Optimal Alpha Modeling

Q Group **Conference**
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PanAgora

Topics of Quantitative Equity Research

- **Statistical methodology – factor returns, IC, IR**
  - Fama, Eugene F and James D. MacBeth. 1973. "Risk, Return, and Equilibrium: Empirical Tests." Journal of Political Economy, 81, 607-636
  - Grinold, R.C. 1989. "The Fundamental Law of Active Management." Journal of Portfolio Management, vol. 15, no. 3 (Spring): 30-37
  - Grinold, Richard C. 1994, "Alpha is Volatility Times IC Times Score." Journal of Portfolio Management, vol. 20, no. 4, pp 9 – 16
  - Grinold, Richard C. And Ronald N. Kahn, 1999. Active Portfolio Management, McGraw-Hill, New York
  - Goodwin, Thomas H. 1998. "The Information Ratio." Financial Analysts Journal, vol. 54, no. 4 (July/August) 34-43

Topics of Quantitative Equity Research

➢Portfolio setting – long-short, constrained long-short

- Clarke, Roger, Harindra de Silva, and Steven Thorley, 2002. "Portfolio Constraints and the Fundamental Law of Active Management," Financial Analysts Journal, vol. 58, no. 5 (Sept/Oct) 48-66
- Clarke Roger, Harindra de Silva, and Steven Thorley. 2004. "Toward More Information Efficient Portfolios." Journal of Portfolio Management. vol. 31, no. 1 (Fall) 54-63
- Grinold, Richard C. and Ronald N. Kahn, 2000. "The Efficiency Gains of Long-short Investing." Financial Analysts Journal, vol. 56, no. 6 (November/December) 40-53
- Jacobs, Bruce I. And Kenneth N. Levy, 2006. "Enhanced Active Equity Strategies." Journal of Portfolio Management, vol. 32, no. 2 (Spring 2006) 45-55
- Sorensen, Eric, Ronald Hua and Edward Qian, "Aspects of Constrained Long/short Equity Portfolios." Journal of Portfolio Management, vol. 33, no. 2, (Winter 2007), 12-22

Topics of Quantitative Equity Research

➢ Portfolio turnover and portfolio dynamics

- Kahn, Ronald N., And J. S. Shaffer. 2005. “The Surprising Small Impact of Asset Growth on Expected Alpha.” Journal of Portfolio Management, vol. 32, no. 1 (Fall 2005) 49-60
- Sneddon, Leigh, “The Dynamics of Active Portfolios.” Northfield Research Conference Proceedings, 2005
- Grinold, Richard C. “A Dynamic Model of Portfolio Management.” Journal of Investment Management, vol. 4, no. 2
- Coppejans, Mark and Ananth Madhavan, “Active Management and Transactions Costs”, 2006, working paper, BGI
- Qian, Edward, Eric Sorensen and Ronald Hua, “Information Horizon, Portfolio Turnover, and Optimal Alpha Models”. Forthcoming, JPM

Optimal Alpha Modeling – An Outline

Optimal multi-factor models

- Single factor evaluation: risk-adjusted IC, strategy risk, turnover
- Multi-factor IR maximization: IC standard deviation, IC correlation (not factor correlation), orthogonalized "factors"
  - Qian, Edward and Ronald Hua, "Active Risk and Information Ratio", Journal of Investment Management, vol. 2., no. 3, (2004) 20-34
  - Sorensen, Eric, Ronald Hua, Edward Qian and Robert Schoen, "Multiple Alpha Sources and Active Management." Journal of Portfolio Management, vol. 30, no. 2 (Winter 2004) 39-45

Contextual models

- Moving away from one-size-fits-all: piecewise linear models
  - Sorensen, Eric, Ronald Hua and Edward Qian, "Contextual Fundamental, Models, and Active Management." Journal of Portfolio Management, vol. 32, no. 1 (Fall 2005) 23-36

Optimal Alpha Modeling - Continued

Optimal models with turnover constraints

- Turnover endogenous not exogenous
- Integrated modeling approach
    - Qian, Edward, Ronald Hua and John Tilney, "Portfolio Turnover of Quantitatively Managed Portfolios." 2004, Proceeding of the 2nd IASTED International Conference, Financial Engineering and Applications, Cambridge, MA
    - Qian, Edward, Eric Sorensen and Ronald Hua, "Information Horizon, Portfolio Turnover, and Optimal Alpha Models". Forthcoming, JPM
    - Qian, Edward, Ronald Hua and Eric Sorensen, Quantitative Equity Portfolio Management: Modern Techniques and Applications, Forthcoming, CRC press, 2007

The Analytical Framework of Measuring Investment Skill

Skill Measures

- **Goal: Hit rate → IC → IR → α**
- **Hit rate is a basic measure of skill**
  - Play well
  - Play often
  - Play a worthwhile game (dispersion)
- **IC is a statistical measure of skill**
  - Correlation of forecast residual return with ex post residual return
  - Based on well-accepted statistical methods
- **IR is the reward to risk in residual space**
  - Like Sharpe ratio in total risk space
  - Relates skill directly to Capital Market Theory, assuming specific IC properties and investor decision process

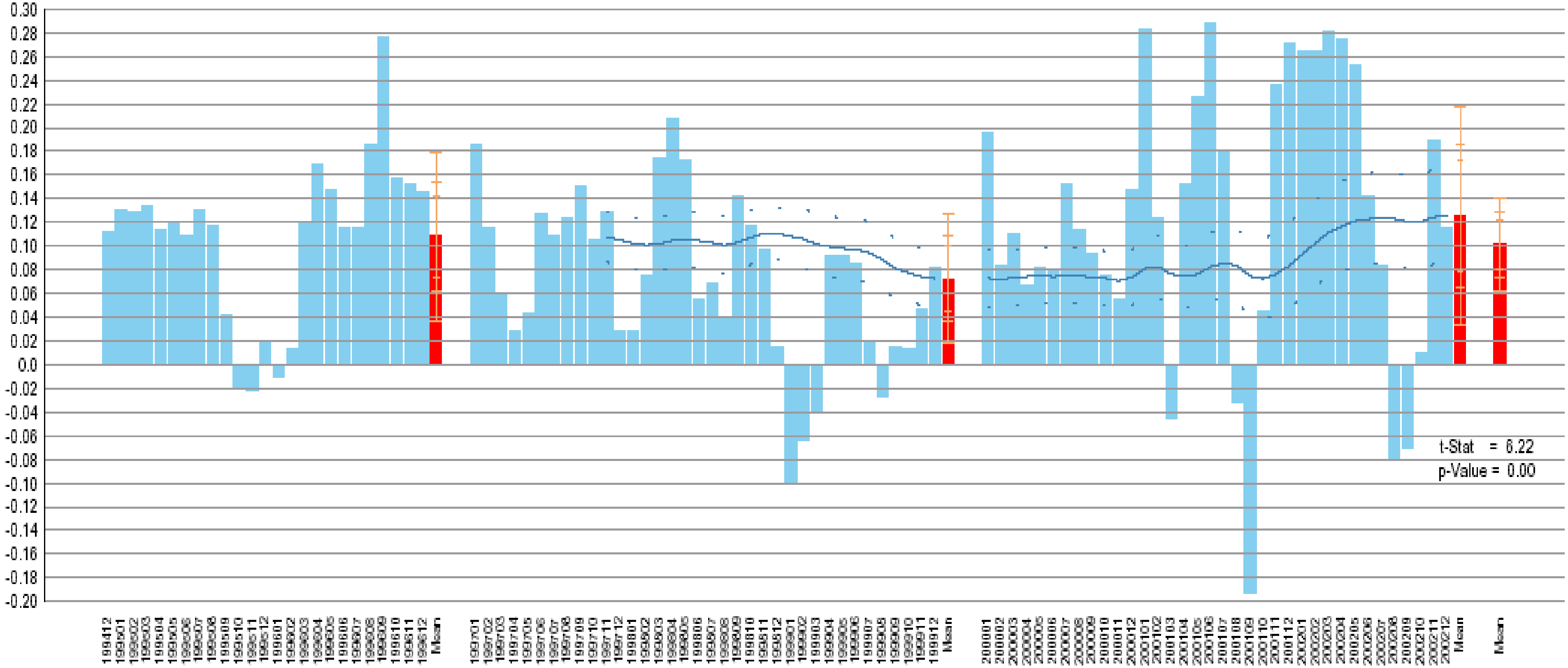
FLAM

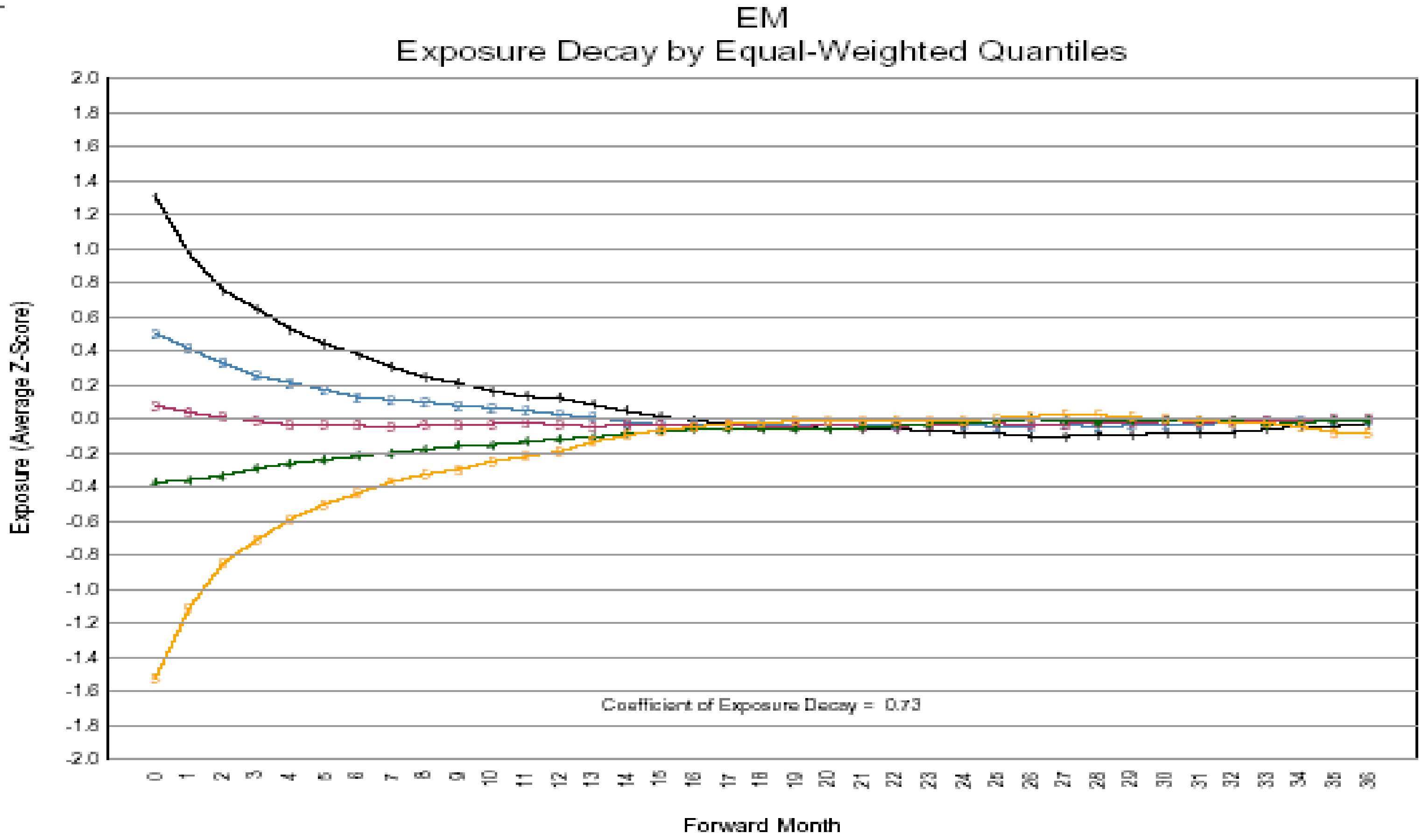
- **The fundamental law of active management (Grinold, 1989)**
  - IR ≈ skill applied to breadth
  - Gives insight, rather than operational
  - Requires several assumptions

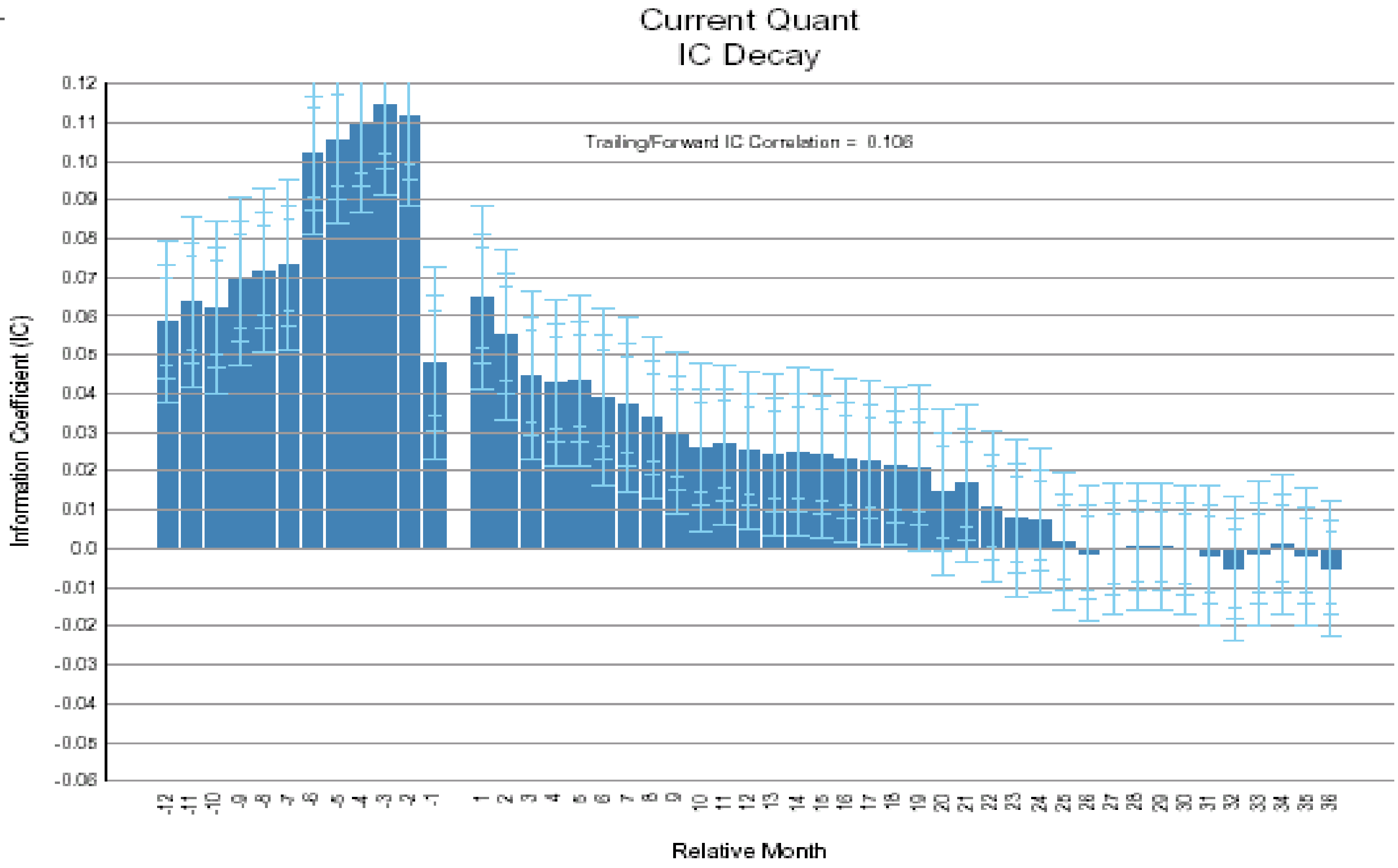
$$IR = IC\sqrt{N}$$

- **Investor behavior assumptions**
  - Manager knows the metric of skill
  - Manager applies (optimizes) skill, according to CAPM
- **Security behavior assumptions**
  - Same skill level applies to all asset choices
  - Sources of information are independent

Current Quant
Equal-Weighted ICs with 3-Year Moving Average




EM
Exposure Decay by Equal-Weighted Quantiles
Exposure (Average Z-Score)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
-1.8
-2.0
Coefficient of Exposure Decay = 0.73
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
Forward Month

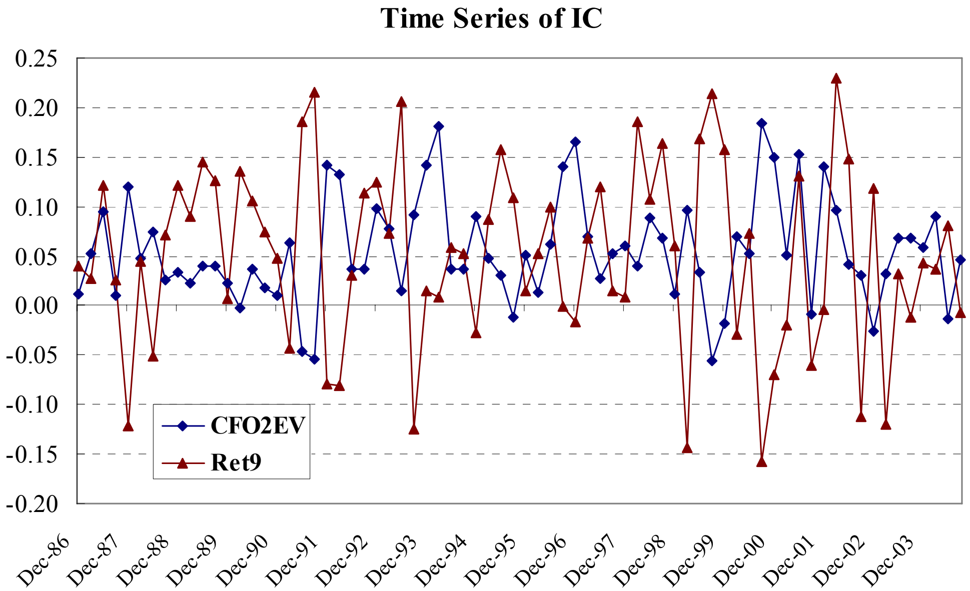


Current Quant
IC Decay
Trailing/Forward IC Correlation = 0.106
Information Coefficient (IC)
Relative Month

Optimal Multi-Factor Models: Maximizing IR

IR *Definition*

- **Fundamental law of active management** $\text{IR} = \text{IC}\sqrt{N}$
  - Do we then maximizing expected IC?
  - What about IC volatility?
  - What defines IC?



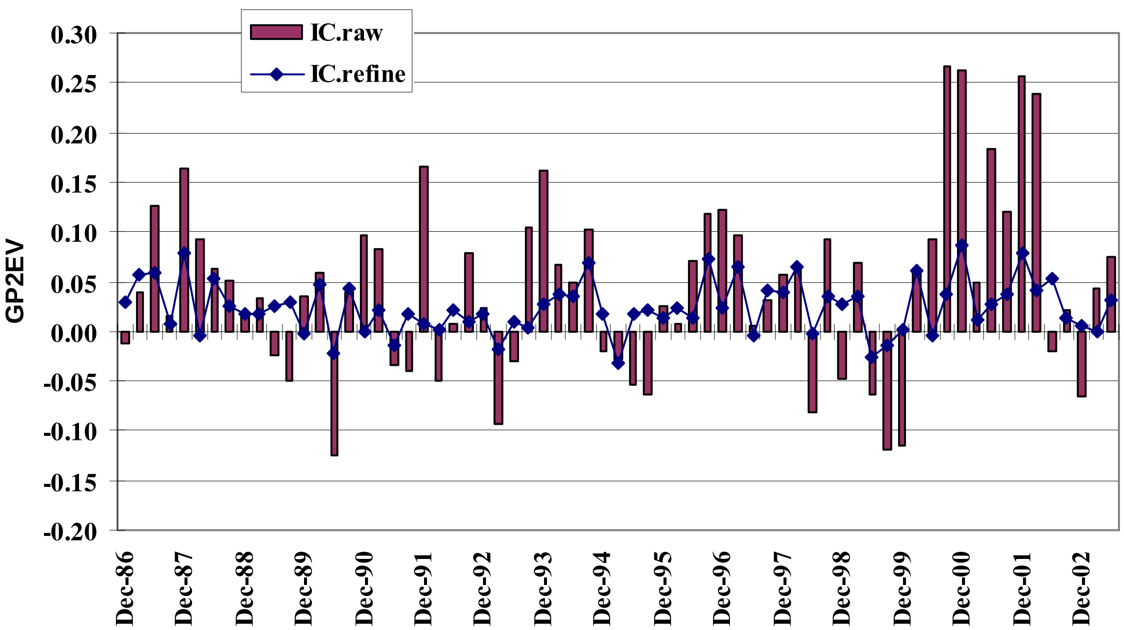
	CFO2EV	Ret9
Avg IC	0.055	0.051
Stdev IC	0.053	0.092
IR	1.04	0.55

Corr	-0.50

IC *Definitions*

- **Raw IC** $\mathrm{IC}_{\mathrm{raw}} = \mathrm{corr}\left(\mathbf{f}, \mathbf{r}\right)$
- **Risk-adjusted IC** $\mathrm{IC}_{\text{risk-adjusted}} = \mathrm{corr}\left(\mathbf{F}_t, \mathbf{R}_t\right)$
- **There could be a big difference**

Raw IC and Risk-adjusted IC



$$F_i = \frac{f_i - l_0 - l_1\beta_{1i} - \cdots - l_K\beta_{Ki}}{\sigma_i}$$

$$R_i = \frac{r_i - m_0 - m_1\beta_{1i} - \cdots - m_K\beta_{Ki}}{\sigma_i}$$

IR *Derivation*

Single Period Analysis

$$\alpha_t = \sum_{i=1}^{N} w_i r_i$$

$$w_i = \lambda^{-1} \frac{f_i - l_0 - l_1 \beta_{1i} - \cdots - l_K \beta_{Ki}}{\sigma_i^2}$$

$$\alpha_t = \sum_{i=1}^{N} w_i r_i = \lambda^{-1} \sum_{i=1}^{N} F_i R_i$$

$$\alpha_t = (N-1)\lambda_t^{-1} \mathrm{corr}(\mathbf{F}_t, \mathbf{R}_t) \mathrm{dis}(\mathbf{F}_t) \mathrm{dis}(\mathbf{R}_t)$$

$$\alpha_t = \mathrm{IC}_t \sqrt{N-1}\, \sigma_{\mathrm{model}} \mathrm{dis}(\mathbf{R}_t) \approx \mathrm{IC}_t \sqrt{N}\, \sigma_{\mathrm{model}}$$

Multi Period Analysis

$$\overline{\alpha_t} = \overline{\mathrm{IC}_t} \sqrt{N}\, \sigma_{\mathrm{model}}$$

$$\sigma = \mathrm{std}(\mathrm{IC}_t) \sqrt{N}\, \sigma_{\mathrm{model}}$$

$$IR \approx \frac{\overline{\mathrm{IC}_t}}{\mathrm{std}(\mathrm{IC}_t)}$$

IR *Results*

- **Information ratio is approximately average IC/standard deviation of IC**
- **True active risk consists of** $\sigma = \text{std}(\text{IC})\sqrt{N}\sigma_{\text{model}}$
  - Risk-model target tracking error
  - Strategy risk $\text{std}(\text{IC})$
  - The strategy risk is different for different factors
- **The fundamental law of active management is true only if**

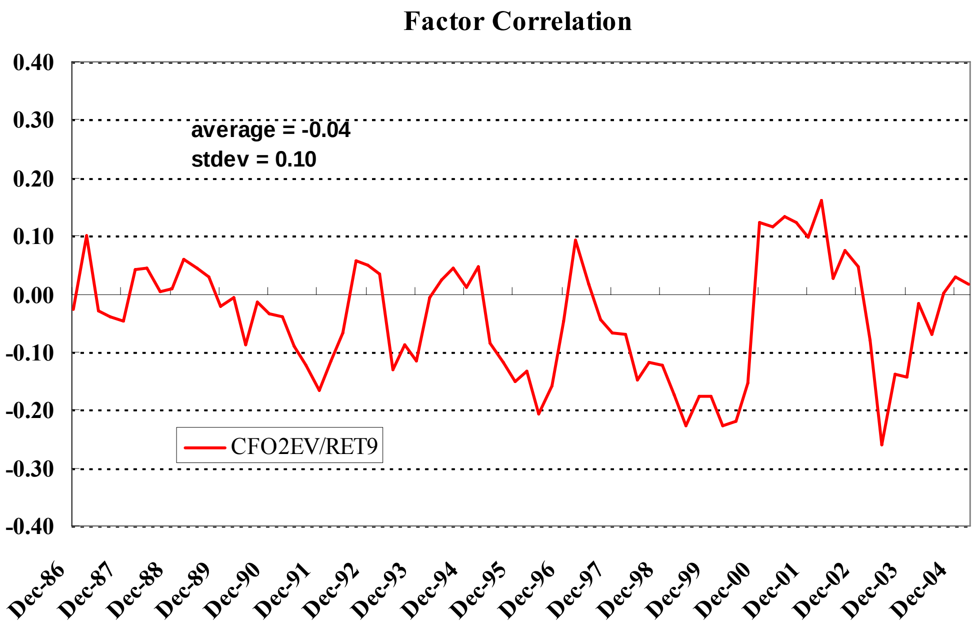
$$\text{std}(\text{IC}) = \frac{1}{\sqrt{N}}$$

  - It is only due to the sampling error, implying IC is time invariant
  - This is not likely to be true in reality

IR *Maximization of Multifactor Models*

- **A quantitative framework for combing multiple factors**
  - Similar to optimal allocation problem for multiple active managers
- **Individual factor (one manager)**
  - Average IC (expected alpha), standard deviation of IC (active risk)
- **Multi factors (managers)**
  - IC correlation: time series correlations between different IC's is key
  - Analogous to correlations between excess returns of different managers
  - The correlations between different factors are much less important
    - Factor correlation is not the same as IC correlation

IC Correlation – Indication of Diversification



IC Correlation

	CFO2EV	Ret9
Avg IC	0.055	0.051
Stdev IC	0.053	0.092
IR	1.04	0.55

Corr	-0.50

IC Correlation

- **In many cases, IC correlations are significantly different from average factor correlations**
- **IC correlations are crucial to maximize multi-period IR**
- **Factor correlations are useful for single-period composite scores**

IR *Maximization of Multifactor Models*

Problem

Maximize

$$\mathrm{IR} = \frac{\mathrm{avg}(\mathrm{IC}_t)}{\mathrm{std}(\mathrm{IC}_t)}$$

Solution

$$\overline{\mathbf{IC}} = \left(\overline{\mathrm{IC}}_1, \overline{\mathrm{IC}}_2, \cdots, \overline{\mathrm{IC}}_M\right)' \quad \mathbf{\Sigma}_{IC} = \left(\rho_{ij,\mathrm{IC}}\right)_{i,j=1}^{M}$$

$$\mathbf{w}^* \propto \mathbf{\Sigma}_{IC}^{-1}\overline{\mathbf{IC}}$$

Correlated Factors

- **Correlated factors in general leads to correlated ICs**
  - High IC correlation can lead to unstable factor weights
- **Correlated factors also present a problem in return attribution**
- **Fama-MacBeth formulation leads to information loss**

Coefficient should be interpreted as the residual influence netting out other factors.

$$\mathbf{r} = \boldsymbol{\alpha} + \beta_1 \mathbf{f}_1 + \beta_2 \mathbf{f}_2 + \boldsymbol{\varepsilon},$$

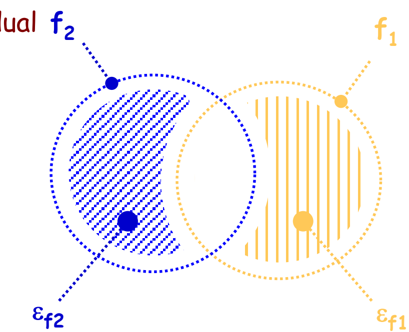
$$\mathbf{r} = \boldsymbol{\alpha} + \beta_1 \boldsymbol{\varepsilon}_{\mathbf{f}_1} + \boldsymbol{\varepsilon}_1,$$

$$\mathbf{r} = \boldsymbol{\alpha} + \beta_2 \boldsymbol{\varepsilon}_{\mathbf{f}_2} + \boldsymbol{\varepsilon}_2,$$

where

$\boldsymbol{\varepsilon}_{\mathbf{f}_1}$: is the residual portion of $\mathbf{f}_1$ uncorrelated with $\mathbf{f}_2$, i.e. $\mathbf{f}_2 = \rho \mathbf{f}_1 + \boldsymbol{\varepsilon}_{\mathbf{f}_2}$, and

$\boldsymbol{\varepsilon}_{\mathbf{f}_2}$: is the residual portion of $\mathbf{f}_2$ uncorrelated with $\mathbf{f}_1$, i.e. $\mathbf{f}_1 = \rho \mathbf{f}_2 + \boldsymbol{\varepsilon}_{\mathbf{f}_2}$.



Sequential Orthogonalization

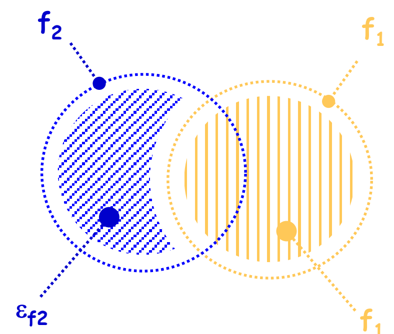
- **Utilize full information set for the alpha model**
- **Make factor correlations stable – always zero**

Identical estimations in one single regression.

$$\mathbf{r} = \boldsymbol{\alpha} + \beta_1^* \mathbf{f}_1 + \beta_2^* \boldsymbol{\varepsilon}_{\mathbf{f}_2} + \boldsymbol{\varepsilon},$$

$$\mathbf{r} = \boldsymbol{\alpha} + \beta_1^* \mathbf{f}_1 + \boldsymbol{\varepsilon}_1,$$

$$\mathbf{r} = \boldsymbol{\alpha} + \beta_2^* \boldsymbol{\varepsilon}_{\mathbf{f}_2} + \boldsymbol{\varepsilon}_2,$$



where

$\mathbf{r}$: is the vector of cross-sectional security returns,

$\mathbf{f}_1, \mathbf{f}_2$: are vectors of cross-sectional factor scores,

$\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2$: are vectors of regression residuals, and

$\boldsymbol{\varepsilon}_{\mathbf{f}_2}$: is the residual portion of $\mathbf{f}_2$ uncorrelated with $\mathbf{f}_1$, i.e. $\mathbf{f}_2 = \rho \mathbf{f}_1 + \boldsymbol{\varepsilon}_{\mathbf{f}_2}$.

Contextual Models – A Unique Model for Every Stock

Contextual Models

➢ Theoretical advances

- Conditional asset pricing

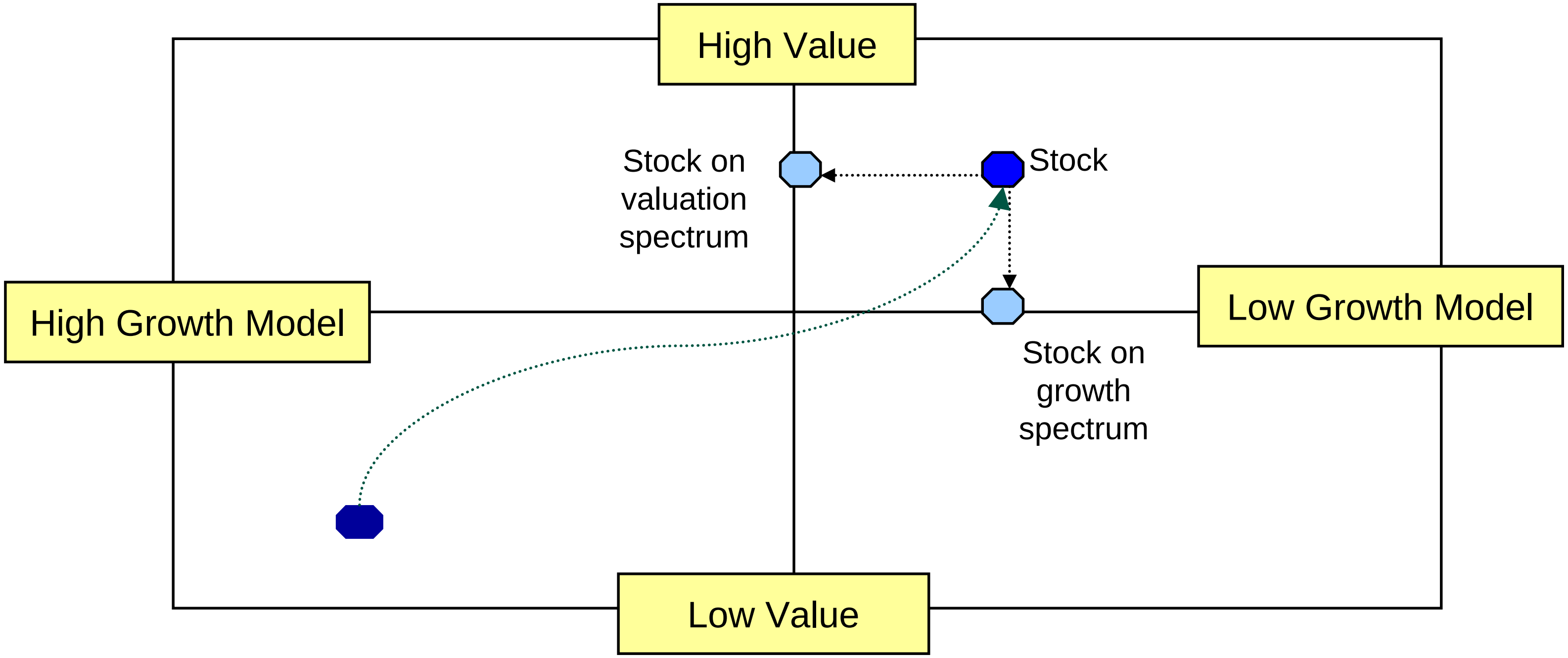
➢ Practical approaches

- Style investing
- Sector models

➢ Contextual modeling

- A piecewise linear model
- Partitioning the security universe according to risk / attributes
- It follows business cycle of individual stocks

Contextual Model – A Two Dimensional Example



Contextual Alpha Modeling – Factor Weights

			Value					Fundamental					Momentum	
universe	**high/low**	**riskfactor**	**d2p**	**b2p**	**e2p**	**c2p**	**holt**	**oe**	**fs**	**eq**	**capx**	**noag**	**pm**	**em**
large	high	growth	4.4%	-0.2%	-0.3%	6.2%	3.0%	20.5%	14.7%	17.1%	4.1%	8.7%	7.5%	13.2%
large	low	growth	7.1%	10.9%	5.0%	13.8%	11.3%	4.2%	-1.6%	16.1%	7.2%	9.8%	1.7%	11.2%
large	high	value	13.0%	2.7%	1.1%	10.1%	7.0%	5.2%	6.8%	12.5%	8.7%	20.6%	3.4%	8.9%
large	low	value	2.4%	2.2%	1.3%	9.2%	6.2%	24.4%	9.0%	15.1%	2.2%	6.7%	8.7%	12.6%
large	high	earnyld	22.5%	0.2%	4.0%	17.2%	8.5%	4.5%	1.8%	11.0%	6.3%	15.2%	3.9%	4.8%
large	low	earnyld	1.5%	2.1%	-3.3%	9.9%	11.6%	16.0%	3.7%	17.6%	6.1%	5.7%	9.1%	13.2%
large	high	earnvar	12.4%	1.0%	-0.2%	7.3%	2.5%	14.3%	12.6%	13.9%	10.4%	18.7%	2.9%	3.8%
large	low	earnvar	0.6%	0.3%	1.4%	10.2%	12.5%	27.5%	5.8%	16.3%	2.8%	5.4%	5.5%	11.5%
large	high	predBeta	5.0%	-1.4%	0.5%	6.5%	3.3%	25.2%	11.1%	20.8%	4.2%	8.4%	5.0%	8.6%
large	low	predBeta	3.7%	3.1%	2.1%	7.7%	7.8%	14.6%	1.7%	27.1%	3.0%	10.7%	4.0%	14.4%
small	high	growth	9.1%	1.5%	2.0%	8.9%	2.9%	14.1%	9.7%	13.6%	7.0%	18.1%	6.5%	6.7%
small	low	growth	5.1%	4.0%	3.5%	12.5%	4.3%	9.6%	7.2%	13.7%	8.6%	10.9%	9.7%	10.9%
small	high	value	8.2%	3.0%	1.1%	8.9%	1.3%	8.4%	12.0%	14.9%	8.9%	15.3%	7.8%	10.2%
small	low	value	3.4%	1.5%	1.9%	8.3%	2.0%	15.9%	9.4%	21.7%	3.4%	15.0%	8.7%	8.8%
small	high	earnyld	4.8%	3.1%	6.2%	17.7%	1.5%	11.5%	8.6%	18.2%	7.8%	7.0%	6.7%	6.8%
small	low	earnyld	7.1%	2.7%	4.4%	10.6%	3.6%	13.7%	5.9%	13.2%	3.7%	16.9%	8.4%	9.9%
small	high	earnvar	7.8%	1.9%	1.3%	6.4%	0.3%	8.9%	7.8%	24.9%	9.1%	15.8%	7.3%	8.5%
small	low	earnvar	1.2%	1.5%	2.1%	13.6%	3.0%	15.6%	12.8%	17.7%	9.0%	14.1%	4.6%	4.7%
small	high	predBeta	7.0%	1.7%	1.0%	8.0%	1.4%	19.7%	16.2%	16.2%	5.3%	16.9%	4.0%	2.6%
small	low	predBeta	2.1%	2.6%	3.4%	11.3%	5.8%	10.6%	13.7%	13.8%	8.6%	6.3%	8.1%	13.8%

20 models

Factor Weights

Source: PanAgora Asset Management   Table is shown for illustrative purposes only.

Company's Contextual Dimensions

B: Model Weights

Category			IBM	GM	TYC	VSAT
Large	Growth	High	1%	-	**94%**	-
		Low	4%	1%	0%	-
	Value	High	1%	**48%**	-	-
		Low	10%	-	-	-
	Earnings Yield	High	2%	**50%**	0%	-
		Low	-	-	-	-
	Earnings Variability	High	-	1%	-	-
		Low	**83%**	-	-	-
	Beta	High	-	-	5%	-
		Low	-	-	0%	-
Small	Growth	High	-	-	-	**95%**
		Low	-	-	-	-
	Value	High	-	-	-	-
		Low	-	-	-	-
	Earnings Yield	High	-	-	-	-
		Low	-	-	-	-
	Earnings Variability	High	-	-	-	-
		Low	-	-	-	1%
	Beta	High	-	-	-	2%
		Low	-	-	-	-

IBM : large, stable earnings
GM : large, cheap
TYC : large, high growth
VSAT : small, high growth

Source: PanAgora Asset Management
Table is shown for illustrative purposes only.

Unique Factor Weights for Each Stock

Factor Values

Stock	d2p	b2p	e2p	c2p	holt	oe	fs	eq	capx	noag	pm	em
IBM	1.62	0.00	1.45	1.31	0.07	1.52	0.91	0.83	-0.92	1.45	-0.69	-0.41
GM	-0.55	1.39	1.29	-1.33	-1.34	-1.70	-1.66	-1.25	0.23	-1.56	-0.35	1.01
TYC	1.17	-0.74	0.00	0.35	-0.58	0.89	0.23	1.08	1.29	1.36	1.40	1.18
VSAT	0.80	-0.45	-0.35	-0.08	0.06	1.35	1.35	1.33	1.15	1.23	0.37	1.02

Factor weightings are unique for each stock to provide the best return forecast.

Factor Weights

Stock	d2p	b2p	e2p	c2p	holt	oe	fs	eq	capx	noag	pm	em
IBM	2%	1%	2%	10%	12%	**26%**	6%	16%	3%	6%	6%	**12%**
GM	**18%**	2%	3%	**14%**	8%	5%	4%	12%	8%	**18%**	4%	7%
TYC	4%	0%	0%	6%	3%	**20%**	14%	**17%**	4%	9%	**7%**	**13%**
VSAT	9%	1%	2%	9%	3%	**14%**	10%	**13%**	7%	18%	**6%**	**6%**

Scores

Stock	Score
IBM	0.75
GM	-0.84
TYC	0.88
VSAT	0.93

IBM : efficiency of operations and positive earnings revisions
GM : share buybacks (debt pay-downs), and cash flow yield
TYC : efficiency of operations, high earnings quality, and momentum (very little valuation)
VSAT : same as TYC, except valuation

Source: PanAgora Asset Management Table is shown for illustrative purposes only.

Optimal Alpha Models with Turnover Constraints: Maximize Net IR

Portfolio Turnover of Quantitative Factors

$$T = \frac{1}{2}\sum_{i=1}^{N}\left|w_i^{t+1} - w_i^t\right|$$

$$T = \sqrt{\frac{\mathrm{N}}{\pi}}\sigma_{\text{model}}\sqrt{1-\rho_f}\,\mathrm{E}\left(\frac{1}{\sigma}\right) \qquad \rho_f = \mathrm{corr}\left(\tilde{F}^{t+1}, \tilde{F}^{t}\right)$$

σ_{model} - targeted risk $\Uparrow$	$T \uparrow$
N the number of stocks $\Uparrow$	$T \uparrow$
ρ_f factor autocorrelation $\Downarrow$	$T \uparrow$
σ specific risk $\Downarrow$	$T \uparrow$

- **Turnover is a function of the targeted risk, the number of stocks, the forecast autocorrelation, and the average specific risk**

Portfolio Turnover of Quantitative Factors

Category	Factors	Avg(ρ_f)
Momentum	EarnRev9	0.64
	Ret9Monx1	0.60
	LtgRev9	0.37
Value	E2PFY0	0.96
	B2P	0.93
	CFO2EV	0.84
Quality	RNOA	0.89
	XF	0.76
	NCOinc	0.80

- **Momentum factors have a lowest autocorrelation (highest turnover)**
- **Value factors have a highest autocorrelation (lowest turnover)**

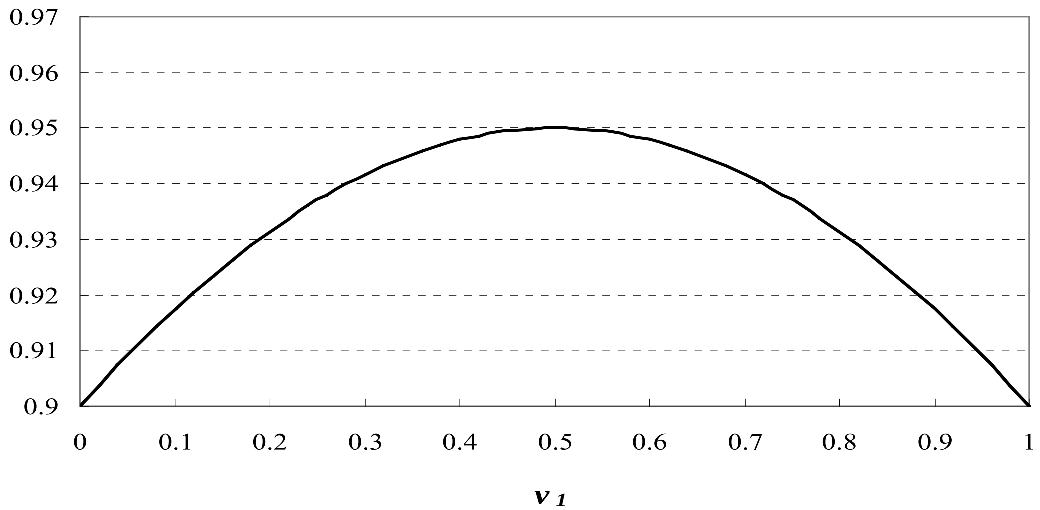
Reducing Turnover

- **Brute force – turnover constraint in portfolio optimization**
- **Integrated approach – optimal models with turnover targets**
  - More value, less momentum
  - <u>*Use moving average of factors*</u>
- **Do the lagged factors forecast future return?**
  - Lower turnover at the cost of alpha?
  - What is the right tradeoff?

Reducing Turnover

Figure 8.2 Serial autocorrelation of forecast moving average with $L = 2$, and

$\rho_f(1) = 0.90,\ \rho_f(2) = 0.81$.



- **Moving average – MA(2)** $\mathbf{F}_{ma}^{t} = v_0\mathbf{F}^{t} + v_1\mathbf{F}^{t-1}$
- **Reduction rate – 70%** $\sqrt{1-0.95} \approx 71\%\sqrt{1-0.9}$

Lagged Information Coefficients

Conventional IC

- Factors known at time *t*
- Subsequent return from t to *t*+1

$$IC_{t,t} = \text{corr}\left(\mathbf{F}_t, \mathbf{R}_t\right)$$

Lagged IC

- Factors known at time *t-l*
- Subsequent return from t to *t*+1
- Information decay

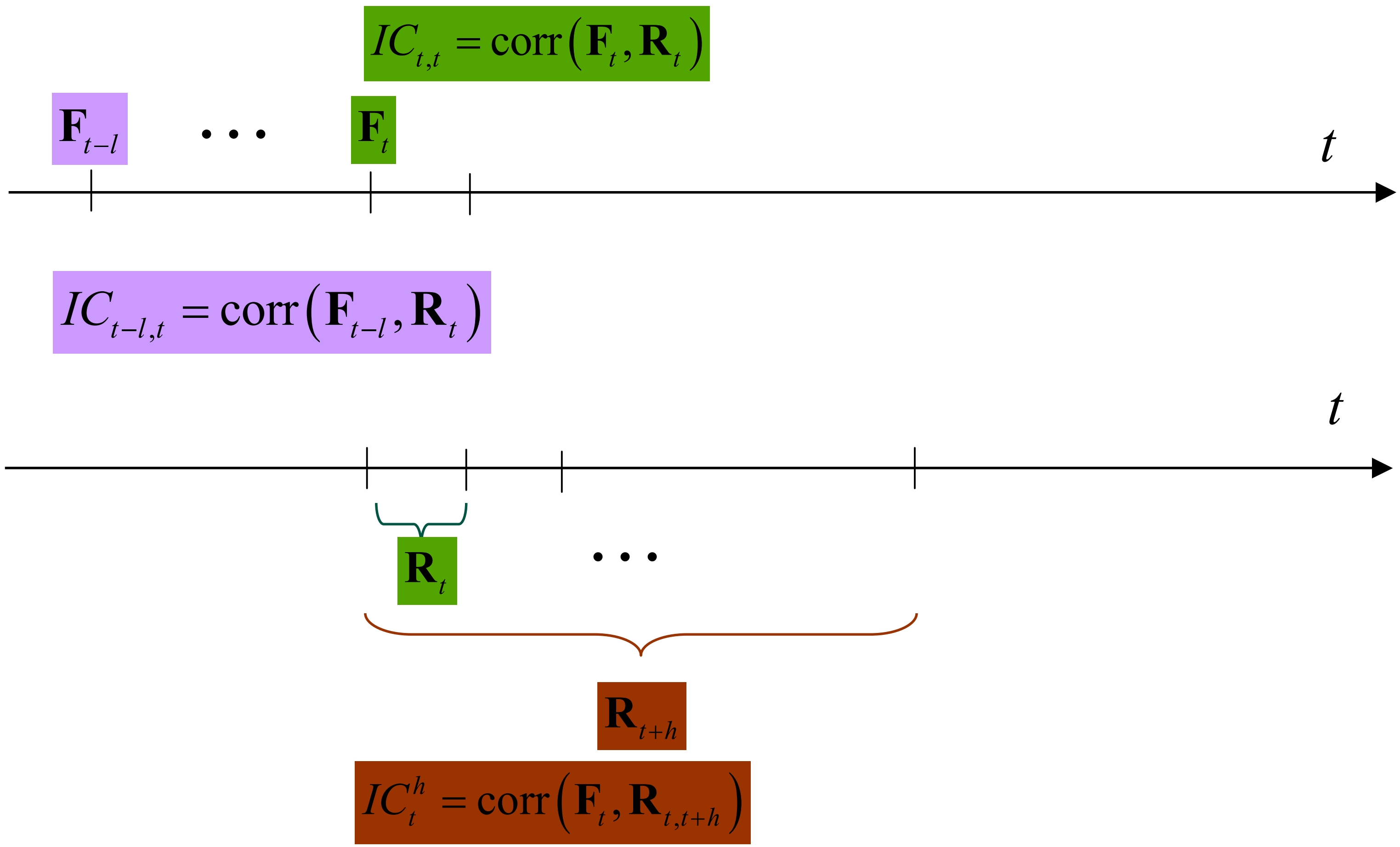
$$IC_{t-l,t} = \text{corr}\left(\mathbf{F}_{t-l}, \mathbf{R}_t\right)$$

Horizon IC

- Factors known at time *t*
- Subsequent return from t to *t*+*h*

$$IC_t^h = \text{corr}\left(\mathbf{F}_t, \mathbf{R}_{t,t+h}\right), \quad h = 0,1,\cdots,H$$

ICs

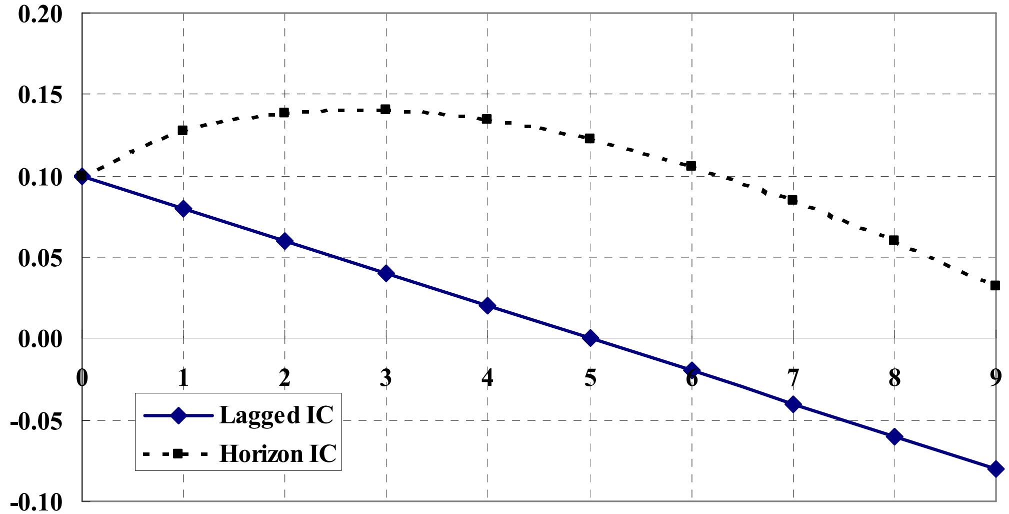

$IC_{t,t} = \text{corr}\left(\mathbf{F}_t, \mathbf{R}_t\right)$
$\mathbf{F}_{t-l}$
...
$\mathbf{F}_t$
t
$IC_{t-l,t} = \text{corr}\left(\mathbf{F}_{t-l}, \mathbf{R}_t\right)$
t
$\mathbf{R}_t$
...
$\mathbf{R}_{t+h}$
$IC_t^h = \text{corr}\left(\mathbf{F}_t, \mathbf{R}_{t,t+h}\right)$

Lagged IC and Horizon IC

- **Relationship between ICs**

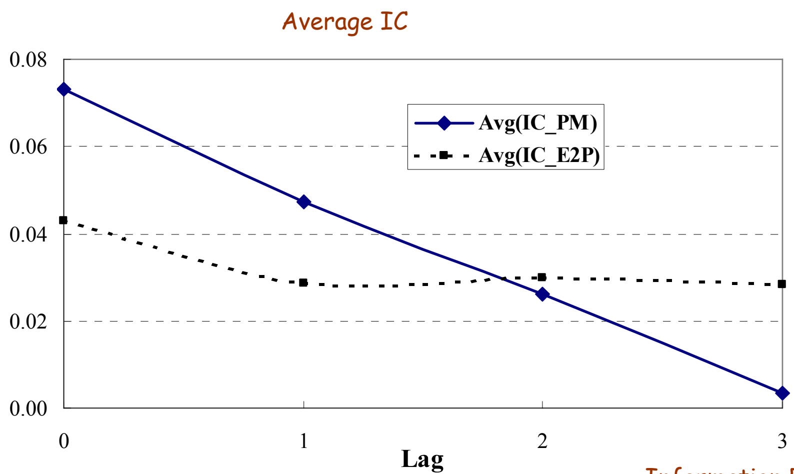
$$IC_t^h \approx \frac{IC_{t,t} + IC_{t,t+1} + \cdots + IC_{t,t+h}}{\sqrt{h+1}} = \text{avg}\left(IC\right)\sqrt{h+1}$$

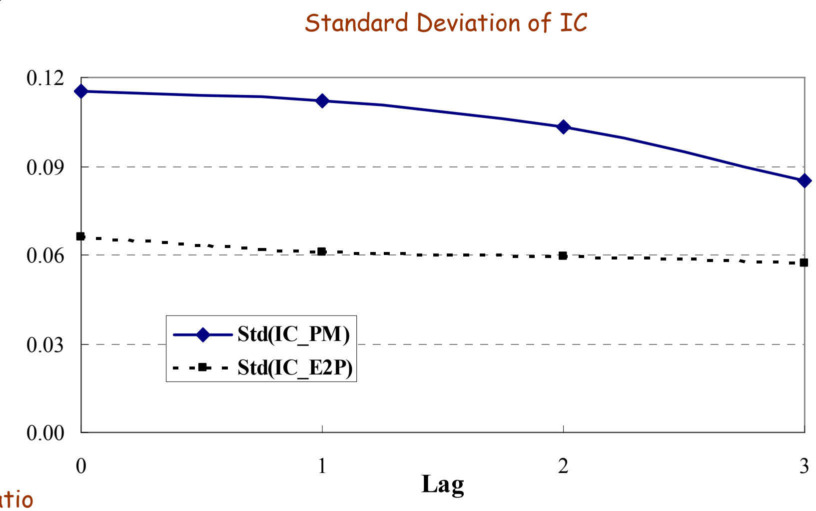
- **Horizon IC typically increases with horizon**

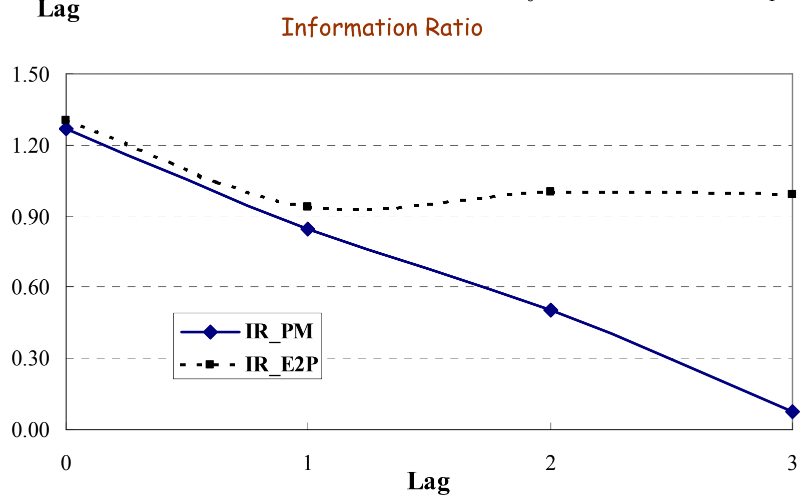


Different Decay Rates

- **Two factors: E2P, PM (Ret9x1)**







Optimal Alpha Models With Lagged Factors

- **Objective: maximize model IR utilizing current and lagged factors while controlling portfolio turnover**

$$\mathbf{F}_{c,ma}^{t} = v_{01}\mathbf{F}_1^{t} + v_{02}\mathbf{F}_2^{t} + v_{11}\mathbf{F}_1^{t-1} + v_{12}\mathbf{F}_2^{t-1} + \cdots +$$

- **Constrained optimization to find the optimal weights**

$$\text{Maximize: } \mathrm{IR} = \frac{\mathbf{v}' \cdot \overline{\mathbf{IC}}}{\sqrt{\mathbf{v}' \cdot \boldsymbol{\Sigma}_{IC} \cdot \mathbf{v}}}$$

$$\text{subject to: } \rho_{f_{c,ma}} = \rho_{\text{target}}\left(\mathbf{v}, \boldsymbol{\Sigma}_{\mathbf{F}}\right)$$

IC Covariance Matrix

Average Factor Covariance Matrix

IC Correlation Matrix

- **Two factor example:** Σ_{IC}

Table 8.2 The IC correlation matrix of current and lagged values for the price momentum and earning yield factor

	PM_0	E2P_0	PM_1	E2P_1	PM_2	E2P_2	PM_3	E2P_3
PM_0	1.00	-0.42	0.86	-0.37	0.78	-0.26	0.61	-0.19
E2P_0	-0.42	1.00	-0.44	0.92	-0.31	0.84	-0.29	0.78
PM_1	0.86	-0.44	1.00	-0.45	0.88	-0.36	0.71	-0.30
E2P_1	-0.37	0.92	-0.45	1.00	-0.33	0.94	-0.30	0.86
PM_2	0.78	-0.31	0.88	-0.33	1.00	-0.28	0.83	-0.22
E2P_2	-0.26	0.84	-0.36	0.94	-0.28	1.00	-0.28	0.94
PM_3	0.61	-0.29	0.71	-0.30	0.83	-0.28	1.00	-0.30
E2P_3	-0.19	0.78	-0.30	0.86	-0.22	0.94	-0.30	1.00

Average Factor Correlation Matrix

- **Two factor example** Σ_F

Table 8.3 The factor correlation matrix of current and lagged values for the price momentum and earning yield factor

	PM_0	E2P_0	PM_1	E2P_1	PM_2	E2P_2	PM_3	E2P_3	PM_4	E2P_4
PM_0	1.00	-0.08	0.68	0.00	0.40	0.05	0.09	0.08	0.07	0.09
E2P_0	-0.08	1.00	-0.09	0.94	-0.06	0.84	0.01	0.73	0.03	0.61
PM_1	0.68	-0.09	1.00	-0.08	0.68	0.00	0.40	0.05	0.09	0.08
E2P_1	0.00	0.94	-0.08	1.00	-0.09	0.94	-0.06	0.84	0.01	0.73
PM_2	0.40	-0.06	0.68	-0.09	1.00	-0.08	0.68	0.00	0.40	0.05
E2P_2	0.05	0.84	0.00	0.94	-0.08	1.00	-0.09	0.94	-0.06	0.84
PM_3	0.09	0.01	0.40	-0.06	0.68	-0.09	1.00	-0.08	0.68	0.00
E2P_3	0.08	0.73	0.05	0.84	0.00	0.94	-0.08	1.00	-0.09	0.94
PM_4	0.07	0.03	0.09	0.01	0.40	-0.06	0.68	-0.09	1.00	-0.08
E2P_4	0.09	0.61	0.08	0.73	0.05	0.84	0.00	0.94	-0.08	1.00

Optimal Alpha Model Weights

- **Maximize IR while targeting model autocorrelation**

ρ_f	IR	PM_0	E2P_0	PM_1	E2P_1	PM_2	E2P_2	PM_3	E2P_3
0.85	2.30	45%	55%	0%	0%	0%	0%	0%	0%
0.86	2.33	43%	57%	0%	0%	0%	0%	0%	0%
0.87	2.36	41%	59%	0%	0%	0%	0%	0%	0%
0.88	2.38	39%	61%	0%	0%	0%	0%	0%	0%
0.89	**2.39**	36%	64%	0%	0%	0%	0%	0%	0%
0.90	2.38	34%	65%	2%	0%	0%	0%	0%	0%
0.91	2.37	31%	65%	4%	0%	0%	0%	0%	0%
0.92	2.36	28%	65%	7%	0%	0%	0%	0%	0%
0.93	2.33	24%	65%	10%	0%	0%	0%	0%	1%
0.94	2.28	21%	58%	12%	4%	0%	1%	0%	4%
0.95	2.21	18%	50%	12%	8%	0%	4%	0%	8%
0.96	2.09	15%	42%	11%	10%	2%	7%	2%	10%
0.97	1.88	11%	32%	8%	14%	5%	12%	5%	14%

Highest IR

Lagged Factor Weights

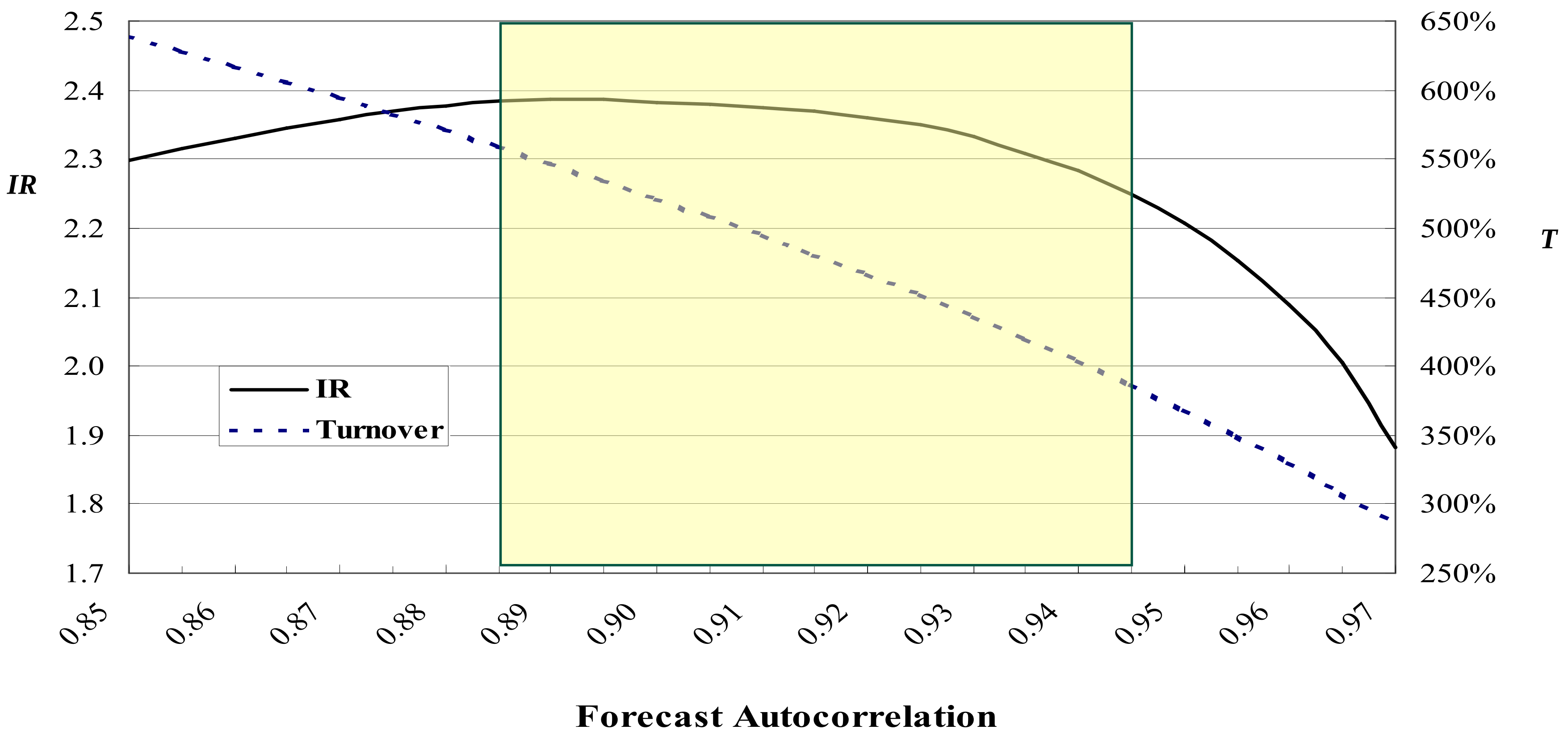
Optimal Alpha Model Weights

➢ Optimal weights - aggregated

ρ_f	IR	PM	E2P	w_0	w_1	w_2	w_3
0.85	2.30	45%	55%	100%	0%	0%	0%
0.86	2.33	43%	57%	100%	0%	0%	0%
0.87	2.36	41%	59%	100%	0%	0%	0%
0.88	2.38	39%	61%	100%	0%	0%	0%
0.89	**2.39**	36%	64%	100%	0%	0%	0%
0.90	2.38	35%	65%	98%	2%	0%	0%
0.91	2.37	35%	65%	96%	4%	0%	0%
0.92	2.36	35%	65%	93%	7%	0%	0%
0.93	2.33	34%	66%	88%	10%	0%	1%
0.94	2.28	33%	67%	79%	15%	1%	4%
0.95	2.21	30%	70%	68%	20%	4%	8%
0.96	2.09	30%	70%	57%	21%	9%	13%
0.97	1.88	28%	72%	42%	23%	16%	19%

IR *and Turnover Tradeoff*

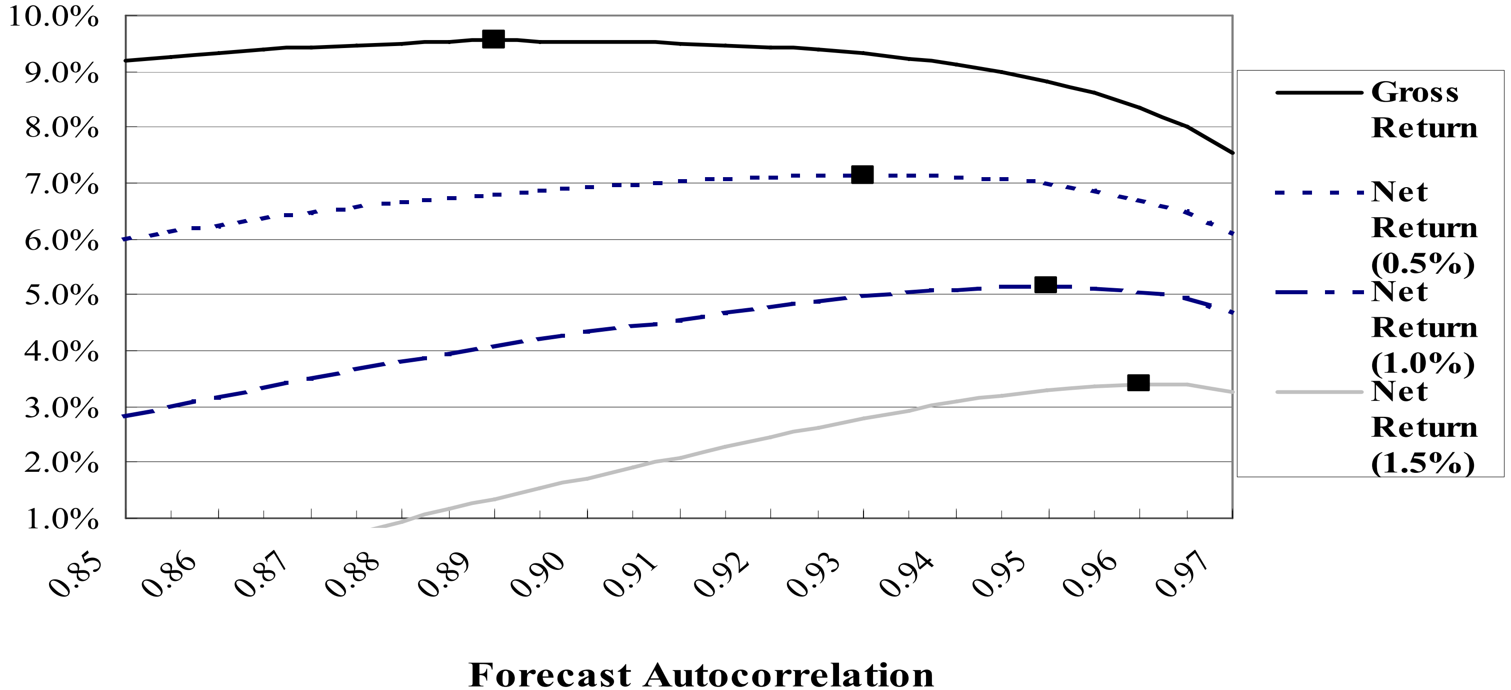
- **IR declines slowly while turnover decreases more rapidly**



Optimal Alpha Models of Net Returns

- **They have higher forecast autocorrelations and utilize lagged factors**

Figure 8.7 The gross excess return and net excess returns under different transaction cost assumption for portfolios with $N = 3000$, target risk $\sigma_{\text{model}} = 4\%$, and stock specific risk $\sigma_0 = 30\%$.



Summary - Advances in Multifactor Models

- **Correct skill measure – risk adjusted IC**
  - Bridge the gap between model and actual performance
- **Optimal modeling framework – maximizing IR**
  - Maximize IR not IC
  - Incorporate IC volatility and IC correlation
- **Contextual modeling – not one-size-fits-all**
  - Increase the depth of quant model
  - Know where the market efficiency is
- **Optimal models with costs constraints – maximizing net IR**
  - Integrate alpha model with implementation

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