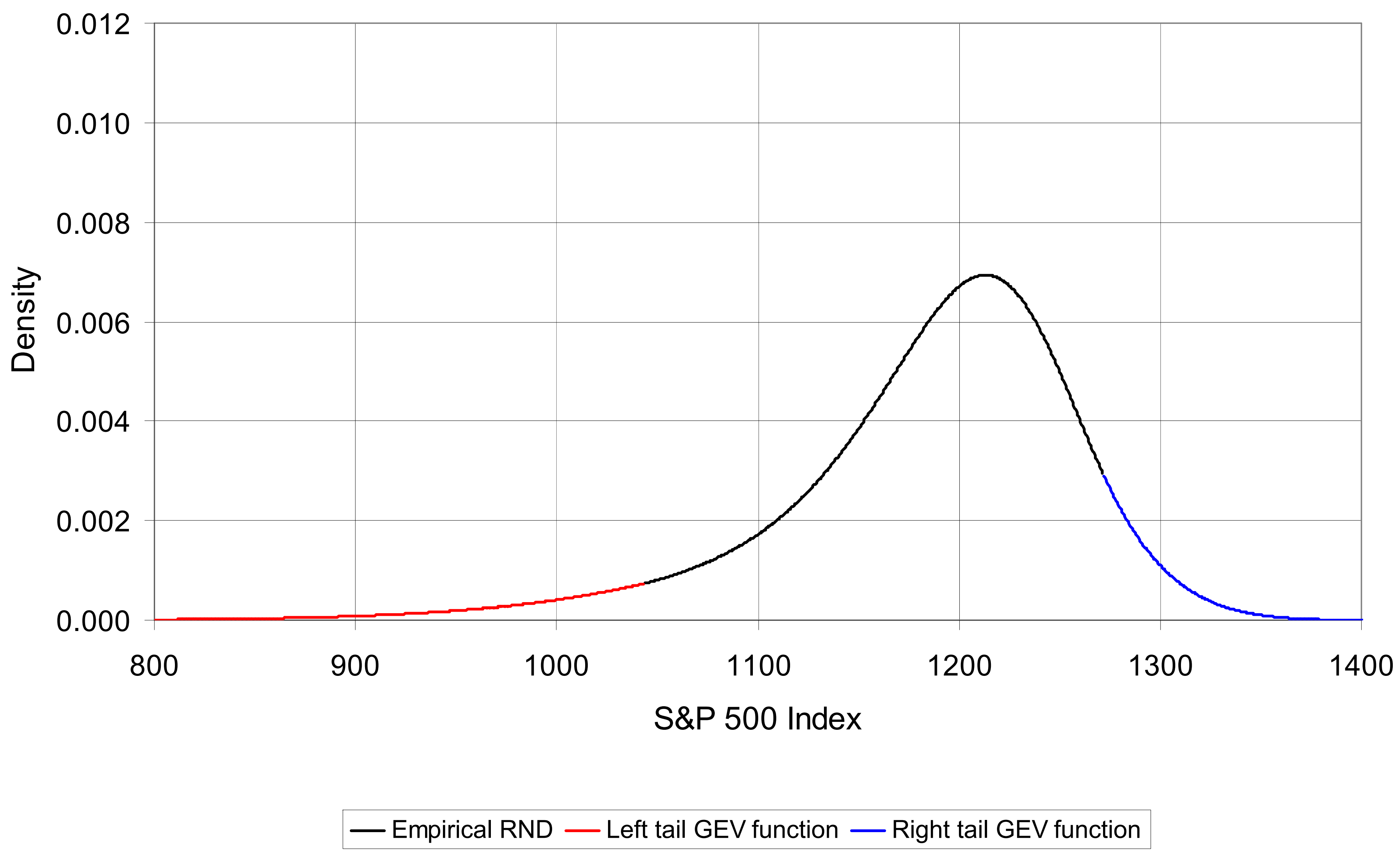
ξ = 0 indicates a Gumbel distribution with tails like the normal;

ξ < 0, the density is a Weibull, with a finite tail that does not extend to infinity.



Risk Neutral Density and Fitted GEV Tail Functions
0.012
0.010
0.008
0.006
0.004
0.002
0.000
Density
800
900
1000
1100
1200
1300
1400
S&P 500 Index
2%
5%
95%
98%
Empirical RND
Left tail GEV function
Right tail GEV function
Attachment points

Figure 10: Full Estimated Risk Neutral Density Function for Jan. 5, 2005



Data: Options data

Risk neutral volatilities: RNDs are fitted using the above procedure on daily closing bid and ask prices for S&P 500 index call and put options with a full set of strike prices, that satisfy the following conditions:

- observation date and expiration date are both between 1/4/1996 and 4/29/2011 ;
- OptionMetrics has calculated an implied volatility > 0 for the option;
- bid price >.50 ;
- days to expiration between 13 and 200 days.
- implied volatility curves were sufficiently regular that a 4th order spline could be fitted
- options data from the market covered at least the 10th to the 90th RND percentiles
- the RND for the previous day could also be computed, to allow lags.

This produced a sample of 10,694 RNDs, one for each date-expiration pair in the sample.

Data: Explanatory Variables

Option market prices are determined by interplay between supply and demand from investors and liquidity provision by market makers.

Both investors and market makers care about future volatility, although possibly in different forms and over different horizons.

* investors: where underlying could be at expiration; possibility of early unwind
* market makers: intraday fluctuations, price jumps, changes in implied volatility

Relevant variables for gauging future volatility include:

* historical volatility, current GARCH volatility forecast
* returns for date t and earlier periods, both magnitude and sign
* trading range, both intraday (hedging costs) and lifetime (unwind possibility)
* frequency and size of extreme tail events

Factors that might influence risk premia required by investors

* "confidence"
* risk premia in other financial markets, e.g., the P/E ratio for the S&P 500
* "forecastability" of volatility
* previous period volatility risk premium

Data: Explanatory Variables

The explanatory variables are divided into three classes:

I. Volatility-related variables that do not require looking back over past data.

II. Volatility-related variables computed from past data over a horizon that must be specified.

III. Variables related to risk attitudes, that may influence risk neutralization.

Set 1: Date t Explanatory Variables

Date t return: 100 x log(S_t / S_{t-1}), where S_t denotes the date t S&P 500 index level. SPX data were downloaded from the Yahoo.com Finance website.

Date t absolute return: Absolute value of the Date t return.

GARCH to expiration: Out of sample variance forecasts from a GARCH(1,1) model with an asymmetry term (i.e., the Glosten, Jagannathan, Runkle (1993) version of GARCH) were provided by Robert Engle's VLAB. The model was refitted each day using only historical data up to that date. GARCH variance forecasts for date t+1 through option expiration were computed and converted to equivalent annualized percent volatilities.

Date t trading range: 100 x log(1 + ($S_{t,HI}$ - $S_{t,LOW}$) / (($S_{t,HI}$ + $S_{t,LOW}$) /2)))

Left tail return: 100 x (Date t return), conditional on it being in the 2% left tail. The tail cutoff is defined as -2.054 x (date t-1 1-day ahead GARCH forecast).

Right tail return: 100 x (Date t return), conditional on it being in the 2% right tail.

Set 2: Explanatory Variables from Historical Data

These variables are backward looking, measuring the average or cumulative effect of a factor over the recent past. The correct horizon to use is unknown. After some experimentation with different lag values, lags were selected that made sense and appeared to work well in terms of explanatory power.

Last year return: $100 \times \log(S_{t-1} / S_{t-252})$

Last year absolute return: Absolute value of Last year return.

Historical volatility: Return variance over the past 150 trading days (computed assuming zero mean). The variable is converted to the equivalent annual percentage volatility.

Average daily trading range: The daily trading range as defined for variable Set 1, averaged over the last 10 trading days.

Past trading range over option life: $100 \times \log(1 + (S_{HI} - S_{LOW}) / ((S_{HI} + S_{LOW}) /2)))$ where S_{HI} and S_{LOW} are the high and low values for SPX over a period ending on date t-1 that is equal in length to the period remaining from date t to expiration date.

Set 2: Explanatory Variables from Historical Data, continued

Number of left tail events: Left tail returns are defined as above. This is the number observed in the year ending on date t-1.

Average size of left tail events: Over the past year, the average return conditional on it being in the 2% left tail.

Number of right tail events: The number of right tail returns in the last year.

Average size of right tail events: Equivalent to average left tail events.

Set 3: Explanatory Variables Related to Risk Attitudes

Michigan consumer sentiment: University of Michigan index of Consumer Sentiment. The series is monthly. We use the same value for every day within the month.

S&P 500 Price/Earnings ratio: Monthly series downloaded from Robert Shiller's website

BAA-AAA bond yield spread: The daily yield spread between high and low investment grade corporate bonds, downloaded from the St. Louis Federal Reserve FRED system.

GARCH model error over option life: As a measure of the GARCH model's current accuracy, this is the average error over a period of the same length as the option's remaining life, ending at date t. The GARCH error for each day is defined as (r^2 x 252) – (GARCH 1-day ahead variance forecast).

GARCH model RMSE: This is the GARCH model root mean squared error over the period described above.

RND volatility risk premium date t-1: Date t-1 (RND volatility – GARCH volatility forecast to expiration).

Correlations

Correlations among dependent and explanatory variables in the sample.

The first two columns indicate the strength of univariate relationships between the RND volatility, the realized volatility from date t to expiration, and the explanatory variables. Among the explanatory variables, most correlations are fairly low, except for a few easy to understand exceptions, highlighted in blue.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1 RND volatility	1	0.216	-0.017	0.173	0.353	0.254	-0.023	0.041	-0.204	0.224	0.311	0.316	-0.101	0.219	-0.244	0.137	0.215	-0.076	-0.234	0.226	0.160	0.239	0.877
2 Realized volatility to expiration	0.216	1	-0.076	0.345	0.660	0.520	-0.076	0.052	-0.440	0.346	0.438	0.619	0.366	0.344	-0.366	0.252	0.261	-0.253	-0.381	0.451	0.336	0.466	-0.010
3 Date t return	-0.017	-0.076	1	-0.050	-0.059	-0.087	0.272	0.342	0.039	-0.014	0.022	0.026	0.019	-0.012	0.001	0.021	0.024	-0.007	-0.003	0.013	-0.010	0.005	-0.007
4 Date t absolute return	0.173	0.345	-0.050	1	0.562	0.788	-0.296	0.363	-0.333	0.278	0.351	0.476	0.296	0.207	-0.243	0.180	0.210	-0.150	-0.238	0.332	0.340	0.391	0.013
5 GARCH to expiration	0.353	0.660	-0.059	0.562	1	0.763	-0.097	0.086	-0.623	0.513	0.666	0.930	0.551	0.340	-0.442	0.329	0.409	-0.377	-0.399	0.697	0.605	0.800	0.011
6 Date t trading range	0.254	0.520	-0.087	0.788	0.763	1	-0.229	0.252	-0.475	0.384	0.488	0.702	0.421	0.293	-0.356	0.235	0.290	-0.196	-0.340	0.465	0.455	0.550	0.006
7 Date t left tail return	-0.023	-0.076	0.272	-0.296	-0.097	-0.229	1	0.010	0.010	-0.019	-0.001	-0.021	-0.004	-0.073	0.018	-0.022	-0.001	-0.007	0.003	0.012	-0.090	-0.031	-0.012
8 Date t right tail return	0.041	0.052	0.342	0.363	0.086	0.252	0.010	1	-0.064	0.030	0.060	0.099	0.051	0.029	-0.029	0.063	0.030	-0.039	-0.025	0.045	0.063	0.048	0.014
9 Last year return	-0.204	-0.440	0.039	-0.333	-0.623	-0.475	0.010	-0.064	1	-0.331	-0.752	-0.594	-0.513	0.029	0.469	-0.539	-0.508	0.450	0.621	-0.742	-0.244	-0.634	0.007
10 Last year absolute return	0.224	0.346	-0.014	0.278	0.513	0.384	-0.019	0.030	-0.331	1	0.631	0.473	0.497	0.270	-0.433	0.312	0.490	-0.277	-0.480	0.586	0.215	0.585	0.066
11 Historical volatility	0.311	0.438	0.022	0.351	0.666	0.488	-0.001	0.060	-0.752	0.631	1	0.643	0.670	0.294	-0.690	0.511	0.769	-0.430	-0.750	0.839	0.234	0.800	0.090
12 Avg daily range last 10 days	0.316	0.619	0.026	0.476	0.930	0.702	-0.021	0.099	-0.594	0.473	0.643	1	0.556	0.352	-0.460	0.311	0.390	-0.260	-0.434	0.620	0.579	0.727	-0.005
13 Trading range, past option lifespa	-0.101	0.366	0.019	0.296	0.551	0.421	-0.004	0.051	-0.513	0.497	0.670	0.556	1	0.259	-0.503	0.364	0.550	-0.337	-0.497	0.592	0.352	0.716	-0.282
14 # left tail events last year	0.219	0.344	-0.012	0.207	0.340	0.293	-0.073	0.029	0.029	0.270	0.294	0.352	0.259	1	-0.413	0.080	0.098	0.042	-0.172	0.147	0.208	0.273	0.125
15 Avg left tail return last year	-0.244	-0.366	0.001	-0.243	-0.442	-0.356	0.018	-0.029	0.469	-0.433	-0.690	-0.460	-0.503	-0.413	1	-0.413	-0.681	0.151	0.705	-0.448	-0.166	-0.519	-0.099
16 # right tail events last year	0.137	0.252	0.021	0.180	0.329	0.235	-0.022	0.063	-0.539	0.312	0.511	0.311	0.364	0.080	-0.413	1	0.434	-0.198	-0.459	0.367	0.156	0.360	0.047
17 Avg right tail return last year	0.215	0.261	0.024	0.210	0.409	0.290	-0.001	0.030	-0.508	0.490	0.769	0.390	0.550	0.098	-0.681	0.434	1	-0.486	-0.663	0.595	0.118	0.510	0.083
18 Michigan consumer sentiment	-0.076	-0.253	-0.007	-0.150	-0.377	-0.196	-0.007	-0.039	0.450	-0.277	-0.430	-0.260	-0.337	0.042	0.151	-0.198	-0.486	1	0.005	-0.656	-0.089	-0.378	0.026
19 S&P 500 P/E ratio	-0.234	-0.381	-0.003	-0.238	-0.399	-0.340	0.003	-0.025	0.621	-0.480	-0.750	-0.434	-0.497	-0.172	0.705	-0.459	-0.663	0.005	1	-0.502	-0.126	-0.511	-0.093
20 BAA-AAA bond yield spread	0.226	0.451	0.013	0.332	0.697	0.465	0.012	0.045	-0.742	0.586	0.839	0.620	0.592	0.147	-0.448	0.367	0.595	-0.656	-0.502	1	0.266	0.797	0.002
21 GARCH model error recent period	0.160	0.336	-0.010	0.340	0.605	0.455	-0.090	0.063	-0.244	0.215	0.234	0.579	0.352	0.208	-0.166	0.156	0.118	-0.089	-0.126	0.266	1	0.529	-0.020
22 GARCH model RMSE	0.239	0.466	0.005	0.391	0.800	0.550	-0.031	0.048	-0.634	0.585	0.800	0.727	0.716	0.273	-0.519	0.360	0.510	-0.378	-0.511	0.797	0.529	1	-0.023
23 RND volatility premium date t-1	0.877	-0.010	-0.007	0.013	0.011	0.006	-0.012	0.014	0.007	0.066	0.090	-0.005	-0.282	0.125	-0.099	0.047	0.083	0.026	-0.093	0.002	-0.020	-0.023	1

Correlations – Univariate Relationships (First Two Columns)

	RND volatility	Realized volatility
RND volatility	1	0.216
Realized volatility to expiration	0.216	1
Date t return	-0.017	-0.076
Date t absolute return	0.173	0.345
GARCH to expiration	0.353	0.660
Date t trading range	0.254	0.520
Date t left tail return	-0.023	-0.076
Date t right tail return	0.041	0.052
Last year return	-0.204	-0.440
Last year absolute return	0.224	0.346
Historical volatility	0.311	0.438
Avg daily range last 10 days	0.316	0.619
Trading range, past option lifespan	-0.101	0.366
# left tail events last year	0.219	0.344
Avg left tail return last year	-0.244	-0.366
# right tail events last year	0.137	0.252
Avg right tail return last year	0.215	0.261
Michigan consumer sentiment	-0.076	-0.253
S&P 500 P/E ratio	-0.234	-0.381
BAA-AAA bond yield spread	0.226	0.451
GARCH model error recent period	0.160	0.336
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Regressions on All Variables

To gauge the marginal influence of each of these explanatory variable, I combined them in a single grand regression.

The RND volatility risk premium shows considerable persistence. We don't include the previous day risk premium in most of the regressions, since it is a large part of what we want to explain. But to show the effect of including the previous day RND volatility risk premium on the other explanatory variables, the third pair of columns in the next table presents results from that regression.

The mixed time series cross section structure of the sample induces cross-correlation between the residuals from any two observations whose underlying contract maturities overlap. A variant of the Newey-West approach is used to fit robust standard errors for the coefficients.

Specification 3: RND volatility regressed on all variables

	RND volatility		Realized volatility		RND volatility	
Variable	coef	t-stat	coef	t-stat	coef	t-stat
Constant	30.015	2.652	37.905	2.374	3.860	2.055
Date t return	-0.078	-0.515	-0.150	-1.191	0.196	2.480
Date t absolute return	-0.884	-3.920	-0.882	-5.550	-0.890	-5.994
GARCH to expiration	0.782	2.553	0.961	4.976	1.086	21.718
Date t trading range	0.644	1.799	0.959	2.922	0.543	3.894
Date t left tail return	1.561	3.480	0.187	0.431	1.114	4.283
Date t right tail return	1.000	1.930	-0.101	-0.383	-0.032	-0.091
Last year return	-0.004	-0.100	0.006	0.138	-0.006	-0.863
Last year absolute return	0.132	2.039	0.029	0.488	0.027	2.562
Historical volatility	0.692	3.380	-0.362	-1.738	0.067	1.790
Avg daily range last 10 days	0.382	0.238	-0.222	-0.205	-0.465	-1.522
Trading range, past option lifespan	-1.579	-10.952	0.036	0.665	-0.224	-4.686
# left tail events last year	1.900	4.176	1.240	1.682	0.269	3.164
Avg left tail return last year	0.242	0.582	-0.255	-0.539	0.021	0.363
# right tail events last year	0.018	0.071	0.246	0.687	0.015	0.399
Avg right tail return last year	1.344	2.304	-1.534	-2.784	0.055	0.543
Michigan consumer sentiment	-0.108	-1.502	-0.201	-1.860	-0.014	-1.280
S&P 500 P/E ratio	-1.149	-1.560	-3.042	-2.829	-0.195	-1.624
BAA-AAA bond yield spread	-8.845	-3.891	0.835	0.385	-1.132	-2.808
GARCH model error recent period	0.010	1.177	0.001	0.232	0.001	1.059
GARCH model RMSE	0.004	2.207	-0.001	-1.342	0.001	1.737
RND volatility premium date t-1			-0.006	-0.595	0.956	33.492
RND volatility date t			-0.011	-0.852		
R^2	0.369		0.526		0.886	
NOBS	10926		10694		10694	

Regressions on All Variables

These regressions feature some Key Results, some Weird Results, and a few Major Differences between the behavior of Risk Neutral volatility and Realized volatility.

Key Results:

- A large date t absolute return decreases both volatility measures. (also "Weird")
- The volatility forecast from the GARCH model is highly significant for both RND volatility and future Realized volatility
- The date t trading range is significant. A wider range is associated with a wider RND.
- The return over the past year has no impact on either RND or Realized volatility
- Strength in the Michigan Consumer Sentiment index and the S&P 500 P/E ratio are both associated with lower volatilities, although statistical significance is marginal for RND volatility.
- If the GARCH model's recent RMSE was large, RND volatility is higher. The sign of the errors may also matter, with volatility underprediction having a larger effect than overprediction.
- Neither the date t RND volatility nor the date t-1 premium of RND volatility over the GARCH forecast had any explanatory power for future Realized volatility.

Specification 3: RND volatility regressed on all variables

Key result

	RND volatility		Realized volatility	
Variable	coef	t-stat	coef	t-stat
Constant	30.015	2.652	37.905	2.374
Date t return	-0.078	-0.515	-0.150	-1.191
Date t absolute return	-0.884	-3.920	-0.882	-5.550
GARCH to expiration	0.782	2.553	0.961	4.976
Date t trading range	0.644	1.799	0.959	2.922
Date t left tail return	1.561	3.480	0.187	0.431
Date t right tail return	1.000	1.930	-0.101	-0.383
Last year return	-0.004	-0.100	0.006	0.138
Last year absolute return	0.132	2.039	0.029	0.488
Historical volatility	0.692	3.380	-0.362	-1.738
Avg daily range last 10 days	0.382	0.238	-0.222	-0.205
Trading range, past option lifespan	-1.579	-10.952	0.036	0.665
# left tail events last year	1.900	4.176	1.240	1.682
Avg left tail return last year	0.242	0.582	-0.255	-0.539
# right tail events last year	0.018	0.071	0.246	0.687
Avg right tail return last year	1.344	2.304	-1.534	-2.784
Michigan consumer sentiment	-0.108	-1.502	-0.201	-1.860
S&P 500 P/E ratio	-1.149	-1.560	-3.042	-2.829
BAA-AAA bond yield spread	-8.845	-3.891	0.835	0.385
GARCH model error recent period	0.010	1.177	0.001	0.232
GARCH model RMSE	0.004	2.207	-0.001	-1.342
RND volatility premium date t-1			-0.006	-0.595
RND volatility date t			-0.011	-0.852
R2	0.369		0.526	
NOBS	10926		10694	

Regressions for short and long maturities

Weird Results and Differences:

- Date t absolute return is significantly negative
- A date t return in the 2% left tail significantly narrows the RND (because the return and coefficient are both negative)
- Historical volatility is a significant positive influence on RND volatility, but the coefficient in the Realized volatility equation is negative and significant.
- Trading over a wide range in the recent past is associated with a highly significant but anomalous negative coefficient for RND volatility, and it does not affect realized volatility either way.
- The occurrence of left tail events in the past increases RND volatility, but their size did not matter. For right tail events, the number didn't matter but the size was significant for both RND and Realized volatility, but with different signs.
- The BAA-AAA bond yield spread is highly significant for RND volatility, but the sign is a puzzle. Wider risk premia in the bond market seem to be associated with narrower RNDs. There is no apparent effect on Realized volatility.

Specification 3: RND volatility regressed on all variables

Weird/ Different Result

	RND volatility		Realized volatility	
Variable	coef	t-stat	coef	t-stat
Constant	30.015	2.652	37.905	2.374
Date t return	-0.078	-0.515	-0.150	-1.191
Date t absolute return	-0.884	-3.920	-0.882	-5.550
GARCH to expiration	0.782	2.553	0.961	4.976
Date t trading range	0.644	1.799	0.959	2.922
Date t left tail return	1.561	3.480	0.187	0.431
Date t right tail return	1.000	1.930	-0.101	-0.383
Last year return	-0.004	-0.100	0.006	0.138
Last year absolute return	0.132	2.039	0.029	0.488
Historical volatility	0.692	3.380	-0.362	-1.738
Avg daily range last 10 days	0.382	0.238	-0.222	-0.205
Trading range, past option lifespan	-1.579	-10.952	0.036	0.665
# left tail events last year	1.900	4.176	1.240	1.682
Avg left tail return last year	0.242	0.582	-0.255	-0.539
# right tail events last year	0.018	0.071	0.246	0.687
Avg right tail return last year	1.344	2.304	-1.534	-2.784
Michigan consumer sentiment	-0.108	-1.502	-0.201	-1.860
S&P 500 P/E ratio	-1.149	-1.560	-3.042	-2.829
BAA-AAA bond yield spread	-8.845	-3.891	0.835	0.385
GARCH model error recent period	0.010	1.177	0.001	0.232
GARCH model RMSE	0.004	2.207	-0.001	-1.342
RND volatility premium date t-1			-0.006	-0.595
RND volatility date t			-0.011	-0.852
R2	0.369		0.526	
NOBS	10926		10694	

The Preferred Parsimonious Specification

	RND volatility	
Variable	coef	t-stat
Constant	29.232	2.615
Date t return		
Date t absolute return	-0.908	-3.781
GARCH to expiration	0.825	4.370
Date t trading range	0.668	1.669
Date t left tail return	1.526	3.259
Date t right tail return	0.936	1.924
Last year return		
Last year absolute return	0.132	2.346
Historical volatility	0.704	3.851
Avg daily range last 10 days		
Trading range, past option lifespa	-1.572	-11.634
# left tail events last year	1.795	5.479
Avg left tail return last year		
# right tail events last year		
Avg right tail return last year	1.193	2.205
Michigan consumer sentiment	-0.108	-1.501
S&P 500 P/E ratio	-1.087	-1.680
BAA-AAA bond yield spread	-8.702	-3.885
GARCH model error recent perioc	0.010	1.243
GARCH model RMSE	0.004	2.285
RND volatility premium date t-1		
RND volatility date t		
R2	0.369	
NOBS	10926	

Is Risk Neutral Volatility Forward Looking?

Regressions on Future Realized Volatility over Different Horizons

Theoretically, option prices are a function of the expected volatility from date t through expiration. But traders who plan to hold the option only until the market moves to some target value will have shorter forecasting horizons.

To explore to what extent RND volatility is a forward looking forecast rather than just aggregating past information, realized volatility was regressed on the three main volatility measures, RND volatility, GARCH, and Historical volatility, each separately and in combination over periods ranging from 1 day to the full life of the option.

The table also reports the coefficients on these 3 variables in regressions with all of the other explanatory variables included.

Is Risk Neutral Volatility Forward Looking?

Key Results:

- In a univariate regression with RND volatility alone, the coefficient is positive and highly significant at all horizons
- The size of the coefficient and its t-statistic, and the regression R^2 suggest that the horizon over which RND volatility has the best predictive power is about two weeks, distinctly shorter than option maturity.
- The GARCH model and historical volatility forecasts also seem to do best at about the same 1-2 week horizon.
- In pairwise comparisons, GARCH dominates both RND volatility and historical volatility. Neither contributes anything close to significant explanatory power beyond what is in GARCH.
- Combining RND volatility and historical volatility, both are highly significant. RND volatility receives a much smaller weight, but nearly the same t-statistic.

An observation: If GARCH is about the best one can do in forecasting future realized volatility and RND volatility impounds GARCH but also variations in risk attitudes, we should expect these results, with RND volatility being a strong predictor on its own, but being completely dominated by GARCH in a combined regression.

Regressions on Future Realized Volatility over Different Horizons

	1-Day		2-Day		3-Day		5-Day		7-Day		10-Day		15-Day		To expiration	
Variable	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat
Coefficients from Regressions on Volatility Variables Only																
RND volatility date t	0.126	5.537	0.139	6.551	0.144	6.840	0.144	6.905	0.144	6.876	0.142	6.816	0.139	6.626	0.108	5.002
R^2	0.028		0.046		0.055		0.065		0.070		0.072		0.071		0.046	
RND volatility date t	0.010	0.485	0.001	0.049	-0.003	-0.217	-0.006	-0.371	-0.005	-0.335	-0.006	-0.367	-0.004	-0.289	-0.010	-0.590
GARCH to expiration	0.984	7.714	1.180	11.235	1.252	10.988	1.277	10.583	1.266	10.516	1.260	10.458	1.218	10.589	1.004	9.389
R^2	0.193		0.362		0.461		0.561		0.595		0.614		0.598		0.436	
RND volatility date t	0.063	3.934	0.066	3.871	0.066	3.753	0.066	3.663	0.066	3.588	0.065	3.548	0.063	3.440	0.045	2.674
Historical volatility	0.423	4.538	0.498	4.391	0.522	4.183	0.527	4.025	0.528	3.957	0.524	3.909	0.515	3.872	0.424	3.648
R^2	0.095		0.169		0.210		0.251		0.271		0.278		0.278		0.199	
RND volatility date t	0.009	0.470	0.000	0.024	-0.003	-0.221	-0.005	-0.351	-0.005	-0.336	-0.006	-0.362	-0.005	-0.319	-0.010	-0.640
GARCH to expiration	0.965	5.589	1.170	8.563	1.251	8.783	1.283	8.637	1.265	8.554	1.260	8.524	1.208	8.457	0.994	6.959
Historical volatility	0.021	0.310	0.011	0.173	0.001	0.014	-0.007	-0.090	0.001	0.016	-0.001	-0.012	0.012	0.142	0.010	0.132
R^2	0.193		0.362		0.461		0.561		0.595		0.614		0.598		0.436	
GARCH to expiration	0.991	8.810	1.184	12.200	1.246	11.600	1.272	11.023	1.264	10.938	1.255	10.889	1.216	11.090	0.992	10.182
R^2	0.192		0.363		0.460		0.559		0.595		0.613		0.597		0.435	
Historical volatility	0.462	4.847	0.540	4.660	0.563	4.451	0.569	4.290	0.569	4.223	0.565	4.174	0.555	4.135	0.453	3.887
R^2	0.089		0.160		0.199		0.237		0.256		0.263		0.263		0.192	
GARCH to expiration	0.969	5.939	1.178	8.983	1.247	9.021	1.284	8.811	1.268	8.724	1.260	8.688	1.208	8.661	0.986	7.218
Historical volatility	0.024	0.340	0.007	0.105	-0.001	-0.021	-0.013	-0.166	-0.005	-0.060	-0.005	-0.061	0.008	0.096	0.006	0.082
R^2	0.193		0.363		0.460		0.559		0.595		0.613		0.597		0.435	
Selected Coefficients from All-Variables Regressions																
RND volatility date t	0.013	0.568	0.004	0.214	0.001	0.060	-0.003	-0.174	-0.004	-0.243	-0.005	-0.321	-0.005	-0.303	-0.016	-0.939
GARCH to expiration	0.368	1.167	0.438	1.668	0.526	2.165	0.617	2.695	0.656	2.930	0.697	3.190	0.739	3.417	0.953	4.987
Historical volatility	-0.148	-0.522	-0.167	-0.726	-0.202	-0.921	-0.218	-1.020	-0.216	-1.018	-0.202	-0.957	-0.178	-0.840	-0.366	-1.760
R^2	0.254		0.416		0.522		0.616		0.647		0.664		0.645		0.532	

RND Volatility Regressions for short and long maturities

	Maturity < 75 days				Maturity ≥ 75 days			
Variable	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat
Constant	24.919	1.760	17.731	1.500	7.347	2.521	7.548	1.759
Date t return	0.124	0.548			-0.316	-8.594		
Date t absolute return	-1.277	-2.700	-2.078	-4.834	-0.419	-5.538	-0.443	-5.887
GARCH to expiration	0.801	1.777	1.702	6.249	0.333	4.152	0.403	5.700
Date t trading range	1.700	2.537	1.837	2.732	0.605	6.059	0.681	5.236
Date t left tail return	2.543	3.141	3.089	3.895	0.458	3.977	0.105	0.979
Date t right tail return	0.834	1.235	1.402	2.065	0.733	1.237	0.293	0.511
Last year return	-0.099	-1.346			0.038	2.376		
Last year absolute return	0.219	2.812	0.211	2.961	-0.009	-0.446	0.033	1.573
Historical volatility	1.066	4.201	1.418	6.541	0.345	5.742	0.483	5.381
Avg daily range last 10 days	7.131	2.454			0.401	1.040		
Trading range, past option lifespan	-3.390	-18.405	-3.210	-17.720	-0.018	-0.604	0.028	0.802
# left tail events last year	2.382	3.806			0.905	7.199		
Avg left tail return last year	0.285	0.493			0.176	1.321		
# right tail events last year	-0.020	-0.050			-0.295	-3.078		
Avg right tail return last year	0.710	0.742			0.875	5.465		
Michigan consumer sentiment	-0.064	-0.732	-0.027	-0.379	-0.064	-3.635	-0.074	-2.953
S&P 500 P/E ratio	0.209	0.217	0.055	0.070	0.167	0.702	0.542	1.340
BAA-AAA bond yield spread	-11.771	-3.207	-13.256	-3.966	-1.857	-2.587	-3.927	-3.555
GARCH model error recent period	0.007	0.756	0.011	1.199	0.001	0.302	0.0000	0.015
GARCH model RMSE	0.006	2.333	0.004	1.679	0.0003	0.765	-0.0003	-0.382
RND volatility premium date t-1								
RND volatility date t								
R^2	0.409		0.394		0.633		0.581	
NOBS	5639		5639		5287		5287	

RND Volatility Regressions for short and long maturities

Among these numbers there are some Key Results, some Weird Results, and a few major differences between long and short maturity options.

The Key Results and Weird Results are similar to those in the full sample.

Key Results:

- The volatility forecast from the GARCH model is highly significant
- Historical volatility is also highly significant
- The date t trading range is significant. A wider range is associated with a wider RND.

RND Volatility Regressions for short and long maturities

Key results

	Maturity < 75 days				Maturity ≥ 75 days			
Variable	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat
Constant	24.919	1.760	17.731	1.500	7.347	2.521	7.548	1.759
Date t return	0.124	0.548			-0.316	-8.594		
Date t absolute return	-1.277	-2.700	-2.078	-4.834	-0.419	-5.538	-0.443	-5.887
GARCH to expiration	0.801	1.777	1.702	6.249	0.333	4.152	0.403	5.700
Date t trading range	1.700	2.537	1.837	2.732	0.605	6.059	0.681	5.236
Date t left tail return	2.543	3.141	3.089	3.895	0.458	3.977	0.105	0.979
Date t right tail return	0.834	1.235	1.402	2.065	0.733	1.237	0.293	0.511
Last year return	-0.099	-1.346			0.038	2.376		
Last year absolute return	0.219	2.812	0.211	2.961	-0.009	-0.446	0.033	1.573
Historical volatility	1.066	4.201	1.418	6.541	0.345	5.742	0.483	5.381
Avg daily range last 10 days	7.131	2.454			0.401	1.040		
Trading range, past option lifespan	-3.390	-18.405	-3.210	-17.720	-0.018	-0.604	0.028	0.802
# left tail events last year	2.382	3.806			0.905	7.199		
Avg left tail return last year	0.285	0.493			0.176	1.321		
# right tail events last year	-0.020	-0.050			-0.295	-3.078		
Avg right tail return last year	0.710	0.742			0.875	5.465		
Michigan consumer sentiment	-0.064	-0.732	-0.027	-0.379	-0.064	-3.635	-0.074	-2.953
S&P 500 P/E ratio	0.209	0.217	0.055	0.070	0.167	0.702	0.542	1.340
BAA-AAA bond yield spread	-11.771	-3.207	-13.256	-3.966	-1.857	-2.587	-3.927	-3.555
GARCH model error recent period	0.007	0.756	0.011	1.199	0.001	0.302	0.0000	0.015
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RND volatility premium date t-1								
RND volatility date t								
R^2	0.409		0.394		0.633		0.581	
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RND Volatility Regressions for short and long maturities

Weird Results:

- Date t absolute return is significantly negative
- A date t return in the 2% left tail significantly narrows the RND
- The occurrence of left tail events in the past increases RND volatility, but the bigger they were, the smaller the increase. For longer maturity options, past right tail events reduced RND volatility, but a bigger average size of a right tail return increased RND volatility.

RND Volatility Regressions for short and long maturities

Weird results

Variable	Maturity < 75 days				Maturity ≥ 75 days			
	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat
Constant	24.919	1.760	17.731	1.500	7.347	2.521	7.548	1.759
Date t return	0.124	0.548			-0.316	-8.594		
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Date t right tail return	0.834	1.235	1.402	2.065	0.733	1.237	0.293	0.511
Last year return	-0.099	-1.346			0.038	2.376		
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RND Volatility Regressions for short and long maturities

Major differences between long and short maturity options:

- A large positive return over the last year reduced the RND volatility for nearby options but raised it significantly for distant ones
- Wide daily trading ranges in the recent past significantly increased short maturity RND volatility but had little effect on longer term RND volatility.
- The wide trading range over a past period of the same length as the option's remaining lifetime (anomalously) significantly narrowed the RND for nearby contracts but had little impact on distant contracts.
- Positive Consumer sentiment, from the University of Michigan survey, narrowed the RND for all maturities by very similar amounts, but the effect was much more statistically significant for longer maturities
- The size of recent GARCH model residuals was highly significant for near maturity RNDs but not for longer ones.

RND Volatility Regressions for short and long maturities

Variable	Maturity < 75 days				Maturity ≥ 75 days			
Differences	coef	t-stat	coef	t-stat	coef	t-stat	coef	t-stat
Constant	24.919	1.760	17.731	1.500	7.347	2.521	7.548	1.759
Date t return	0.124	0.548			-0.316	-8.594		
Date t absolute return	-1.277	-2.700	-2.078	-4.834	-0.419	-5.538	-0.443	-5.887
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R^2	0.409		0.394		0.633		0.581	
NOBS	5639		5639		5287		5287	

Conclusions:

Tentative answers to the broad questions mentioned above:

What return and volatility-related factors are most important to investors in forming forecasts of the empirical probability density that are embedded in the RND?

Simple correlations indicating the strength of the direct univariate relationships between explanatory variables and both RND and realized volatility were largely of the expected sign.

- Correlation between the two volatility measures was only 0.216.
- Most correlations were stronger with realized future volatility than with RND volatility.
- Greatest correlation was with GARCH forecasts, historical volatility, the trading range variables, and left tail events especially for realized volatility.
- Correlation with yesterday's volatility risk premium (RND vol – GARCH vol) was 0.877 for RND volatility but -0.010 for realized volatility.

Conclusions, continued

Is the market primarily forward-looking or backward-looking? What are the relevant time horizons it focuses on?

<u>Backward looking:</u>

- Key backward looking variables include GARCH model forecasts and the errors in past GARCH forecasts, and historical volatility. The past trading range and some tail-related variables were statistically significant but not easy to interpret.

<u>Forward looking:</u>

- The GARCH model had the best predictive power at all horizons;
- RND volatility alone or combined with historical volatility was highly significant, but contributed nothing when combined with GARCH.
- All three volatility measures appear to have the greatest marginal explanatory power for future volatility over the next 5 to 10 trading days.
- In combined regressions with all variables, of the three only GARCH was significant. The coefficient on Historical volatility was uniformly negative and insignificant.

Conclusions, continued

What factors go into the process of risk neutralization?

- The previous day volatility risk premium as measured by yesterday's RND volatility minus GARCH model volatility was extremely powerful in the RND volatility equations, with t-statistics over 30. Adding this variable to a specification with 20 other variables increased R^2 from around 37% to 89%.
- Strength in the Michigan Consumer Sentiment variable reduced RND volatility at a significance level better than 10% but not 5%. A high P/E ratio for the stock market also reduced RND volatility, at a slightly higher significance level.
- The past error from a GARCH model was important, both in absolute value and to a lesser extent as a signed variable. Large past errors of either sign increased risk neutral volatility, but underpredicting volatility had a greater impact.
- The yield spread in the bond market showed highly significant but anomalous positive signs in the RND volatility equation but negative signs in explaining realized volatility.
- Variables that were significant and much bigger in the realized volatility equation than in the RND volatility equation were the date t trading range, the S&P P/E ratio, and especially the date t-1 RND volatility risk premium.
- Variables that were significant and much more important in the RND volatility equation than in the realized volatility equation include historical volatility, the past trading range, the number of past left tail events and the size of past right tail events, and the bond yield spread.

Final Conclusions

THANKS!

for your attention, questions, and anticipated post-seminar feedback!