Liability-Relative Investing II: Surplus Optimization with Beta, Alpha, and an Economic View of the Liability

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Abstract[1]

This paper revisits and updates the technology for calculating surplus efficient frontiers and surplus asset allocation, to separately incorporate both systematic and unsystematic, or beta and alpha, characteristics. We develop and incorporate an economic view of the liability, also in beta and alpha terms. This gives us a measure of the liability that is more relevant to the asset allocation problem than provided by standard approaches. With these tools, we can provide better risk control for pension plans, through controlled hedging of the assets against the liability. Further, they present the opportunity to support, as appropriate, the inclusion of alpha and active risk from active management.

SURPLUS OPTIMIZATION AND SURPLUS EFFICIENT FRONTIERS

Most efforts to use optimization for developing investment policies simply try to find the desired weights of asset classes in an asset-only context. Surplus optimization hasn't yet become common practice, probably because the liability of a defined-benefit pension plan is still widely measured only in its accounting form rather than in its market, or economic form, and in that form it is difficult to work properly into an economic process such as asset class optimization.[2] But a liability is a financial instrument, a set of future cash flows, its value and its market-related risks are capable of estimation, and as such it can be treated in an optimizer as a part of the total portfolio, as a "short" asset class that the plan sponsor is constrained to hold. This economic version of the liability can in fact be more readily and usefully estimated than its accounting counterpart, and when used in surplus optimization, it better helps the sponsor manage the natural hedge between the assets and the liabilities.

Further, some investors have begun demanding in special cases that we include expected alpha from active management in an asset-liability or investment policy study, while traditionally investment policy has mostly been a beta-only process.[3] What would

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[2] While this article is written in the context of defined benefit pension plans, the general approach is equally applicable to any investor whose assets exist to fund some liability. This includes individuals funding retirement accounts, banks, insurance companies, etc.

[3] For those who accept that markets are efficient, *expected* alpha is zero; Sharpe [1964]. In *realization*, alpha obviously may differ from zero. This essay assumes that markets are not perfectly

an optimizer look like if it were a *total return* optimizer, dealing correctly with both beta and alpha?

These improvements would mean a great deal. They would provide the ability to build a portfolio that is optimal in the presence of a liability. To better control liability-relative risk is a key objective for most defined benefit plan sponsors and many other investors, and "surplus optimization" is the tool of choice for that purpose (where "surplus" refers to assets minus liabilities, often actually a deficit). And the total return character of such an optimizer would allow the investor to appropriately structure the allocations to investment *managers* at the same time as conducting the surplus optimization at the asset-class level.

The best practically usable prior effort at helping the community to implement surplus optimization in practice is no doubt that of Sharpe [1990], and Sharpe and Tint [1990], restated in Sharpe [2002].[4] But despite the elegance, simplicity, and power of this method, it has been only lightly used in practice. It is the logical starting point for this piece, and a good one; but the aspiration is to make it more general and more usable.

Specifically, this paper's goals are to:

1) Explicitly measure and incorporate the liability in *economic* terms rather than in regulatory or book value terms;[5]

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efficient, that some skillful players do exist, and that therefore the *conditional* expected alpha can be nonzero. See Waring and Siegel [2003].

[4] I'm not sure who first invented the concept of surplus optimization, *per se*. Jack Treynor, writing as "Walter Bagehot" [1972], and Treynor, Regan, and Priest [1976] incorporated the liability into the investment analysis, but not explicitly in a surplus optimization framework. However, their comments may have made the notion of surplus optimization sufficiently obvious to avoid the need for further discovery. The earliest explicit reference I have found to surplus optimization or surplus asset allocation is in a number of works by Martin Leibowitz; see Fabozzi [1992] for a republished collection of several exemplars, for example Leibowitz [1987]. Arnott and Bernstein [1988] published an excellent conceptual paper on the topic, still a classic. Mulvey [1989] discussed pension management in a surplus context, and reports to the author in private correspondence that he was using the concept with pension plans in the early 1980s; and Sharpe [1990], and Sharpe and Tint [1990], set out a most elegant and highly streamlined mean-variance optimization formulation of the surplus efficient frontier; see also Sharpe [2004]. Elton and Gruber [1992] contributed a very nice econometric evaluation of the characteristics of a surplus efficient portfolio.

[5] The economic nature of the liability is well established in the literature, but has been little seen in actual practice. The notion appears to originate with Treynor, et al., [1976], page 56. It appears again in Bookstaber and Gold [1988]; in Mulvey [1989]; and in Michaud [1989] and [1998, chapter 10]. A recent example is Ryan and Fabozzi [2002].

2) Show how the asset beta and the liability beta combine to form a *surplus beta*, which reflects the net market risk exposure of the pension plan;

3) Refocus Sharpe and Tint's basic surplus approach from an *asset*-centric orientation to a *liability*-centric one appropriate to the pension policy problem;

4) Divide total return and total risk between beta and alpha, thus adding the capability of integrating the active management hiring and retention decision to the optimization.

Many worked examples of the approach that we're suggesting can be found in Waring [2004b].

CHOOSING A DEFINITION FOR "SURPLUS RETURN"

Our first task is to define a "surplus return" for meaningfully tracking and measuring changes in the net wealth of a pension plan.

The usual meaning of *surplus*, *S,* is the dollar difference between the value of the assets, *A,* and the value of the liability*, L.* Of course, it might be negative. Indicating beginning period values with subscript *0*:

$$S_0 = A_0 - L_0 . \tag{1}$$

We can also think about the *change* in the value of the surplus during the period as the change in value of the assets, net of the change in value of the liability:

$$S_1 - S_0 = \left(A_1 - A_0\right) - \left(L_1 - L_0\right). \tag{2}$$

A "return," R , is the rate of growth of beginning value that equates it to ending value, so equation (2) is equivalently stated as follows:

$$S_1 - S_0 = A_0 \cdot R_A - L_0 \cdot R_L . \tag{3}$$

It is natural to similarly represent the change in the value of the surplus as the product of the beginning surplus and a "surplus return:"

$$S_0 \cdot R_S = S_1 - S_0 = A_0 \cdot R_A - L_0 \cdot R_L . \tag{4}$$

Dividing through by beginning surplus we arrive at a definition of surplus return:

$$R_S = \frac{S_1 - S_0}{S_0} = \left[\frac{A_0}{S_0} \cdot R_A - \frac{L_0}{S_0} \cdot R_L \right]. \qquad (5)$$

Unfortunately this isn't a robust measure of surplus return and will break down under commonly occurring conditions. The worst problem is that when the beginning surplus is zero, there is a zero divide condition and the function is not continuous. There are other, related problems: As surplus gets small, small changes in the value of surplus generate very large numbers for surplus return, so that the surplus return can't be understood without some knowledge of the asset/liability ratio; the return is no longer a useful, single-number summary of performance. And if the surplus goes from positive to negative (a surplus to a deficit), or vice versa, during a period, this definition gives a nonsensical surplus return. Thus, while "pure" in some sense, the definition of surplus return in equation (5) is quite impractical because of the likelihood that it will "break."[6]

So we need to scale, or normalize, the surplus return so that it will not break. There is no absolute reason why the change in value of the surplus, $S_1 - S_0$, needs to be scaled to beginning surplus. It could be scaled against some other useful valuation reference point, one numerically large and stable enough to avoid zero-divide and related difficulties, but sufficiently connected to the problem to provide intuition and information. Because the purpose of pension assets is solely to fund the liability, this is a liability-centric problem and so I have chosen to divide through by the liability to get a *"return of the surplus relative to the liability,"* this particular scaling of the surplus return definition being indicated by the parenthetical subscript (L):

$$L_0 \cdot R_{S(L)} = S_1 - S_0 = A_0 \cdot R_A - L_0 \cdot R_L \qquad (6)$$

$$R_{S(L)} = \frac{S_1 - S_0}{L_0} = \left[\frac{A_0}{L_0} \cdot R_A - R_L \right]. \qquad (7)$$

Equation (7) is the definition for surplus return used in the balance of this paper. Leibowitz et al [1991; 1992 p. 266] also used this same liability-centric scaling, but Sharpe and Tint [1990] chose to rescale the surplus return to the assets, preferring to be asset-centric. That is the more general solution, as most investors are also asset-centric, but for pension plans, the desire to provide funding for the *liability* dominates the thinking.[7] After all, the task of these assets is to fund the liability, and we're interested in how well they are doing at that task.

An illustration of the usefulness of the liability-centric version of surplus return is in its interpretation: It tells us by how much the return on assets has changed the surplus, or the dollar level of funding of the plan, stated in the form of a rate expressed as a percentage of the liability – the pension plan's ultimate reference mark.

Other definitions are possible. For example, one could define the return of the surplus as the simple difference between the returns of the assets and of the liabilities, $R_A - R_L$. But to calculate the return on any composite portfolio (in this case one that is long in assets and short in the liability), we need the portfolio *weights* – which are missing from this formulation – as well as the returns.

There is one other method of calculating a surplus return that is sometimes suggested: plugging the A/L ratio into a standard-form return formula. Such a return definition has initial appeal because it does not have the sign change or zero-divide problem (as it is a ratio that is always positive and non-zero for any realistic situation):

$$R_{S\left(\frac{A}{L}\right)} = \frac{A_1 / L_1}{A_0 / L_0} - 1 \quad (8)$$

Dividing the denominator of equation (8) into the numerator:

$$R_{S\left(\frac{A}{L}\right)} = \frac{A_1 / A_0}{L_1 / L_0} - 1$$

[6] See, e.g., Elton and Gruber [1992] for examples of research done using this apparently more natural definition of surplus return. By confining itself to problems where these problems don't come up, this measure was usefully applied.

[7] The asset-centric approach of Sharpe and Tint [1990] has a very nice characteristic when it is time to construct a utility function: If the surplus return is scaled to the assets, then the liability terms of the utility function would all drop out as the liability goes to zero, leaving us with a conventional asset-only utility function; this is why I say it may be more useful as the general case, outside the pension problem. All of this is consistent with the interesting observation made by Martin Leibowitz and Roy D. Henriksson, that surplus optimization is just the asset-only case expanded to allow for short positions. In other words, surplus optimization can be thought of as the general case, with asset-only optimization being the special case relevant only where there is no anticipated payout. Leibowitz [1988, 1992 p. 157].

$$= \frac{1 + R_A}{1 + R_L} - 1 . \qquad (9)$$

Equation (9) reveals that this definition of surplus return is, in reality, just a nicely disguised geometric difference between the return of the assets and the return of the liabilities. Because optimization requires arithmetic inputs, not geometric, we would have to convert this back to an arithmetic return if we wanted to use it. And despite the fact that in its initial form it appears to include the relative weights of the assets and liabilities, equation (9) reveals that it does not. It isn't a great definition for surplus return.

So in this work we will measure and manage net pension wealth, assets minus the liability (the "surplus"); and we will define the return on this surplus as a return relative to the liability, $R_{S(L)}$. After this point, however, we'll drop the subscript *(L)* notation unless it is necessary for clarity in particular applications, assuming that our definition of surplus return is agreed upon and that the distinction does not need to be further called out.

SEPARATING BETA AND ALPHA IN THE SURPLUS

We can model the surplus return in more detail, reflecting our interest in decoupling market risk and return from active risk and return – that is, in separately considering beta and alpha. This lays the groundwork for total return optimization.

The difference between beta and alpha is important: market or beta risk is inherently rewarded, in that more beta risk implies a higher expected return (think of the upwardly sloping security market line), but active risk is only rewarded conditionally on special skill – its unconditional expected return is zero. Further, beta exposures cost very little to maintain (because of the low cost of index funds and futures contracts), while strategies which have a positive expected alpha can be expected to be much more expensive (see Waring and Siegel [2003]). Thus alpha and beta have very different characteristics, and we want to track and manage them differently.

Market-related returns and a broadly-defined measure of "beta"

For now, we'll model returns and risks in a manner consistent with the Capital Asset Pricing Model, a single-beta model of security returns (later, we'll use a multi-beta model). This approach separates market or beta risks and returns from residual or alpha risks and returns.

The choice of a reference portfolio is important. The term "beta" is usually used to refer to an equity-only beta, often calculated solely by reference to the S&P 500 U.S. stock market index or some similar index. But for the reasons pointed out by Roll [1977], such a beta is too limited, and the "correct" beta would have to be calculated relative to the return on all of the capital assets in the world, including such untraded assets as human capital and residential real estate. So while this ideal world market portfolio is not perfectly identifiable, the "next best" approach is to proxy it as the world portfolio of liquid and traded assets, in market capitalization weights.[8] The universe of return-generating assets includes not only domestic equities but also foreign equities, and not just equities, but also bonds.

I'll follow the convention of referring to this best approximation of the "consensus portfolio" as "portfolio Q." Note that the return on portfolio Q contains interest rate risk as well as equity risk, and that a default-free U.S. Treasury bond will therefore have a non-zero beta.

A model of *asset* returns

The return of a portfolio of assets, A, can always be expressed in terms of its market, or beta, terms and its residual terms:[9]

$$R_A = R_F + \beta_A \mu_Q + \alpha_A . \qquad (10)$$

where R_F is the risk-free rate;[10] β_A is the beta of the assets relative to portfolio Q, μ_Q is the return of portfolio Q in excess of the risk-free rate (the risk premium); and α_A is the residual component of returns.

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[8] Roll [1977]. This is better than an equity-only beta, but still not completely ideal. The measurement of returns for a traded version of the world market portfolio is imperfect because of crossholdings, ambiguity regarding how many shares or bonds are actually outstanding, and other issues in defining the relevant set of assets.

[9] See Grinold and Kahn [2000], Appendix to their chapter 4, for the inspiration for this approach.

[10] It has been argued persuasively that the correct risk-free asset isn't a government bond but rather should be an inflation protected bond of the appropriate horizon, such as the U.S. Treasury's TIPS bonds or the U.K. 'linkers.' See Campbell and Viciera [2002]. It is still difficult to work that very good idea into a standard form of single- or multi-beta models of returns, so I am avoiding it here. However, I do presume that there will be an asset class for inflation protected bonds, which should give the investment

The residual component – alpha – is what we ordinarily think of as the value added from active management. The beta component is very close to what we think about when we do conventional strategic asset allocation work – i.e., allocating across fully diversified representations of the major asset-class markets. So this framework fits our task well.

A model of *liability* returns

So a liability is just another asset, albeit one held short by the plan sponsor (and long by the participant community). Like any other asset, the return of the liability is also composed of market and non-market components, and can be represented using the same model that we used for the assets:

$$R_L = R_F + \beta_L \mu_Q + \alpha_L \,, \tag{11}$$

where β_L is the beta of the liability-relative to portfolio Q, and α_L is the residual component of the liability return. I'll discuss both of these in more detail, below.

Combining assets and liabilities: Modeling surplus returns

Combining equations (10) and (11), the surplus return of equation (7) expands to the following beta-alpha version. First directly, then combining like terms:

$$\begin{aligned} R_S &= \frac{A_0}{L_0}\left(R_F + \beta_A \mu_Q + \alpha_A\right) - \left(R_F + \beta_L \mu_Q + \alpha_L\right) \\ &= \underbrace{\left(\frac{A_0}{L_0} - 1\right) R_F}_{\text{Risk-free return}} + \underbrace{\left(\frac{A_0}{L_0}\beta_A - \beta_L\right)\mu_Q}_{\text{Surplus beta return}} + \underbrace{\left(\frac{A_0}{L_0}\alpha_A - \alpha_L\right)}_{\text{Surplus residual return}} \end{aligned} \tag{12}$$

We can see that the term $\left(\frac{A_0}{L_0}\beta_A - \beta_L\right)$ is the beta, or market risk summary measure, of the *surplus,* β_S . We'll call it "surplus beta."

policy choice the same opportunity to choose to be inflation protected. This may actually be a better way of accommodating the concern, as it appears to come wholly from a liability-centered view of investment strategy in any event.

The risk models

Given the return stream for the assets in equation (10), the associated risk models for the assets and liabilities can be written as:

$$\sigma_A^2 = \beta_A^2 \sigma_Q^2 + \omega_A^2 . \tag{13}$$

$$\sigma_L^2 = \beta_L^2 \sigma_Q^2 + \omega_L^2 . \tag{14}$$

In this case, omega (ω) is the residual risk of the assets or the liabilities; it is the standard deviation of the alpha term, α.

But we can write the risk of the surplus, the assets and liabilities combined, directly from equation (12) ($\omega_{A,L}$ is the covariance of the asset and liability residuals):

$$\sigma_S^2 = \underbrace{\left(\frac{A_0}{L_0} \beta_A - \beta_L \right)^2 \sigma_Q^2}_{\text{Surplus beta risk}} + \underbrace{\left(\left(\frac{A_0}{L_0} \right)^2 \omega_A^2 - 2 \frac{A_0}{L_0} \omega_{A,L} + \omega_L^2 \right)}_{\text{Surplus residual risk}} \tag{15}$$

An interesting result is apparent: The lowest risk, "minimum surplus variance" asset portfolio has a surplus beta of zero, and for that to happen, $\beta_A^* = \frac{L_0}{A_0} \beta_L$: The asset beta of an underfunded plan *must be more aggressive than the liability beta*, simply to "keep up."

THE SURPLUS UTILITY FUNCTION

The objective function is to maximize expected return *of the surplus* at a given level of *surplus* risk. A standard-form mean-variance utility function suggests the following general structure for implementing that objective:

$$Max(U_S) = R_S - \lambda \sigma_S^2 \tag{16}$$

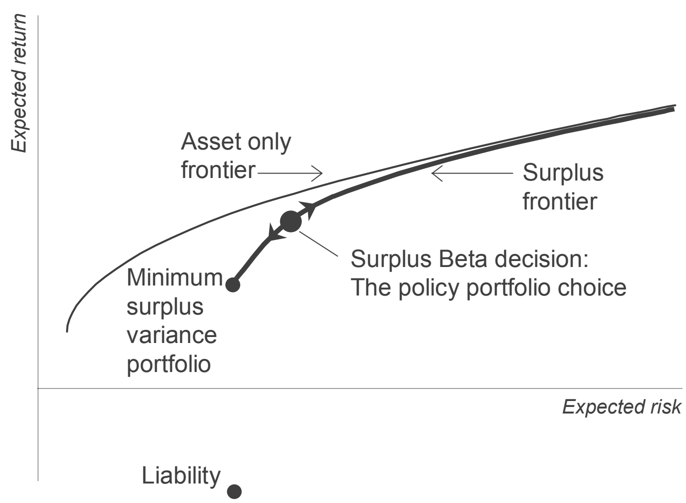
We use λ, or "lambda," to represent the investor's aversion to taking on surplus risk. This term effectively transforms risk into a return-equivalent measure, allowing surplus risk to be viewed as a penalty to be charged against surplus return. Surplus utility can be interpreted then as a *risk-adjusted return*.

Populating equation (16) with equations (12) and (15) for surplus return and surplus risk respectively, collecting terms, and simplifying, we have a "beta-alpha total return surplus optimization function," equation (17):

$$
\begin{aligned}
Max\left(U_S\right) = & \underbrace{\left(\frac{A_0}{L_0} - 1\right) R_F}_{\text{risk-free return}} \\
& + \underbrace{\left(\frac{A_0}{L_0}\beta_A - \beta_L\right)\mu_Q - \lambda_\beta\left(\frac{A_0}{L_0}\beta_A - \beta_L\right)^2 \sigma_Q^2}_{\text{Surplus beta utility}} \\
& + \underbrace{\left(\frac{A_0}{L_0}\alpha_A - \alpha_L\right) - \lambda_\omega\left(\left(\frac{A_0}{L_0}\right)^2 \omega_A^2 - 2\frac{A_0}{L_0}\omega_{A,L} + \omega_L^2\right)}_{\text{Surplus residual utility}}
\end{aligned} \tag{17}
$$

Because we have built this model explicitly in terms of surplus beta risks and residual or alpha risks, this is a total return utility function and conveniently allows us to incorporate into our thinking both the market (asset class) and active management interrelationships between the asset allocation policy and the underlying economic liability.

Figure 1
Beta-Only Surplus Efficient Frontier (in *asset* space)

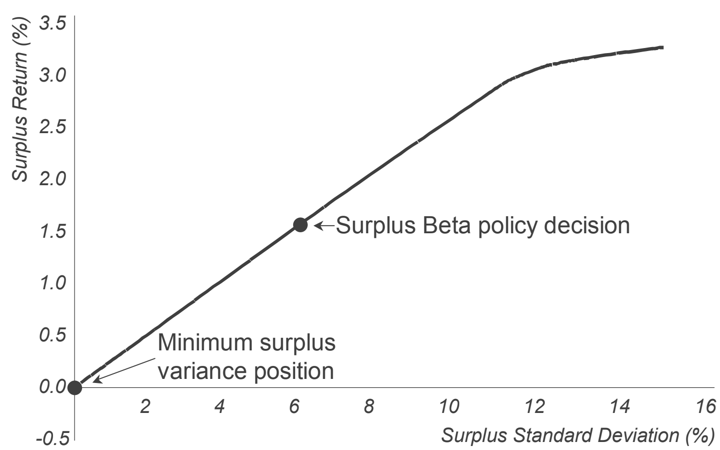


Equation (17) can be abbreviated, for clarity:

$$
Max\left(U_S\right) = \left(\frac{A_0}{L_0} - 1\right) R_F + \underbrace{\beta_S \mu_Q - \lambda_\beta \beta_S^2 \sigma_Q^2}_{\text{Beta-related surplus utility}} + \underbrace{\alpha_S - \lambda_\omega \omega_S^2}_{\text{Residual surplus utility}} \tag{18}
$$

Figure 1 is a stylized illustration of the difference between an asset-only efficient frontier and the surplus frontier, in a typical policy (beta-only) form. Figure 2 shows an actual beta-only surplus frontier calculated using the procedures found in the Appendix, but in "surplus space," i.e., on a set of axes scaled in terms of surplus return and surplus risk, rather than asset return and asset risk. This is more useful space for managing the net wealth of the plan. And Figure 3 shows the result of adding active managers as alpha sources to this frontier – assuming skill in manager selection, of course.

Figure 2
Surplus Efficient Frontier (in *Surplus* Space; A/L=1.0)



Removing the constants: Sharpe-Tint redux

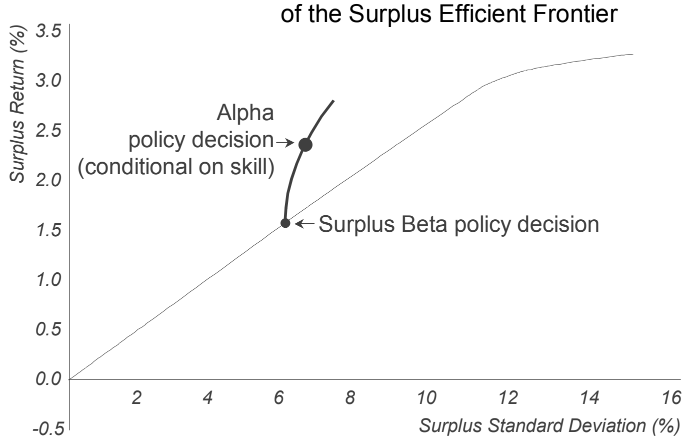
To simplify the equation, we could drop the many constant terms related to the liability, as Sharpe and Tint [1990] did. The constants can't affect the outcome, and mathematically it is elegant to discard terms that have no bearing on the result. To remind us that the actual value for utility resulting from this abbreviated calculation is no longer the same as surplus utility, and that the remaining return and risk terms no longer represent the full values for surplus return or surplus risk, I'll rename it, as they did, from U_S to Z_S:[11]

[11] Sharpe and Tint [1990] effectively provide for an additional differential lambda that allows investors to choose their relative aversion to asset-only risk versus surplus risk. Their "k" variable is

$$
\begin{aligned}
Max\left(Z_S\right) = \frac{A_0}{L_0}\beta_A\mu_Q - \lambda_\beta\left(\left(\frac{A_0}{L_0}\beta_A\right)^2\sigma_Q^2 - \underbrace{2\frac{A_0}{L_0}\beta_A\beta_L\sigma_Q^2}_{\text{LHC}}\right) \\
+\frac{A_0}{L_0}\alpha_A - \lambda_\omega\left(\left(\frac{A_0}{L_0}\right)^2\omega_A^2 - \underbrace{2\frac{A_0}{L_0}\omega_{A,L}}_{\text{Residual LHC}}\right)
\end{aligned} \qquad (19)
$$

Unfortunately, while *Z* is no different than *U* for optimization, the detailed *output data* describing any mixes evaluated using *Z* will have lost its natural scaling, and both surplus return and surplus risk will be difficult to interpret, at best. To retain "natural" units for risk and return, then, we'll retain the constants and use equation (17) or (18) rather than equation (19).

Figure 3
Adding Active Managers: The Alpha vs. Active Risk Extension of the Surplus Efficient Frontier



multiplied by the covariance term remaining between the assets and the liabilities, what they call the "liability hedging credit." This k variable is allowed a value between 0 and 1. A zero value eliminates the covariance term, effectively indicating a zero aversion to surplus risk, and a one value preserves the original lambda which indicates that surplus risk is as important as asset-only risk. We haven't included the k factor here, it being our experience that plan sponsors who are educated in the reasons for considering surplus optimization are usually committed to a full surplus approach. In some circumstances it might be useful to pull this tool out of the toolkit, however, and where that is the case it is easy enough to work it back into the math. I note that Sharpe [2004] continues to use a similar term, expressing it as the desire to hedge.

ECONOMIC VIEWS OF THE LIABILITY

It seems like a blinding flash of the obvious – of course a liability, like any other financial asset or instrument, can be thought of in terms of its beta and alpha characteristics! There is no reason why the liability would not be subject to the same laws of financial gravity about the nature of return, risk, and valuation as is any other stream of cash flows.

To be sure, the liability's cash flows are complex. But they're no more complex than the streams of cash flows for a large publicly-traded company being valued by investment bankers for merger or sale, and that is a routine task. In those situations, incredibly complex cash flows, from many different and often unrelated divisions or business units, are boiled down to a present value based on market-related or beta discount rates, and people are very comfortable with the result.

Note that since we are making an economic decision – deciding the best asset allocation policy – both A and L are economic, rather than accounting or regulatory, quantities. Ideally, then, A should include all the assets, including the present value of planned future contributions,[12] or PVPFC, and L should include the fullest measure of the liabilities, a measure that includes benefits not yet accrued, including those expected to be accrued for future hires.[13] We're trying to manage the total wealth of the plan, and the economically real but "hidden" assets and liabilities should be considered. They will affect the evolution of the reported valuations as they flow into the plan year by year. On this basis, the plans for most healthy organizations are close to fully funded, so $A/L \Rightarrow 1$.[14]

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[12] Waring [2004b] argues that today's ad hoc contribution policies need to be replaced with formal contribution policies.

[13] The present value of all future costs, including wage costs, is an important component of any complete valuation of a business. In pension practice, however, we have by habit limited ourselves to the present value of all future wage costs *for identified individuals only*, because that is what is required for reporting purposes, and because the "benefit security" part of the liability is the only part that needs to be specifically set aside in a pension *fund*. But for asset allocation work, an inherently economic task, the complete valuation is more appropriate. That being said, I recognize that many practitioners will continue to use the more narrow definition, partly because of legitimate concern over estimation problems with respect to these components, and partly from their long standing habit of using only current assets and actuarial liabilities to define the surplus.

[14] In other words, the PVPFC for financially healthy and well-managed institutions is sufficient to make up most of the difference between the value of the investment assets in the pension plan and the value of the economic liability; the balance is expected to come from investment earnings.

Alphas in the liability

The liability alpha, or residual return, deserves description. There are two parts to it. The first is the truly idiosyncratic component of the liability's return, an organization-specific residual return component that is unrelated to the return of the market. It represents growth in the liability resulting from population growth, or from salary or benefit growth, where that growth is specific to the organization and otherwise *uncorrelated with market factors*.

An example of a liability alpha would be the growth of the liability at a company or organization that is raising benefits generously, with motivations completely unrelated to what is happening in the larger economy or in the security markets, a truly idiosyncratic or unsystematic change in benefit levels. Another example is a public fund that has a *fixed* 4% per year "cost of living allowance" or COLA. Not being in the slightest related to inflation or any other beta or market factor, this "COLA" is in fact a liability alpha (maybe it's an un-COLA!). Another example would be an experience of mortality rates that is different than predicted by the actuarial tables used by the plans. Mortality risk is clearly not correlated to market returns. Another example is from the common practice among cash balance plans of paying a crediting rate that is "three year Treasury bonds plus 2%." The "plus 2%" is an alpha.

Good pension management practice involves managing all risks that can be managed, and much of this idiosyncratic liability residual risk can be reduced through tighter management, by justifying benefits more rigorously as required for attracting, retaining, and motivating employees in a competitive marketplace.

This alpha term and the volatility of it would be very difficult to estimate because the historical data all come from the accounting system, which includes distortions caused by smoothing, artificial discount rates, etc. (see Waring [2004b]). But the liability alpha estimates, being constants, are not absolutely essential to the surplus optimization problem. It's a "nice to have," not a "have to have," that would help describe the full pension impact.

The second type of liability residual is more technical, and results from the fact that the liability is not an optimal mix of assets, nearly always being below, or interior to, the natural efficient frontier, the capital market line. This type deals with asset classes, which we can treat as betas. This form of liability residual is dealt with in the appendix.

Beta in the liability: Hedging the liability's risks with the asset portfolio

There is an immediate benefit to separately accounting for beta and alpha in the utility function. Because beta returns are *correlated* return streams by definition, we can use the beta characteristics of the assets (including the beta-related residuals) to reduce total plan risk, *by choosing them so that they hedge away beta risks that naturally occur in the liability*. The truly idiosyncratic portion of the liability residual, or liability alpha, in contrast, is uncorrelated with market exposures (and is likely to be uncorrelated with asset alpha), and cannot be hedged. In fact, *there is nothing that can be done through investment policy to hedge any of the idiosyncratic residual returns and risks in the liability*.[15]

This is a valuable insight because it says that the *only* characteristic of the liability that is important for deciding policy weights among the fully diversified asset classes is the beta, or market-related, characteristic of the liability. Alpha, both on the asset side and the liability side, is important – and valuable – but the big lever for *risk control* is beta.

Among these market-related characteristics, or betas, of any asset or liability are real interest-rate duration and inflation duration (see Siegel and Waring [2004]). Since it is difficult to incorporate duration directly into an optimization solution (because bonds of different durations are highly correlated, and the yield curve changes depending on the number of periods included), it is necessary to follow, alongside the surplus optimization solution proposed here, the additional, separate process for dual-duration matching described in Waring [2004a].

Estimating the liability model, $\boldsymbol{\beta}_L$

An economic approach to managing the liability depends on estimating a useful beta model for the liability, for purposes of assessing the economically correct discount

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[15] As with any strong statement, there are some exceptions. In cash balance plans, for example, a crediting rate might be tied to the return of an active mutual fund. Thus the liability has an alpha that

rate for valuation and for surplus optimization. This is, at heart, just a cost of capital estimation problem, which in turn is about estimating market-related risks (the discount rate is the same as the expected return is the same as the cost of capital). The cost of capital can be estimated in many ways, so I have presented a stylized example of one way in **Table 1**. This method involves first parsing the liability into a few different beneficiary groups, and then estimating a vector of appropriate beta exposures representing the market-related risk in the cash flows for that group; thus in practice, we use a mix of asset class betas, a multiple-beta rather than a single beta approach. (We use vector-matrix notation, set in Roman bold face type for consistency with the appendix, to condense the expressions for weighted-average expected returns. For example: $\boldsymbol{\beta}_{\mathbf{q}}\mathbf{R}_{\mathbf{q}} = \beta_1 \cdot R_1 + \beta_2 \cdot R_2 + ... + \beta_n \cdot R_n$).

Table 1. Modeling Liability Beta

	Beneficiary Groups				
Asset class:	*Retired Lives*	*Active Accrued*	*Active Future Accruals*	*Future Lives*	Expected Return $\mathbf{R}_{\mathbf{q}}$
Long bonds	100.00%[16]	50.00%	30.00%	10.00%	5.00%
Long TIPS	—	50.00	60.00	70.00	5.25
Domestic equities	—	—	5.00	10.00	8.25
Foreign equities	—	—	5.00	10.00	8.25
Discount rate $\boldsymbol{\beta}_{\mathbf{q}}\mathbf{R}_{\mathbf{q}}$	5.00	5.12	5.48	5.83	

The retired lives and inactive lives, and even the accrued portion of the active lives, are very bond-like and are principally modeled as allocations among long nominal bonds and long inflation-protected bonds (i.e., beta weights). But for the future accruals on the active lives, and all accruals of the future lives, things start getting a bit more interesting. In those cases, the strategist has to think about whether some part of the market-related risk might be equity-like instead of just bond-like, with growth related to

would be correlated to the alpha of any assets that might be invested in that fund. And I'm assuming that the risk preference term, or terms, are fixed.

[16] If there were a COLA policy, the retired lives would be modeled as all or part TIPS bonds.

some of the macroeconomic factors that affect equity markets. Consequently, the estimated beta vector will likely contain allocations to equities in addition to the bonds.

For each of the beneficiary groups (denoted by *i*) composing the liability, the discount rate is simply the weighted average return of the asset classes in its model, calculated using the same return assumptions as are used for the asset classes in the optimization process ($\mathbf{R_{Li}} = R_F + \boldsymbol{\beta}_{\mathbf{Li}}\mathbf{r_q}$).

Once the discount rate for each beneficiary group is estimated, it is a simple matter of first using that discount rate to value the benefit payment cash flow stream for that group, to get the vector of valuations across the beneficiary groups ($\mathbf{L_i}$), and then taking the valuation-weighted average of the component beta vectors to obtain the overall liability beta vector, or model, ($\boldsymbol{\beta}_{\mathbf{L}} = \boldsymbol{\beta}_{\mathbf{Li}}\mathbf{L_i}$). The overall liability discount rate is then simply the weighted average discount rate of this overall liability model, $R_L = R_F + \boldsymbol{\beta}_{\mathbf{L}}\mathbf{r_q}$.

Is this a perfect method for modeling the liability? No, of course not. There is plenty of room for educated judgment in estimating these betas, and in judgments there are errors. But, unlike other methods, this method uses the *right* variables – the beta risk factors – to model the liability for use in an asset-liability or asset allocation task. In that spirit, it will be "more perfect" than conventional methods.

RISK TOLERANCE IN THE PRESENCE OF A LIABILITY

How much surplus beta?

Surplus beta, $\beta_S = \left(\frac{A_0}{L_0} \beta_A - \beta_L \right)$, is a new and more relevant summary measure of market-related risk for pension plans, a beta of the assets *relative* to that of the liability. Note that surplus beta risk is any market-related or investment risk taken by the pension plan beyond that necessary to hedge the market-related risks present in the liability.

How much surplus beta risk should be taken? After all, in some sense – and as we have been saying here – the liability is the ultimate reference point for the assets.

Any investment policy that takes on more investment risk than that inherent in the liability needs to be justified on some basis.

This is just the two-fund theorem in the presence of a liability: the investor should hold only two portfolios, a combination of the portfolio matching the beta characteristics of the liability, and the market portfolio, with the relative weights being dependent on the investor's tolerance for surplus risk and desire to seek market returns.

There is no one right answer, of course, but there are a couple of landmarks that help us frame the risk tolerance decision:

1). Zero surplus beta: If the market risk of the assets (after being properly scaled to account for the A/L ratio) *matches* the market risk of the liability, then the pension plan has a surplus beta of zero. This is the position taken by those who cash flow match or duration match (successful only if fully funded). It is also consistent with the Black [1980] and Tepper [1981] tax arbitrage arguments for corporate pension plans, and it is consistent with the corporate finance notion that the shareholders and taxpayers don't need the sponsor to hold equities for them, since they can hold equities themselves. This is the most conservative argument, a polar position.

2). Market portfolio: If one has an average desire to hedge the liability, one should hold the beta of the market portfolio, which is presumably priced to yield an expected return that compensates fairly for its risk. (The world market portfolio, in assets investable by foreigners, is about 50% equities and 50% bonds.) This position is taken by Sharpe, in a draft of his internet book [2004]. He continues the argument, saying that a given liability-centric investor should tilt away from the market portfolio based, among other things, on the difference between his or her desire to hedge and the desire to hedge of the average investor.[17]

3). History of prior investment policies: Historically, it has only been in the last ten years that pension policies became as aggressive as they are today. Going back twenty and thirty years, where the data seem mostly to be anecdotal, it appears that typical pension plan investment policies were less than 50% in equities, often much less. So one has to wonder whether today's typical 70-30 equity-bond allocation doesn't

[17] Sharpe argument appears to rely on the observation that in a world with long and short investors (or assets and liabilities), the CAPM yields this conclusion.

really reflect some element of momentum investing, even perhaps market timing, no doubt subliminal, as a result of the bull markets of 1982-2000. If so, then long term risk tolerance is probably better represented by a lower surplus beta than is implicit in 70-30.

For what it is worth, this author's opinion is that surplus beta, the policy portfolio, should probably be dialed back to a somewhat lower risk level than is prevalent today.

But the beauty of the surplus optimization approach is not that it provides a particular answer, but that it enables the plan sponsor consciously to set the surplus beta position for the plan, controlling pension plan financial risk in the process. Risk tolerance is ultimately an individual decision.[18]

Relating surplus risk to accounting risks

What does taking more or less surplus beta risk mean? If "risk happens" to the economic surplus, a number of things happen as a direct consequence: The liability and asset values move apart, reducing the funding ratio. As a consequence, contributions go up, and for corporations, pension expense goes up.

Each of these resulting effects is filtered and smoothed in some degree, as the economic or "real" values flow through the regulatory and accounting rules into the reported books and records, the "book" values. But no matter – accounting will always follow economics, albeit sometimes with some delay.

The bottom line is that if we've controlled the risks of the plan's economic surplus, we've necessarily also controlled the risk in the plan's accounting values for funded ratio, contributions, and expense.

How much alpha?

Whatever the decision about how much surplus beta risk to take, it is expressed in the utility function as a lambda, a term expressing the degree of aversion to risk. I have used a subscript to differentiate the lambda for market-related risks (as reflected in asset allocation policy), λ_β, from the lambda for active management, λ_ω. We might expect to use a separate, and presumably higher, lambda for that portion of residual risk,

λ_ω, related to active forecasts than we would use for market-related risk, λ_β, consistent with observations made by Grinold and Kahn [2000] and Waring *et al.* [2000, Appendix 2].

There is a contrary point of view that "returns are returns," "risks are risks," and that therefore "lambda is lambda," and that these two lambdas should have the same value. Kneafsey [2003] is consistent with this view. But to hold this view, one has to very strongly believe that his or her *conditional* forecasts of positive expected alphas are really estimated on the same scale as one's *unconditional* forecasts for the market returns. If the magnitudes of the numbers generated for these two different types of expectations aren't properly related to one another, this will generate a bias, usually towards more active risk, arising from the tendency of investors to forecast alphas relatively more generously than betas. Differential lambdas help correct for this. For well-crafted estimates of the return to market risk and to active management, either approach can be correct, and will result in an appropriate division of a total "risk budget" between beta risk and manager active risk.

Either way, if an organization believes it has skill at picking managers that have skill at picking securities, it will be willing to (1) make forecasts of manager alphas, and (2) hold them in the portfolio. See Waring, et al., [2000] and Waring and Siegel [2003]. The relative sizes of the risk budget for active risk and for surplus beta risk depend on that assessment of manager selection skill, on the predicted values of expected alpha, and on the risk aversion terms, λ_β and λ_ω.

CONCLUSION

It is widely agreed that defined benefit pension plans are the best vehicles for delivering a secure retirement to plan participants. But to be comfortable continuing to sponsor these wonderful plans, plan sponsors must have better control over pension investment risk. The tools are available to achieve this ambitious goal. Hedging liability risk with asset risk to maximize risk reduction – making conscious decisions about how much beta risk to take, *surplus beta* – is a critical part of this process. And sponsors can

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[18] The risk to the surplus from different investment positions can be usefully compared using the

close the loop by incorporating their expectations for active management with appropriate return, risk and active lambda assumptions. Using these robust tools, both public and private plan sponsors can build properly hedged plans with acceptable (and well understood) levels of risk that will endure turbulent markets.

APPENDIX: An Optimizable Form of the Utility Function for Practical Applications

This appendix is the "user's manual" for putting these concepts into practice. If we're going to follow our prescription, then we have to sit down and write a disciplined optimizer routine that will actually "do it."

We will focus here first on the beta policy decision, allocating market-related risks and returns, including the market-related residuals. This is the form appropriate for conventional strategic asset allocation processes. We will end by showing how to include alpha and residual risk from active managers, guidance for the increasing number of plan sponsors who want to do this.[19]

Multiple-beta models versus single-beta models

Note that β_A, β_L, and β_S are single-beta representations of market-related risks. Since the ability to explain residual risk increases as we add factors, we might prefer to describe portfolio Q, the assets, and the liabilities with a multiple-beta market model rather than a single beta model. Asset allocation work is typically done across asset classes, and that implies a model in which the asset class weights are implicitly also beta weights.

Working with the residuals

There are many benefits to parsing the returns and risk into market-related and residual components. But there is one downside, in that one must learn to work with the *beta-related* residuals. We're used to thinking that all of the residual is from active

surplus return distributions of Waring [2004b].

[19] The author has a working paper that is much more fully generalized, showing how to include not only alpha from active managers but also alpha generated from asset class timing insights. This effort relies heavily on the beta-alpha framework set forth here.

management, but in fact the total residual terms need to be seen in two parts, one related to betas, and one related to independent sources such as active managers:

$$\begin{aligned}\alpha_{total} &= \alpha_{\text{beta-related residual return}} + \alpha_{\text{manager-sourced residual return}} \\ \omega_{total}^2 &= \omega_{\text{beta-related residual variance}}^2 + \omega_{\text{manager-sourced residual variance}}^2\end{aligned} \tag{A1}$$

(I'm assuming that the two pieces are uncorrelated, which is a fair assumption if one describes the managers' beta exposures as suggested below in equation (A16).)

What is a residual? The efficient frontier – in its natural form the capital market line (CML) – consists of the market portfolio held in some linear proportion relative to the investor's portfolio, levered up or down with the risk-free asset. This "natural" efficient frontier is subject only to the full investment constraint (it is long-only for classes other than the risk-free asset, but not by constraint).

A portfolio that is interior to the CML isn't linearly proportional to the market portfolio. So one can imagine that the market-related residuals that we are discussing are the *differences* between a given set of portfolio holdings and the holdings of the market portfolio (having leveraged the market portfolio up or down to get the right beta).

And in practice, many or most portfolios are interior portfolios. One example is that any time we put a constraint on the efficient frontier such that it becomes a *curved* frontier (the usual case in practice), most or all portfolios on that curved frontier will be interior portfolios relative to the CML. On the liability side, the liability model would only be efficient, i.e., on the CML, by chance, and so it is almost certainly interior.

Expressing the beta-related residuals. We solve for these beta-related residuals by manipulation of any of the equations defining returns, (10), (11), or (12), so as to highlight the residual term. We'll start with an asset example, equation (10), before switching to surplus:

$$\alpha_A = \left(R_A - R_F\right) - \beta_A \mu_Q$$

$$\alpha_A = r_A - \beta_A \mu_Q \tag{A2}$$

Since portfolio Q is the market portfolio, it is on the CML, so the residual return is seen as the portion of the actual return not explained by the beta-adjusted return of portfolio Q. Let's re-write this scalar version in vector-matrix form. For notation, we'll indicate the (1 × q) vector of asset class weights, really betas, as $\boldsymbol{\beta}_{\mathbf{A}}$, and of the market portfolio as $\boldsymbol{\beta}_\mathbf{q}$. The (q × 1) vector of excess returns across the asset classes will be $\mathbf{r}_\mathbf{q}$, so that $\mu_Q = \boldsymbol{\beta}_\mathbf{q}\mathbf{r}_\mathbf{q}$. The (q × q) variance-covariance matrix across the asset classes will be $\mathbf{V}_\mathbf{q}$.

$$\alpha_A = \underbrace{\boldsymbol{\beta}_\mathbf{A}\mathbf{r}_\mathbf{q}}_{r_A} - \underbrace{\frac{\boldsymbol{\beta}_\mathbf{A}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}{\boldsymbol{\beta}_\mathbf{q}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}\boldsymbol{\beta}_\mathbf{q}\mathbf{r}_\mathbf{q}}_{\beta_A\mu_Q} \tag{A3}$$

$$\alpha_A = \left(\boldsymbol{\beta}_\mathbf{A} - \frac{\boldsymbol{\beta}_\mathbf{A}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}{\boldsymbol{\beta}_\mathbf{q}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}\boldsymbol{\beta}_\mathbf{q}\right)\mathbf{r}_\mathbf{q} \tag{A4}$$

This is easier to read if we compress the full vector-matrix version of the single-factor overall beta into a scalar beta:

$$\alpha_A = \left(\boldsymbol{\beta}_\mathbf{A} - \beta_A\boldsymbol{\beta}_\mathbf{q}\right)\mathbf{r}_\mathbf{q}. \tag{A5}$$

The surplus residual is entirely parallel to this asset residual. Using the same derivation (which starts from equation (12) showing the surplus return), we get

$$\alpha_S = \left[\boldsymbol{\beta}_\mathbf{S} - \beta_S\boldsymbol{\beta}_\mathbf{q}\right]\mathbf{r}_\mathbf{q}, \tag{A6}$$

where the net surplus holdings vector $\boldsymbol{\beta}_\mathbf{S}$ is the (1 × q) vector of differences between the factor weights of the multi-factor liability benchmark vector and the corresponding factor weights of the asset classes:

$$\boldsymbol{\beta}_\mathbf{S} = \frac{A_0}{L_0}\boldsymbol{\beta}_\mathbf{A} - \boldsymbol{\beta}_\mathbf{L} \tag{A7}$$

and the scalar value for surplus beta is in standard form, the ratio of covariance over variance:

$$\beta_S = \frac{\boldsymbol{\beta}_\mathbf{S}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}{\boldsymbol{\beta}_\mathbf{q}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}. \tag{A8}$$

While we're dealing with the residuals, let's figure out the parallel definition of surplus residual *risk*. Let's start by solving the market-related portion of surplus risk, described in equation (15), for the residual:

$$\omega_S^2 = \sigma_S^2 - \left(\frac{A_0}{L_0}\beta_A - \beta_L\right)^2\sigma_Q^2 \tag{A9}$$

$$\omega_S^2 = \boldsymbol{\beta}_\mathbf{S}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{S}^\mathbf{T} - \beta_S^2\boldsymbol{\beta}_\mathbf{q}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T} \tag{A10}$$

Equation (A10) can be equivalently written as:

$$\omega_S^2 = \left(\boldsymbol{\beta}_\mathbf{S} - \beta_S\boldsymbol{\beta}_\mathbf{q}\right)\mathbf{V}_\mathbf{q}\left(\boldsymbol{\beta}_\mathbf{S} - \beta_S\boldsymbol{\beta}_\mathbf{q}\right)^\mathbf{T} \tag{A11}$$

The utility function in vector-matrix form

We can rewrite the surplus utility function now, equation (18), in vector-matrix form using this notation and the residual definitions just developed. (I've included terms for any remaining residuals, from the liability and from active management, for consistency with the point made in equation (A1).)

$$\begin{aligned} Max\left(U_S\right) = {} & \left(\frac{A_0}{L_0} - 1\right)R_F \\ & + \underbrace{\beta_S\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}\mathbf{r}_\mathbf{q} - \lambda_\beta\beta_S^2\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}}_{\text{Beta-related surplus returns and risks -- on the CML}} \\ & + \underbrace{\left(\boldsymbol{\beta}_\mathbf{S} - \beta_S\boldsymbol{\beta}_\mathbf{q}\right)^\mathbf{T}\mathbf{r}_\mathbf{q} - \lambda_{\omega|\beta}\left(\boldsymbol{\beta}_\mathbf{S} - \beta_S\boldsymbol{\beta}_\mathbf{q}\right)\mathbf{V}_\mathbf{q}\left(\boldsymbol{\beta}_\mathbf{S} - \beta_S\boldsymbol{\beta}_\mathbf{q}\right)^\mathbf{T}}_{\text{Residual returns and risks from betas off of the CML}} \\ & \underbrace{\left(\frac{A_0}{L_0}\alpha_A - \alpha_L\right) - \lambda_\omega\left(\left(\frac{A_0}{L_0}\right)^2\omega_A^2 - 2\frac{A_0}{L_0}\omega_{A,L} + \omega_L^2\right)}_{\text{Residual returns and risks from sources other than beta (active management; liability)}} \end{aligned} \tag{A12}$$

(A12)[20]

This is the version that we would use for ordinary strategic asset allocation work (dropping the final line for other sources of residual). The output variable is $\boldsymbol{\beta}_\mathbf{A}$, which is the (1 × q) factor weight vector, or equivalently, the strategic asset allocation policy weight vector, to be adopted. $\boldsymbol{\beta}_\mathbf{A}$ is hidden in this abbreviated version, as it is a component term of $\boldsymbol{\beta}_\mathbf{S} = \frac{A_0}{L_0}\boldsymbol{\beta}_\mathbf{A} - \boldsymbol{\beta}_\mathbf{L}$ and also of the scalar beta, $\beta_S = \frac{A_0}{L_0}\beta_A - \beta_L$, where it is included in the term $\beta_A = \frac{\boldsymbol{\beta}_\mathbf{A}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}{\boldsymbol{\beta}_\mathbf{q}\mathbf{V}_\mathbf{q}\boldsymbol{\beta}_\mathbf{q}^\mathbf{T}}$.

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[20] The author will make available his spreadsheet-based research optimizer implementing equation (A12), enhanced by the efforts of Zephyr Associates' master modeler Thomas Idzorek, to readers interested in seeing an example of the workings of the math.

Adding active managers to the utility function

With this groundwork laid, it is relatively straightforward to add residual terms from active managers to this optimization process for those that would like to do so in order to incorporate expected alpha – that is, alpha from active management, alpha as we usually think about it.

The first step, adding in the alphas and active risks, is easy – we simply take the product of the (n × 1) manager weight vector $\mathbf{h_n}$ (the new output vector) times the (n × 1) vector of those manager's alphas, $\boldsymbol{\alpha}_\mathbf{n}$, to get the weighted average portfolio alpha sourced in active management, α_N, and we take a parallel product involving the (n × n) variance-covariance matrix $\mathbf{V_n}$ across those manager weights to get the comparable portfolio residual risk term for active managers, ω_N^2:

$$\alpha_N = \mathbf{h_n}\boldsymbol{\alpha}_\mathbf{n}^\mathbf{T} \tag{A13}$$

$$\omega_N^2 = \mathbf{h_n}\mathbf{V_n}\mathbf{h_n^T} \tag{A14}$$

We add these to the return and risk sides of equation (A12), getting an additional utility component:

$$\begin{aligned} Max(U_S) = & \left(\frac{A_0}{L_0} - 1\right) R_F \\ & + \underbrace{\beta_S \boldsymbol{\beta}_\mathbf{q}^\mathbf{T} \mathbf{r_q} - \lambda_\beta \beta_S^2 \boldsymbol{\beta}_\mathbf{q}^\mathbf{T} \mathbf{V_q} \boldsymbol{\beta}_\mathbf{q}}_{\text{Beta-related surplus returns and risks -- on the CML}} \\ & + \underbrace{\left(\boldsymbol{\beta}_\mathbf{S} - \beta_S \boldsymbol{\beta}_\mathbf{q}\right)^\mathbf{T} \mathbf{r_q} - \lambda_{\omega|\beta} \left(\boldsymbol{\beta}_\mathbf{S} - \beta_S \boldsymbol{\beta}_\mathbf{q}\right) \mathbf{V_q} \left(\boldsymbol{\beta}_\mathbf{S} - \beta_S \boldsymbol{\beta}_\mathbf{q}\right)^\mathbf{T}}_{\text{Residual returns and risks from betas off of the CML}} \\ & + \underbrace{\frac{A_0}{L_0} \mathbf{h_n} \boldsymbol{\alpha}_\mathbf{n}^\mathbf{T} - \lambda_{\omega|\alpha} \left(\frac{A_0}{L_0}\right)^2 \mathbf{h_n} \mathbf{V_n} \mathbf{h_n^T}}_{\text{Residual returns and risks from active managers}} \\ & \underbrace{-\alpha_L - \lambda_{\omega|L} \left(-2 \frac{A_0}{L_0} \omega_{A,L} + \omega_L^2\right)}_{\text{Residual returns and risks from the liability}} \end{aligned} \tag{A15}$$

The terms on the bottom row are the remaining terms of the utility function (equation (17)) that have not yet been discussed in vector-matrix form.[21] They are the liability alphas discussed in the main text, and are of necessity scalar values and difficult to estimate. They are constants and can be omitted without affecting the asset allocation results, but their omission means that the return and risk values reported will be off by that amount.

But there is also a second step: Each manager also comes with some beta exposure or exposures, and these need to be integrated with the underlying beta allocation decision.[22]

There is a very direct and simple substitution that will accomplish this goal. The total portfolio beta exposures, $\boldsymbol{\beta}_{\mathbf{A}}$, is the composite, or weighted average, of all these manager beta exposures. This weighted equation average is obtained by multiplying the manager holdings vector $\mathbf{h}_{\mathbf{n}}$ times the (n × q) matrix $\mathbf{B}_{\mathbf{n,q}}$ of the manager beta vectors, in which each row represents the beta weights for one of the managers across the *q* asset classes:

$$\boldsymbol{\beta}_{\mathbf{A}} = \mathbf{h}_{\mathbf{n}}\mathbf{B}_{\mathbf{n,q}} \tag{A16}$$

So wherever the term $\boldsymbol{\beta}_{\mathbf{A}}$ appears, we simply substitute our (A16) definition.

The output variable is now the manager holding vector $\mathbf{h}_{\mathbf{n}}$, and the optimizer will find the mix of managers that simultaneously maximizes expected alpha and that brings with it the optimal beta, all appropriate at each value for the risk aversion parameters, lambda.

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[21] I've made a minor notation distinction between the lambdas for the different parts of the residual terms, using $\omega|\beta$, $\omega|\alpha$, and $\omega|L$ as subscripts – indicating that the residual risk is sourced in beta, in alpha, and in the liability. Some argue that these lambdas might have different values; the discussion is outside the scope of this paper.

[22] I'm using the term "manager" to represent a unit of whatever it is that is held. Usually that is an actual active investment manager, but in this usage it also includes index funds and also any equivalent derivative positions, possibly including short beta positions in derivatives. Sufficient managers must be available to the optimizer to provide full coverage of all beta (asset class) types.

Constraints and "Portable Beta"

For policy development use (not including active managers), this optimizer will normally be written incorporating budget and perhaps long-only constraints on the beta exposures.

If active managers are to be included, there are additional constraints to consider, and some interesting opportunities, the complete detail being beyond the scope of this paper. For example, if one sets up a "manager" that represents short exposure to one or more betas, as for example a short S&P 500 Index futures contract, under appropriate constraints one could be over-exposed to large cap equity managers because one likes their alpha expectations, while correcting the beta overweight through the short position (requiring an exception to the long-only constraint for this holding). This is full alpha-beta separation, what we like to call "portable beta."

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