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Singapore – London – Nice

# Optimal Asset Allocation For Resource Based Sovereign Wealth Funds

Q-Group Meeting  
March 2010

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with the support of

Deutsche Bank



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## Outline

- Types of Sovereign Wealth Funds
- SWF 101 - A Static Model
- Resource Uncertainty
- Predictability and Horizon Dependent Hedging
- Asset Allocation and Optimal Extraction Policies

## Types of Sovereign Wealth Funds

- **FX funds: transfers of assets from foreign exchange reserves.**
  - Exchange rate policies create current account surplus and/or capital account deficit and hence significant reserve accumulation.
  - Country transfers “excess” foreign exchange reserves to stand-alone funds.
  - BUT, reserve accumulation must generally be sterilized, so excess reserve funds can be thought of as being financed through borrowed funds.
- **Resource based funds**
  - Commodity funds are financed through earnings on commodity exports (taxed by the government).

## Oil Price Volatility



**Daily oil price movements from January 1982 to September 2008.** The underlying total wealth position of an oil rich country can vary dramatically over time and needs management to smooth intergenerational consumption patterns.

## Why Do Resource Based Funds Exist?

- The obvious return seeking argument: “oil to equity transformation”.
- There is also a risk management argument. We have evidence that managing macroeconomic risks increases growth. Oil price movements are unpredictable and volatile with extremely wide confidence intervals. This provides a powerful argument to reduce the volatility of oil related revenues with a positive impact of consumption smoothing on total welfare. The usual routes to consumption smoothing, i.e. borrowing funds or hedging revenue risk are not available due to limited access to international debt markets (precautionary savings motive) or incomplete markets for oil price hedging instruments (size, liquidity, contract choice).
- There is also an economic diversification argument. Diversify an economy away from its vulnerability to oil price shocks: macroeconomic diversification (developing a competitive non-oil sector) and investment diversification (setting up an international SWF). To an economist, macroeconomic diversification runs counter to specialization advantages and takes long to implement. Investment diversification (SWF) is faster and easier to implement as the preferred route of self insurance

## The Basic Problem

Let  $\theta$  denote the fraction of financial SWF wealth relative to total wealth (financial wealth plus oil wealth). If the SWF has a size of 1 monetary unit, while the market value of oil reserves amounts to 5 monetary units this translates into  $\theta = \frac{1}{1+5} = \frac{1}{6}$  weight for the SWF asset and  $1 - \theta = 1 - \frac{1}{6} = \frac{5}{6}$  weight for oil revenues.

$$\tilde{r} = \theta w \tilde{r}_a + (1 - \theta) \tilde{r}_o.$$

Note that  $1 - w$  represents the implied cash holding that carries a zero risk premium and no risk in a one period consideration. Expressing returns as risk premium has got the advantage that we do not need to model cash holdings. These simply become the residual asset that ensures portfolio weights add up to one without changing risk or (excess) return.

## The Basic Solution

The optimal solution for this problem can be found from

$$\max_w \left( \theta w \mu_a - \frac{\lambda}{2} \left[ \theta^2 w^2 \sigma_a^2 + (1 - \theta)^2 \sigma_o^2 + 2w\theta(1 - \theta) \rho \sigma_a \sigma_o \right] \right)$$

$$w^* = w_s^* + w_h^* = \frac{1}{\theta} \frac{\mu_a}{\lambda \sigma_a^2} - \frac{1 - \theta}{\theta} \frac{\rho \sigma_o}{\sigma_a}$$

Speculative demand: In the case of uncorrelated assets and oil resources the optimal solution is equivalent to a leveraged (with factor  $\frac{1}{\theta}$ ) position in the asset only maximum SHARPE-ratio portfolio or in other words  $w_s^*$ .

Hedging demand: is given as the product of leverage and oils asset beta,  $\beta_{o,a} = \frac{\rho \sigma_o}{\sigma_a}$ . The later is equivalent to the slope coefficient of a regression of (demeaned) asset returns against (demeaned) oil returns, i.e. of the form  $(r_o - \bar{r}_o) = \beta_{o,a} (r_a - \bar{r}_a) + \varepsilon$ . Hedge demand is only zero if oil price risk is purely idiosyncratic.

## Hedging Recession Risks

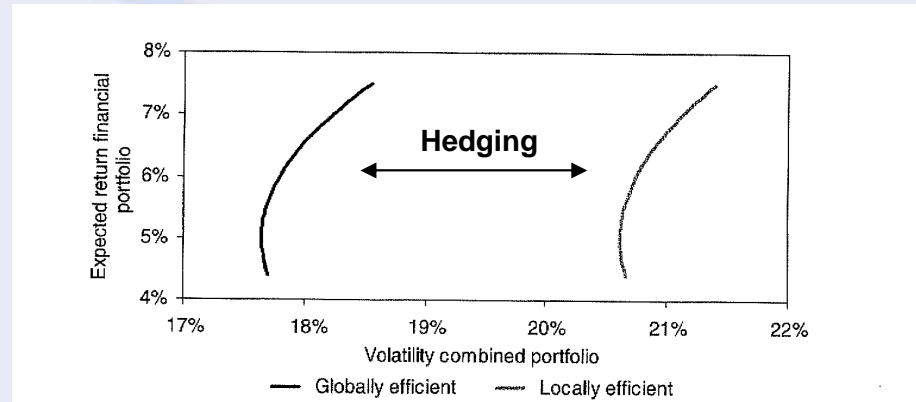
Frequency	US Treasury Bond							Global Equities
	1 year	1-3 year	3-5 year	5-7 year	7-10 year	10-20 year	20 plus year	
Monthly	-8,28%	-2,38%	1,16%	2,26%	2,75%	1,18%	0,16%	9,19%
	-0,99	-0,43	0,21	0,41	0,49	0,21	0,03	1,66
Quarterly	-9,29%	-22,59%	-20,08%	-21,00%	-19,60%	-23,92%	-24,42%	-9,34%
	-0,65	-2,42	-2,14	-2,24	-2,09	-2,57	-2,63	-0,98
Annual	-38,81%	-40,70%	-50,94%	-56,52%	-59,48%	-55,04%	-51,29%	24,10%
	-1,52	-2,36	-3,13	-3,63	-3,92	-3,49	-3,16	1,31

**Correlation of asset returns with percentage oil price changes.** The table uses global equities (MSCI World in USD) and US government bonds (Lehman US Treasury total return index for varying maturities) and oil (Crude Oil-Brent Cur Month FOB from Thompson) for the period from January 1997 to September 2008. This translates into 142 monthly, 48 quarterly and 13 annual data points. For each data frequency, the first line shows the correlation coefficient, while the second line provides its  $t$ -value. We calculate  $t$ -values according to  $t = \rho \sqrt{\frac{n-2}{1-\rho^2}}$ , where  $n$  represents the number of data points and  $\rho$  the estimated correlation coefficient. Critical values are given by the  $t$ -distribution with  $n - 2$  degrees of freedom. For example the critical value for 13 annual data points at the 95% level is 2.2. All significant correlation coefficients are grey shaded.

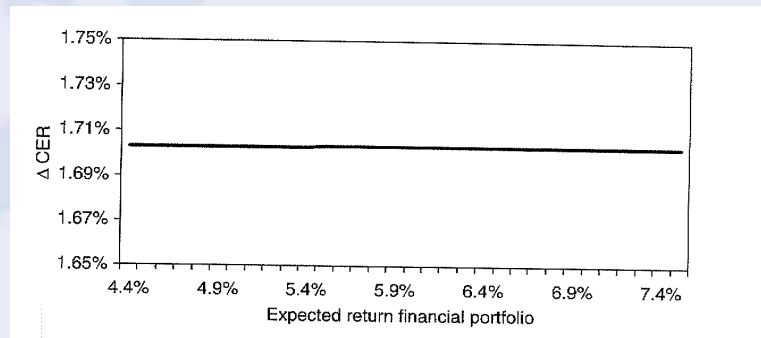


## The Costs of Ignoring Advice

- GINTSCHEL/SCHERER (2004) - Reduction in total volatility



- GINTSCHEL/SCHERER (2004) – Increase in security equivalent



## Why Is Advice Ignored?

- Political meddling
- Institutional separation of oil wealth and financial wealth
  - Competing for growth
  - No joint optimization
  - Loss in security equivalent is nowhere accounted for

## Adding Resource Uncertainty

- There is a vast literature on background risk.
- How can we translate this idea into our framework for finding the optimal allocation for a SWF?
- So far we have assumed that (i.e. the fraction of SWF assets to total sovereign wealth) is known with certainty in , i.e. that the value of oil reserves is known to the decision maker.
- However the size of an oil field is not known with great precision. Additionally government claims are sometimes legally disputed (among neighbouring countries) and new undiscovered fields might yet be to be found. Hence might be best thought of as a random variable.

## Extending Our Basic Model

$\theta$  might be best thought of as a random variable. We assume the fraction of financial wealth relative to total (financial and oil) wealth follows a uniform distribution around the governments estimate of  $\bar{\theta}$ . More precisely we assume

$$\tilde{\theta} \sim U(\bar{\theta} - \varepsilon, \bar{\theta} + \varepsilon)$$

It seems natural to further assume independence between background risk on the level of available oil reserves and asset risk. The joint probability density function can then be written down as

$$f(\theta, r_a) = f(\theta)f(r_a) = \frac{1}{\sigma_a\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{r_a - \mu_a}{\sigma_a}\right)^2} \frac{1}{(\bar{\theta} + \varepsilon) - (\bar{\theta} - \varepsilon)}$$

We are looking for

$$Var(\tilde{\theta}\tilde{r}_a) = E[(\tilde{\theta}\tilde{r}_a)^2] - E[\tilde{\theta}\tilde{r}_a]^2$$

in order to calculate portfolio risk.

## Extending Our Basic Model

This amounts to integrating over the joint probability density where

$$E\left[(\tilde{\theta}\tilde{r}_a)^2\right] = \int_{-\infty}^{\infty} \int_{\bar{\theta}-\varepsilon}^{\bar{\theta}+\varepsilon} (\theta r_a)^2 f(\theta, r_a) d\theta dr_a = \frac{1}{3}(\varepsilon^2 + 3\bar{\theta}^2)(\mu_a^2 + \sigma_a^2)$$

$$\begin{aligned} Var(\tilde{\theta}\tilde{r}_a) &= \frac{1}{3}(\varepsilon^2 + 3\bar{\theta}^2)(\mu_a^2 + \sigma_a^2) - \mu_a^2\bar{\theta}^2 \\ &= \bar{\theta}^2\sigma_a^2 + \varepsilon^2 \frac{\mu_a^2 + \sigma_a^2}{3} \end{aligned}$$

The reader will notice that for  $\varepsilon \rightarrow 0$  we converge to the well known expression known from undergraduate statistics  $Var(\tilde{\theta}\tilde{r}_a) = \bar{\theta}^2\sigma_a^2$ .

## Extending Our Basic Model

What does this imply? As long as we have background risk in the form of uncertainty around the size of oil reserves the optimal asset allocation for the SWF (we focus on the case with uncorrelated assets and oil returns for simplicity) becomes

$$w_{br}^* = \frac{1}{\lambda} \frac{\bar{\theta} \mu_a}{\bar{\theta}^2 \sigma_a^2 + \varepsilon^2 \frac{\mu_a^2 + \sigma_a^2}{3}}$$

We now compare this with the solution in absence of background risk  $w^* = \frac{1}{\theta} \frac{\mu_a}{\lambda \sigma_a^2}$  by building the quotient.

$$\frac{w^*}{w_{br}^*} = 1 + \frac{\varepsilon^2 (\mu_a^2 + \sigma_a^2)}{\bar{\theta}^2 \sigma_a^2} > 1$$

which will always be greater than 1.

## Conclusions

- An increase in background risk will lead to a decrease in risk taking for the SWF.
- The effect becomes stronger the more volatile our risky asset is. Empirically we should observe that SWF's with larger resource uncertainty should invest less aggressively and vice versa.
- Also we would expect that economies with low reserves relative to financial wealth are less affected by resource uncertainty.

## Idea

- Extending GINTSCHEL/SCHERER (2004, 2008), we apply the portfolio choice problem for a sovereign wealth funds (SWF) in a CAMPBELL/VICEIRA (2002) strategic asset allocation framework.
- We derive closed form solutions for two sources of inter-temporal hedging demand. More precisely we derive a three fund separation that splits the optimal portfolio for a SWF into speculative demand as well as hedge demand against oil price shocks and shocks to the short term risk-free rate.
- We provide a small empirical example for equities, bonds and listed real estate to illustrate our framework.



## Setup

We can write the  $n$  period return,  $\mu_{t,SWF}^{(n)}$ , and risk,  $\sigma_{t,SWF}^{(n)2}$ , as a function of our decision vector of portfolio weights,  $w_t^{(n)}$ , according to

$$\begin{aligned}\mu_{t,SWF}^{(n)} &= \theta w_t^{(n)T} \left( \mu_a^{(n)} + \frac{1}{2} \sigma_a^2 \right) - \frac{1}{2} \theta w_t^{(n)T} \Sigma_{aa} \theta w_t^{(n)} \\ \sigma_{t,SWF}^{(n)2} &= \theta^2 w_t^{(n)T} \Sigma_{aa}^{(n)} w_t^{(n)} + 2\theta(1-\theta) w_t^{(n)T} \Sigma_{ao}^{(n)} + 2\theta w_t^{(n)T} \Sigma_{ac}^{(n)}\end{aligned}$$

Note that we (in accordance with the literature) implicitly assume  $w_t^{(n)}$  to remain constant, which equates to assuming an investor with constant time horizon  $n$ . Assuming CRRA we maximize  $\frac{1}{1-\gamma} (W_{SWF,t+n})^{1-\gamma}$  and the optimization problem becomes

$$\max_{w_t^{(n)}} \left[ \mu_{t,SWF}^{(n)} + \frac{1}{2} (1-\gamma) \sigma_{t,SWF}^{(n)2} \right].$$

$$\mu^{(n)} = n^{-1} E \left[ \sum_{i=1}^n x_{t+n} \mid s_t \right] \quad \Sigma^n = n^{-1} Var \left( \sum_{i=1}^n x_{t+n} \mid s_t \right)$$

## Solution

Differentiating we get the first order conditions

$$\theta \left( \mu_a^{(n)} + \frac{1}{2} \sigma_a^2 \right) - \theta^2 \Sigma_{aa} w_t^{(n)} + \frac{(1-\gamma)}{2} \theta^2 2 \Sigma_{aa}^{(n)} w_t^{(n)} + \frac{(1-\gamma)}{2} 2\theta (1-\theta) \Sigma_{ao}^{(n)} + \frac{(1-\gamma)}{2} 2\theta \Sigma_{ac}^{(n)} = 0$$

We divide by  $\theta$ , collect all terms involving  $w_t^{(n)}$  on the left side and use the identity that  $(1-\gamma) = -(\gamma-1)$  to finally arrive at

$$w_t^{(n)} = \frac{1}{\gamma} \left( \frac{1}{\gamma} \Sigma_{aa} + \frac{(\gamma-1)}{\gamma} \Sigma_{aa}^{(n)} \right)^{-1} \left[ \frac{1}{\theta} \left( \mu_a^{(n)} + \frac{1}{2} \sigma_a^2 \right) - (\gamma-1) \frac{(1-\theta)}{\theta} \Sigma_{ao}^{(n)} - \frac{1}{\theta} (\gamma-1) \Sigma_{ac}^{(n)} \right]$$

## Solution

$$w_{spec}^{(n)} = \left(\frac{1}{\theta}\right)\left(\frac{1}{\gamma}\right)\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{\gamma-1}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\left(\mu^{(n)} + \frac{1}{2}\sigma_a^2\right)$$

$$w_{hedge,cash}^{(n)} = -\left(\frac{1}{\theta}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{\gamma-1}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\Sigma_{ac}^{(n)}$$

$$w_{hedge,oil}^{(n)} = -\left(\frac{1-\theta}{\theta}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{\gamma-1}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\Sigma_{ao}^{(n)}$$

For  $\theta = 1$  we arrive at the optimal solution for an investor with financial wealth only, while for  $\gamma \rightarrow \infty$  we get  $w_{hedge,cash}^{(n)} = -\left(\frac{1}{\theta}\right)[\Sigma_{aa}^n]^{-1}\Sigma_{ac}^{(n)}$  and  $w_{hedge,oil}^{(n)} = -\left(\frac{1-\theta}{\theta}\right)[\Sigma_{aa}^{(n)}]^{-1}\Sigma_{ao}^{(n)}$ .

Also it is interesting to note that for  $\gamma = 1$ , i.e. a log investor, we arrive at the familiar solution that there is no hedging demand and the optimal solution degenerates to the one period myopic speculative demand  $w_{spec}^{(n)} = \frac{1}{\theta}\Sigma_{aa}^{-1}\left(\mu^{(n)} + \frac{1}{2}\sigma_a^2\right)$ . Note that this framework is extremely general and its application is not limited to equities or bonds, but can easily be applied to hedge funds, private equity investments etc.

## Data Generating Process

We start with assuming a first order vector auto-regression as our data generating process (DGP):

$$z_{t+1} = a + Bz_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \Omega)$$

where  $z_t = [x_t \quad s_t]^T$ ,  $\Omega$  represents the residual covariance matrix of our VAR with a vector of constants,  $a$ , and a coefficient matrix  $B$ . From above we can easily work out the multi-period expressions for risk and return

$$\Sigma^{(n)} = \sum_{i=1}^n \left[ \left( \sum_{j=0}^{i-1} B^j \right) \Omega \left( \sum_{j=0}^{i-1} B^j \right)^T \right]$$

where again  $\Sigma^{(n)}$  denotes the covariance matrix of  $n$  period returns and  $B^0 = I$ . Correlation, variance and covariance can be easily picked from the elements in  $\Sigma^{(n)}$ . Plotting variances, correlations, etc. against  $n$  provides a term structure of risk.

## Data

	Symbol	Mean ( $\mu$ )	Volatility ( $\sigma$ )	Sharpe ( $\frac{\mu-rc}{\sigma}\sqrt{4}$ )	Min	Max	Skew	Kurtosis
<b>Assets</b>								
Oil	$ro_t$	-0.37	19.23	-0.04	-0.89	0.82	-0.27	6.33
Tips	$rr_t$	0.42	0.60	1.40	-0.02	0.02	-0.94	2.29
T-Bill	$rc_t$	1.29	0.59		0.00	0.03	0.32	0.12
Long Bonds	$rb_t$	1.33	4.97	0.54	-0.09	0.17	0.51	0.37
Equities	$re_t$	1.69	8.14	0.42	-0.28	0.18	-0.79	1.46
Real Estate	$ri_t$	1.42	7.03	0.40	-0.17	0.19	0.00	0.05
<b>State Variables</b>								
Nominal	$yn_t$	7.15	2.56		0.04	0.15	0.96	0.36
Credit Spread	$cs_t$	1.96	0.50		0.01	0.04	1.04	0.67
Time Spread	$ts_t$	1.75	1.25		-0.01	0.05	-0.05	-0.62
Dividend Yield	$dy_t$	-3.66	34.24		-4.55	-2.97	-0.21	-0.58

**Summary Statistics.** We report descriptive statistics for all endogenous (assets and state variables) variables in our VAR. Only the SHARPE-ratios have been annualized.

## Estimated DGP

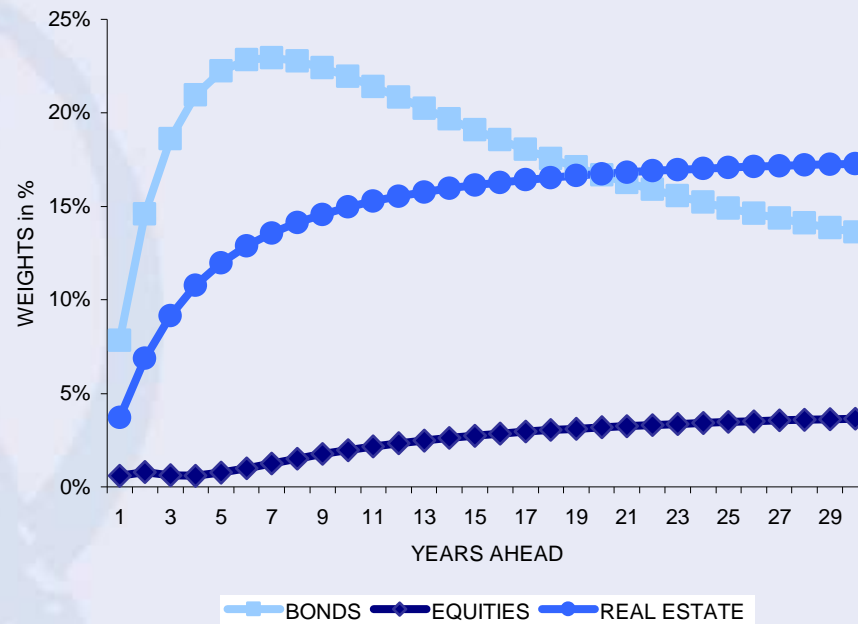
	$z_t$										
	$x_t$						$s_t$				
	$rr_t$	$rc_t$	$rb_t$	$re_t$	$ro_t$	$ri_t$	$yn_t$	$cs_t$	$ts_t$	$dy_t$	
$a$	<b>0.04</b>	-0.00	<b>-0.41</b>	<b>0.57</b>	0.62	-0.02	<b>0.01</b>	<b>-0.01</b>	<b>0.05</b>	<b>-1.30</b>	
	<b>0.01</b>	0.00	<b>0.11</b>	<b>0.18</b>	0.46	0.17	<b>0.01</b>	<b>0.00</b>	<b>0.01</b>	<b>0.36</b>	
	<b>3.23</b>	-0.61	<b>-3.73</b>	<b>3.08</b>	1.34	-0.12	<b>2.13</b>	<b>-2.02</b>	<b>4.76</b>	<b>-3.64</b>	
$rr_{t-1}$	0.08	<b>-0.06</b>	<b>2.28</b>	<b>-4.29</b>	-5.77	-1.13	0.06	-0.02	0.14	-1.61	
	0.13	<b>0.03</b>	<b>1.14</b>	<b>1.90</b>	4.76	1.77	0.07	0.05	0.12	3.70	
	0.65	<b>-1.97</b>	<b>2.00</b>	<b>-2.26</b>	-1.21	-0.64	0.85	-0.37	1.24	-0.44	
$rc_{t-1}$	-0.22	1.45	<b>-8.42</b>	-5.27	-7.36	<b>-12.23</b>	0.65	-0.22	<b>-5.65</b>	15.38	
	0.44	0.11	<b>3.97</b>	6.59	16.53	<b>6.16</b>	0.24	0.17	<b>0.40</b>	12.83	
	-0.51	13.51	<b>-2.12</b>	-0.80	-0.45	<b>-1.99</b>	2.76	-1.36	<b>-14.04</b>	1.20	
$rb_{t-1}$	-0.02	<b>-0.01</b>	0.10	<b>0.42</b>	<b>-1.25</b>	0.13	<b>-0.10</b>	<b>0.03</b>	<b>-0.03</b>	-0.32	
	0.01	<b>0.00</b>	0.11	<b>0.18</b>	<b>0.44</b>	0.16	<b>0.01</b>	<b>0.00</b>	<b>0.01</b>	0.34	
	-1.51	<b>-5.14</b>	0.95	<b>2.43</b>	<b>-2.86</b>	0.78	<b>-16.22</b>	<b>7.52</b>	<b>-2.88</b>	-0.94	
$re_{t-1}$	-0.01	0.00	0.12	0.05	-0.12	0.04	0.00	<b>-0.01</b>	0.00	-0.05	
	0.01	0.00	0.07	0.11	0.29	0.11	0.00	<b>0.00</b>	0.01	0.22	
	-1.71	0.10	1.75	0.45	-0.41	0.36	0.28	<b>-3.41</b>	0.71	-0.24	
$ro_{t-1}$	-0.00	0.00	0.02	<b>0.12</b>	-0.09	0.04	0.00	-0.00	0.00	0.09	
	0.00	0.00	0.03	<b>0.05</b>	0.12	0.05	0.00	0.00	0.00	0.09	
	-0.71	0.75	0.52	<b>2.55</b>	-0.75	0.98	0.12	-0.19	0.54	1.01	
$ri_{t-1}$	0.00	0.00	-0.16	-0.24	0.08	-0.01	0.01	<b>-0.01</b>	-0.01	0.04	
	0.01	0.00	0.09	0.15	0.37	0.14	0.01	<b>0.00</b>	0.01	0.28	
	0.50	0.67	-1.84	-1.63	0.23	-0.11	1.03	<b>-2.16</b>	-0.88	0.13	
$yn_{t-1}$	-0.11	<b>-0.11</b>	<b>3.33</b>	-0.26	-0.96	2.71	<b>0.80</b>	0.08	<b>1.17</b>	-1.12	
	0.11	<b>0.03</b>	<b>0.95</b>	1.58	3.96	1.48	<b>0.06</b>	0.04	<b>0.10</b>	3.07	
	-1.05	<b>-4.33</b>	<b>3.51</b>	-0.17	-0.24	1.83	<b>14.13</b>	1.92	<b>12.16</b>	-0.37	
$cs_{t-1}$	-0.02	<b>-0.07</b>	-1.12	-0.04	1.53	1.42	0.09	<b>0.82</b>	0.12	0.31	
	0.11	<b>0.03</b>	0.99	1.65	4.13	1.54	0.06	<b>0.04</b>	0.10	3.21	
	-0.18	<b>-2.52</b>	-1.13	-0.02	0.37	0.92	1.61	<b>19.89</b>	1.17	0.10	
$ts_{t-1}$	0.06	<b>0.18</b>	-1.17	-0.80	-1.97	-1.91	<b>0.14</b>	-0.04	<b>-0.60</b>	6.74	
	0.10	<b>0.02</b>	0.90	1.50	3.77	1.40	<b>0.05</b>	0.04	<b>0.09</b>	2.92	
	0.63	<b>7.33</b>	-1.29	-0.53	-0.52	-1.36	<b>2.53</b>	-1.11	<b>-6.59</b>	2.31	
$dy_{t-1}$	<b>0.01</b>	-0.00	<b>-0.09</b>	<b>0.12</b>	0.11	-0.00	<b>0.00</b>	<b>-0.00</b>	<b>0.01</b>	<b>0.71</b>	
	<b>0.00</b>	0.00	<b>0.02</b>	<b>0.04</b>	0.10	0.04	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.08</b>	
	<b>2.51</b>	-0.98	<b>-3.63</b>	<b>2.86</b>	1.09	-0.07	<b>2.28</b>	<b>-2.88</b>	<b>4.24</b>	<b>8.86</b>	
$R^2$	0.35	0.96	0.26	0.22	0.14	0.11	0.99	0.88	0.88	0.83	
$\bar{R}^2$	0.28	0.96	0.18	0.14	0.04	0.01	0.99	0.86	0.86	0.81	

**Results from first order VAR: Parameter Estimates.** We report the coefficients of the VAR together with standard errors and  $t$ -values. The last two rows contain the "R-square" as well as "adjusted R-square" for each individual regression. Coefficients significant at the 5% level are presented in bold.

	$rr_t$	$rc_t$	$rb_t$	$re_t$	$ro_t$	$ri_t$	$yn_t$	$cs_t$	$ts_t$	$dy_t$
$rr_t$	<b>0.47</b>	0.05	-0.45	-0.28	0.54	-0.40	0.20	-0.01	0.09	0.05
$rc_t$	0.05	0.12	-0.08	-0.07	0.09	-0.16	0.08	-0.19	-0.33	-0.08
$rb_t$	-0.45	-0.08	4.28	0.10	-0.22	0.26	-0.28	0.02	-0.24	0.04
$re_t$	-0.28	-0.07	0.10	7.11	-0.31	0.55	-0.15	-0.01	0.11	-0.27
$ro_t$	0.54	0.09	-0.22	-0.31	17.84	-0.28	-0.08	-0.01	-0.21	-0.02
$ri_t$	-0.40	-0.16	0.26	0.55	-0.28	6.65	-0.17	0.01	0.02	-0.31
$yn_t$	0.20	0.08	-0.28	-0.15	-0.08	-0.17	0.25	-0.54	0.47	0.18
$cs_t$	-0.01	-0.19	0.02	-0.01	-0.01	0.01	-0.54	0.18	-0.19	-0.21
$ts_t$	0.09	-0.33	-0.24	0.11	-0.21	0.02	0.47	-0.19	0.43	0.03
$dy_t$	0.05	-0.08	0.04	-0.27	-0.02	-0.31	0.18	-0.21	0.03	13.85

**Results from first order VAR: Residual Covariance Matrix.** We report the error covariance matrix,  $\Omega$ , of the VAR. The main diagonal contains (quarterly) volatility while off diagonal entries represent correlations.

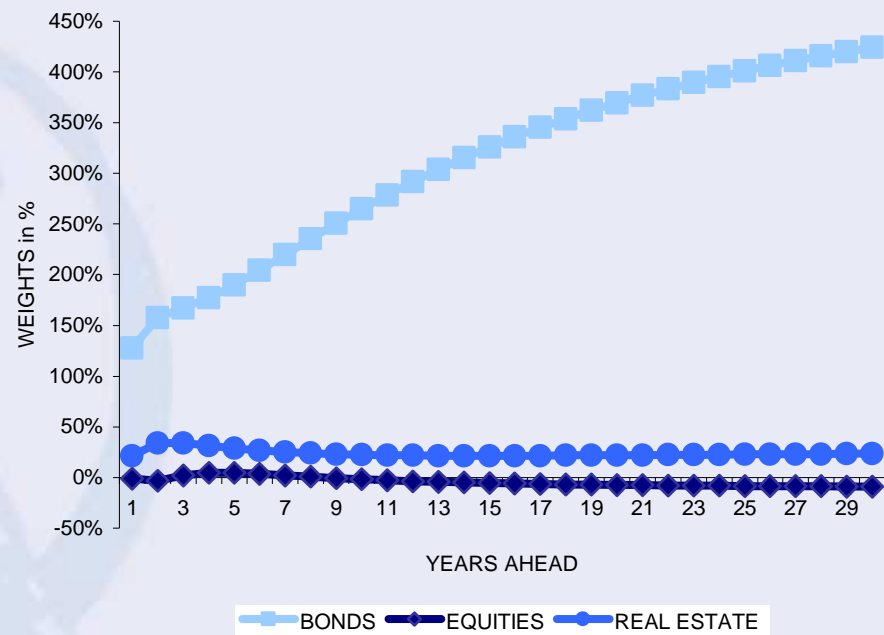
## Term Structure of Hedging Demand



**Hedging time variation in short term interest rates.** We represent the hedge portfolio according to

$$w_{hedge,cash}^{(n)} = -\frac{1}{\theta} [\Sigma_{aa}^n]^{-1} \Sigma_{ac}^{(n)} \text{ for } n = 1, \dots, 30 \text{ and } \theta = \frac{1}{2}$$

## Term Structure of Hedging Demand



**Figure 1. Hedging oil price shocks over time.** We represent the hedge portfolio according to

$$w_{hedge,oil}^{(n)} = -\left(\frac{1-\theta}{\theta}\right) \left[\Sigma_{aa}^{(n)}\right]^{-1} \Sigma_{ao}^{(n)} \text{ for } n = 1, \dots, 30 \text{ and } \theta = \frac{1}{2} .$$



## Conclusion

- We show how to derive three fund separation in the CAMPBELL/VICEIRA (2002) framework for a SWF.
- We provide a small empirical example for equities, bonds and listed real estate to illustrate our framework and conclude that a SWF should hold a considerable amount of assets in long US government bonds in order to hedge both the negative effects of oil price shocks as well as deteriorating short rates.
- Anecdotal evidence on the investment behaviour of SWF confirms our view.