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Optimal Portfolio Choice over the Lifecycle with Social Security

Motivation

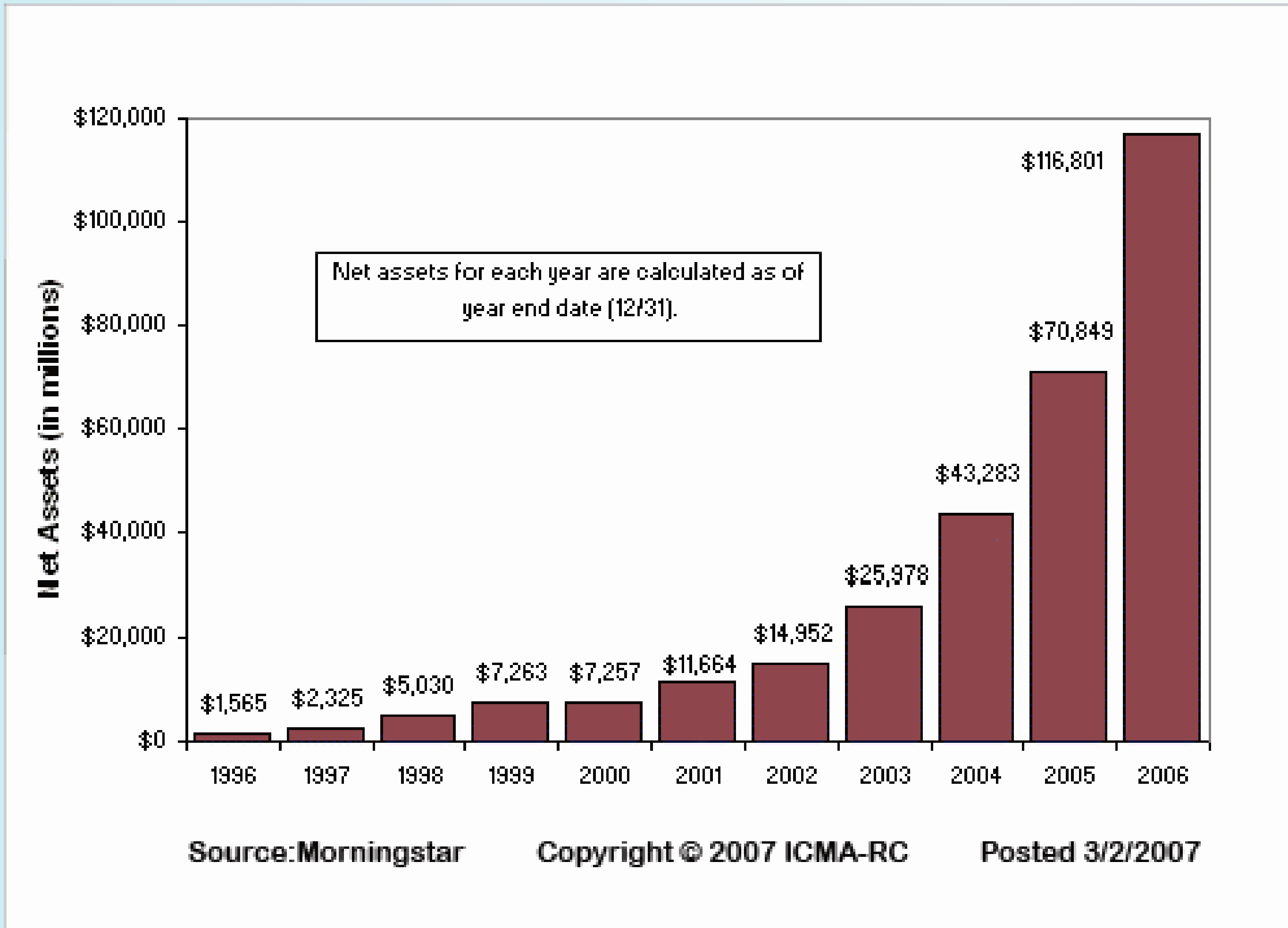
Optimal portfolio choice over the lifecycle has a large impact on expected utility; hence, highly studied

Central to financial planning, especially for non-institutional accounts (households)

The wrong guidance can have a substantial impact on lifetime utility, much like a large tax

The growth of Target Date Funds assumes we actually know the answer – I take a contrarian view

Asset Growth in Target-Date Funds



Data is before Pension Protection Act. TDF’s will dominate DC’s.

Motivation

Evolution of literature

- Inelastic, riskless labor supply
- Elastic (riskless) labor supply
- Inelastic, risky labor supply
- Inelastic, risky labor supply; other imperfections

Core problem: existing portfolio models fail to explain the household-level empirical evidence

Evolution of Literature (1)

Inelastic, riskless labor supply: seminal papers by Samuelson (1969) and Merton (1969)

- Only PV of labor income matters
- Portfolio choice is the same as single-period investors and hence, is constant.

Elastic, risky labor supply: first investigated by Bodie, Merton and Samuelson (1992)

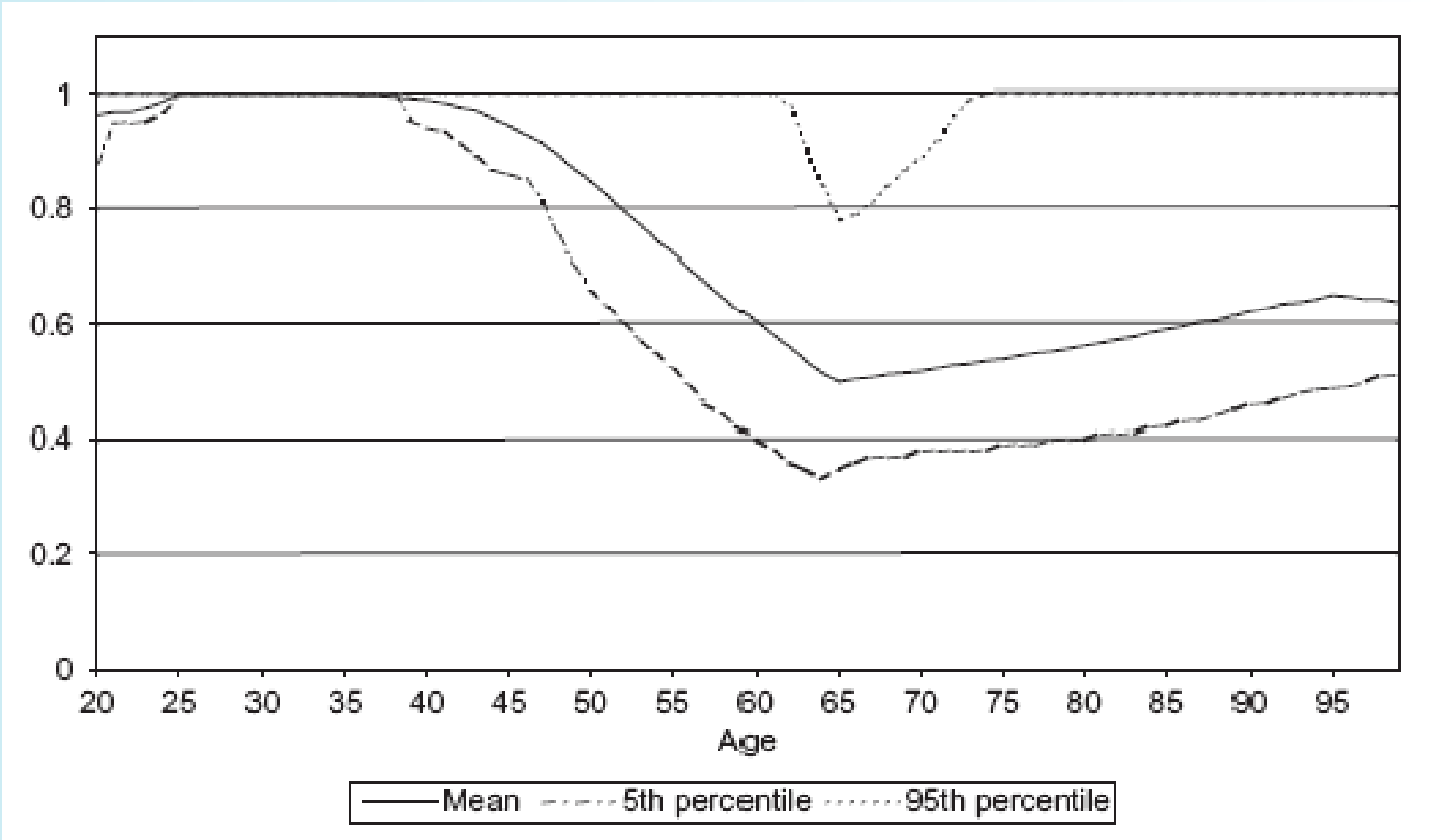
- Future labor income (human capital) riskless (bond-like) but the source (ability) less elastic over lifecycle
- Optimal share in stocks declines over lifecycle

Evolution of Literature (2)

Inelastic, risky labor supply: Koo(1998); Heaton and Lucas (1997); Cocco, Gomes and Maenhout (2004)

- Although labor supply inelastic, cannot simply count on the present value of future labor income since it is risky; the *expected* PV is not a sufficient statistic either in the presence of prudence ($u''' > 0$)
- Imposes a "natural" borrowing limit equal to the PV of the guaranteed *minimum* of future wage income
- *Trend* of wage income diminishes over lifecycle
- Result: optimal portfolio share diminishes over lifecycle; potential U shape with borrowing limit

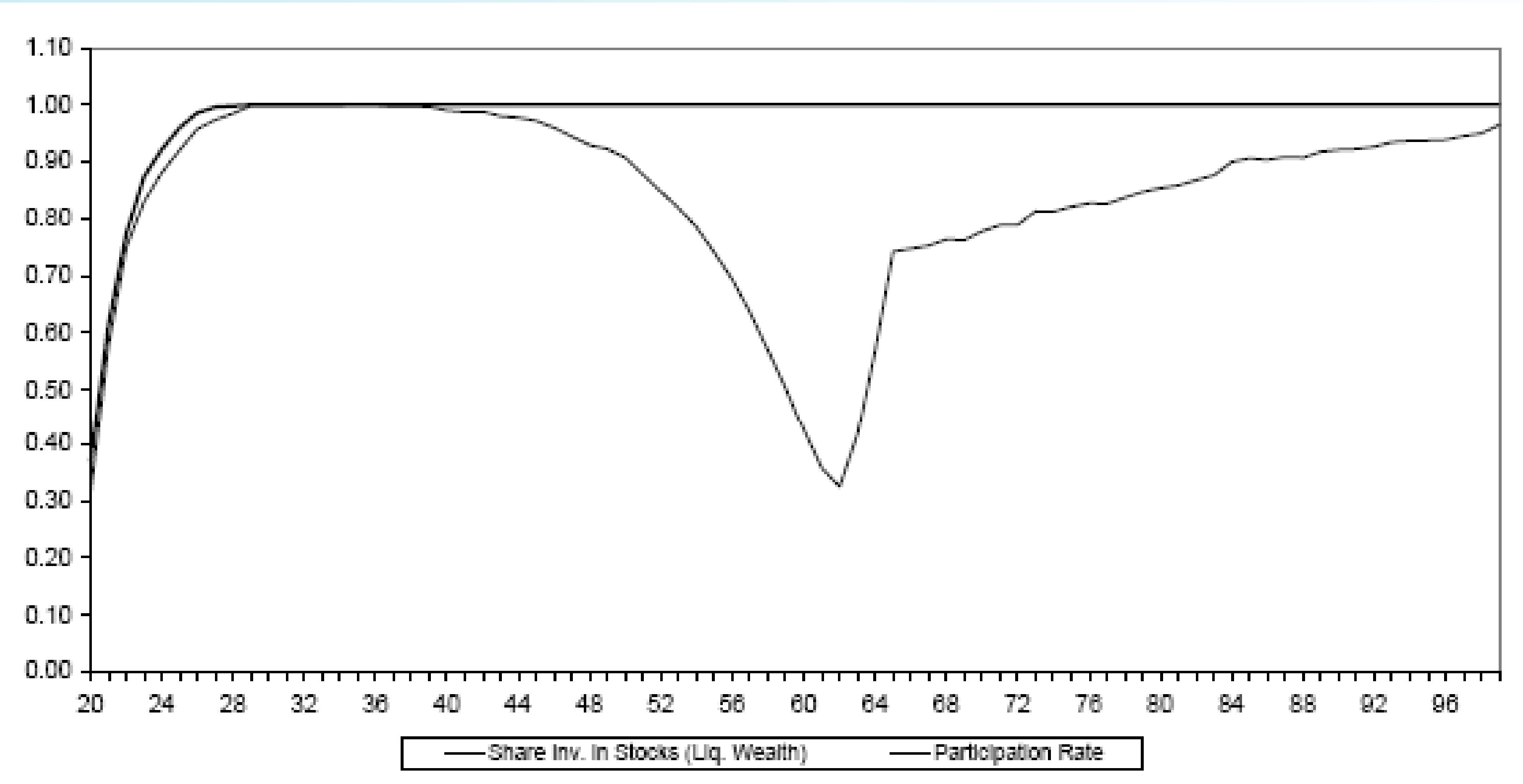
Ex1. Cocco, Gomes and Maenhout (2004)



Ex2. Gomes, Michaelides and Polkovnichenko (2006): examined taxable vs. tax-deferred accounts

Age	Taxable	Tax-Def.	Total
< 36	0.98	1.00	0.99
36 - 50	1.00	0.84	0.96
51 - 65	0.98	0.61	0.81
> 65	0.85	0.57	0.82

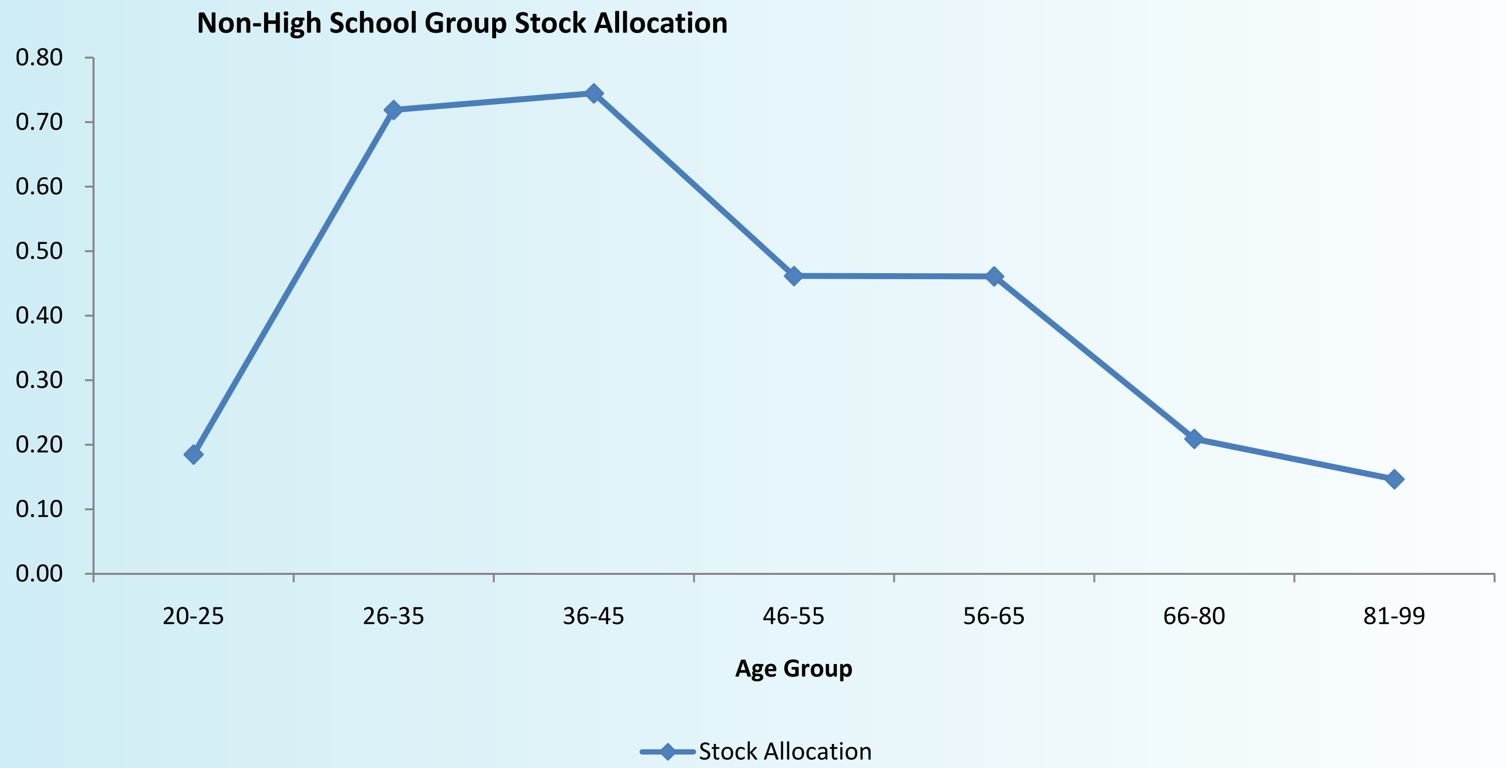
Ex3. Campbell, Cocco, Gomes and Maenhout (1999): Retirement wealth invested 50/50 in risky/safe asset & fixed cost of entering stock market



Empirical Evidence (SCF) (1)

Education Group 1: No High School Education

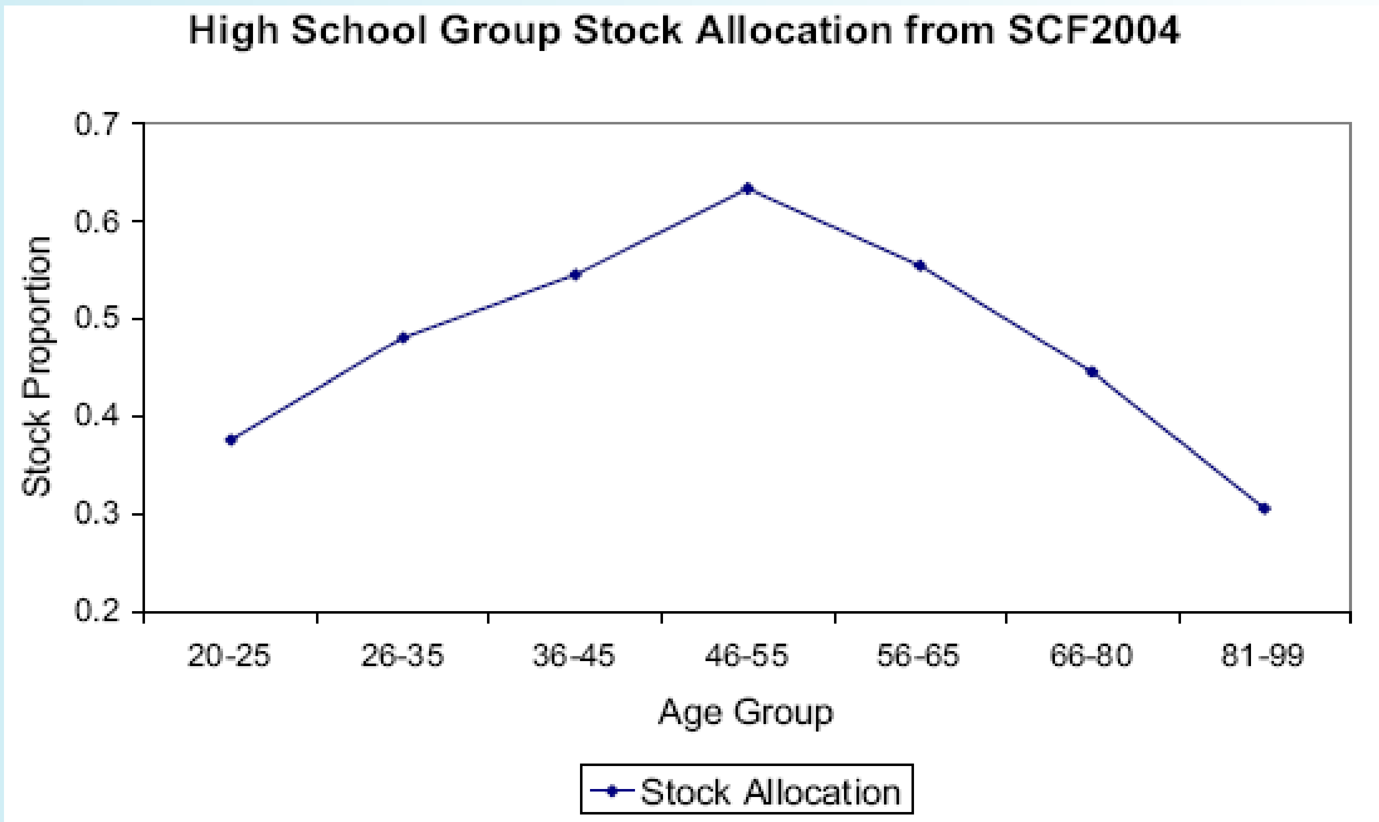
% of Portfolio in Stocks



Empirical Evidence (SCF) (2)

Education Group: High School Education

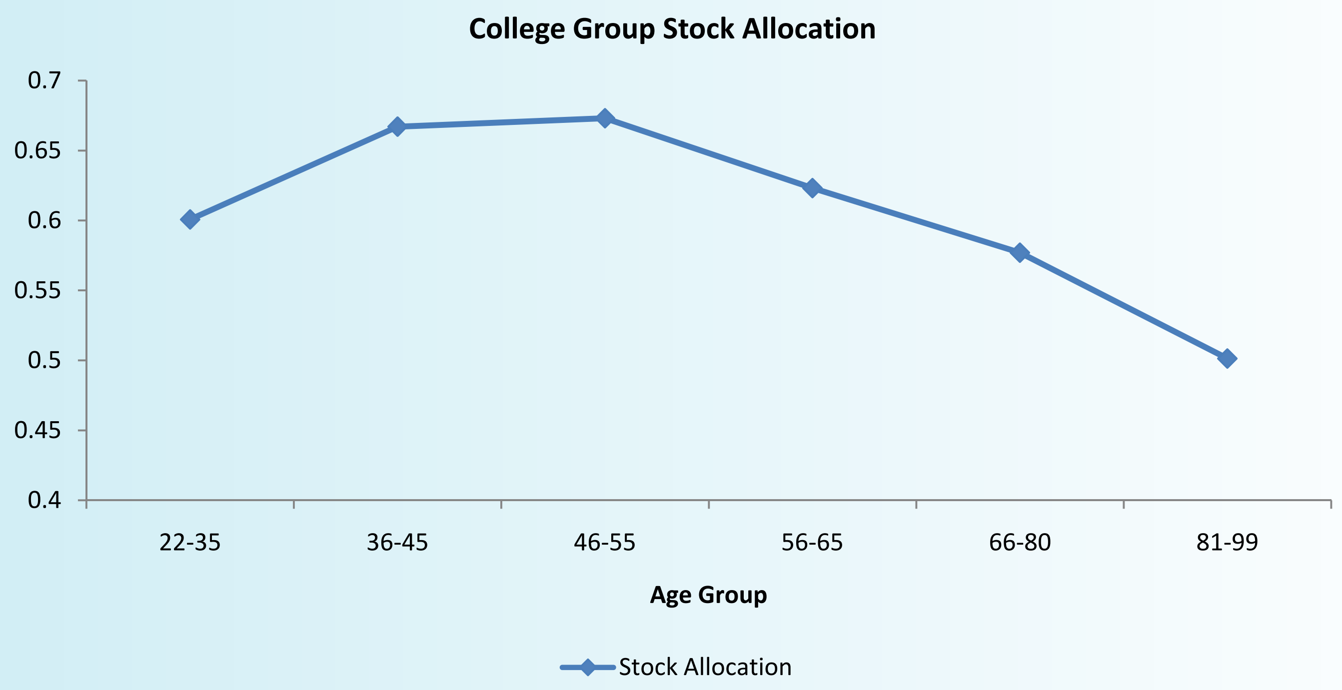
% of Portfolio in Stocks



Empirical Evidence (SCF) (3)

Education Group: College Education

% of Portfolio in Stocks



Three Key "Stylized Facts"

1) The share of a household's portfolio invested in equities is much less than 100% for most households

2) The lifetime poor invest in fewer equities than richer households

3) The share of portfolio invested in risky assets tend to be "hump shape" (∩) in age

Previous papers also found these: Amerkis and Zeldes 2000; Heaton and Lucas 2000; Poterba and Samwick, 2002. AZ, however, warn about data quality.

Evolution of Literature (3)

Inelastic, risky labor supply, other incomplete markets:

- Liquidity constraints (Brown 1990; Amerkis and Zeldes 2000)
- Saving for illiquid assets such as a house (Faig and Shum 2002)
- Incomplete trading markets between generations in a real business cycle economy where wages and stock returns are perfectly correlated (Storesletten, Telmer, and Yaron, 2007)

Other: Habit persistence (Polkovnichenko, 2007)

This Paper

Keep standard model but allow for more realistic fiscal institutions.

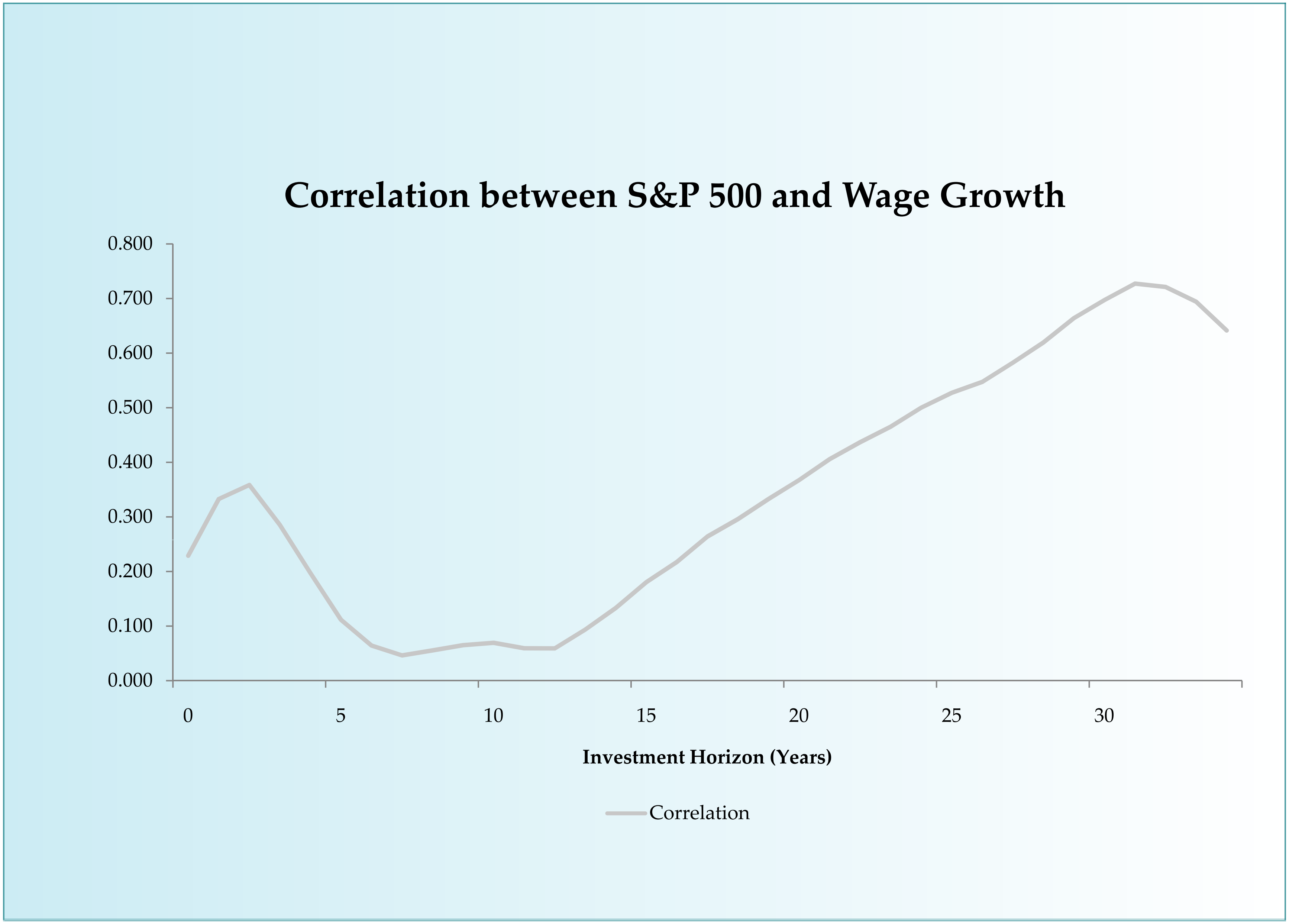
In particular, we investigate the role of *wage-indexed* and *progressive* Social Security benefits

- Wage indexation: previous wages adjusted upward by the economy-wide average growth rate. High correlation (aggregate and low frequency)
- Progressivity: more wage-indexed benefits for poor

Positive vs. Normative Interpretation (?)

Example of Wage Indexing (Only 10 Years)

Year	Economy Avg	Index	Person X	AIYE	(AIME)
2000	$40,000	**1.55**	$ 50,000	$77,566	$6,464
2001	$42,000	**1.48**	$ 45,000	$66,485	$5,540
2002	$44,100	**1.41**	$ 65,000	$91,462	$7,622
2003	$46,305	**1.34**	$ 63,000	$84,426	$7,036
2004	$48,620	**1.28**	$ 74,000	$94,445	$7,870
2005	$51,051	**1.22**	$ 35,000	$42,543	$3,545
2006	$53,604	**1.16**	$ 47,000	$54,408	$4,534
2007	$56,284	**1.10**	$ 39,000	$42,998	$3,583
2008	$59,098	**1.05**	$ 42,000	$44,100	$3,675
2009	$62,053	**1.00**	$ 44,000	$44,000	$3,667
			AVG	**$64,243**	**$5,354**


Correlation between S&P 500 and Wage Growth
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
0
5
10
15
20
25
30
Investment Horizon (Years)
Correlation

Progressivity

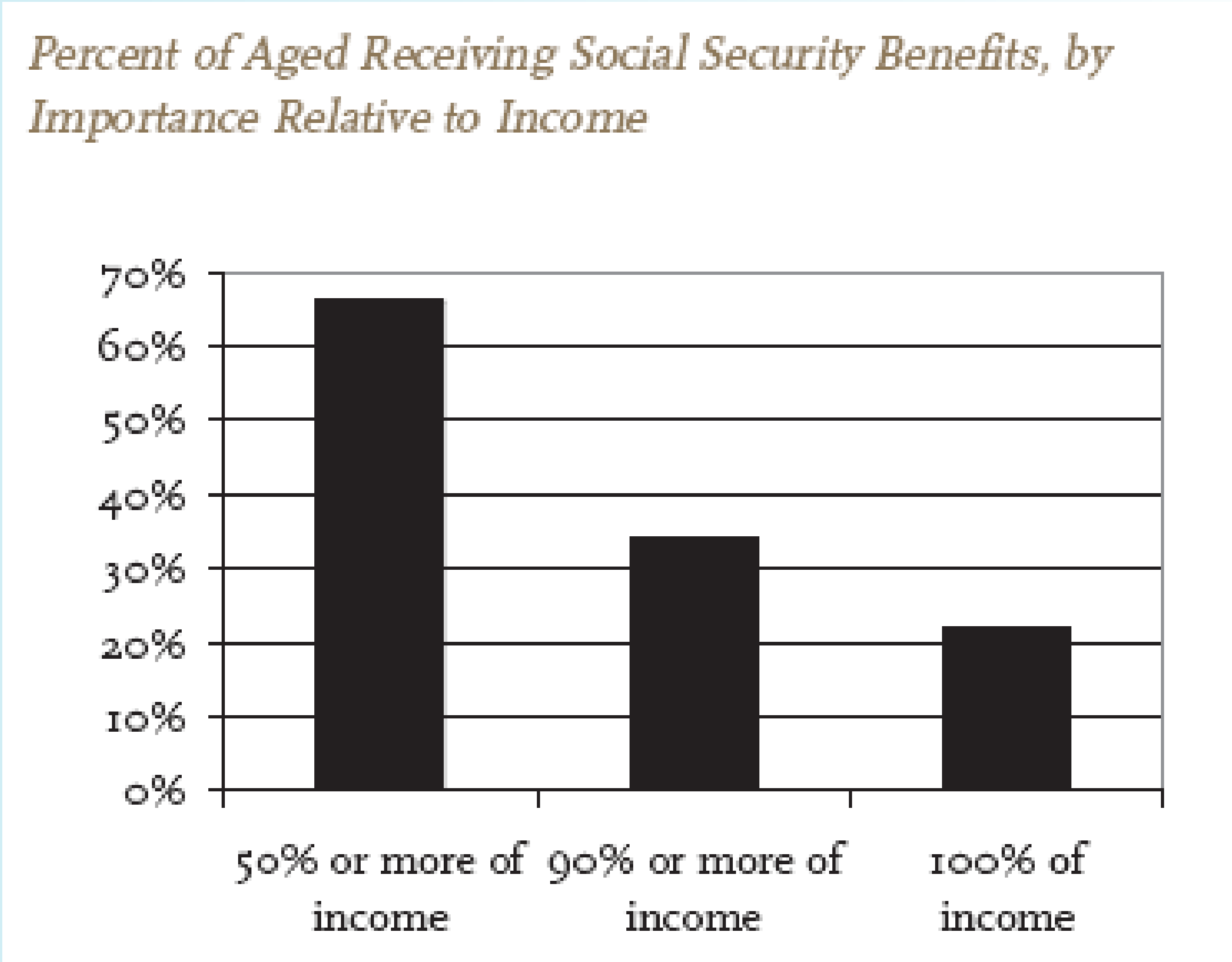
PIA = F(AIYE)

= 0.90 x First $7344

+ 0.32 x (AIYE over $7,344 through $44,268)

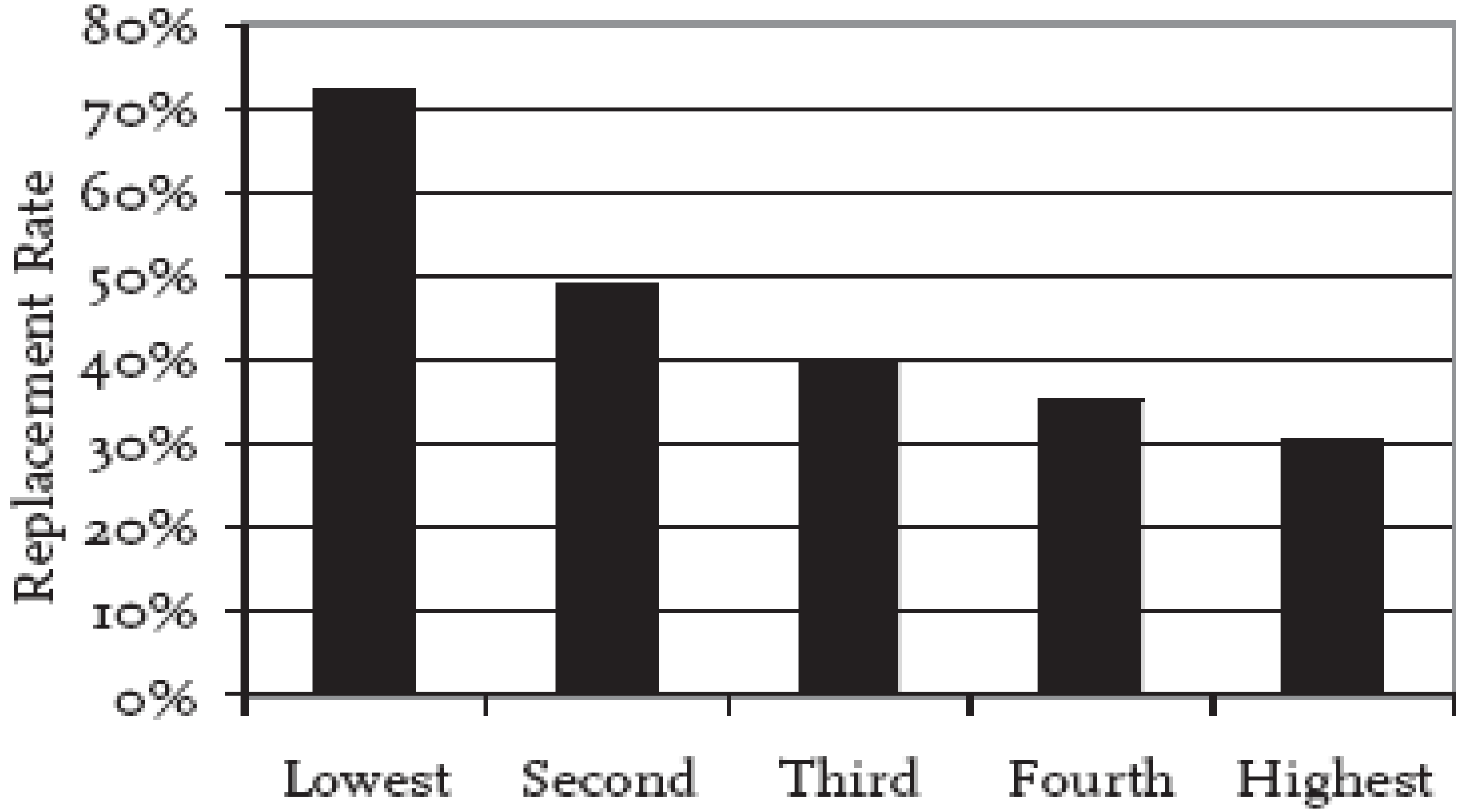
+ 0.15 x (AIYE over $44,268)

Note: Social Security PIA is defined as a function of AIME (monthly basis). We use AIYE (yearly basis) to reduce # of computations.



Source: Munnell and Soto (2005)

Median Social Security Replacement Rates of New Retired-Worker Beneficiaries, by Quintiles of AIME



Source: Munnell and Soto (2005)

Summary and Intuition

Produces “correct” results pre-retirement; post-retirement shares are off

- Wage indexation creates correlation with stock returns at low frequency (Result 1)
- Progressivity breaks homotheticity (i.e., standard wealth separation does not hold) (Result 2)
- Correlation large early in life; then shifts toward bonds later in life => *upward* sloping stock allocation early in life. But human capital also depreciates later in life => downward sloping (Result 3)
- Post-retirement differences against data: “contamination” of traditional financially planning? (normative interpretation of our results)

Model

$$\max E_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \left(\prod_{j=t_0-1}^{t-2} p_j \right) \left(p_{t-1} \frac{C_{i,t}^{1-\gamma}}{1-\gamma} + b(1-p_{t-1}) \frac{W_{i,t}^{1-\gamma}}{1-\gamma} \right)$$

$$R_t^S = R^f + \mu + \eta_t$$

$$Y_{i,t} = \exp(y_{i,t})$$

$$y_{i,t} = g(t, F_{i,t}) + \omega_{i,t} + z_{i,t}$$

$$z_{i,t} = \theta z_{i,t-1} + \xi_t + \phi_{i,t}$$

$$\begin{pmatrix} \eta \\ \xi \end{pmatrix} \xrightarrow{d} N \left(\begin{matrix} \mu_\eta \\ \mu_\xi \end{matrix} , \begin{matrix} \sigma_\eta^2 & \rho_{\xi\eta}\sigma_\xi\sigma_\eta \\ \rho_{\xi\eta}\sigma_\xi\sigma_\eta & \sigma_\xi^2 \end{matrix} \right)$$

$$
\begin{aligned}
Y^d_{i,t} &= (1-\tau_{SS})(1-\tau_w[Y_{i,t}])Y_{i,t} \quad \text{for } t < M \\
&= 0 \quad \text{for } t \geq M
\end{aligned}
$$

$$
\begin{aligned}
\overline{Y}_{i,M} &= \frac{Y_{i,t_0}\frac{Y^A_M}{Y^A_{t_0}} + Y_{i,t_0+1}\frac{Y^A_M}{Y^A_{t_0+1}} + \cdots + Y_{i,M-1}\frac{Y^A_M}{Y^A_{M-1}}}{M-t_0} \\
&= \frac{\left(\left(Y_{i,t_0}\frac{Y^A_{t_0+1}}{Y^A_{t_0}} + Y_{i,t_0+1}\right)\frac{Y^A_{t_0+2}}{Y^A_{t_0+1}} + Y_{i,t_0+2}\right)\cdots}{M-t_0}
\end{aligned}
$$

$$Y_{t+1}^{A} = Y_{t}^{A} \cdot \exp \xi_{t+1}$$

$$\frac{Y_{t+1}^{A}}{Y_{t}^{A}} = \exp \xi_{t+1}$$

$$\overline{Y}_{i,t+1} = \frac{(t - t_0)\overline{Y}_{i,t} \cdot \exp \xi_{t+1} + Y_{i,t+1}}{t - t_0 + 1}$$

$$\begin{aligned} SS_{i,t} &= Q\left(\overline{Y}_{i,M}\right) \text{ for } t \geq M \\ &= 0 \text{ for } t < M \end{aligned}$$

Aggregate wage-stock correlation:

$$\rho_{\xi\eta} = \frac{cov(\xi_t, \eta_t)}{\sigma_\xi \cdot \sigma_\eta}$$

Idiosyncratic wage-stock correlation:

$$\rho_{y\eta} = \frac{cov(\xi_t, \eta_t)}{\sigma_\eta \cdot \sqrt{\sigma_\xi^2 + \sigma_\phi^2 + \sigma_\omega^2}} \approx \frac{cov(\xi_t, \eta_t)}{\sigma_\eta \cdot \sqrt{16 \cdot \sigma_\omega^2}} = \frac{cov(\xi_t, \eta_t)}{4 \cdot \sigma_\xi \cdot \sigma_\eta}$$

Distinction crucial for calibration at low and high frequency

$$X_{i,t} = W_{i,t} + (1 - h_t)Y_{i,t}^d + SS_{i,t}$$

$$\begin{aligned} R_{i,t} &= \left[\alpha_{i,t} \cdot R_{i,t}^S + (1 - \alpha_{i,t}) \cdot R_f - 1\right] \cdot (1 - \tau_d) + 1 \quad \text{for } R_{i,t}^S > 1 \\ &= \alpha_{i,t} \cdot R_{i,t}^S + (1 - \alpha_{i,t}) \cdot \left[(R_f - 1) \cdot (1 - \tau_d) + 1\right] \quad \text{for } R_{i,t}^S \leq 1 \end{aligned}$$

four state variables, $\left\{t, X_{i,t}, z_{i,t}, \overline{Y}_{i,t+1}\right\}$

$$V_{i,t}(X_{i,t}, z_{i,t}) = \max_{C_{i,t},\alpha_{i,t}} \left\{ \frac{(C_{i,t})^{1-\gamma}}{1-\gamma} + \beta \left[p_t \cdot E_t V_{i,t+1}(X_{i,t+1}, z_{i,t+1}) + b(1-p_t) \cdot E_t \left(\frac{(W_{i,t+1})^{1-\gamma}}{1-\gamma} \right) \right] \right\}$$

$$\alpha_{i,t} \in [0,1]$$

$$X_{i,t+1} = R_{i,t}(X_{i,t} - C_{i,t}) + (1-h_{t+1})Y^d_{i,t+1} + SS_{i,t}$$

$$0 \le \alpha_{i,t}(X_{i,t} - C_{i,t}) \notin (0,m)$$

Numerical solution requires use of a super-computer due to state space and constraints. About 600,000,000 optimization problems. Shape preservation and other factors important for creating small Euler errors.

Calibration (1)

Common problem with structural "data fitting" exercises: lots of deep parameters and few observable moments, i.e., too many degrees of freedom

• This point is a little strong since data fitting is still hard when sufficient *structure* imposed (this isn't reduced-form econometrics like the DRI model). Still, want to make sure that the model does not just fit some observables without hitting others (not perfect)

Calibration (2)

Use wealth-income ratios as "over-identifying" restrictions, i.e., we don't match on portfolio shares.

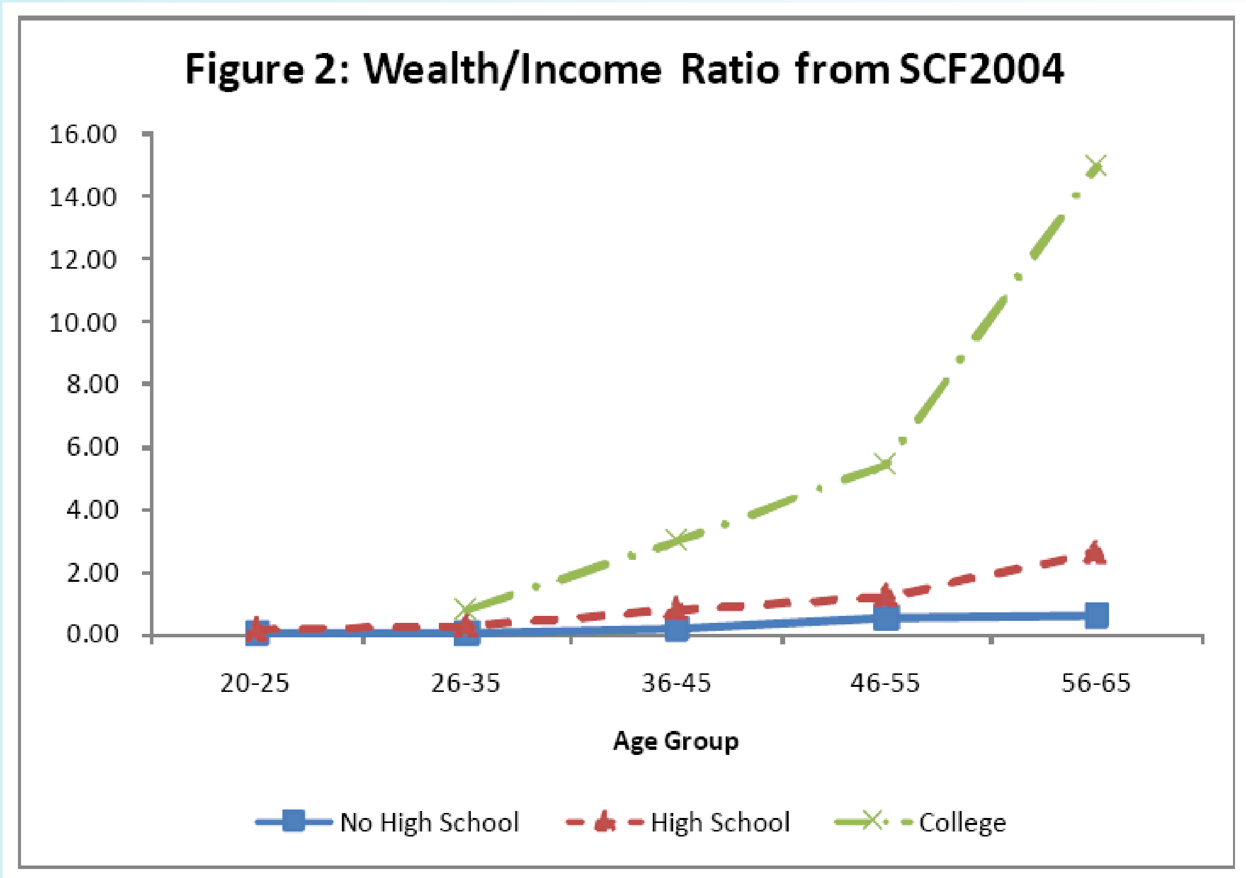


Table 1: Common Parameters in All Simulations

Description	Parameter Value
Start working age (t_0)	20/22
Retirement age (M)	65
Discount factor (β)	see text
Coefficient of relative risk aversion (γ)	see text
Riskless returns (R_f)	1.02
Mean risky returns ($R_f + \mu$)	1.06
Variance of transitory income shocks (σ_ω^2)	0.01
Variance of permanent income shocks (σ_u^2)	0.01
Standard deviation of stock returns (σ_η)	0.157
Correlation between stock returns and income shocks ($\rho_{\xi\eta}$)	0.15
Social security tax rate (τ_{SS})	0.124
Investment return tax rate (τ_{SS})	0.20
Bequest intensity (b)	5
Minimum investment required in stock account (m)	\$3000 (when on)
Marital status	single
Family size	1

Table 2: Labor Income Process

Coefficient of characteristic variables for labor income	No high school	High school	College
Constant	2.6275	2.7004	2.3831
Marital status	0.4008	0.4437	0.4831
Family size	-0.0176	-0.0236	-0.0228
Coefficient of age dummies for labor income			
Constant	-2.1361	-2.1700	-4.3148
Age	0.1684	0.1682	0.3194
$\text{Age}^2/10$	-0.0353	-0.0323	-0.0577
$\text{Age}^3/100$	0.0023	0.0020	0.0033

Table 3: 2004 Progressive Federal Personal Income Tax Rates

Tax Rate	Single Filers
10%	Up to $7,150
15%	$7,151 - $29,050
25%	$29,051 - $70,350
28%	$70,351 - $146,750
33%	$146,751 - $319,100
35%	$319,101 or more

Table 4: Calibrated Parameters for High School Group in 4 Cases

	No Minimum Investment $m = \$0$	Minimum Investment $m = \$3000$
$\rho_{\xi\eta} = 0.15$	(i): $\beta = 0.76$, $\gamma = 6$	(ii): $\beta = 0.76$, $\gamma = 6$
$\rho_{\xi\eta} = 0.80$	(iii): $\beta = 0.86$, $\gamma = 4$	(iv): $\beta = 0.86$, $\gamma = 4$

Table 5: Calibrated Parameters for College Group in 4 Cases

	No Minimum Investment $m = \$0$	Minimum Investment $m = \$3000$
$\rho_{\xi\eta} = 0.15$	(i): $\beta = 0.60$, $\gamma = 6$	(ii): $\beta = 0.60$, $\gamma = 6$
$\rho_{\xi\eta} = 0.80$	(iii): $\beta = 0.74$, $\gamma = 4$	(iv): $\beta = 0.74$, $\gamma = 4$

Table 6: Calibrated Parameters for No-High School Group in 4 Cases

	No Minimum Investment $m = \$0$	Minimum Investment $m = \$3000$
$\rho_{\xi\eta} = 0.15$	(i): $\beta = 0.86$, $\gamma = 6$	(ii): $\beta = 0.86$, $\gamma = 6$
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Results

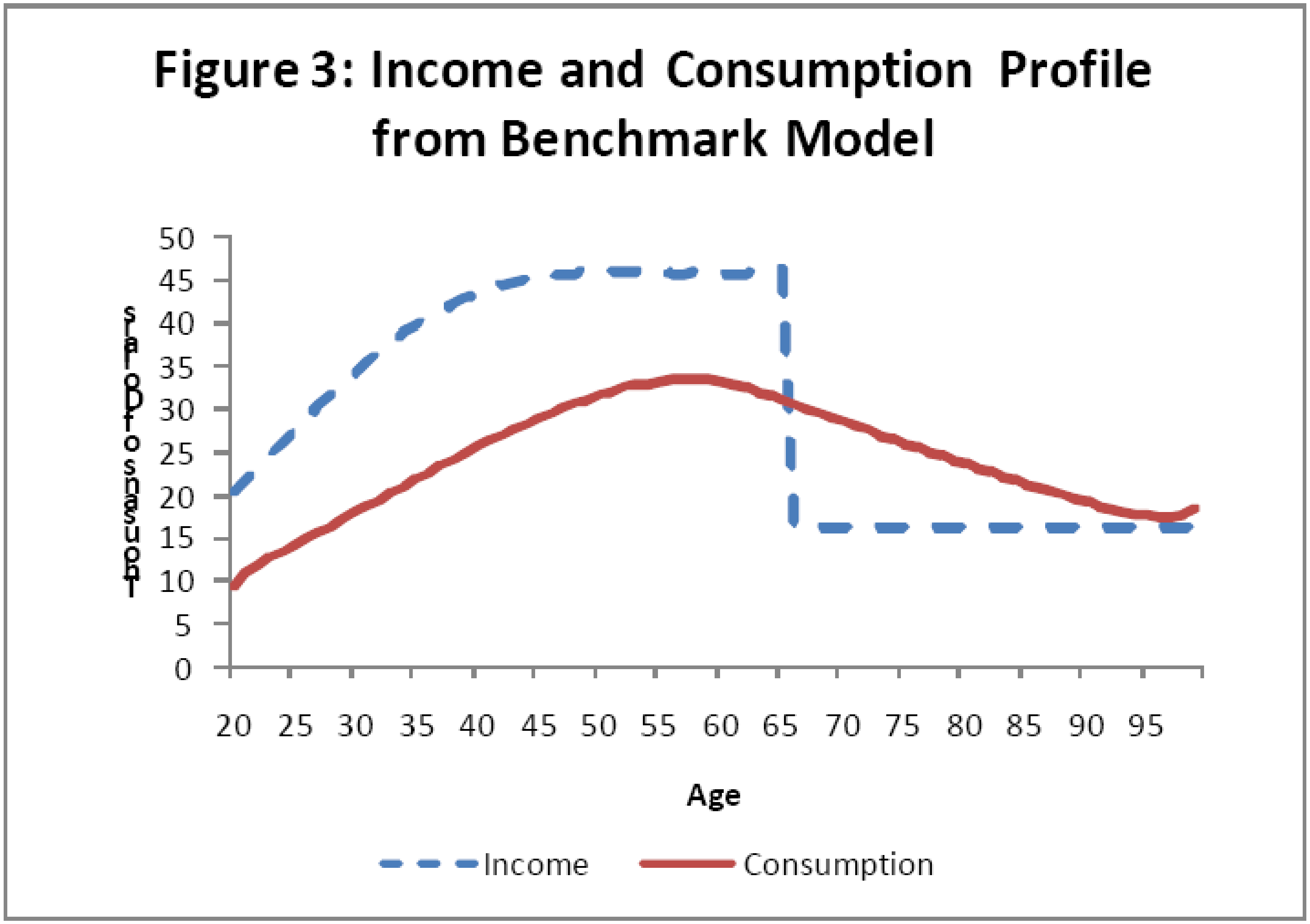

Figure 3: Income and Consumption Profile from Benchmark Model
50
45
40
35
30
25
20
15
10
5
0
20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Age
Income
Consumption

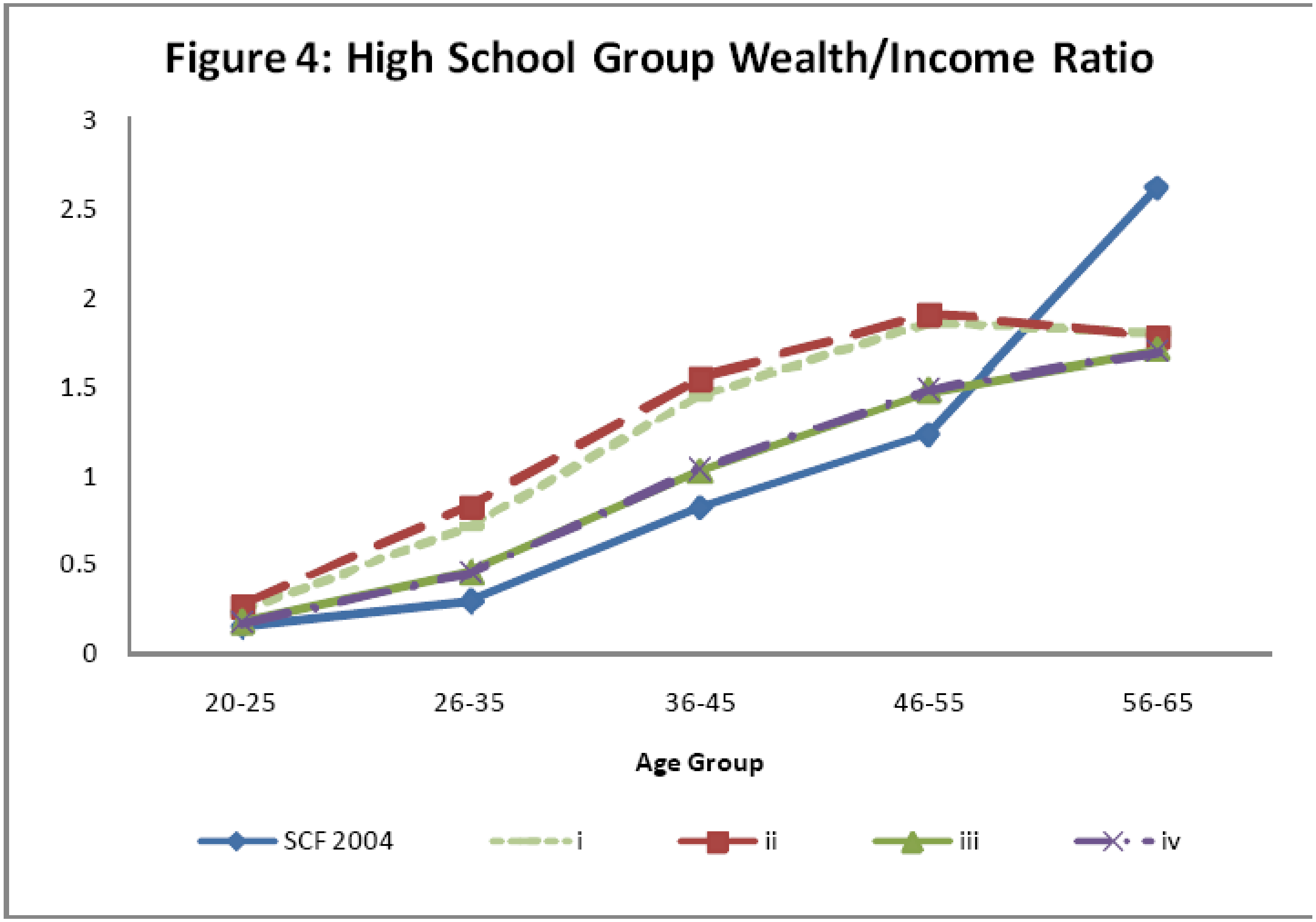
Figure 4: High School Group Wealth/Income Ratio
3
2.5
2
1.5
1
0.5
0
20-25
26-35
36-45
46-55
56-65
Age Group
SCF 2004
i
ii
iii
iv

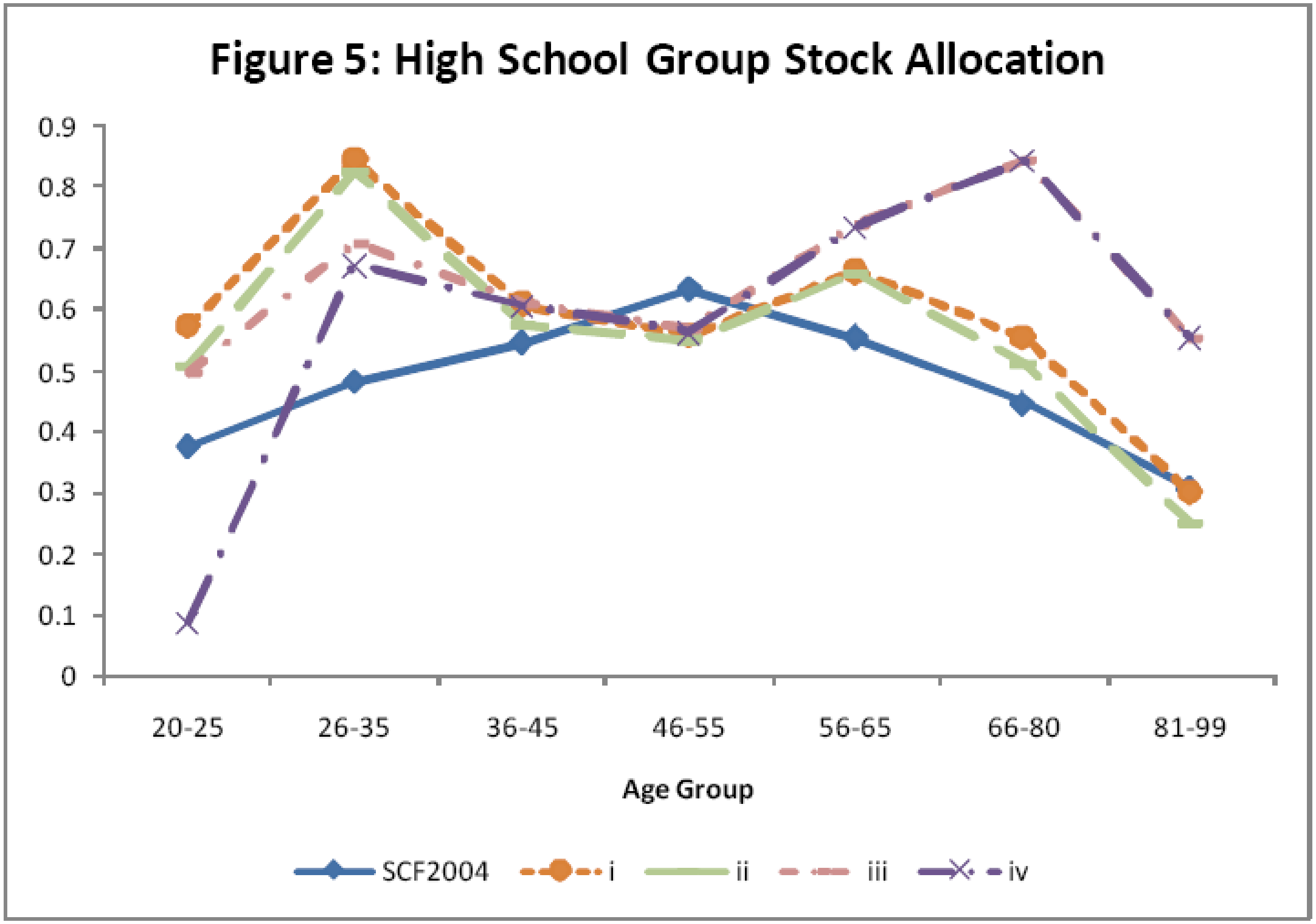
Figure 5: High School Group Stock Allocation
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20-25
26-35
36-45
46-55
56-65
66-80
81-99
Age Group
SCF2004
i
ii
iii
iv

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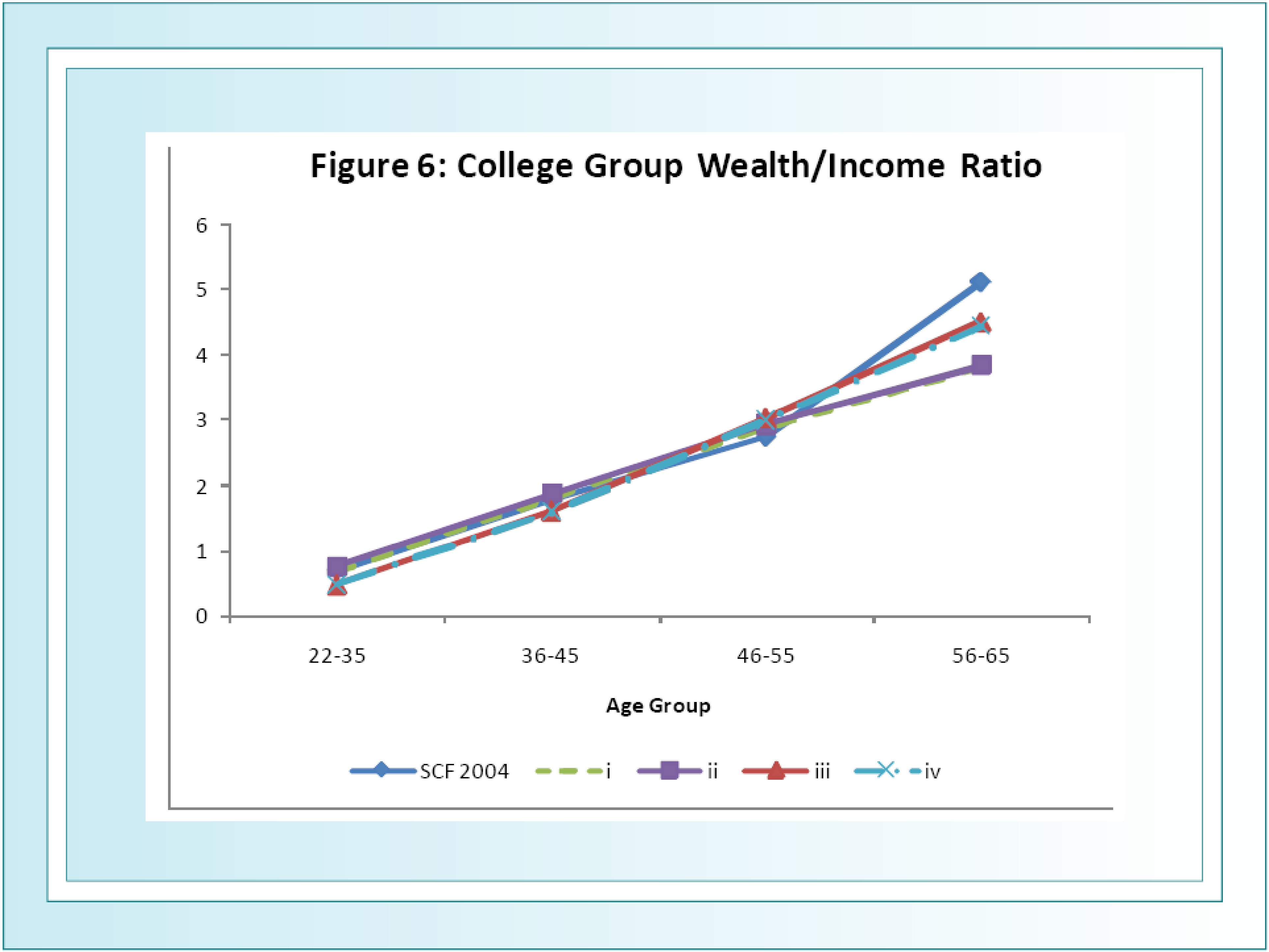

Figure 6: College Group Wealth/Income Ratio
6
5
4
3
2
1
0
22-35
36-45
46-55
56-65
Age Group
SCF 2004
i
ii
iii
iv

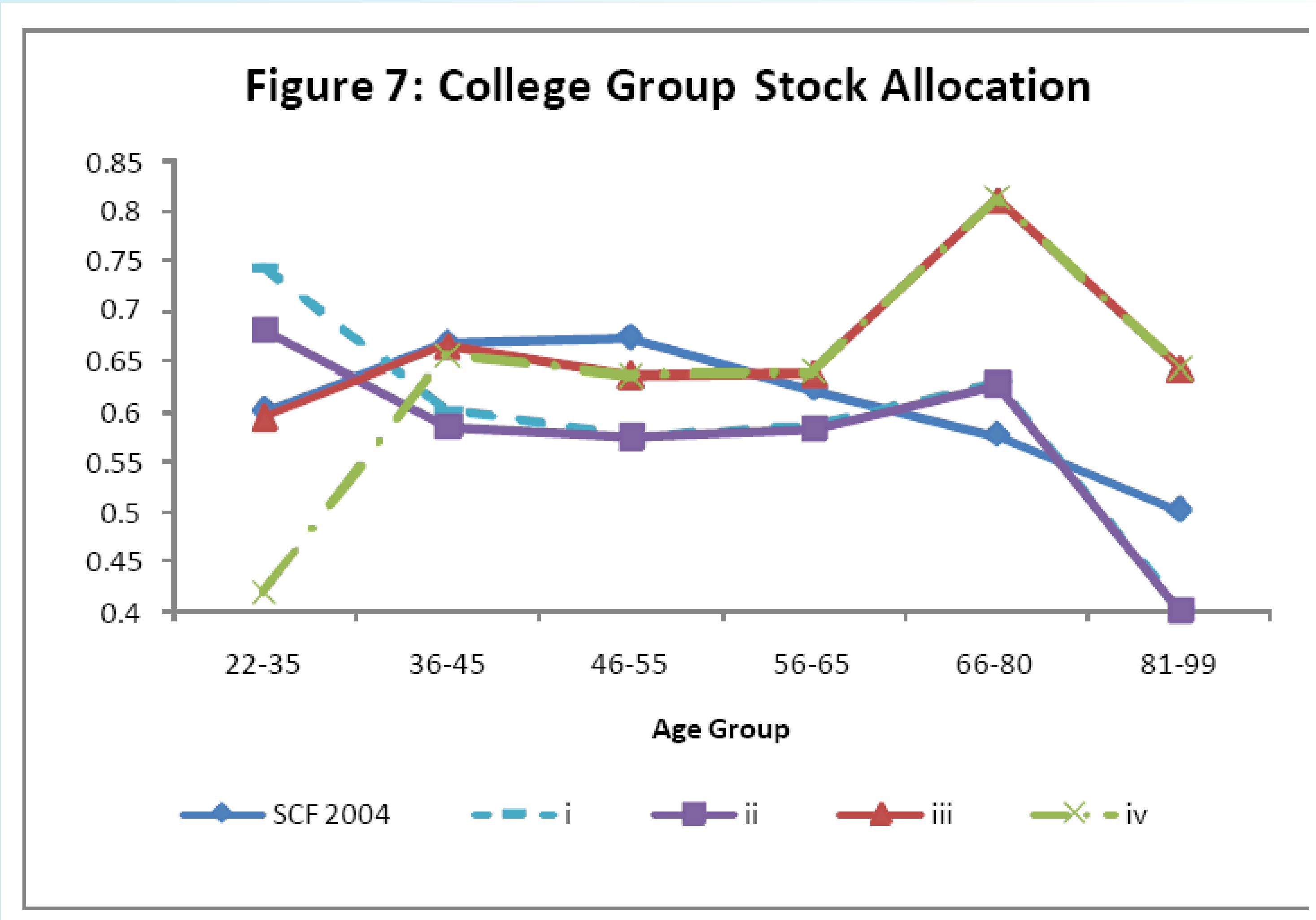
Figure 7: College Group Stock Allocation
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
22-35
36-45
46-55
56-65
66-80
81-99
Age Group
SCF 2004
i
ii
iii
iv

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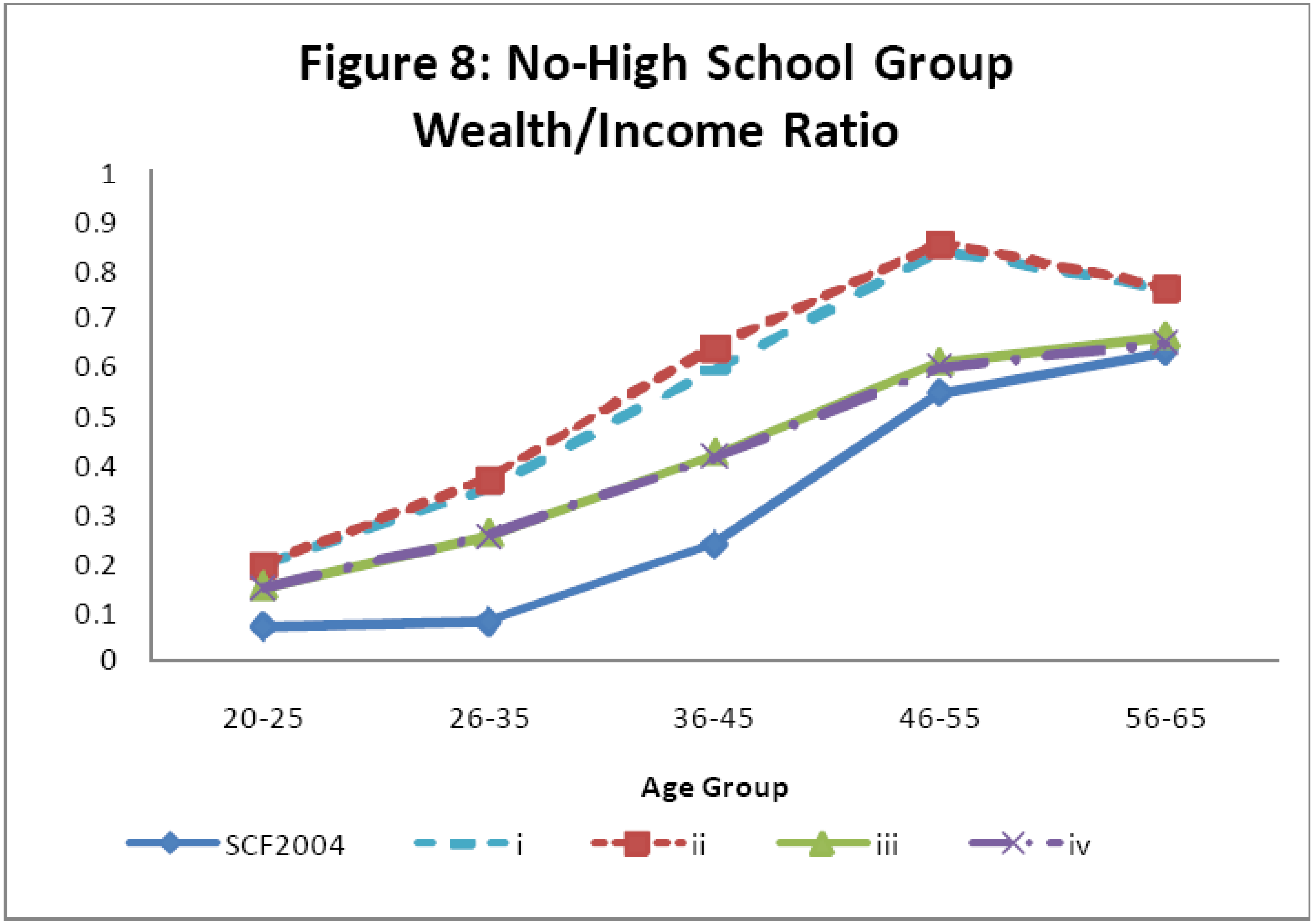
Figure 8: No-High School Group
Wealth/Income Ratio
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20-25
26-35
36-45
46-55
56-65
Age Group
SCF2004
i
ii
iii
iv

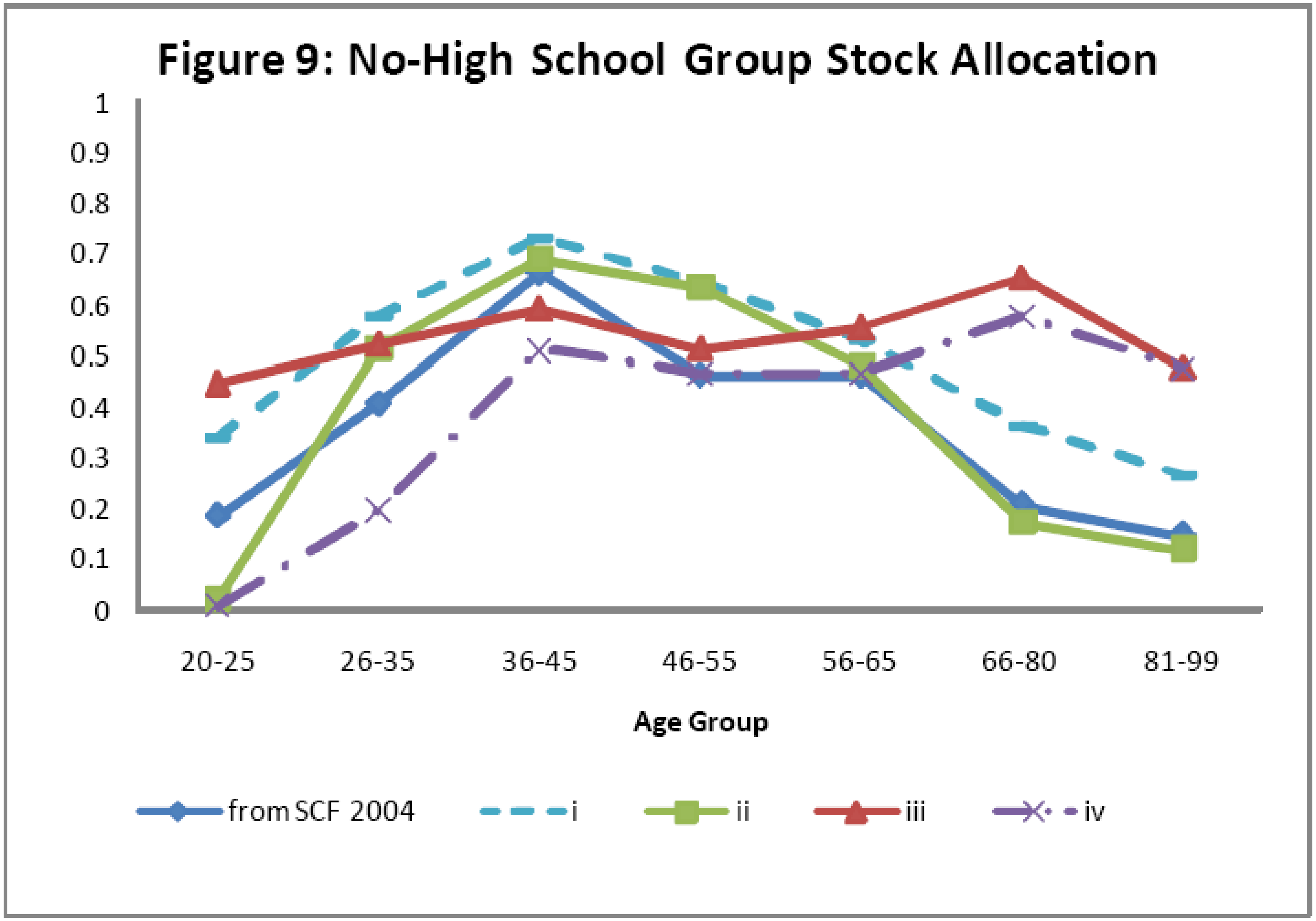
Figure 9: No-High School Group Stock Allocation
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20-25
26-35
36-45
46-55
56-65
66-80
81-99
Age Group
from SCF 2004
i
ii
iii
iv

Table 6: Calibrated Parameters for No-High School Group in 4 Cases

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$\rho_{\xi\eta} = 0.15$	(i): $\beta = 0.86$, $\gamma = 6$	(ii): $\beta = 0.86$, $\gamma = 6$
$\rho_{\xi\eta} = 0.80$	(iii): $\beta = 0.94$, $\gamma = 4$	(iv): $\beta = 0.94$, $\gamma = 4$

Conclusions

Role of fiscal policy in distorting both savings and portfolio choice

Results can be interpreted both positively and normatively

Standard financial planning rules (e.g., bond % equal to your age) might be very off at both sides of the age spectrum: should hold fewer stocks early in life and more stocks after retirement.