Common Failings: How Corporate Defaults Cluster

Sanjiv Das

Darrell Duffie

Nikunj Kapadia

Leandro Saita

Paper: `http://scumis.scu.edu/~srdas/ddks.pdf`

Correlated Default

- Bond funds, credit portfolios, junk pools.
- Basket default swaps.
- Collateralized Default Obligations (CDOs).
- Bank balance-sheets.

How Corporate Defaults Cluster?

- Systematic risk: common factors.
- Contagion - domino or cascade effect.
- Frailty - unobservable common variables - learning from default.

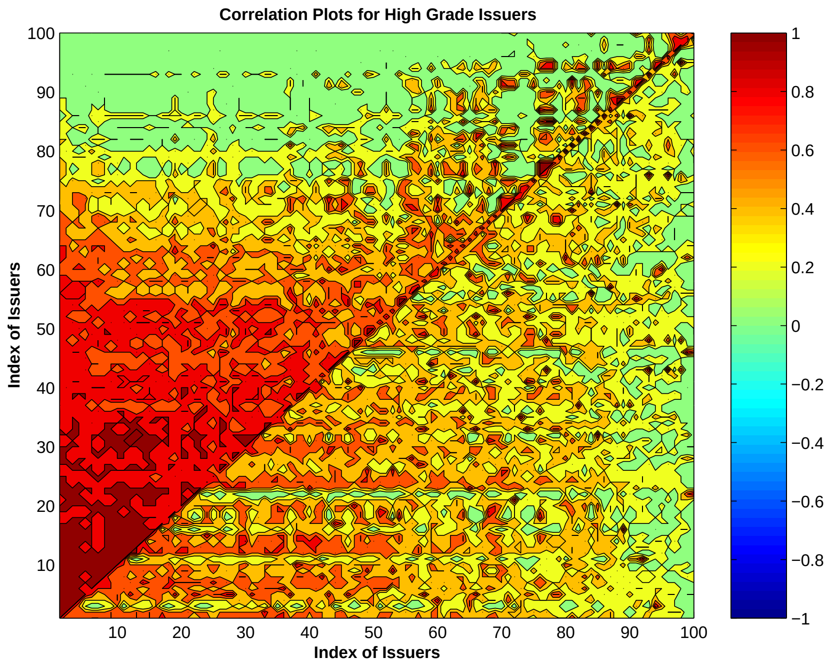
Default leads to spread increases: Collin-Dufresne, Goldstein & Helwege (2003), Zhang (2004).

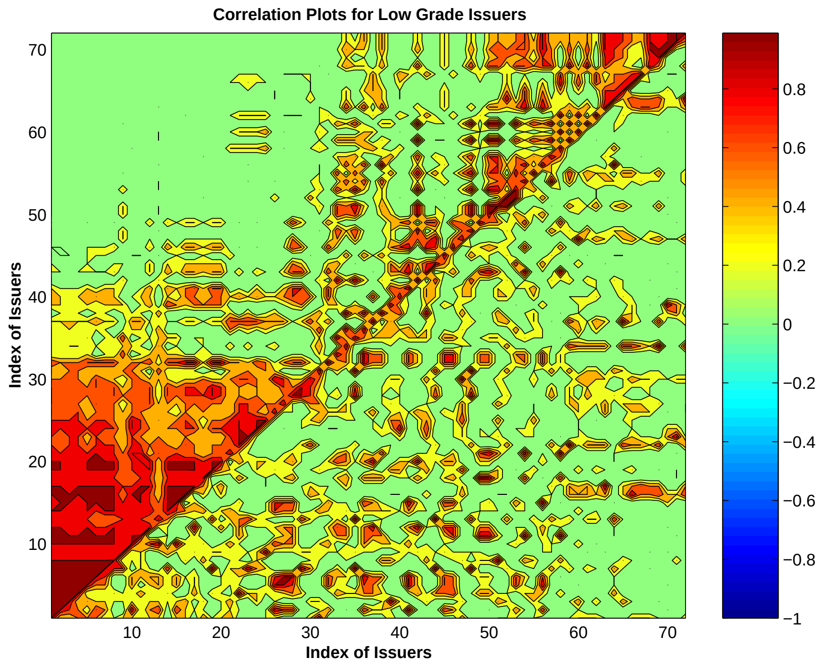
Conditional default probabilities or risk premia may increase (Kusuoka 1999).

Cox Process Framework

- Key ingredient - "intensity" model (as opposed to "structural" models driven by leverage and volatility).
- Intensity is based on state variables $\mathbf{X}$.
- Default arrival by jump N_t such that $\lambda(X_t)$ is the $\mathcal{F}_t$-intensity of N.
- The "doubly stochastic" process. Two-fold:
  - Process 1: Default intensity: $\lambda(t) = \lim_{h \to 0} \frac{s(t)-s(t+h)}{s(t)h} = -\frac{s'(t)}{s(t)}$.
  - Process 2: Conditional default probability: $Pr[D_i = 1|\lambda]$, $\forall i$.
- Doubly stochastic assumption is that processes in 2 are independent $\Longrightarrow$ defaults are Poisson after conditioning on intensities.
- Plenty of data for Process 1, much less for Process 2.
- Strong evidence that Process 1 evidences correlation across intensities.
- Not much known about Process 2.

Conditional Correlation of Intensities





Heat Maps

Relevance

- Simulation models may be mis-specified.
- Risk management of portfolios of corporate debt.
- Maintaining capital adequacy by banks, at high confidence levels of 99.97%. Tail properties are sensitive to clustering.
- Very large business: annual growth rate of synthetic CDOs alone is over 130% in 2003 (BoA). In 2004 BIS estimated synthetic CDO volume of $673 billion.
- Critical for holders of tranches in CDOs.

Some Related Literature

1. Lang and Stulz (1992) - default contagion in equity prices.
2. Jarrow, Lando & Yu (1999), Jarrow & Yu (2001) - diversifiable default risk.
3. Correlation in Process 1 - Das, Freed, Geng and Kapadia (2001) - driven by market volatility, regime dependence (macro clustering). Also Lopez (2002).
4. Renault and deServigny (2002) - default correlation using rating transitions, no test of clustering.
5. Collin-Dufresne, Goldstein and Helwege (2003) - defaults associated with spread increases, may come from (a) updated intensities or (b) increased default premia (Kusuoka 1999).
6. Frailty: learning from default by updating on unobservable covariates - CGH 2003, Giesecke 2002, Schönbucher 2004.
7. Duffie-Lando (2000), Yu (2004) - reduction in measured precision of accounting variables results in spread increases.
8. Clustering and copulas: Das and Geng (2004).
9. Macro influence on correlation: Lucas & Klaassens (2003).

Data

- Intensities based on Duffie, Saita and Wang (2005).
-
$$\lambda_i(t) = e^{\beta_0+\beta_1 X_{i1}(t)+\beta_2 X_{i2}(t)+\gamma_1 Y_1(t)+\gamma_2 Y_2(t)}, \tag{1}$$

  where

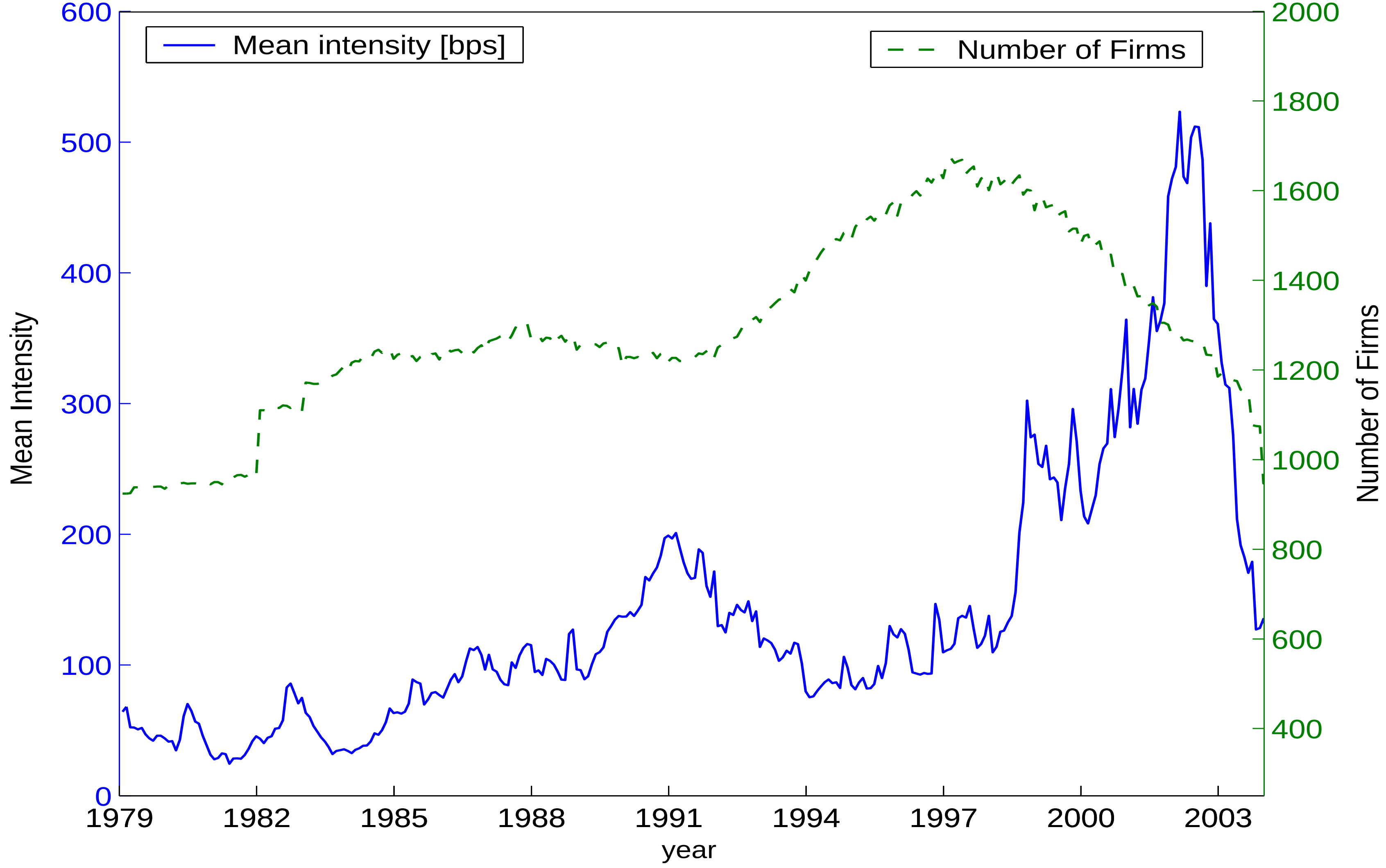
  $X_{i1}(t)$: distance to default of firm i.

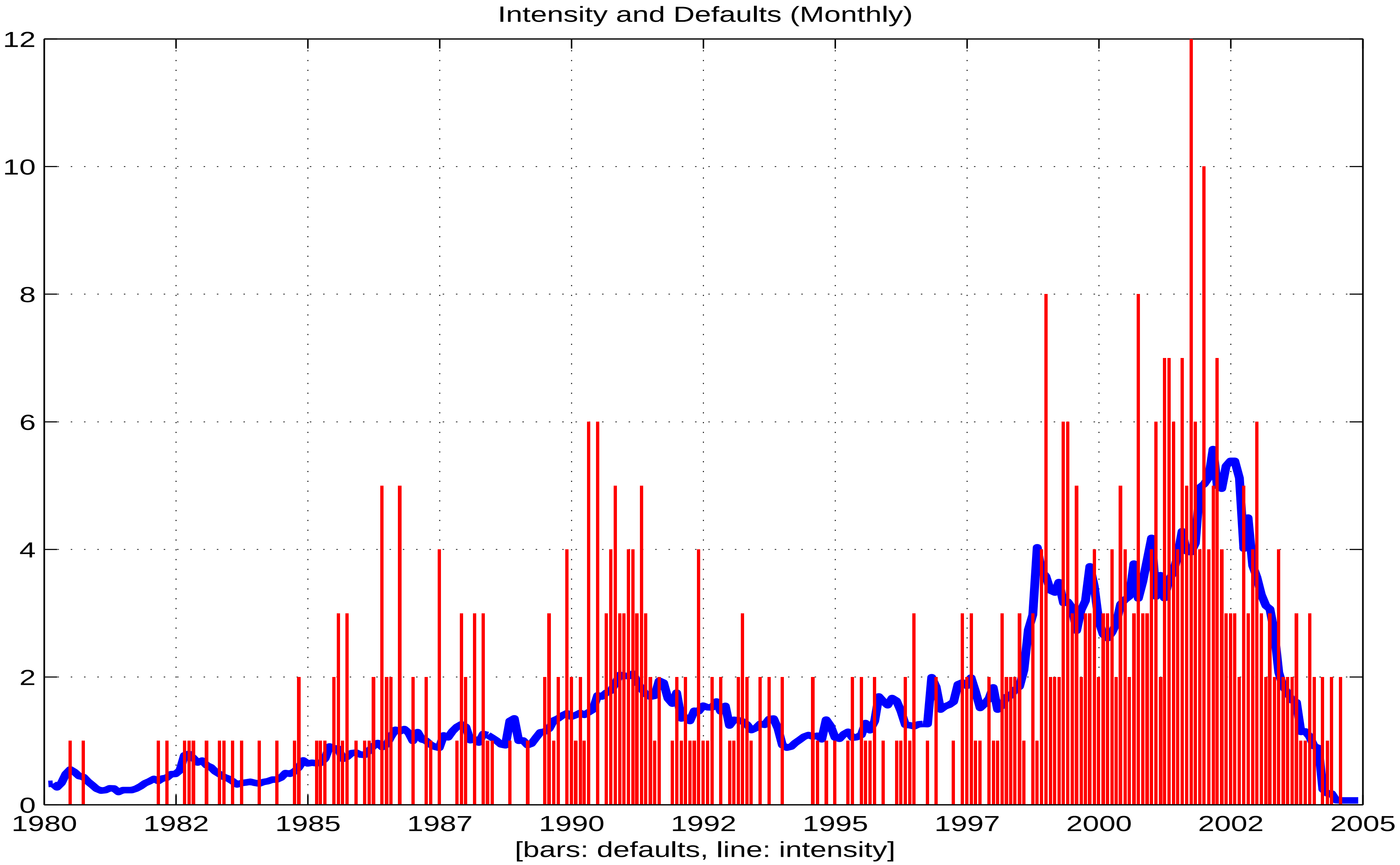
  $X_{i2}(t)$: trailing one-year stock return of firm i.

  $Y_1(t)$: U.S. 3-month Treasury bill rate.

  $Y_2(t)$: trailing one-year return of the S&P500.

- Parsimonious - accuracy ratios (CAR) of upto 88%.
- 1979-2004: 2770 firms, 392,404 firm months.
- Default data: Moodys, 495 defaults.


Mean intensity [bps]
Number of Firms
Mean Intensity
Number of Firms
year
0
100
200
300
400
500
600
400
600
800
1000
1200
1400
1600
1800
2000
1979
1982
1985
1988
1991
1994
1997
2000
2003


Intensity and Defaults (Monthly)
12
10
8
6
4
2
0
1980
1982
1985
1987
1990
1992
1995
1997
2000
2002
2005
[bars: defaults, line: intensity]

PROPOSITION:

Suppose that $(\tau_1, \ldots, \tau_n)$ is doubly stochastic with intensity $(\lambda_1, \ldots, \lambda_n)$. Let $K(t) = \#\{i : \tau_i \leq t\}$ be the cumulative number of defaults by t, and let $U(t) = \int_0^t \sum_{i=1}^n \lambda_i(u) 1_{\{\tau_i > u\}}\, du$ be the cumulative aggregate intensity of surviving firms, to time t. Then $J = \{J(s) = K(U^{-1}(s)) : s \geq 0\}$ is a Poisson process with rate parameter 1.

Poisson property:

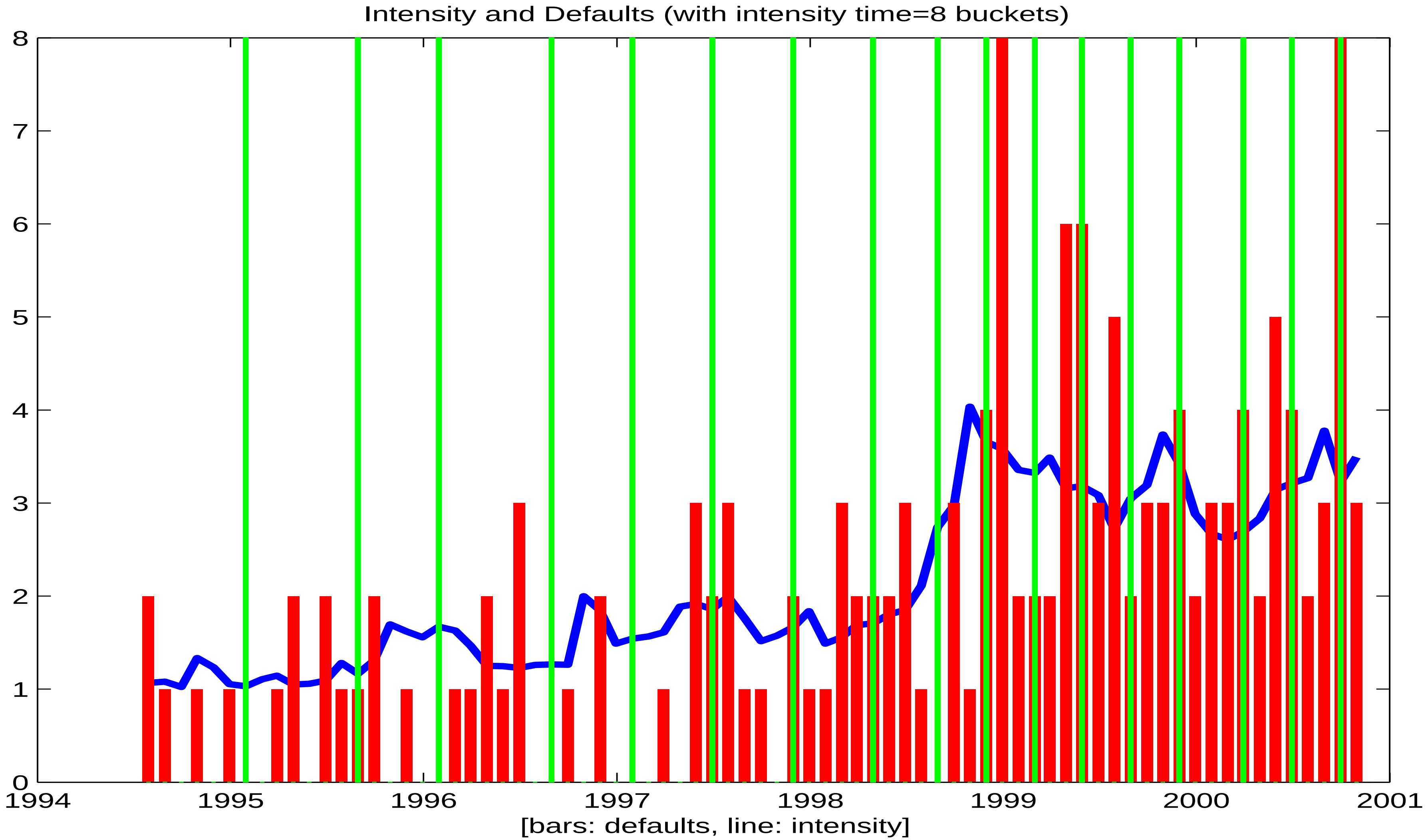
For any $c > 0$, the random variables

$$J(c), J(2c) - J(c), J(3c) - J(2c), \ldots$$

are *iid* Poisson with parameter c.

Test strategy:

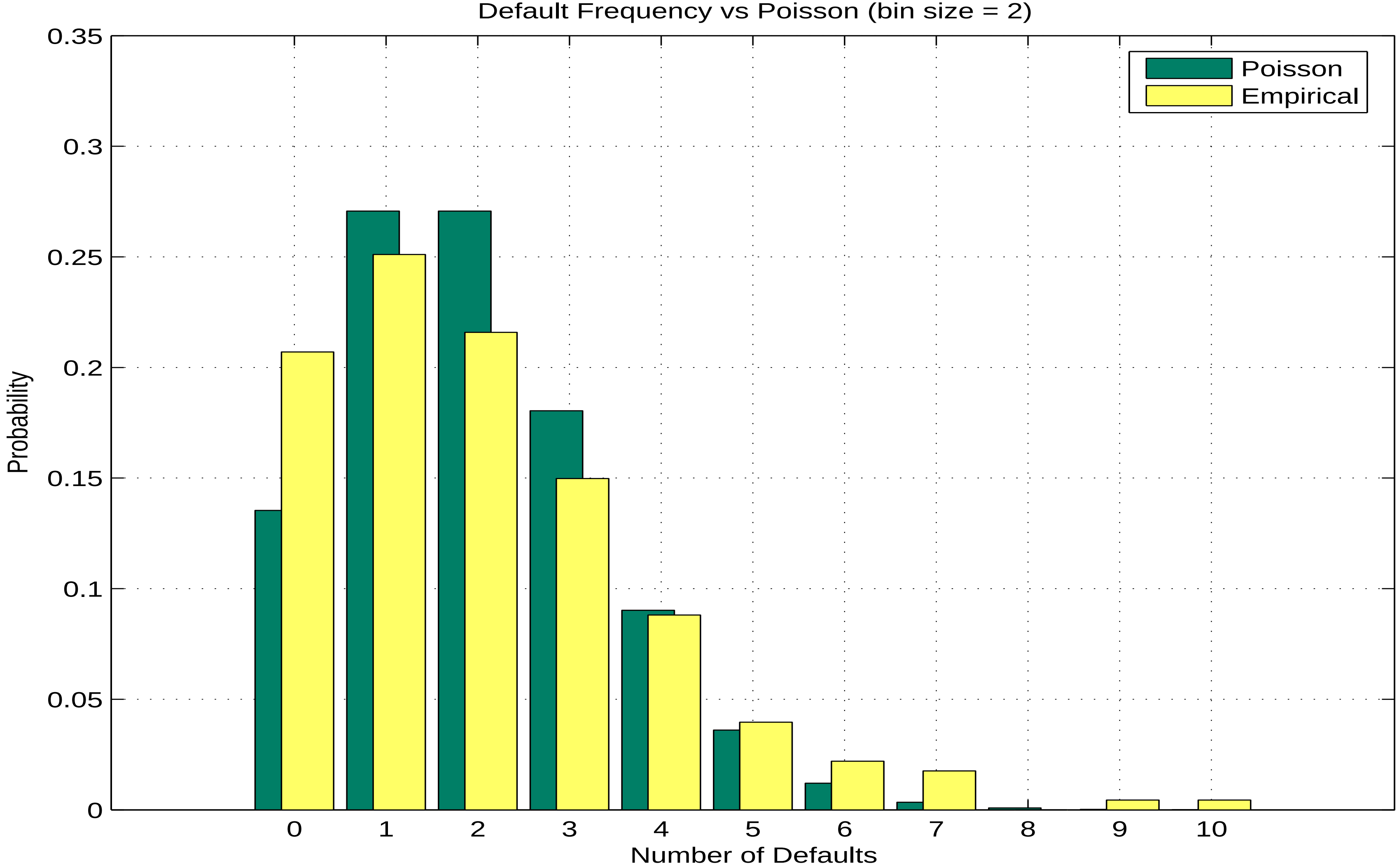
We divide our sample period into “bins” that each have an equal cumulative aggregate intensity of c, then test whether the numbers of defaults in successive bins are independent Poisson with common parameter c.

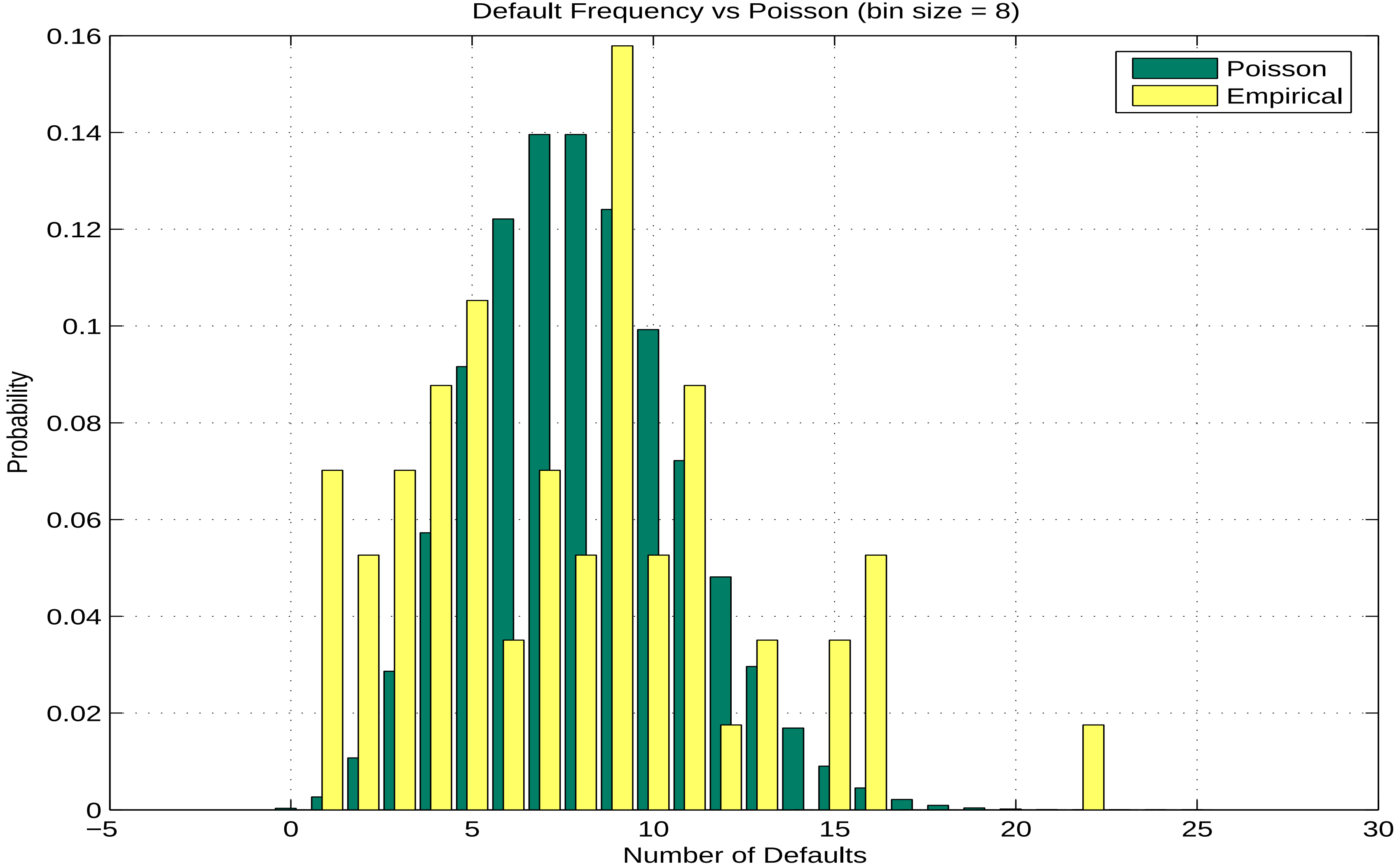

Intensity and Defaults (with intensity time=8 buckets)
0
1
2
3
4
5
6
7
8
1994
1995
1996
1997
1998
1999
2000
2001
[bars: defaults, line: intensity]

Moments

Bin Size	Mean	Variance	Skewness	Kurtosis
2	2.04	2.04	0.70	3.49
(230)	2.12	2.92	1.30	6.20
4	4.04	4.04	0.50	3.25
(116)	4.20	5.83	0.44	2.79
6	6.04	6.04	0.41	3.17
(77)	6.25	10.37	0.62	3.16
8	8.04	8.04	0.35	3.12
(58)	8.33	14.93	0.41	2.59
10	10.03	10.03	0.32	3.10
(46)	10.39	20.07	0.02	2.24

Means fit; empirical variances bigger than Poisson.


Default Frequency vs Poisson (bin size = 2)
Poisson
Empirical
Probability
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10
Number of Defaults


Default Frequency vs Poisson (bin size = 8)
Probability
Number of Defaults
Poisson
Empirical
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
−5
0
5
10
15
20
25
30

Fisher's Dispersion Test

Fixing the bin size c, under the null,

$$W = \sum_{i=1}^{K} \frac{(X_i - c)^2}{c}, \tag{2}$$

is χ^2 with $K - 1$ degrees of freedom.

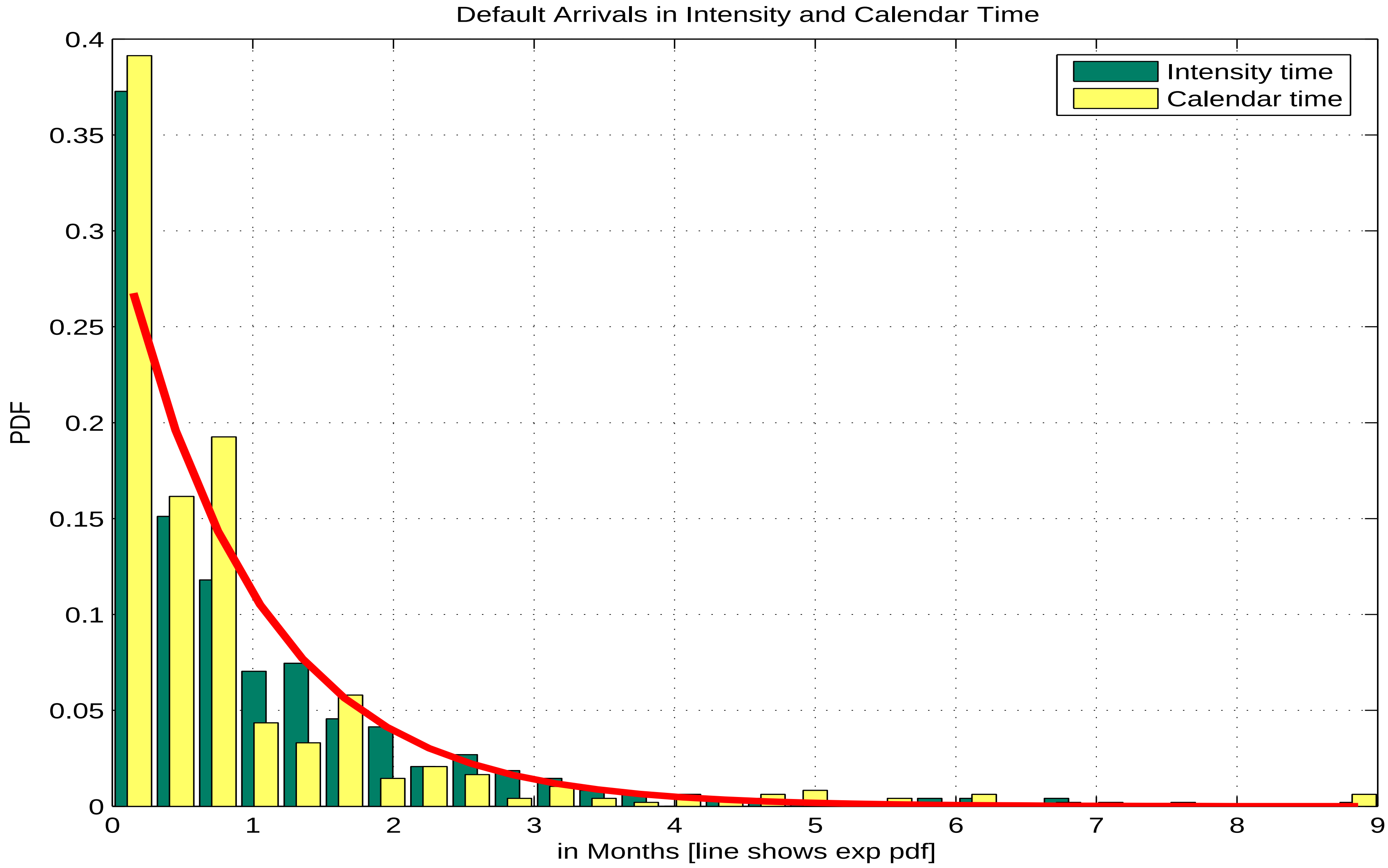
Bin Size	K	W	p-value
2	230	336.00	0.0000
4	116	168.75	0.0008
6	77	132.17	0.0001
8	58	107.12	0.0001
10	46	91.00	0.0001

Result: Rejection of the null hypothesis for all bin sizes.

Upper Tail Tests

Bin	Mean of Tails		p-value	Median of Tails		p-value
Size	Data	Simulation		Data	Simulation	
2	4.00	3.69	0.00	4.00	3.18	0.00
4	7.39	6.29	0.00	7.00	6.01	0.00
6	9.96	8.95	0.02	9.00	8.58	0.06
8	12.27	11.33	0.08	11.50	10.91	0.19
10	16.08	13.71	0.00	16.00	13.25	0.00
All			0.0018		0.0003	

"All" is the probability, under the hypothesis that time-changed default arrivals are Poisson with parameter 1, that there exists at least one bin size for which the mean (or median) of number of defaults per bin exceeds the corresponding empirical mean (or median).


Default Arrivals in Intensity and Calendar Time
Intensity time
Calendar time
PDF
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8
9
in Months [line shows exp pdf]

Interarrival Intensity

Moment	Intensity time	Calendar time	Exponential
Mean	0.95	0.95	0.95
Variance	1.17	4.15	0.89
Skewness	2.25	8.59	2.00
Kurtosis	10.06	101.90	6.00

The associated K-S statistic is 3.14 (intensity time), (p-value $= 0.000$). For calendar time, K-S $= 4.03$ (p-value $= 0.000$).

Prahls (1999) Test of Clustered Defaults (across bin sizes)

Prahl's test statistic is based on the fact that, in the new time scale under which default arrivals are those of a Poisson process (with rate parameter 1), the inter-arrival times $Z_1, Z_2, \ldots$ are iid exponential of mean 1.

Letting C^* denote the sample mean of $Z_1, \ldots, Z_n$, Prahl shows that

$$M = \frac{1}{n} \sum_{\{Z_k < C^*\}} \left(1 - \frac{Z_k}{C^*}\right). \qquad (3)$$

is asymptotically (in n) normal with mean $e^{-1} - \alpha/n$ and variance β^2/n,

$$\alpha \simeq 0.189, \quad \beta \simeq 0.2427.$$

Empirical results in Prahl's test

1. Values under the null:

$$\mu(M) = \frac{1}{e} - \frac{\alpha}{n} = 0.3675$$
$$\sigma(M) = \frac{\beta}{\sqrt{n}} = 0.0109.$$

2. In intensity time:

   Using our data, for $n = 495$ default times, $M = 0.4055$ Evidence of default clustering.

3. In calendar time: $M = 0.4356$. This is evidence of a violation.

Copulas

$$C(u_1, u_2, \ldots, u_n) = C[F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots, F_n^{-1}(u_n)]$$

Procedure

1. Fix correlation r and cumulative-intensity bin size c.
2. For issuer i, bin k, increase in cumulative intensity $C_i^{c,k}$.
3. For 5,000 independent scenarios, draw bin k, at random (equally likely), and draw joint-standard-normal $X_1, \ldots, X_n$ with corr$(X_i, X_m) = r$.
4. $U_i = F(X_i)$, F standard-normal cumulative distribution function, "default" for name i in bin k if $U_i > \exp(-C_i^{c,k})$.
5. Match mean of the upper quartile of the simulated distribution (across scenarios) of the number of defaults per bin - *implied* correlation.

Residual Copula Correlation

Bin Size	Mean of Upper quartile (data)	Mean of Simulated Upper Quartile Copula Correlation				
		$r = 0.00$	$r = 0.01$	$r = 0.02$	$r = 0.03$	$r = 0.04$
2	4.00	3.87	**4.01**	4.18	4.28	4.48
4	7.39	6.42	6.82	7.15	**7.35**	7.61
6	9.96	8.84	9.30	9.74	**10.13**	10.55
8	12.27	11.05	11.73	**12.29**	12.85	13.37
10	16.08	13.14	14.01	14.79	15.38	**16.05**

Akhavein, Kocagil, and Neugebauer (2005), estimate a Gaussian copula correlation parameter of approximately 19.7% within sectors, and 14.4% across sectors.

Test of Independence of Successive Defaults

Estimates of an auto-regressive model for a range of bin sizes

Bin deviations regression : $X_k = A + BX_{k-1} + \epsilon_k$

Bin Size	No. of Bins	A (t_A)	B (t_B)	R^2
2	230	2.091	0.019	0.0004
		0.506	0.286	
4	116	2.961	0.304	0.0947
		-2.430	3.438	
6	77	4.705	0.260	0.0713
		-1.689	2.384	
8	58	5.634	0.338	0.1195
		-2.090	2.733	
10	46	7.183	0.329	0.1161
		-1.810	2.376	

(t-statistics are shown below the estimates).

PD Mis-specification

Fix bin size c. Defaults in a bin in excess of the mean, $Y_k = X_k - c$, regression:

$$Y_k = \alpha + \beta_1 \; GDP_k + \beta_2 \; IP_k + \epsilon_k, \tag{4}$$

Bin Size	No. Bins	Intercept	GDP	IP	R^2 (%)
2	230	0.27 (0.17)	-4.57 (-0.83)	-35.70 (-1.68)	2.31
4	116	0.53 (1.41)	-5.08 (-0.50)	-103.27 (-2.51)	5.73
6	230	0.91 (1.58)	-18.09 (-1.18)	-124.09 (-1.42)	8.98
8	58	1.35 (1.77)	-0.08 (-0.00)	-357.20 (-3.47)	18.63
10	46	1.96 (1.80)	-41.45 (-1.38)	-205.15 (-2.08)	11.78

Upper-tail regressions.

Regression of the number of defaults observed in the upper quartile less the mean of the upper quartile of the theoretical distribution (with Poisson parameter equal to the bin size).

Bin Size	K	Intercept	Previous Qtr GDP	Previous Month IP	$R^2(\%)$
2	77	0.16	8.99	-76.80	6.94
		(1.04)	(1.04)	(-2.11)	
4	48	0.29	-22.15	-65.26	3.88
		(0.79)	(-0.02)	(-1.14)	

Bin Size	K	Intercept	Current Bin GDP	Current Bin IP	$R^2(\%)$
2	77	0.36	0.98	-50.28	2.84
		(1.23)	(0.10)	(-1.56)	
4	48	0.63	-7.85	-62.55	18.63
		(1.78)	(-0.74)	(-2.30)	

Additional test: Clustering remains with re-calibrated intensities including IP.

In Conclusion

1. A new test of the *doubly stochastic* model of default. Uses a *time-change technique* for the joint test of correctly specified intensities and the doubly stochastic assumption.

2. The doubly stochastic property is *violated*. Hence, care is required in simulations of credit risk.

3. Small amounts of *additional copula correlation* may be required to match implied default correlations.

4. Inclusion of a promising *additional macroeconomic covariate* (IP) does not resuscitate the DS model.