On the Size of the Active Management Industry

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The "Active Management Puzzle"?

- Track record of the active management (AM) industry is poor
  - active equity mutual funds in aggregate: $\hat{\alpha} :< 0$ with $t \approx -2$
- Nonetheless, active management remains popular:
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- Decreasing returns to scale in the active management industry
  - industry's α depends on industry's size:
  - α becomes more elusive as more money chases it
  - past underperformance $\Rightarrow$ industry should shrink, but
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  - uncertainty about decreasing returns $\Rightarrow$ large confidence interval for the size we should expect
- In contrast, industry's size would seem puzzling if it were known that returns to scale are constant

What We Do

- Develop an equilibrium model of active management (AM) with
  - competing utility-maximizing investors
  - competing fee-maximizing fund managers
- Solve for equilibrium size, alpha, and fees in AM industry
- Relate size of AM industry to past performance
- Analyze learning about returns to scale

What We Find

- AM industry can be large even if the track record is poor
- Investors' learning about returns to scale is endogenous
  - what they invest depends on what they've learned
  - what they learn depends on what they invest
- Investors never learn the degree of returns to scale exactly
- Industry size can be suboptimal for a long time
- Other features of the model
  - industry size crucially depends on the degree of competition
  - $\alpha > 0$
  - "investment externality"

Model

- Two types of agents:
  - M managers of active funds (have skill but no capital)
  - N investors in active funds (have capital but no skill)
- Benchmark-adjusted return to investors from fund $i = 1, \ldots, M$:

$$
\begin{aligned}
r_i &= \alpha_i + u_i \\
u_i &= x + \epsilon_i
\end{aligned}
$$

$\sigma_x > 0 \Rightarrow$ cannot fully diversify risk by buying many funds

Model: Expected Profits

- Benchmark-adjusted expected total profit to fund i's manager and investors:

$$\pi_i = s_i \left(a - b \frac{S}{W} \right)$$

  $s_i \ldots$ size of manager i's fund

  $S = \sum_{i=1}^{M} s_i \ldots$ aggregate size of the AM industry

  $W \ldots$ total investable wealth of the N investors

- Manager i charges a proportional fee $f_i \Rightarrow$ investors expect benchmark-adjusted rate of return

$$\alpha_i = a - b \frac{S}{W} - f_i$$

- Decreasing returns to aggregate scale: $b > 0$

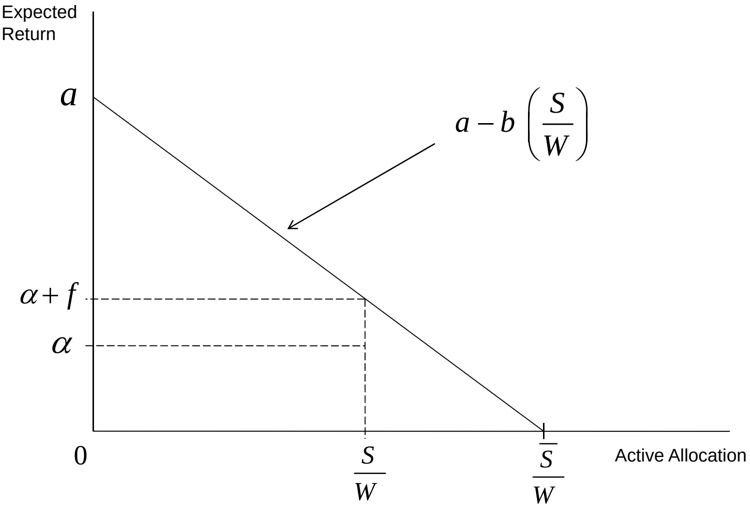


Figure 1. Decreasing returns to scale in the active management industry.

Model: Optimization

- Each manager i chooses f_i to maximize fee revenue:

$$\max_{f_i} f_i s_i$$

- Investors know f_i's before making investment decisions
- Each investor j chooses weights δ_j on the M funds to maximize

$$\max_{\delta_j} \left\{ \delta_j' \mathrm{E}(r|D) - \frac{\gamma}{2} \delta_j' \mathrm{Var}(r|D) \delta_j \right\}$$

  - All investors have same risk aversion $\gamma > 0$, same wealth, and same information set D
  - Unrestricted allocations to benchmarks and T-bill
  - No short sales of funds ($\delta_j \geq 0$)

Equilibrium with Known a and b

- We solve for a symmetric Nash equilibrium
  - First for investors' allocations, as a function of fees, then for managers' fees
- In equilibrium, fees and α's are equal across funds ($f_i = f$, $\alpha_i = \alpha$):

$$
\begin{aligned}
f &= \frac{a\gamma\sigma_\epsilon^2}{2\gamma\sigma_\epsilon^2 + (M-1)p} \\
\alpha &= a\left(1 - \frac{\gamma\sigma_\epsilon^2}{2\gamma\sigma_\epsilon^2 + (M-1)p}\right)\left(1 - \frac{Mb}{\gamma\sigma_\epsilon^2 + Mp}\right) \\
\frac{S}{W} &= \frac{Ma}{\gamma\sigma_\epsilon^2 + Mp}\left(1 - \frac{\gamma\sigma_\epsilon^2}{2\gamma\sigma_\epsilon^2 + (M-1)p}\right),
\end{aligned}
$$

$$\text{where} \qquad p = \frac{N+1}{N} b + \gamma\sigma_x^2$$

Equilibrium Fee

- $M \uparrow \Rightarrow f \downarrow$, due to competition among managers
  - For $M = 1$, we have $f = a/2$
  - For $M \to \infty$, we have $f \to 0$
- Note: f is the discretionary component of the total fee
  - $f =$ fee the manager sets while considering its effect on fund size
    (any competitive proportional fee is part of a)
- Also, $f \uparrow$ when $a \uparrow$, $b \downarrow$, $\sigma_\epsilon \uparrow$, $\sigma_x \downarrow$, and $N \uparrow$

Equilibrium Alpha

- In general, the equilibrium alpha is positive,

$$\alpha > 0$$

because investors

1. demand compensation for risk (σ_x and possibly also σ_ϵ)
2. internalize some of the "investment externality"

Equilibrium Alpha & Risk

- Investors demand compensation for two kinds of **risk**:

  σ_ϵ: diversifiable if $M \to \infty$

  σ_x: non-diversifiable even if $M \to \infty$

- When $M \to \infty$, diversifiable risk σ_ϵ drops out:

$$\alpha = a \left(\frac{(1/N)b + \gamma\sigma_x^2}{[(N+1)/N]b + \gamma\sigma_x^2} \right)$$

When $N \to \infty$ as well,

$$\alpha = a \left(\frac{\gamma\sigma_x^2}{b + \gamma\sigma_x^2} \right)$$

Equilibrium Alpha & Number of Investors (N)

- "Investment externality":
  - new investors impose a negative externality on existing investors by diluting their returns
- When N is finite, investors internalize some of the reduction in profits resulting from their own investment

  $\Rightarrow$ $\alpha > 0$ even if there is no risk
- When $M \to \infty$ and $\sigma_x \to 0$,

$$\alpha = \frac{a}{N+1}$$

- Note: $\alpha \downarrow$ when $N \uparrow$

Equilibrium Alpha & Number of Managers (M)

- The effect of M on α is ambiguous; two opposing effects:
  - $M \downarrow \Rightarrow \alpha \downarrow$ because fees $\uparrow$
  - $M \downarrow \Rightarrow \alpha \uparrow$ because investors demand compensation for σ_ϵ

Equilibrium Size of the AM Industry

- When $N \to \infty$, we obtain a familiar mean-variance result:

$$\frac{S}{W} = \frac{E(r_A|D)}{\gamma \text{Var}(r_A|D)}$$

where r_A is the aggregate benchmark-adjusted return

  - When $N \to \infty$ and $M \to \infty$,

$$\frac{S}{W} = \frac{a}{b + \gamma \sigma_x^2} = \frac{\alpha}{\gamma \sigma_x^2}$$

  - When $N \to \infty$ and $M = 1$,

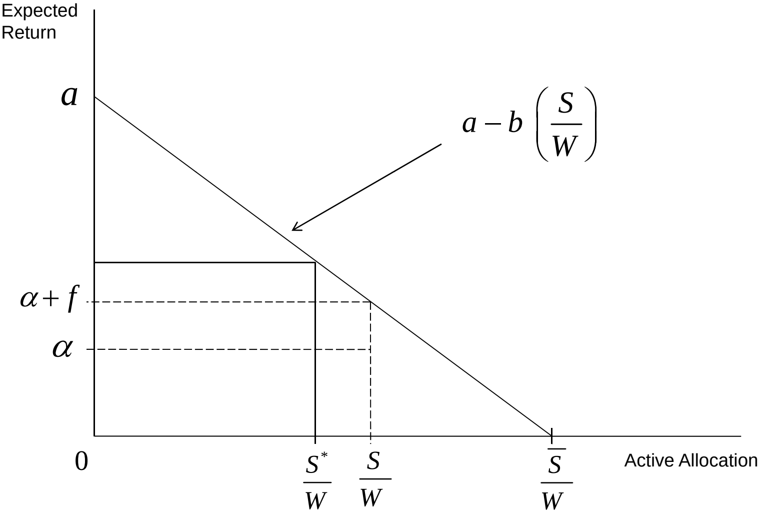
$$\frac{S}{W} = \frac{\alpha}{\gamma(\sigma_x^2 + \sigma_\epsilon^2)}$$

Equilibrium Size of the AM Industry (cont'd)

- Let S^* = size maximizing expected total profit, $S\left(a - b\frac{S}{W}\right)$

$$\frac{S^*}{W} = \frac{a}{2b}$$

- The equilibrium size $S \leq \bar{S} = 2S^*$


Expected
Return
a
$a - b\left(\frac{S}{W}\right)$
$\alpha + f$
α
0
$\frac{S^*}{W}$
$\frac{S}{W}$
$\frac{\bar{S}}{W}$
Active Allocation

Equilibrium Size of the AM Industry (cont'd)

- Let S^* = size maximizing expected total profit, $S\left(a - b\frac{S}{W}\right)$

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- The equilibrium size $S \leq \bar{S} = 2S^*$
- When $M = 1$, there is *underinvestment*, $S \leq S^*$
  - Manager-monopolist charges high fee
  - $S = S^*$ only if $N \to \infty$ and $\sigma_x^2 + \sigma_\epsilon^2 = 0$

Equilibrium Size of the AM Industry (cont'd)

- When $M \to \infty$,

$$\frac{S}{S^*} = \frac{2b}{\frac{N+1}{N}b + \gamma\sigma_x^2}$$

$\Rightarrow$ *underinvestment* $(S < S^*)$ or *overinvestment* $(S > S^*)$

- When $M \to \infty$ and $\sigma_x^2 \to 0$, there is *overinvestment*:

$$\frac{S}{S^*} = \frac{2N}{N+1}$$

  - $N \to \infty \;\Rightarrow\; S \to \bar{S} = 2S^*$

Unknown a and b

- Now suppose a and b are unknown

$$\mathrm{E}\left(\begin{bmatrix} a \\ b \end{bmatrix} \mid D\right) = \begin{bmatrix} \tilde{a} \\ \tilde{b} \end{bmatrix}$$

$$\mathrm{Var}\left(\begin{bmatrix} a \\ b \end{bmatrix} \mid D\right) = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix}$$

- For simplicity, let $M \to \infty$ and $N \to \infty$
  - Then $f \to 0$ and $\alpha = a - b(S/W)$
- Solve for a symmetric Nash equilibrium among investors

Unknown a and b (cont'd)

- In equilibrium, S/W is the (unique) real positive solution to

$$0 = \tilde{a} - \frac{S}{W}\left[\tilde{b} + \gamma(\sigma_a^2 + \sigma_x^2)\right] + \left(\frac{S}{W}\right)^2 2\gamma\sigma_{ab} - \left(\frac{S}{W}\right)^3 \gamma\sigma_b^2$$

as long as $\tilde{a} > 0$. If $\tilde{a} \leq 0$, then $S/W = 0$.

- In this setting,

$$\begin{aligned} \mathrm{E}(r_A|D) &= \tilde{a} - \tilde{b}\frac{S}{W} \\ \mathrm{Var}(r_A|D) &= \sigma_a^2 + \sigma_x^2 - 2\left(\frac{S}{W}\right)\sigma_{ab} + \left(\frac{S}{W}\right)^2 \sigma_b^2 \\ \Rightarrow \quad \frac{S}{W} &= \frac{\mathrm{E}(r_A|D)}{\gamma \mathrm{Var}(r_A|D)} \end{aligned}$$

Prior Beliefs for a and b

- One prior for a, two priors for b
  - **Prior 1:** $b = 0$, known (constant returns to scale)
  - **Prior 2:** $b \geq 0$, unknown (decreasing returns to scale)
- Assume
  - $S/W = 0.9$ is optimal under Prior 2
  - Prior mean of $\alpha = 10\%$ per year at $S/W = 0.9$
  - Risk aversion of $\gamma = 2$
  - Volatility of aggregate active return $\sigma_x = 2\%$ per year
    - Close to empirical estimates for active equity mutual funds

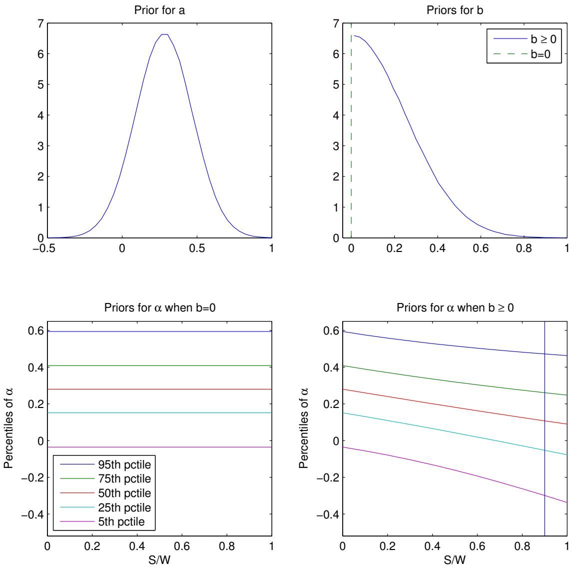

Figure 2. Prior distributions.

Implied Prior Beliefs for α

- Implied prior for α depends on S/W when $b \geq 0$

$$\alpha = a - b(S/W)$$

- **Prior 2 is more pessimistic about** α than Prior 1
  - α is smaller under Prior 2 for any $S/W > 0$
- Nonetheless, we'll see that **Prior 2 investors invest more** in AM than Prior 1 investors after a negative track record

Updated Beliefs

- 300,000 samples of simulated AM returns and allocations
  - For each sample, randomly draw a and b from their priors
- In each year t, beginning with $t = 1$, we perform three steps:
  1. Solve for equilibrium allocation to AM
     - Restrict $(S/W)_t$ between 0 and 1
  2. Construct AM return
     - $r_{A,t} = a - b(S/W)_t + x_t$, where $x_t \sim N(0, \sigma_x^2)$
  3. Update beliefs about a and b
     - Regress r_A on S/W and constant $\Rightarrow$ intercept a, slope $-b$

  ... then back to step 1

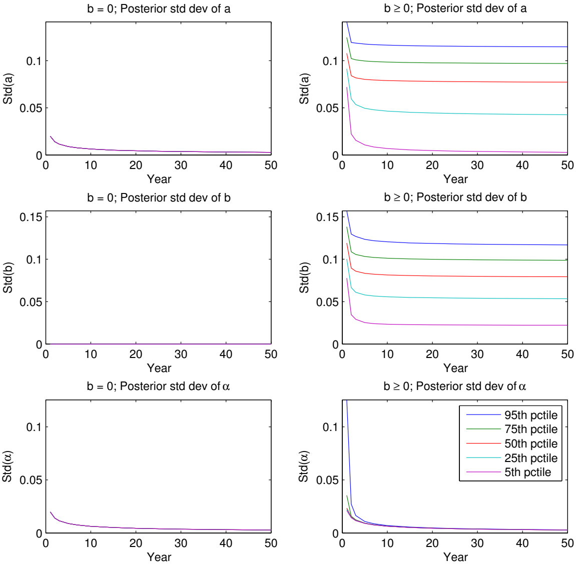

Figure 3. Posterior standard deviations.

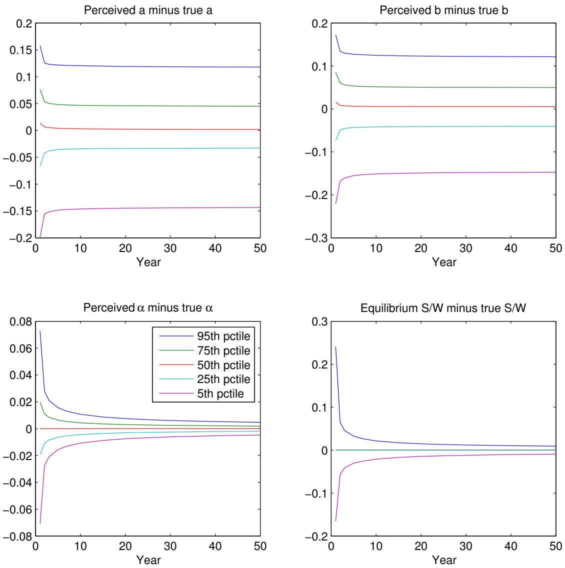

Figure 4. Deviations from true values.

Learning About Returns to Scale

- "Endogeneity": learn $\Rightarrow$ invest $\Rightarrow$ learn $\Rightarrow$ invest $\Rightarrow \dots$
- Learning about the intercept and slope from

$$r_t = a - b(S/W)_t + \epsilon_t$$

  - $(S/W)_{t+1}$ depends on beliefs about a and b at time t
  - If $(S/W)_t$ stops changing, learning about a and b stops
    $\Rightarrow$ Never learn a and b
  - $(S/W)_t$ converges to optimal level quickly when b is high, but it can stay suboptimal for a long time when b is low

Learning When $b \geq 0$

- Investors learn differently under Priors 1 and 2 because $(S/W)_t$ affects learning when $b \geq 0$ but not when $b = 0$
- Representative examples of learning paths: Figure 5
  - Three values of b: "low", "median", "high" (5th, 50th, 95th percentiles of the prior distribution)
  - Given b, pick a such that the "true" $S/W = 0.5$
  - Use (a, b) to generate random samples of returns
- Results:
  - When b is high, investors find the optimal S/W quickly
  - When b is low, investors can get stuck at "wrong" S/W for a long time

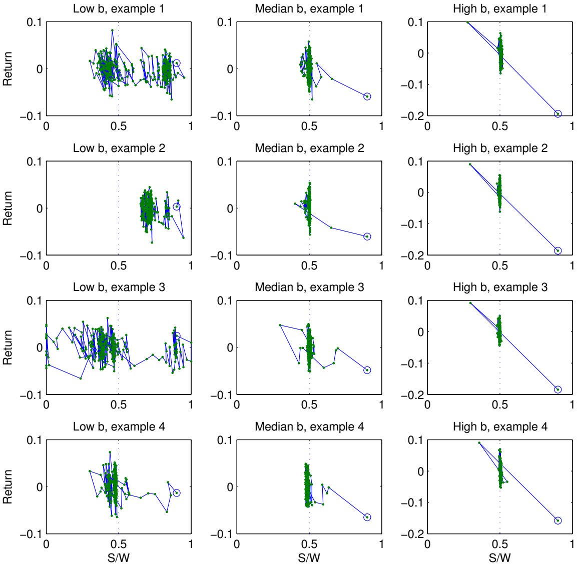

Figure 5. Examples of learning paths.

Role of Historical Performance

- Plot the posterior of the equilibrium S/W conditional on $t(\hat{\alpha})$
  - Note: $t(\hat{\alpha}) = \hat{\alpha}\sqrt{T}/\sigma_x$
- How much is invested in AM when $\hat{\alpha}$ is significantly negative?
- Prior 1 ($b = 0$) investors invest nothing
  - $t(\hat{\alpha})$ is a sufficient statistic for S/W
- Prior 2 ($b \geq 0$) investors can invest a lot
  - despite Prior 2 being more pessimistic about α than Prior 1
  - $t(\hat{\alpha})$ is NOT a sufficient statistic for S/W

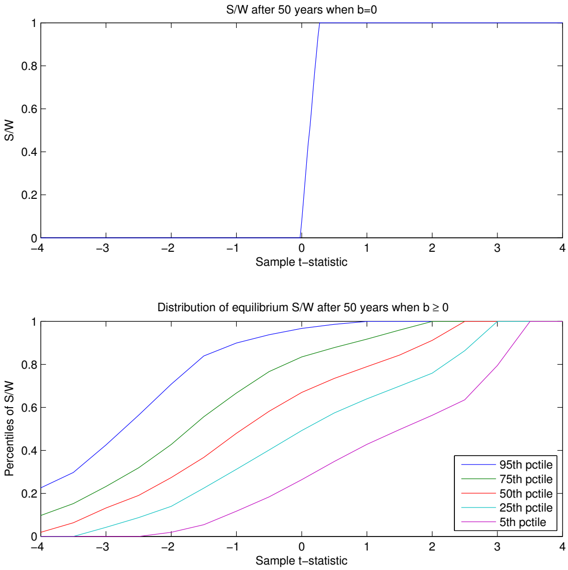

Figure 6. Posterior distribution of the equilibrium allocation to active management conditional on the sample *t*-statistic.

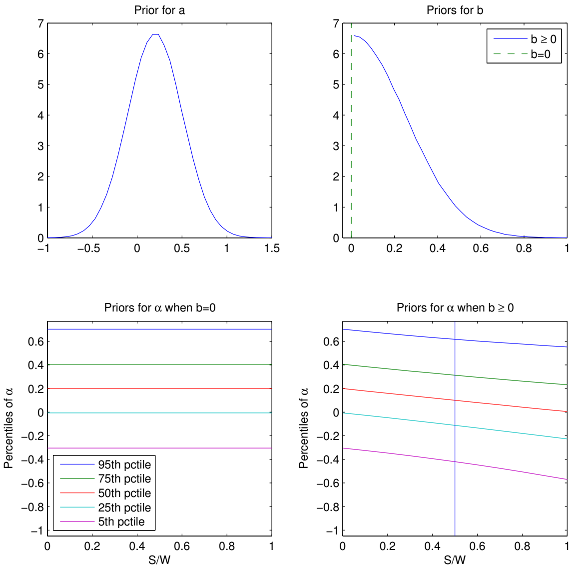

Figure 7. Alternative prior distribution.

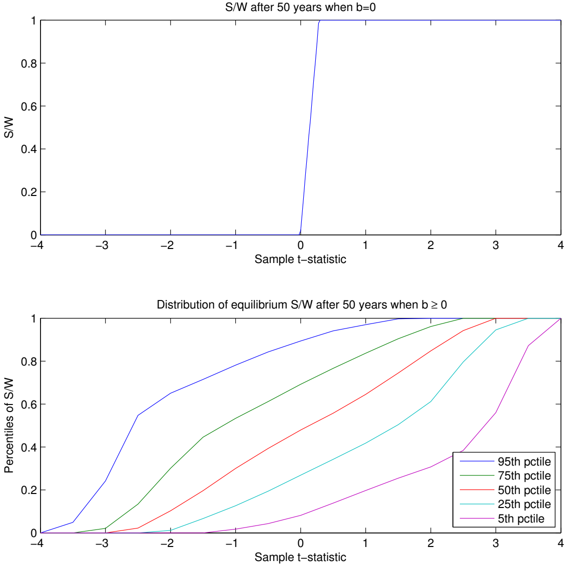

Figure 8. Equilibrium allocation to active management under the alternative prior.

Differences from Berk and Green (2004)

- Focus
  - BG: Capital flows across funds
  - We: Size of the AM industry
- Decreasing returns to scale
  - BG: At individual fund level
  - We: At aggregate industry level
- Parameter uncertainty
  - BG: a unknown, b known
  - We: a and b both unknown
- Fund managers setting fees
  - BG: Monopoly
  - We: Competition

Differences from Berk and Green (2004) (cont'd)

- Equilibrium alpha
  - BG: $\alpha = 0$; no explicit investor optimization
  - We: $\alpha > 0$; equilibrium outcome for optimizing investors
- Our model comes closest to BG when . . .
  - $M = 1$ (a single fund manager) $\Rightarrow$ same fee setting
  - $N \rightarrow \infty$ (many investors) $\Rightarrow$ no investment externality
  - $\sigma_\epsilon = \sigma_x = 0$ (no risk) $\Rightarrow$ no compensation for risk

  Equilibrium then features $\alpha = 0$, $S = S^*$, and $f = a/2$, as in BG

Conclusions

- Size of AM industry can be large even if the track record is poor
  - Due to decreasing returns to scale
- Learning about returns to scale is "endogenous" and slow
  - Never learn the degree of returns to scale exactly
  - Industry size can be suboptimal for a long time
- Interesting features of the model
  1. Industry size crucially depends on the degree of competition
  2. $\alpha > 0$
  3. "Investment externality"