

Deflation Risk

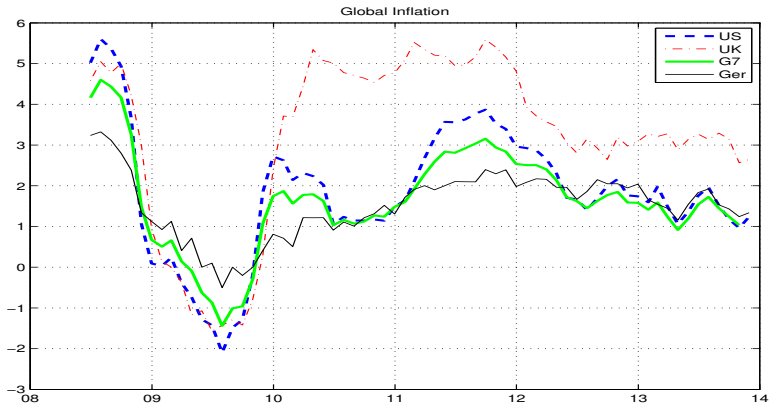
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- Deflation has played a central role during the worst economic meltdowns in U.S. history.
 - Panic of 1837
 - Long Depression of 1874-1896
 - Great Depression of the 1930s
- Growing fears of deflation in the financial press.
 - "Nightmare scenario"
 - "Looming disaster"
 - "Growing threat"
- Mitigating risk of deflation is an explicit motivation behind many recent measures by the Federal Reserve such as the Quantitative Easing Programs.

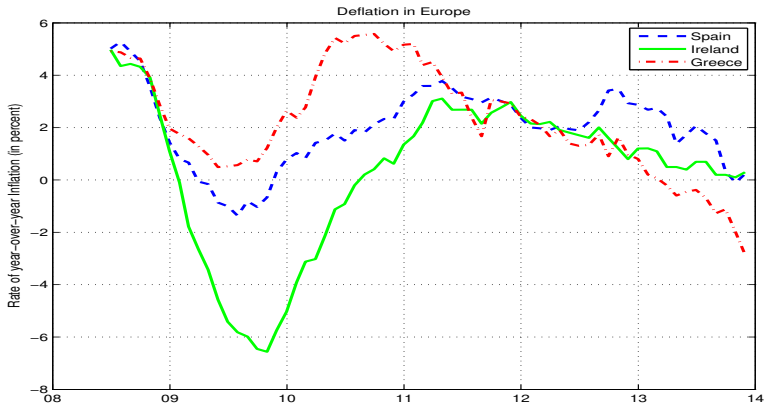
- Relatively little is known about the probability of deflation.
- Reason may be that the distribution of inflation is difficult to measure.
- Ang, Bekaert, and Wei (2007) show that econometric models perform poorly in estimating first moment of inflation.
- Survey data does better, but only looks at first moments, not tail probabilities.

Recent Decline in Global Inflation



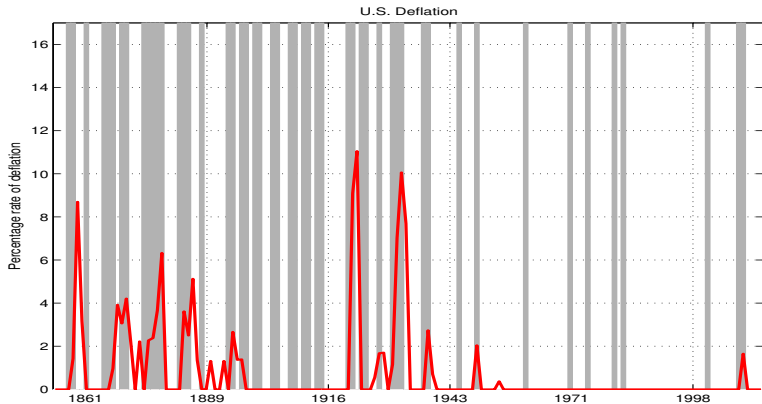
* year-over-year inflation rate in percentages.

Deflation in Europe's Periphery



* year-over-year inflation rate in percentages.

History of U.S. Deflation



* year-over-year deflation rate in percentages. shaded areas are NBER recessions

- We use a new market based approach to measure deflation risk.
 - First we solve for the risk-neutral density of inflation from the market prices of inflation calls and puts.
 - Then we solve for the inflation risk premium via MLE estimation of an affine term structure model using the term structure of inflation swap rates.
 - Finally, we solve for the objective density of inflation by adjusting the pricing measure for the market price of risk.

- Long run expected inflation is about 3.00 percent.
- Average inflation risk premium is close to zero for horizons out to about 20 years.
- Inflation volatility is roughly 1.50 percent.
- Probability of deflation sizable.
 - Bernanke, Aug 27, 2010, "Falling into deflation is not a significant risk."
 - On same date, market-implied probability of deflation was 27.35 percent for two-year horizon, 16.15 percent for five-year horizon, and 6.39 percent for a ten-year horizon.
- Tail risk of deflation priced similarly to other types of tail risk such as catastrophe insurance and corporate bond defaults.
- Deflation correlated with other types of financial and economic tail risk such as systemic credit risk, liquidity risk, and unemployment.
- Inflation risk is priced less severely by the market.

- Objective Measure

$$\begin{aligned}dl &= Xl dt + \sigma l dZ_l \\dX &= \kappa(Y - X) dt + \eta dZ_X \\dY &= (\mu - \xi Y) dt + s dZ_Y\end{aligned}$$

- Risk-Neutral Measure

$$\begin{aligned}dl &= Xl dt + \sigma l dZ_l \\dX &= \lambda(Y - X) dt + \eta dZ_X \\dY &= (\alpha - \xi Y) dt + s dZ_Y\end{aligned}$$

Valuing Inflation Swaps

- The payoff on a T -year inflation swap paying f on the fixed leg is $(1+f)^{10} - I_{10}$.
- The value of the inflation swap at time zero is

$$F(X, Y, T) = \exp(-A(T) - B(T)X - C(T)Y)$$

where

$$A(T) = \frac{\sigma^2}{2\lambda^2} \left(T - \frac{2}{\lambda} (1 - e^{-\lambda T}) + \frac{1}{2\lambda} (1 - e^{-2\lambda T}) \right) \dots$$

$$B(T) = \frac{-(1 - e^{-\lambda T})}{\lambda}$$

$$C(T) = \frac{\lambda}{\xi - \lambda} \left[\frac{1}{\xi} (1 - e^{-\xi T}) - \frac{1}{\lambda} (1 - e^{-\lambda T}) \right]$$

Valuing Inflation Options

- The payoff on a European-style inflation cap at expiration date T is $\max(0, I_T - (1 + K)^T)$. Similarly, for a floor $\max(0, (1 + K)^T - I_T)$.
- The value of the call option at time zero is

$$C(X, Y, V, T) = D(T) E^{Q^*} \left[\max(0, I_T - (1 + K)^T) \right],$$

- The expectation is taken with respect to the adjusted risk-neutral measure Q^* for inflation defined by the following dynamics

$$\begin{aligned} dl &= Xl dt + \sigma l dZ_l \\ dX &= \left[\lambda(Y - X) + \eta^2 B(T - t) \right] dt + \eta dZ_X \\ dY &= \left[\alpha - \xi Y + s^2 C(T - t) \right] dt + s dZ_Y \end{aligned}$$

- Inflation swaps
 - Maturities from one to 55 years for the period from July 23, 2004 to July 29, 2014.
- Inflation caps and floors
 - Strikes from -2% to 6% in increments of 50bps, and maturities from 1 to 30 years for the period from October 5, 2009 to July 29, 2014.
- Inflation surveys
 - University of Michigan Survey of Consumers, the Philadelphia Federal Reserve Bank Survey of Professional Forecasters, and the Livingston Survey for the period from July 2004 to July 2014.
- Measures of financial tail risk
 - One-year Refcorp-Treasury yield spread, 10–15 % CDX IG index tranche prices, one-year Libor-Treasury spread, five-year swap spread, VIX index, Merrill Lynch MOVE index, Baa spread over the five-year Treasury rate, spread for a five-year CDS contract on the U.S. Treasury.
- Macroeconomic variables
 - U.S. industrial production, U.S. unemployment rate, University of Michigan consumer confidence index.

Table 1

Summary Statistics for Inflation Swap Rates. This table reports summary statistics for the inflation swap rates for the indicated maturities. Swap maturity is expressed in years. Inflation swap rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to July 29, 2014.

Swap Maturity	Mean	Standard Deviation	Minimum	Median	Maximum	<i>N</i>
1	1.743	1.244	-4.545	1.862	3.802	2613
2	1.919	0.987	-3.605	1.992	3.460	2613
3	2.077	0.775	-2.047	2.141	3.351	2613
4	2.207	0.626	-1.228	2.268	3.342	2613
5	2.314	0.525	-0.570	2.377	3.310	2613
6	2.394	0.450	-0.080	2.468	3.310	2613
7	2.465	0.389	0.402	2.530	3.229	2613
8	2.522	0.344	0.640	2.580	3.195	2613
9	2.570	0.304	0.904	2.622	3.135	2613
10	2.615	0.272	1.146	2.664	3.145	2613
12	2.671	0.255	1.280	2.713	3.160	2613
15	2.739	0.256	1.161	2.778	3.330	2613
20	2.800	0.266	1.070	2.845	3.360	2613
25	2.841	0.277	1.211	2.887	3.390	2613
30	2.888	0.276	1.455	2.923	3.500	2613
35	2.810	0.156	2.353	2.809	3.119	1023
40	2.769	0.115	1.454	2.828	3.377	1649
45	2.866	0.169	2.220	2.894	3.305	1023
50	2.784	0.139	1.465	2.821	3.500	1649
55	2.878	0.165	2.386	2.861	3.287	1023

Table 2

Summary Statistics for Inflation Caps and Floors. This table reports the average values for inflation caps and floors for the indicated maturities and strikes. The average values are expressed in terms of basis points per \$100 notional. Option Maturity is expressed in years. Ave. denotes the average number of caps and floors available each day from which the risk-neutral density of inflation is estimated. N denotes the number of days for which the risk-neutral density of inflation is estimated. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

Option Maturity	Average Floor Value by Strike								Average Cap Value by Strike								Ave.	N
	-2	-1	0	1	2	3	4	5	-1	0	1	2	3	4	5	6		
1	4	8	17	38	85	161	294	398	237	153	92	38	13	6	3	2	22.5	1210
2	12	19	33	67	151	304	544	749	523	349	201	85	34	16	9	6	22.6	1103
3	14	22	37	76	184	399	752	1053	832	571	346	151	59	27	16	10	23.5	1222
5	23	33	50	103	241	563	1058	1565	1456	1044	658	309	118	56	31	20	23.1	1075
7	23	34	56	115	278	679	1298	1999	2031	1504	1010	516	208	105	58	37	22.9	1124
10	19	31	59	134	335	832	1561	2536	2752	2111	1523	849	395	172	99	58	22.5	1214
12	17	28	58	136	354	910	1704	2863	3117	2419	1803	1030	484	224	113	71	23.0	1218
15	16	27	55	136	372	1014	1920	3346	3532	2788	2164	1260	597	277	141	78	22.8	1198
20	15	25	54	136	399	1182	2259	3959	3975	3266	2667	1582	748	333	186	104	22.4	1121
30	10	20	52	154	533	1755	2867	5896	4511	3883	3607	2227	1144	584	328	197	20.4	789

Maximum Likelihood Estimation

- We assume that the two-year and 30-year inflation swap rates are measured without error.
- Thus, given a parameter vector Θ , X and Y can be expressed as explicit linear functions of the two inflation swap prices $F(X, Y, 2)$ and $F(X, Y, 30)$.

$$\ln F(X, Y, 2) = -A(2) - B(2)X - C(2)Y,$$

$$\ln F(X, Y, 30) = -A(30) - B(30)X - C(30)Y,$$

- The log of the joint likelihood function LLK_t of the two-year and 30-year inflation swap prices is

$$\begin{aligned} &= -\ln\left(2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{XY}}\right) - \frac{1}{2(1-\rho_{XY}^2)} \left[\frac{(X_{t+\Delta t} - \mu_{X_t})^2}{\sigma_X^2} \right. \\ &\quad \left. - 2\rho_{XY} \left(\frac{X_{t+\Delta t} - \mu_{X_t}}{\sigma_X} \right) \left(\frac{Y_{t+\Delta t} - \mu_{Y_t}}{\sigma_Y} \right) + \frac{(Y_{t+\Delta t} - \mu_{Y_t})^2}{\sigma_Y^2} \right] \end{aligned}$$

- We maximize the log likelihood function over the 22-dimensional parameter vector

Table 3

Maximum Likelihood Estimation of the Inflation Swap Model. This table reports the maximum likelihood estimates of the parameters of the inflation swap model along with their asymptotic standard errors. The model is estimated using daily inflation swap prices for the period from July 23, 2004 to July 29, 2014.

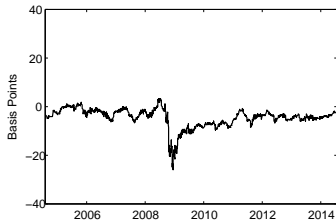
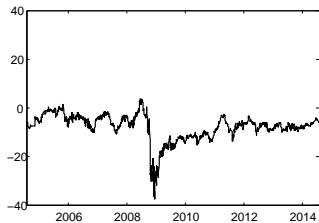
Parameter	Value	Standard Error
κ	0.963868	0.324027
η	0.042007	0.000625
μ	0.006743	0.002709
ξ	0.026974	0.111689
s	0.008720	0.000008
λ	0.866571	0.005804
α	0.001680	0.000001
v_1	0.00000665	0.00000075
v_3	0.00000086	0.00000021
v_4	0.00000180	0.00000035
v_5	0.00000246	0.00000050
v_6	0.00000263	0.00000055
v_7	0.00000287	0.00000064
v_8	0.00000298	0.00000069
v_9	0.00000331	0.00000091
v_{10}	0.00000365	0.00000144
v_{12}	0.00000295	0.00000068
v_{15}	0.00000199	0.00000039
v_{20}	0.00000101	0.00000023
v_{25}	0.00000047	0.00000016

Table 4

Summary Statistics for Inflation Risk Premia. This table reports summary statistics for the estimated inflation risk premia for the indicated horizons. Horizon is expressed in years. The inflation risk premia are measured in basis points. The inflation risk premia are estimated using the period from July 23, 2004 to July 29, 2014.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	<i>N</i>
1	-5.45	4.58	-30.01	-4.80	4.46	2613
2	-7.79	5.43	-37.28	-7.01	3.81	2613
3	-7.68	5.08	-35.50	-6.96	3.07	2613
4	-6.35	4.45	-30.84	-5.72	3.01	2613
5	-4.53	3.83	-25.74	-3.99	3.49	2613
6	-2.64	3.32	-21.06	-2.17	4.26	2613
7	-0.89	2.90	-17.04	-0.48	5.12	2613
8	0.63	2.57	-13.70	1.00	5.92	2613
9	1.86	2.30	-10.99	2.19	6.58	2613
10	2.79	2.08	-8.66	3.08	7.04	2613
12	3.72	1.74	-6.08	3.99	7.26	2613
15	2.91	1.40	-4.99	3.13	5.73	2613
20	-3.55	1.05	-9.47	-3.39	-1.45	2613
25	-15.04	0.83	-19.69	-14.92	-13.36	2613
30	-30.20	0.67	-33.94	-30.10	-28.80	2613

Inflation Risk Premia



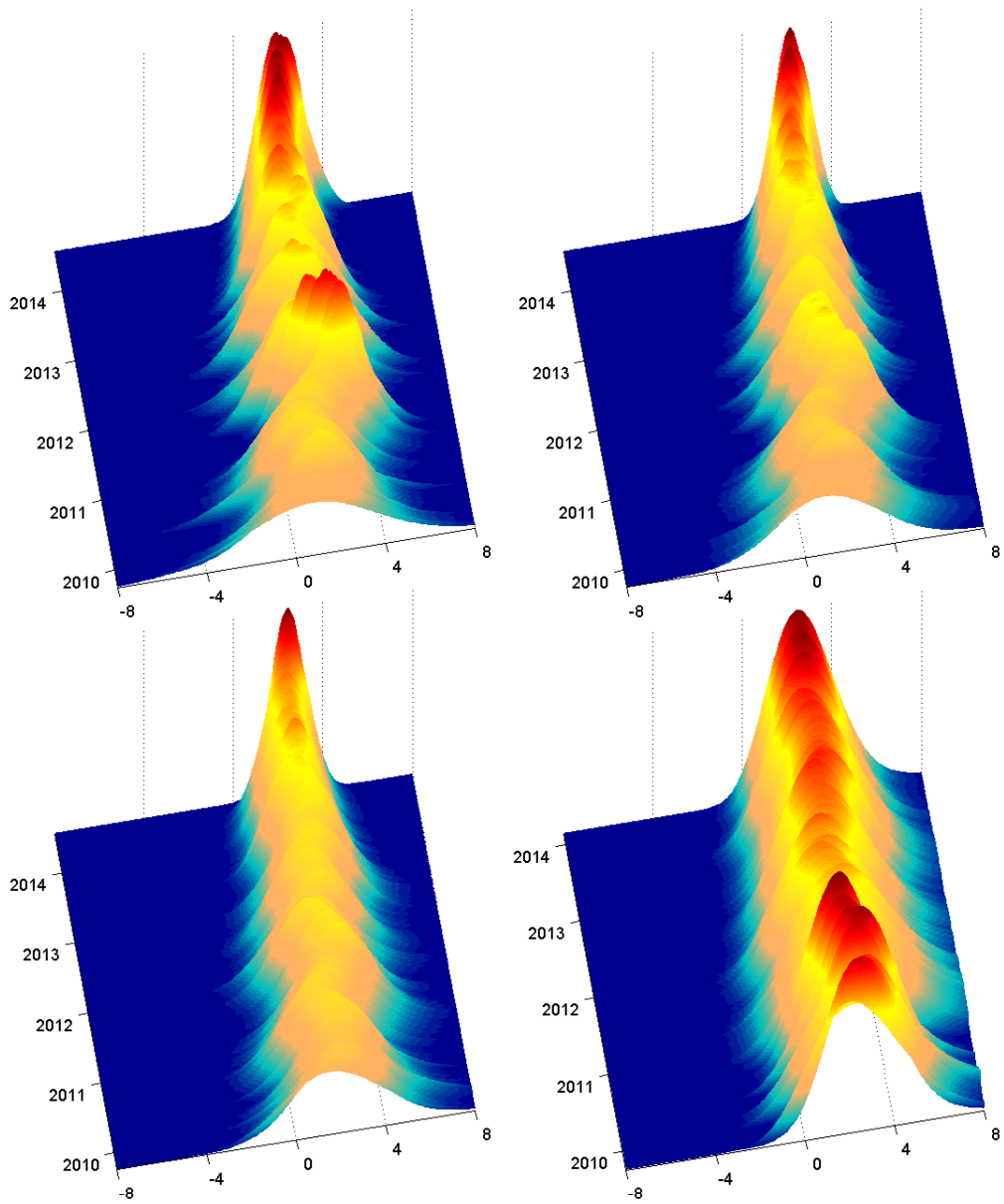


Figure 1. Inflation Densities. This figure plots the time series of inflation densities for horizons of one year (upper left), two years (upper right), five years (lower left), and ten years (lower right).

Table 5

Summary Statistics for Expected Inflation. This table reports summary statistics for the expected inflation rate for the indicated horizons. Horizon is expressed in years. Expected inflation rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to July 29, 2014.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	<i>N</i>
1	1.798	1.201	-4.263	1.914	3.763	2613
2	1.997	0.936	-3.232	2.065	3.421	2613
3	2.154	0.728	-1.692	2.215	3.326	2613
4	2.270	0.586	-0.930	2.326	3.324	2613
5	2.359	0.491	-0.322	2.422	3.288	2613
6	2.421	0.422	0.112	2.488	3.271	2613
7	2.474	0.365	0.572	2.536	3.181	2613
8	2.516	0.324	0.776	2.570	3.163	2613
9	2.551	0.286	0.960	2.600	3.104	2613
10	2.587	0.256	1.184	2.632	3.106	2613
12	2.634	0.243	1.299	2.672	3.113	2613
15	2.710	0.247	1.177	2.747	3.256	2613
20	2.836	0.259	1.139	2.878	3.385	2613
25	2.992	0.271	1.387	3.034	3.532	2613
30	3.190	0.273	1.777	3.224	3.797	2613

Expected Inflation

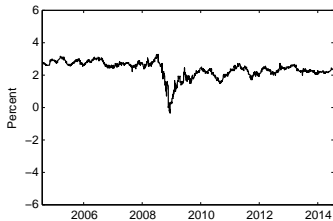
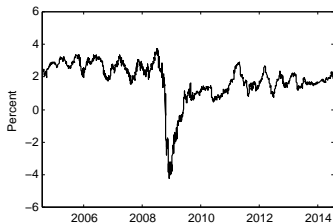


Table 6

Comparison of Survey Forecasts with Market-Implied Forecasts. This table reports the average values of the survey forecasts for the indicated forecast horizon along with the corresponding average of the market-implied expected inflation for the same horizon. Inflation forecasts are expressed as percentages. The sample period is July 2004 to July 2014.

Forecast Horizon	Survey	Frequency	Survey Forecast	Market-Implied Forecast	<i>N</i>
1 Year	SPF	Quarterly	2.12	1.78	40
	Livingston	Semiannual	2.13	1.60	20
	Michigan	Monthly	3.23	1.80	120
5 Years	Michigan	Monthly	2.91	2.31	106
10 Years	SPF	Quarterly	2.40	2.60	120
	Livingston	Semiannual	2.42	2.56	20

Table 7

Summary Statistics for Inflation Volatility. This table reports summary statistics for the volatility of the annualized inflation rate for the indicated horizons. Horizon is expressed in years. Inflation rates are expressed as percentages. The sample consists of daily observations for the period from July 23, 2004 to July 29, 2014.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	<i>N</i>
1	1.385	0.403	0.818	1.373	2.973	1210
2	1.400	0.422	0.715	1.386	2.365	1103
3	1.342	0.356	0.685	1.305	2.231	1222
5	1.351	0.266	0.688	1.399	2.186	1075
7	1.396	0.237	0.775	1.421	2.073	1124
10	1.455	0.232	0.932	1.432	2.032	1214
12	1.445	0.197	1.019	1.448	1.910	1218
15	1.420	0.201	1.072	1.465	1.874	1198
20	1.420	0.206	0.991	1.474	1.842	1121
30	1.397	0.134	0.796	1.414	1.708	789

Table 8

Summary Statistics for Deflation Probabilities. This table reports summary statistics for the probability of the average inflation rate being below zero for the indicated horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

Horizon	Mean	Standard Deviation	Minimum	Median	Maximum	<i>N</i>
1	13.73	10.66	0.14	10.61	41.73	1210
2	11.36	9.06	0.16	8.44	34.00	1103
3	8.32	7.01	0.06	5.93	27.70	1222
5	6.34	4.36	0.04	5.81	22.23	1075
7	5.52	3.18	0.10	5.27	18.27	1124
10	5.23	2.82	0.31	4.51	11.93	1214
12	4.83	2.41	0.66	4.34	11.80	1218
15	4.27	2.37	0.48	3.94	11.77	1198
20	3.96	2.30	0.25	3.83	12.94	1121
30	2.77	1.71	0.00	2.34	9.60	789

Deflation Probabilities

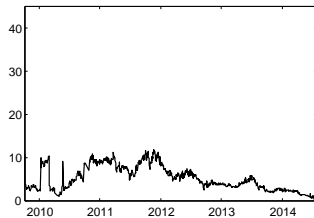
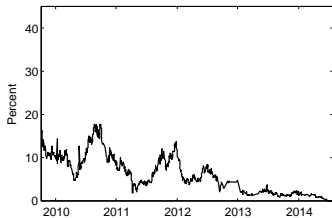
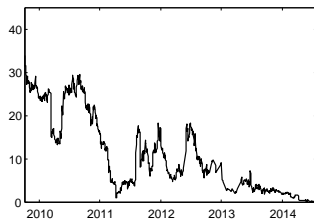
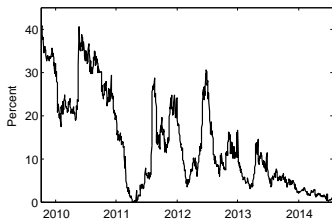


Table 9

Summary Statistics for the Pricing of Deflation Tail Risk. This table reports the means of the ratio of the probability of inflation being below the indicated threshold under the pricing measure divided by the probability of the same event under the actual measure. The mean is taken over only the observations where the probability of the event is greater than 0.01 percent under the actual measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

Horizon	Probability Inflation < 0.00		Probability Inflation < -1.00		Probability Inflation < -2.00	
	Mean	<i>N</i>	Mean	<i>N</i>	Mean	<i>N</i>
1	1.159	1210	1.303	1188	1.404	972
2	1.287	1103	1.518	1039	1.677	835
3	1.373	1222	1.704	1161	1.956	902
5	1.348	1075	1.682	1050	1.880	766
7	1.251	1124	1.495	1107	1.748	883
10	1.139	1214	1.287	1213	1.481	1084
12	1.100	1218	1.214	1218	1.355	1063
15	1.086	1198	1.178	1198	1.269	968
20	1.159	1121	1.261	1119	1.325	895
30	1.713	788	2.052	788	2.294	603

Table 10

Results from the Regression of Monthly Changes in Deflation Probabilities on Financial and Macroeconomic Variables. This table reports the t -statistics and adjusted R^2 s from from the regression of monthly changes in the deflation probabilities for the indicated horizon on the monthly changes in the following variables: the spread between three-month Libor and the overnight index swap (OIS) rate, the five-year swap spread, the VIX volatility index, the CDX North American Investment Grade CDS Index, the return on the CRSP value-weighted stock index, the five-year U.S. Treasury CDS spread, the five-year German CDS spread, industrial production (IP, percentage change), the unemployment rate (Unemp), and the Conference Board's Consumer Confidence Index (Conf). Horizon in measured in years. The t -statistics are based on the Newey-West estimator of the covariance matrix (three lags). The superscript ** denotes significance at the five-percent level; the superscript * denotes significance at the ten-percent level. The sample consists of monthly observations for the period from October 2009 to July 2014.

Horizon	Libor Spread	Swap Spread	VIX	CDX	Stock Return	Trsy CDS	German CDS	IP	Unemp	Conf	Adj. R^2	N
1	1.55	2.04**	-1.35	0.05	-2.40**	-0.96	0.50	1.05	0.24	-2.34**	0.251	56
2	2.29**	2.11**	-2.43**	-0.80	-3.85**	-1.84*	1.81*	1.23	-0.52	-0.62	0.510	56
3	0.00	1.56	-2.91**	-0.76	-5.60**	-1.72*	2.32**	-0.22	-0.79	-0.51	0.455	56
5	0.13	0.22	-2.29**	1.38	-2.69**	0.29	1.52	0.72	-2.27**	-0.96	0.419	54
7	-0.84	0.55	-1.51	1.14	-1.15	-0.29	2.19**	0.14	-1.94	-0.13	0.277	56
10	-1.60	1.46	0.82	-1.78*	-0.31	-0.82	-0.56	1.15	1.64	-0.08	-0.003	56
12	-0.09	-0.39	0.21	1.96*	0.84	0.95	1.83*	-0.66	-1.99*	0.45	0.155	56
15	0.22	0.84	0.24	1.56	1.19	0.79	1.42	0.18	-1.31	-0.03	0.053	56
20	0.59	1.35	-1.70*	1.39	-0.04	-0.47	0.62	1.68*	-1.54	-2.12**	0.204	54
30	1.57	-0.11	0.31	2.30**	1.53	-0.41	-0.40	2.33**	-2.08**	-2.74**	0.492	39

Table 12

Summary Statistics for the Probabilities of Inflationary Scenarios. This table reports summary statistics for the probability of the average inflation rate being above the indicated thresholds for the respective horizons. Horizon is expressed in years. Probabilities are expressed as percentages. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

Horizon	Probability Inflation > 4.00			Probability Inflation > 5.00			Probability Inflation > 6.00			<i>N</i>
	Mean	Min.	Max	Mean	Min.	Max	Mean	Min.	Max	
1	5.14	0.23	22.20	1.40	0.00	8.81	0.35	0.00	4.68	1210
2	11.36	0.16	34.00	6.33	0.21	19.67	1.89	0.00	8.84	1103
3	7.09	0.59	19.56	1.90	0.01	9.88	0.47	0.00	4.31	1222
5	9.37	0.89	23.25	2.53	0.01	12.13	0.55	0.00	5.45	1075
7	11.91	2.36	23.13	3.48	0.06	10.29	0.81	0.00	3.67	1124
10	14.56	5.51	25.67	4.66	0.59	13.04	1.20	0.02	5.58	1214
12	15.03	5.50	24.34	4.72	0.83	10.96	1.15	0.06	4.07	1218
15	15.53	5.09	24.04	4.82	0.65	10.73	1.15	0.04	3.94	1198
20	16.57	4.82	24.10	5.24	0.47	10.64	1.27	0.02	3.79	1121
30	19.80	13.53	26.58	6.30	2.67	11.18	1.42	0.09	3.73	789

Inflation Probabilities

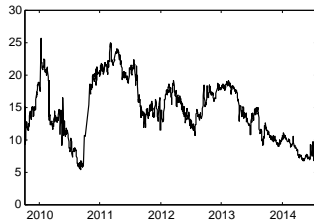
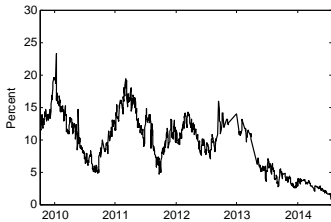
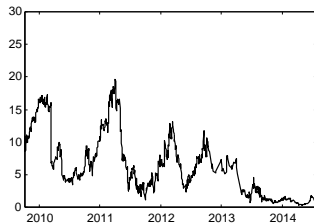
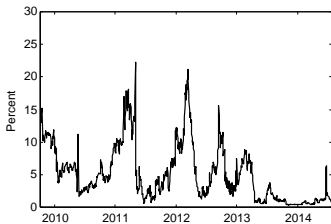


Table 13

Summary Statistics for the Pricing of Inflation Tail Risk. This table reports the means of the ratio of the probability of inflation being above the indicated threshold under the pricing measure divided by the probability of the same event under the actual measure. The mean is taken over only the observations where the probability of the event is greater than 0.01 percent under the actual measure. Horizon is expressed in years. The sample consists of daily observations for the period from October 5, 2009 to July 29, 2014.

Horizon	Probability Inflation > 4.00		Probability Inflation > 5.00		Probability Inflation > 6.00	
	Mean	<i>N</i>	Mean	<i>N</i>	Mean	<i>N</i>
1	1.023	1210	1.099	1160	1.151	863
2	1.043	1103	1.161	1055	1.255	818
3	1.062	1222	1.227	1217	1.372	988
5	1.089	1075	1.262	1071	1.461	933
7	1.097	1124	1.244	1124	1.467	1100
10	1.094	1214	1.204	1214	1.374	1214
12	1.088	1218	1.182	1218	1.322	1218
15	1.065	1198	1.135	1198	1.240	1198
20	0.981	1121	1.001	1121	1.037	1121
30	0.737	789	0.665	789	0.602	789

Conclusion

- We solve for the objective distribution of inflation using the market prices of inflation swap and option contracts and study the nature of deflation risk.
- Market-implied probabilities of deflation are substantial, even though the expected inflation rate is roughly 2.50 to 3.00 percent for horizons of up to 30 years.
- Deflation risk is priced by the market in a manner similar to that of other major types of tail risk such as catastrophic insurance losses or corporate bond defaults.
- Deflation risk is significantly related to measures capturing stress in the financial system and credit risk in the economy.
- Our results support the view that the risk of economic shocks severe enough to result in deflation is fundamentally related to the risk of major systemic shocks in the financial market.