Optimal Asset Allocation and Risk Shifting in Money Management

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(1) Motivation and Objective

(2) Model

(3) Empirical Analysis

(4) Costs of Active Management to Investors

1. Motivation and Objective

- Mutual fund managers' compensation is linked to the value of assets under management
- Implicit incentives due to fund flows to performance relationship
- The flow-performance relationship is
  - positive
  - exhibits convexities
- Question: How does a fund manager respond to these incentives?

Fund Flow - Performance Relationship (Chevalier and Ellison (1997))

RISK TAKING BY MUTAL FUNDS 1177

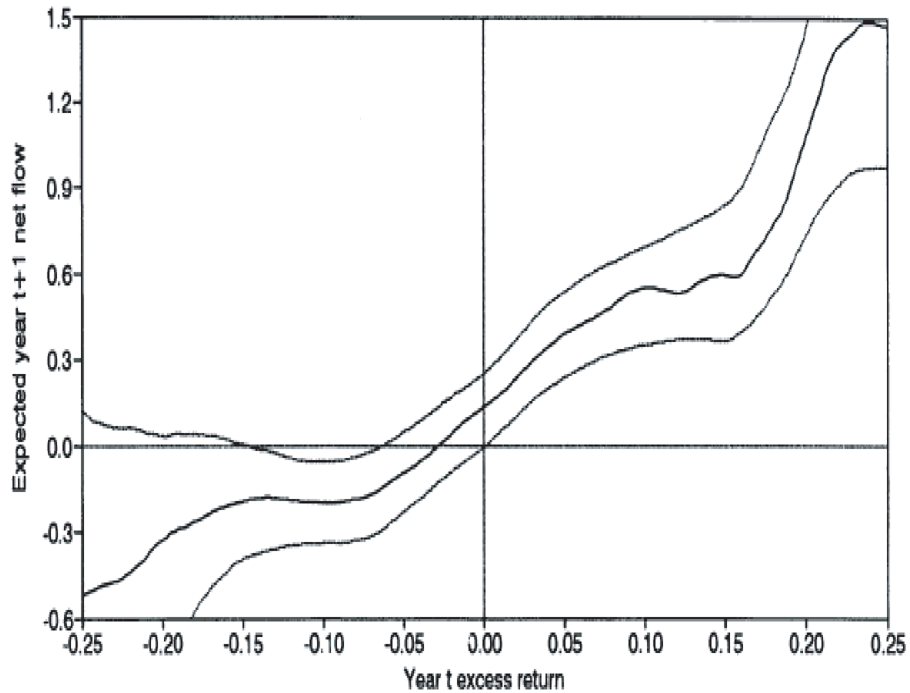
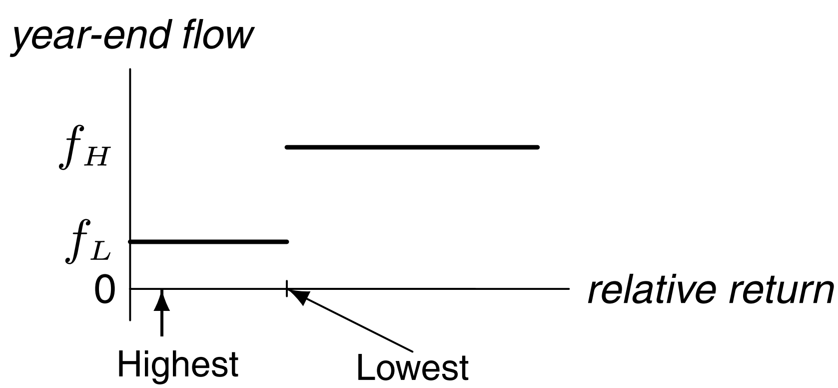


FIG. 1.—Flow-performance relationship $\hat{f}$ for young funds (age 2) with 90 percent confidence bands.

Summary of Main Testable Implications

- Taking risk $\neq$ increasing volatility of portfolio
- Gambling entails either an increase or a decrease in portfolio volatility
  - a sufficiently risk averse manager decreases volatility
  - manager manipulates systematic risk rather than idiosyncratic
  - gambling intensifies towards year-end
- Incentives to gamble are state-dependent. For example, the manager's risk-taking incentives are



Related Literature

- *Risk-taking of fund managers in response to fund flows:*

  Chevalier and Ellison (1997)

- *Managerial incentives and portfolio choice:*

  Brennan (1993), Carpenter (2000), Cuoco and Kaniel (2000), Berk and Green (2005), Hugonnier and Kaniel (2002), Gomez and Zapatero (2003), Ross (2003)

- *Empirical literature on risk-taking by mutual fund managers:*

  Brown, Harlow, and Starks (1996), Busse (2001), Reed and Wu (2005)

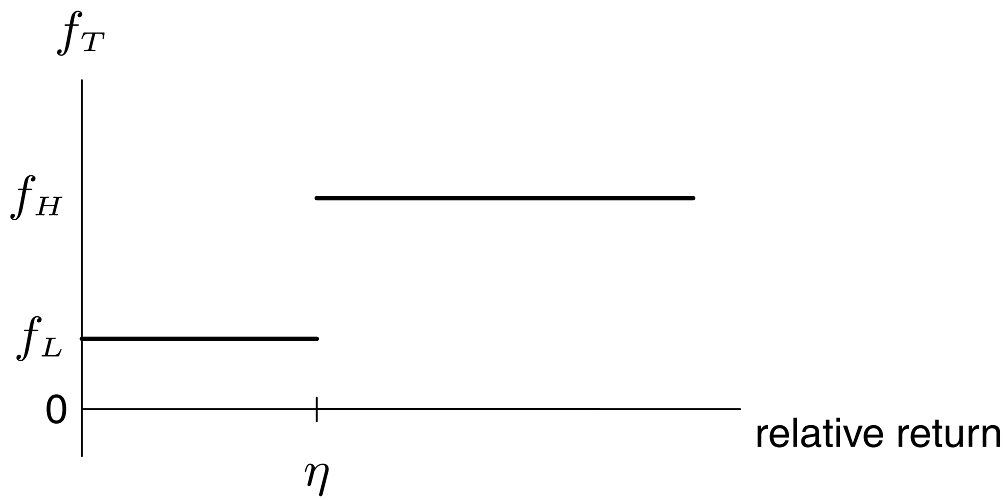
2. Model

- Finite horizon, $[0, T]$, Black-Scholes economy
- Assets:
  - Money market account with rate r
  - Stock follows $dS_t = \mu S_t dt + \sigma S_t dw_t$
- Fund manager:
  - evaluated relative to the index Y_t (fraction β in stock)
  - receives flows at T at rate f_T
  - chooses a trading strategy θ and terminal portfolio value W_T

$$\max_{\theta,\, W_T} E[u(W_T f_T)] = E\frac{(W_T f_T)^{1-\gamma}}{1-\gamma}$$

subject to $$dW_t = [r + \theta_t(\mu - r)]\, W_t dt + \theta_t \sigma W_t dw_t$$

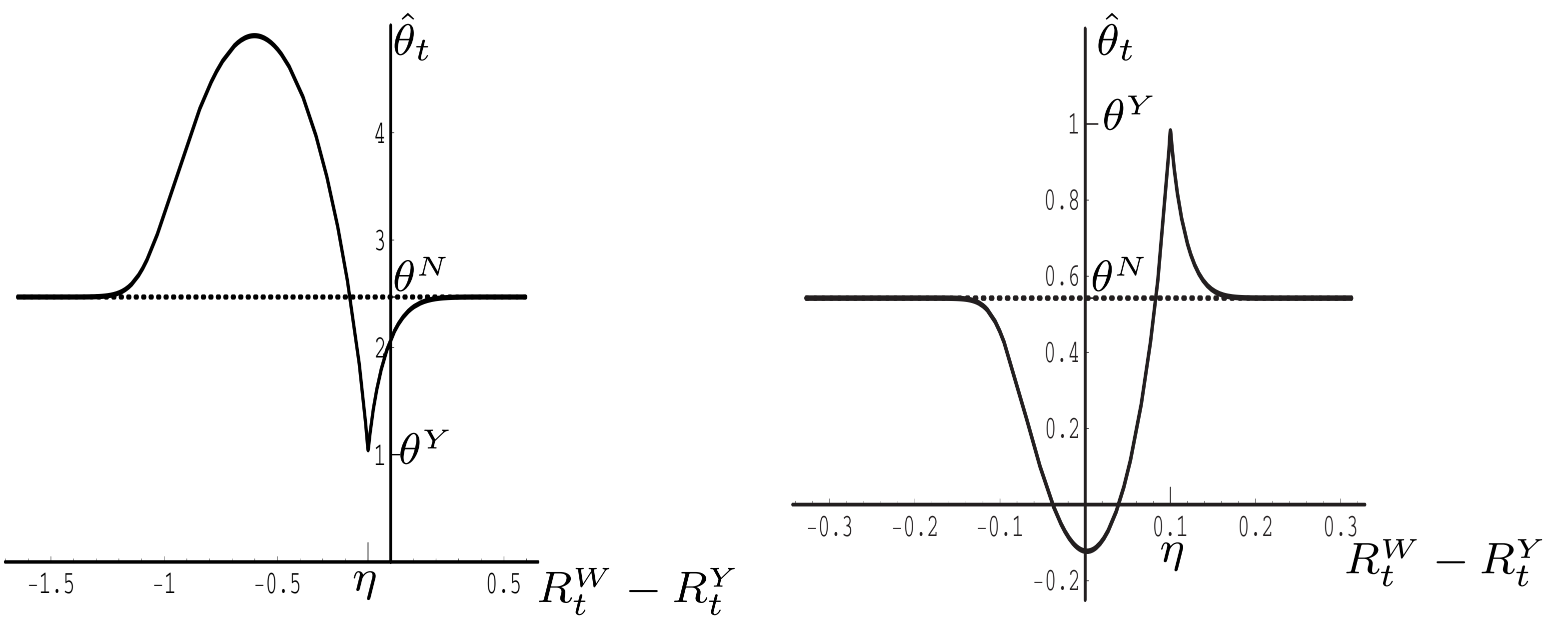
Flow-Performance Relationship (Simplest)



How does one measure risk-taking incentives?

- Conventional view:
  - sensitivity of the payoff's value to volatility (vega): $\frac{\partial V(\sigma_t^W;\, R_t^W - R_t^Y)}{\partial \sigma_t^W}$
- This paper:
  - optimal volatility $\hat{\sigma}_t^W = \hat{\theta}_t \sigma$. That is, $\frac{\partial V(\sigma_t^W;\, R_t^W - R_t^Y)}{\partial \sigma_t^W} = 0 \quad \Rightarrow \quad \hat{\sigma}_t^W$.

Manager's Optimal Risk Exposure

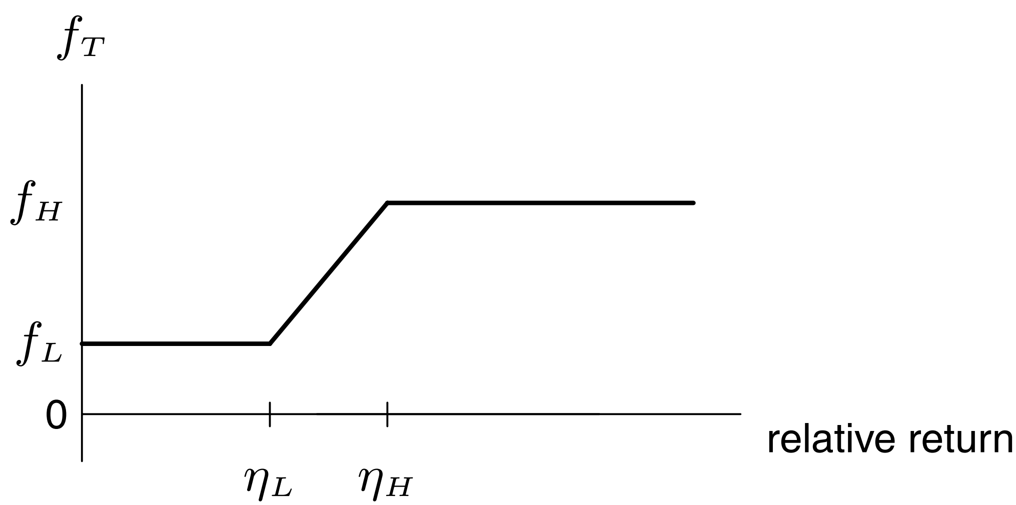


(a) Economies with $\theta^N > \theta^Y$

(b) Economies with $\theta^N < \theta^Y$

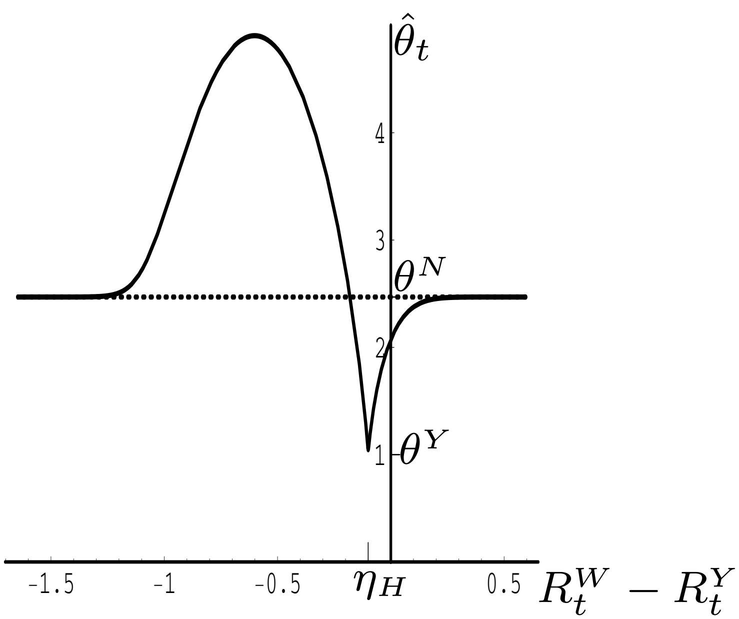
θ^N: risk exposure in Merton's problem, θ^Y: risk exposure of the index

An Alternative Flow-Performance Relationship (Collar-Type)

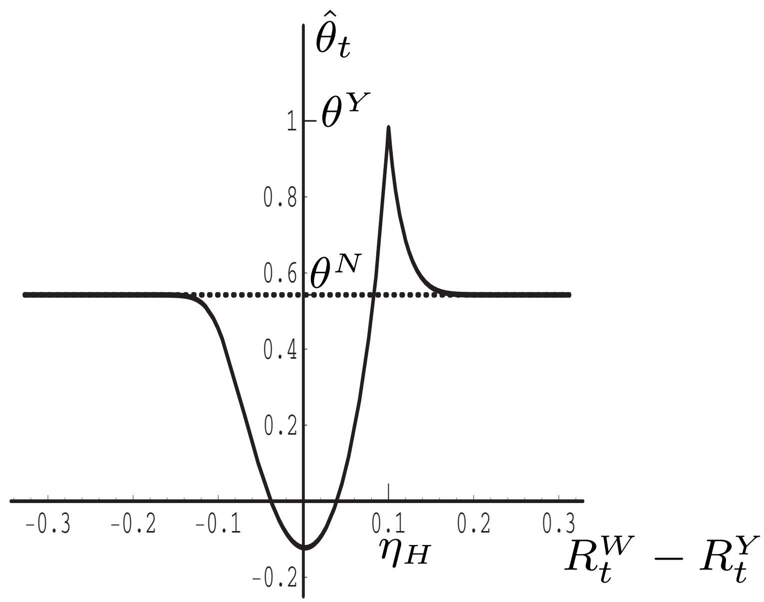


Can also be reinterpreted as an 80/120 annual bonus plan.

Manager's Optimal Risk Exposure (Collar-Type)



(a) Economies with $\theta^N > \theta^Y$

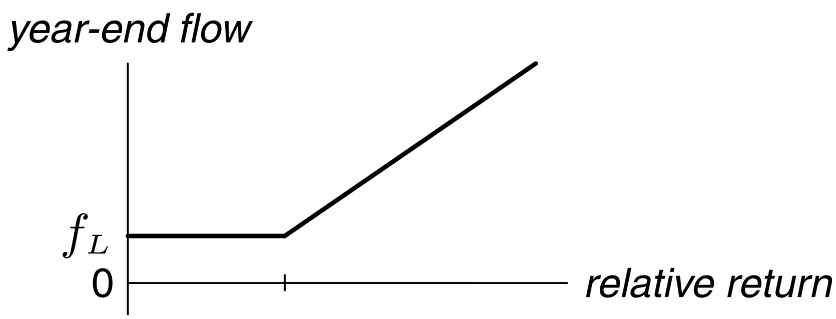


(b) Economies with $\theta^N < \theta^Y$

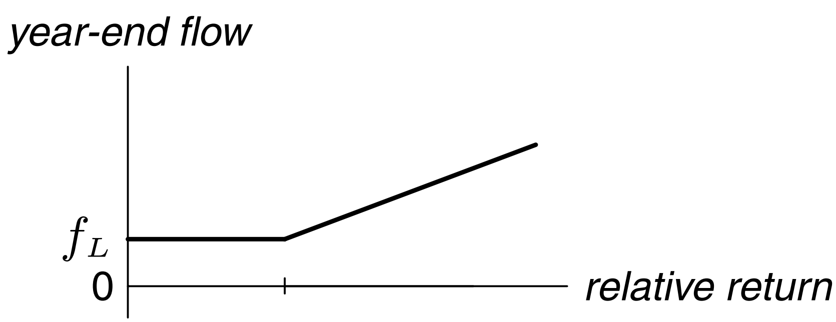
θ^N: risk exposure in Merton's problem, θ^Y: risk exposure of the index

Further Alternative Flow-Performance Specifications

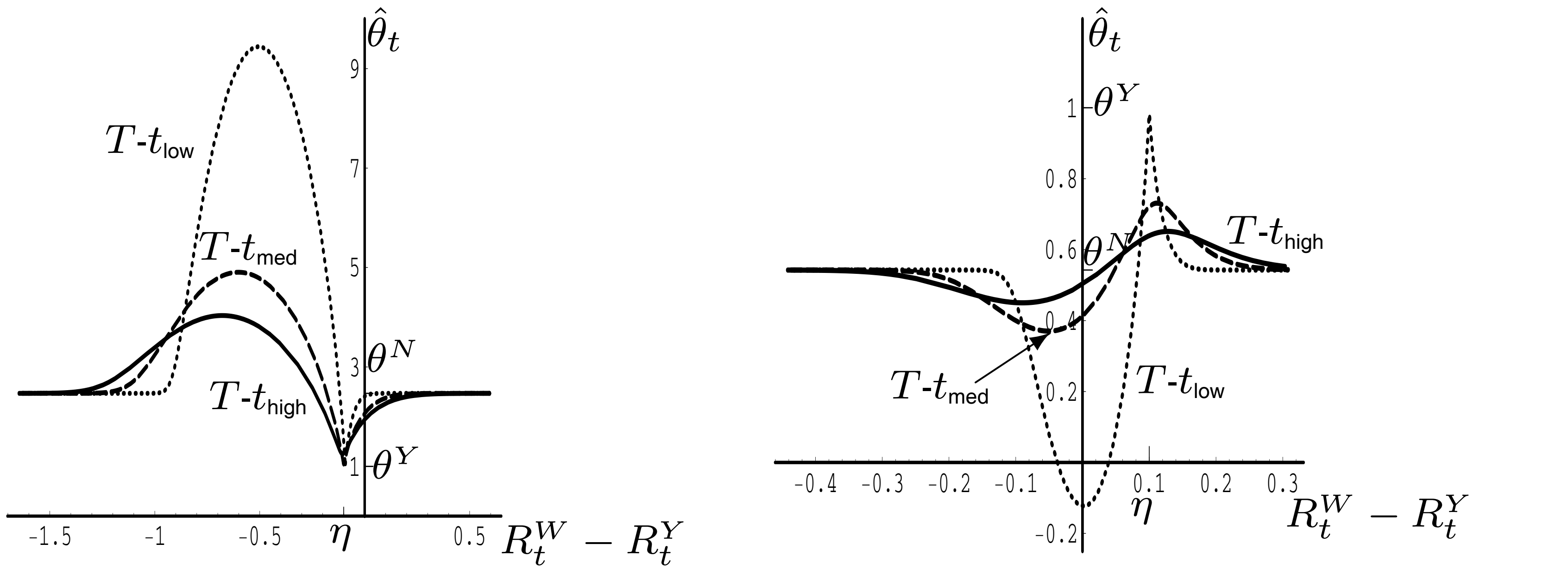
- Linear-convex (Sirri and Tufano (1998))



- Linear-linear (asymmetric fee structure)



Manager's Optimal Risk Exposure: Dynamics

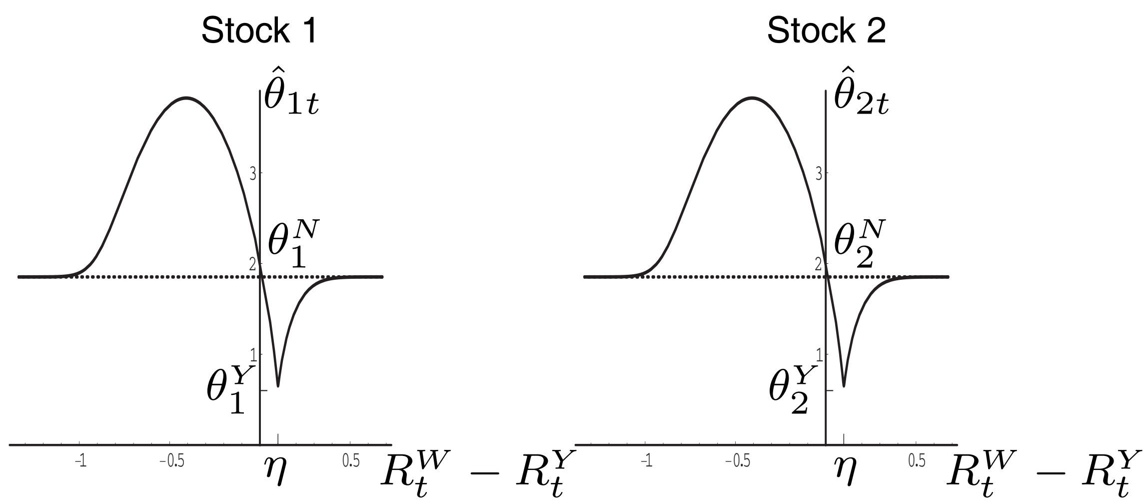


(a) Economies with $\theta^N > \theta^Y$

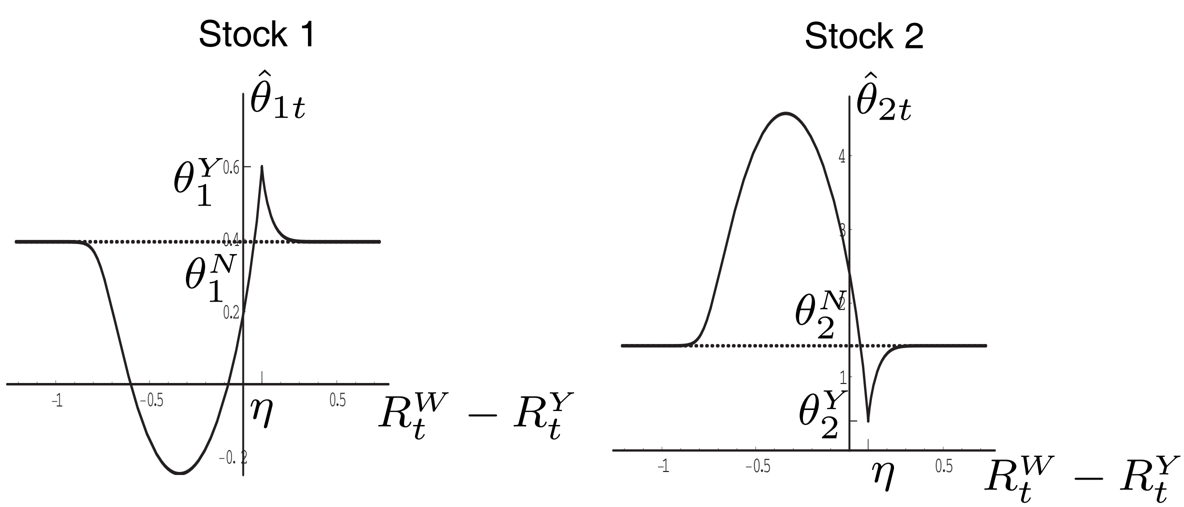
(b) Economies with $\theta^N < \theta^Y$

- Manager engages in risk shifting well before the year-end
- Risk shifting more pronounced as the year-end approaches

Multiple Stocks



(a) Economies with $\theta_1^N > \theta_1^Y$ and $\theta_2^N > \theta_2^Y$



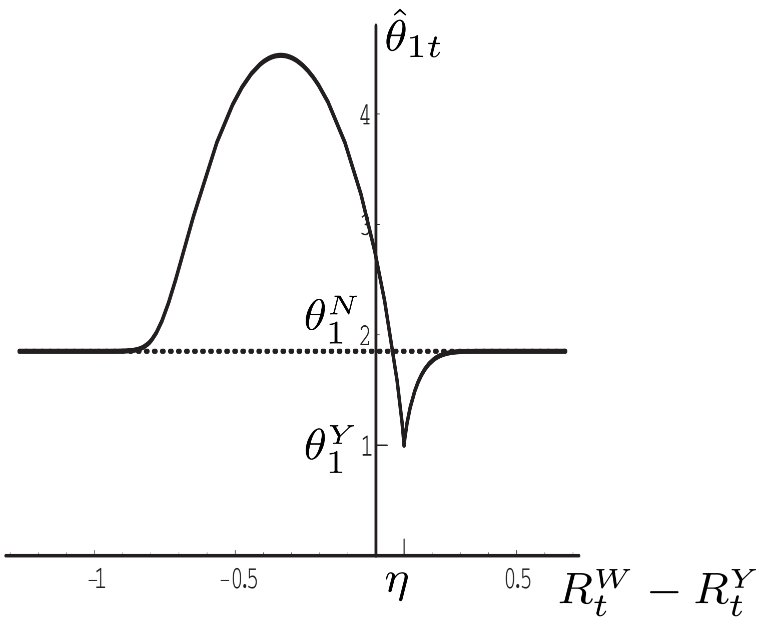
(b) Economies with $\theta_1^N < \theta_1^Y$ and $\theta_2^N > \theta_2^Y$

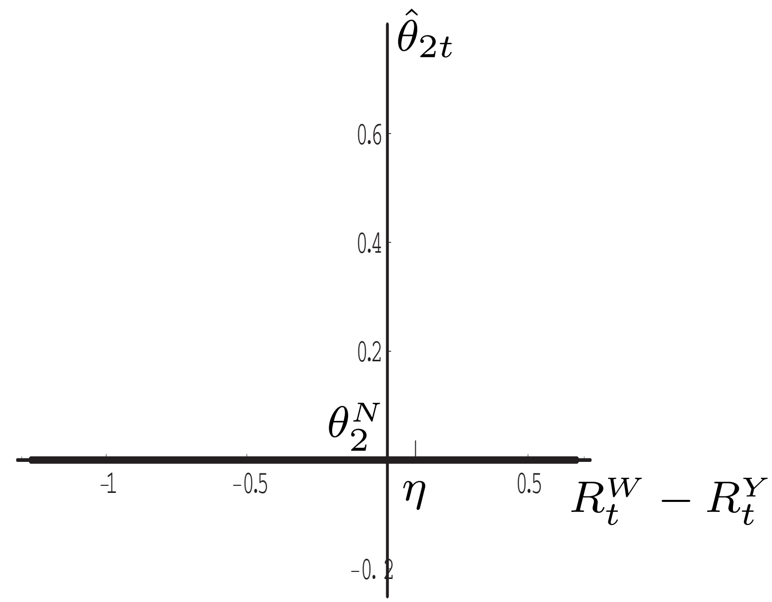
Idiosyncratic versus Systematic Risk

Economic setup:

$$\begin{aligned} dS_{1t} &= \mu_1 S_{1t} dt + \sigma_{11} S_{1t} dw_{1t} + \sigma_{12} S_{1t} dw_{2t} \\ dS_{2t} &= \mu_2 S_{2t} dt + \sigma_{21} S_{2t} dw_{1t} + \sigma_{22} S_{2t} dw_{2t} \end{aligned}$$

$$\mu = \begin{pmatrix} \mu_1 \\ r \end{pmatrix}, \quad \sigma = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}, \quad Y = S_1$$





3. Empirical Analysis

Existing Work

- *Brown, Harlow, and Starks (1996)*
  - find that underperforming managers increase volatility towards the year-end
- *Busse (2001)*
  - shows that the above test fails on daily data
- *Chevalier and Ellison (1997)*
  - look at $\sigma(R^W - R^Y)$ towards the year-end; find an increase
  - use monthly data
- *Reed and Wu (2005)*
  - test the results of this paper on daily data
  - distinguish between tournaments- vs. benchmarking-induced incentives

Data

- Daily mutual fund returns from Will Goetzmann and Geert Rouwenhorst (International Center for Finance at Yale SOM).
- Data range: 1970 through 1998 (comparable with Chevalier-Ellison, Sirri-Tufano).
- Merged with CRSP to find out mutual funds objective codes
  - left only actively managed US equity mutual funds in the aggressive growth, growth and income, and long-term growth categories.
- Used the S&P 500 index as the benchmark.

Tracking error and standard deviation tests

Hypothesis 1: *Tracking error variance is higher for underperforming managers.*

	LHS: $\sigma_m(R^W_{i,t} - R^Y_t)$		LHS: $\sigma_m(R^W_{i,t})$	
	Point Estimate	t-Statistic	Point Estimate	t-Statistic
$\text{OVER}_{i,m} \times 10^3$	-0.1819	-4.73	0.1058	2.38
Fund-year FE	Yes		Yes	
R^2	0.39		0.36	
N	40721		40721	

$\sigma_m(R^W_{i,t} - R^Y_t)$ – standard deviation of tracking error for month m; Y is S&P 500

$\sigma_m(R^W_{i,t})$ – standard deviation of fund returns for month m

$\text{OVER}_{i,m}$ – relative performance indicator prior to month m

Beta tests

Hypothesis 2: *Sufficiently risk-averse managers decrease their portfolio betas when underperforming the market.*

$$R^W_{i,\,t} - R^F_t = a + (b_{Fund-Year} 1_{FY} + b_{Month} 1_M + b_U \text{UNDER}_{i,w})(R^Y_t - R^F_t) + \varepsilon_{i,\,t}$$

	Dependent Variable: $R^W_{i,\,t} - R^F_t$			
	Beta$_T$ below 1		Beta$_{T-1}$ below 1	
	(1)	(2)	(3)	(4)
UNDER$_{i,w} \times (R^Y_t - R^F_t)$	-0.017	-0.020	-0.018	-0.021
	(-6.61)	(-7.72)	(-6.99)	(-8.22)
Month fixed effects	No	Yes	No	Yes
Fund-year fixed effects	Yes	Yes	Yes	Yes
R^2	0.37	0.37	0.37	0.37
Number of observations	808642		816783	

Robustness

- Tried several alternative definitions of the OVER/UNDER indicator
- Included lagged dependent variables to deal with autocorrelation
- Clustered errors by month/day and fund objective code

4. Costs of Active Management to Investors

- Define a measure of gain/loss, $\hat{\lambda}$, in units of investor's initial wealth:

$$V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$$

  - $V^I(\cdot)$ is investor's indirect utility under θ^I
  - $\hat{V}(\cdot)$ is investor's indirect utility under delegation

- Decompose $\hat{\lambda}$ into two components: $1+\hat{\lambda} = (1+\lambda^N)(1+\lambda^Y)$
  - λ^N: gain/loss due to explicit incentives, solves $V^I((1+\lambda^N)W_0) = \hat{V}(W_0;\ f_T\text{=}1)$
  - λ^Y: gain/loss due to implicit incentives

Costs of Active Management in Economies (a) ($\theta^N > \theta^Y$)

Fixed parameter values: $\gamma = 1, \gamma_I = 2, f_L = 0.8, f_H = 1.5, f_L + f_H = 2.3, \beta = 1,$
$\eta_L = -0.08, \eta_H = 0.08, \eta_L + \eta_H = 0, \mu = 0.06, r = 0.02, \sigma = 0.29, W_0 = 1, T = 1.$

Effects of		Cost-benefit measures λ^Y, λ^N $\hat{\lambda}$ (%)				
Managerial risk aversion	γ	0.5	1.0	2.0	3.0	4.0
		-8.13, -4.19	-5.12, -0.47	-3.31, 0.00	-2.56, -0.05	-2.15, -0.11
		-11.98	-5.61	-3.31	-2.61	-2.27
Implicit reward for outperformance	f_H-f_L	0.3	0.5	0.7	0.9	1.1
		-3.33, -0.47	-4.29, -0.47	-5.12, -0.47	-6.01, -0.47	-6.88, -0.47
		-3.79	-4.74	-5.61	-6.46	-7.32
Risk exposure of the benchmark	θ^Y	0.50	0.75	1.00	1.25	1.50
		-5.43, -0.47	-4.63, -0.47	-5.12, -0.47	-6.69, -0.47	-8.45, -0.47
		-5.88	-5.08	-5.61	-7.13	-8.88
Flow threshold differential	η_H-η_L	0.08	0.12	0.16	0.20	0.24
		-4.33, -0.47	-4.70, -0.47	-5.12, -0.47	-5.67, -0.47	-6.21, -0.47
		-4.78	-5.15	-5.61	-6.12	-6.65

Costs of Active Management in Economies (b) ($\theta^N < \theta^Y$)

Fixed parameter values: $\gamma = 1, \gamma_I = 2, f_L = 0.8, f_H = 1.5, f_L + f_H = 2.3, \beta = 1,$
$\eta_L = -0.08, \eta_H = 0.08, \eta_L + \eta_H = 0, \mu = 0.06, r = 0.02, \sigma = 0.29, W_0 = 1, T = 1.$

Effects of		Cost-benefit measures λ^Y, λ^N $\hat{\lambda}$ (%)				
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Risk exposure	θ^Y	<u>0.50</u>	<u>0.75</u>	<u>1.00</u>	<u>1.25</u>	<u>1.50</u>
of the benchmark		-5.43, -0.47	-4.63, -0.47	-5.12, -0.47	-6.69, -0.47	-8.45, -0.47
		-5.88	-5.08	-5.61	-7.13	-8.88
Flow threshold	η_H-η_L	<u>0.08</u>	<u>0.12</u>	<u>0.16</u>	<u>0.20</u>	<u>0.24</u>
differential		-4.33, -0.47	-4.70, -0.47	-5.12, -0.47	-5.67, -0.47	-6.21, -0.47
		-4.78	-5.15	-5.61	-6.12	-6.65

5. Conclusion

- Characterized the manager's optimal behavior in response to incentives induced by the fund flow-performance relationship.
- Identified circumstances in which the manager would like to gamble.
- Gambling may be associated with a decrease in the fund's volatility.
- Adverse incentives of the manager result in an economically significant cost to the investor.