

# The Equilibrium Real Funds Rate: Past, Present and Future

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## I. Introduction

There is a consensus that we're heading towards a "new neutral:" an era of lower equilibrium real Fed funds rate.

- Stagnationists
- Bond market
- Summary of Economic Projections for the FOMC

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There is a consensus that we're heading towards a "new neutral:" an era of lower equilibrium real Fed funds rate.

- Stagnationists

“We may well need, in the years ahead, to think about how we manage an economy in which the zero nominal interest rate is a chronic and systemic inhibitor of economy activity, holding our economies back below their potential.” (Summers (2013b))

- Bond market

- Summary of Economic Projections for the FOMC

## I. Introduction

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- Stagnationists
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  - Nominal funds rate forecast to peak below 2%
- Summary of Economic Projections for the FOMC

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	—Median "longer run"—			Reference:
	Fed funds	PCE inflation	Implied $r$	#Fed funds $\leq 3.50$
Sep 2015	3.50	2.00	1.50	12
Dec 2014	3.75	2.00	1.75	4
Dec 2012	4.00	2.00	2.00	2

## Our goals

- Analyze the past behavior of the real rate, to help form an idea of the prospective equilibrium value of the real rate: the forecast of the real rate 5 or 10 or 12 years from now.
- This will require initial focus on output growth and the equilibrium rate, since conventional wisdom posits a tight link between the two:
  - Theory: consumption growth is tied to the real rate. So on a balanced growth path, trend output growth also tied to the trend (or equilibrium) real rate (e.g., New Keynesian models).
  - Empirically: Laubach and Williams (2003) make trend output growth the central determinant of the equilibrium rate.

## Conclusions

- The equilibrium rate is
  - hard to pin down;
  - highly variable;
  - has many determinants
    - trend output growth plays no special role.
- A vector error correction model that looks only to U.S. and world real rates well captures the behavior of U.S. real rates.
- Looking forward, a plausible range for the equilibrium rate is perhaps: a little above 0 up to 2%.
- A standard model and loss function suggest: uncertainty about equilibrium rate  $\Rightarrow$  Fed should prefer later and steeper normalization of the Fed funds rate.

## Methodology

- “Equilibrium rate”: real safe rate consistent with full employment and stable inflation. Equivalent to:
  - steady state real rate, and
  - forecast of the real rate 5 or 10 or 12 years from now.
- We make no attempt at structural estimation.
- Instead we use rolling averages of time series on the real short term government debt (real Fed funds for post-World War II U.S.).
- Much of our argument is informal.



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V. Narrative evidence on real rates in the U.S.

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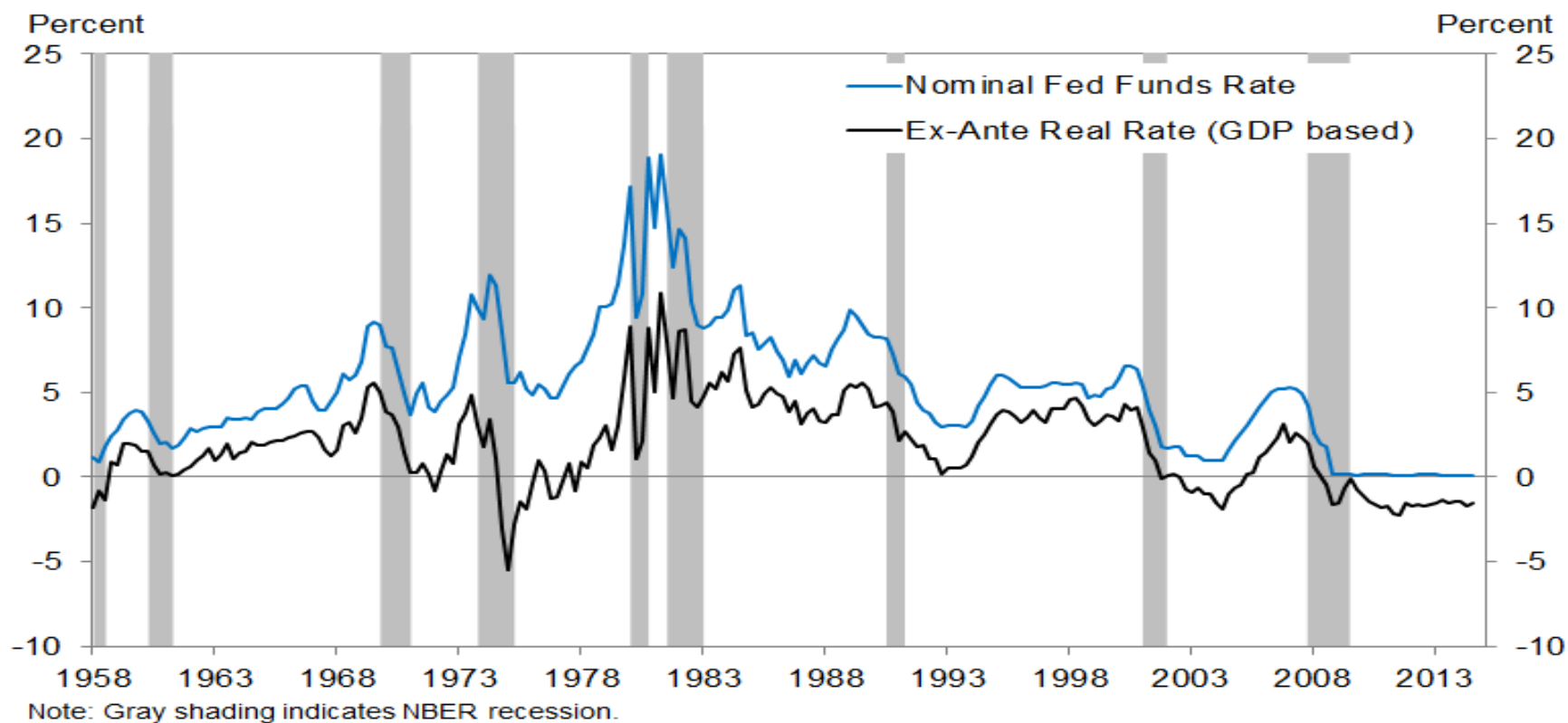
## II. Construction of ex-ante real rates

- Focus is on the U.S.. For the U.S., post-WWII data sources are conventional.
- We also use cross country developed country data:
  - annual data going back 150+ years, up to 17 countries,
  - quarterly data back to 1971, up to 20 countries.

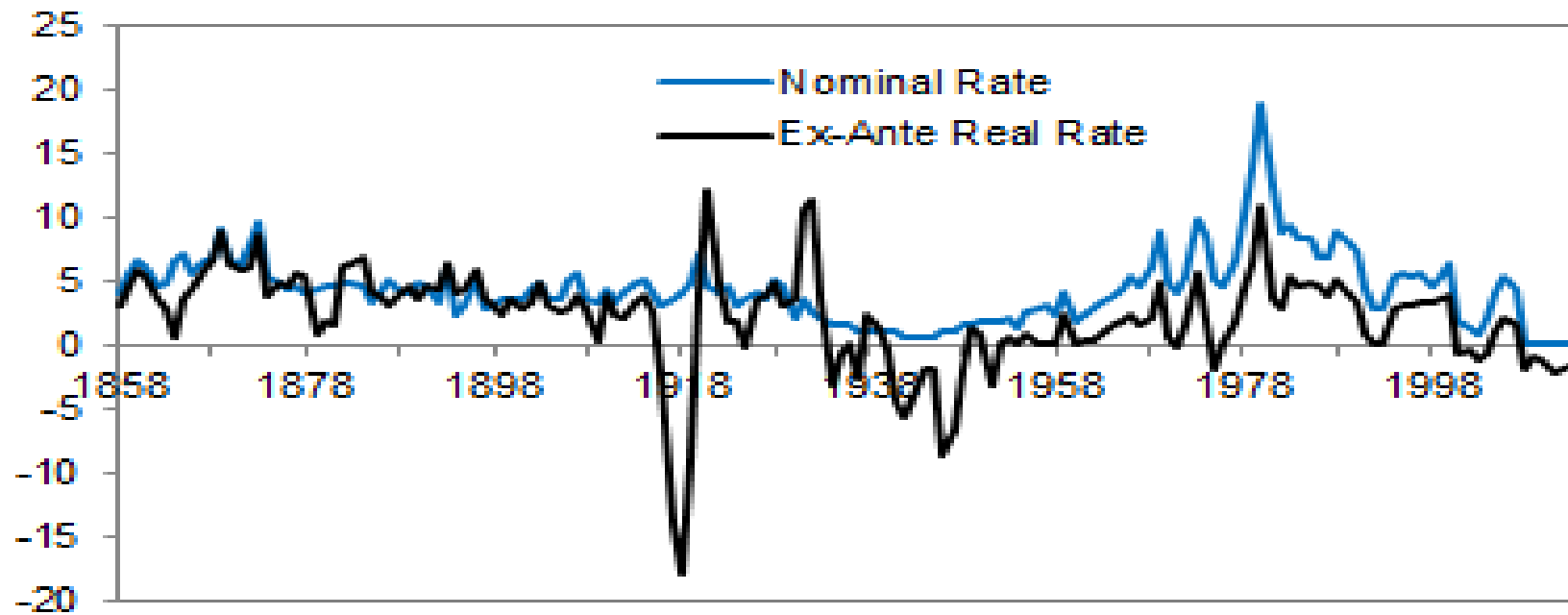
## Construction of ex-ante real rates

- Real rate  $\equiv$  nominal policy rate - expected inflation
- Policy rate:
  - discount rate (countries other than U.S.)
  - commercial paper rate, discount rate, Fed funds rate (U.S.)
- Expected inflation from univariate AR in CPI inflation, rolling samples
  - annual: AR(1), rolling sample = 30 years
  - quarterly: AR(4), rolling sample = 40-80 quarters
- Exception: U.S. uses GDP deflator 1929-2014

Plots of quarterly and annual U.S. rates (see paper for plots from other countries):



	$r$	$i$	$E_t \pi_{t+1}$	$\pi$
mean	1.95	5.27	3.32	3.30
s.d.	2.55	3.60	2.12	2.32



	$r$	$i$	$E_t \pi_{t+1}$	$\pi$
mean	2.15	4.34	2.19	2.26
s.d.	3.89	2.76	3.37	4.82

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### III. The real rate, consumption growth and aggregate growth

- Real rates are often modeled as tightly tied to growth in output or potential output.
  - New Keynesian models and their offshoots, e.g., Laubach and Williams (2003).
  - Discussion of secular stagnation, e.g. Summers (2013a,b).
- Depending on the model, that link works in whole or in part through the link between real rates and consumption.
- In this section, we note that in terms of  $r_t$  and consumption, there is a
  - good theoretical case for a link between the two series, and
  - a poor empirical case for a link between the two series.

## Intertemporal IS

If per period utility is  $C_t^{1-\alpha}/(1-\alpha)$ , the standard intertemporal IS equation is

$$(3.4) \quad r_t \equiv i_t - E_t \pi_{t+1} = \rho + \alpha E_t \Delta c_{t+1}$$

where

$r_t$  = short term real rate on nominally riskless asset,

$i_t$  = short term nominal rate on nominally riskless asset,

$E_t \pi_{t+1}$  = expected inflation,

$\Delta c_{t+1}$  = consumption growth (log difference of  $C$ ),

$\rho$  = per period discount factor.



## $r_t$ and consumption

$$(3.4) \quad r_t \equiv \rho + \alpha E_t \Delta c_{t+1}$$

- In NK models, the average value of  $r_t$  corresponds to the average value of the natural rate of interest: the rate consistent with output at potential and steady inflation. Hence a time series average of  $r_t$  will correspond to the equilibrium real rate.
- It is well known that (3.4) has wildly counterfactual properties for mean real rates—the famous “risk free rate puzzle” of Weil (1989), e.g.:

$$\rho = .04, \alpha = 1, E\Delta c_{t+1} = .02 \Rightarrow Er_t = .06.$$

- Many papers generalize the utility function and environment that lead to (3.4). None work well in explaining  $r_t$  and mean  $r_t$ .
- Conclusion: trend growth in consumption doesn't tell you much about the equilibrium rate.

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## IV. The real rate and aggregate growth: empirical analysis

- Perhaps there will be a clear long-run relationship between trend output growth and the equilibrium rate, despite the weak evidence of such a relationship between consumption and the equilibrium rate.
- We compute the sample correlation between average GDP growth and average real rates over various windows.
- We focus on the sign and magnitude of this correlation. We do not attempt to supply an economic interpretation of the estimated correlation.
- Preview of results: the sign of the correlation is not robust, but instead is sensitive to inclusion or exclusion of a small number of observations. As well, the absolute value of the magnitude of the correlation is small.

## Average GDP growth $y$ vs average $r_t$

- We investigate the correlation between GDP growth and real rates with the following data
  - A. Business cycle (peak to peak) (U.S. data)
  - B. 10 year averages (U.S. data)
  - C. Averages over 10, 20, 30 and 40 years (cross-country data)
- Our view is that we are taking averages over a long enough period that the average rate will closely track the equilibrium rate.

## A. Average GDP growth $y$ vs average $r_t$ : peak to peak (U.S. data)

Unit of observation is (average GDP growth, average  $r_t$ ), computed from a business cycle peak to the next business cycle peak.

### 1. Quarterly

7 data points,

1960:2-1969:4 delivers first observation,

2001:1-2007:4 delivers last observation.

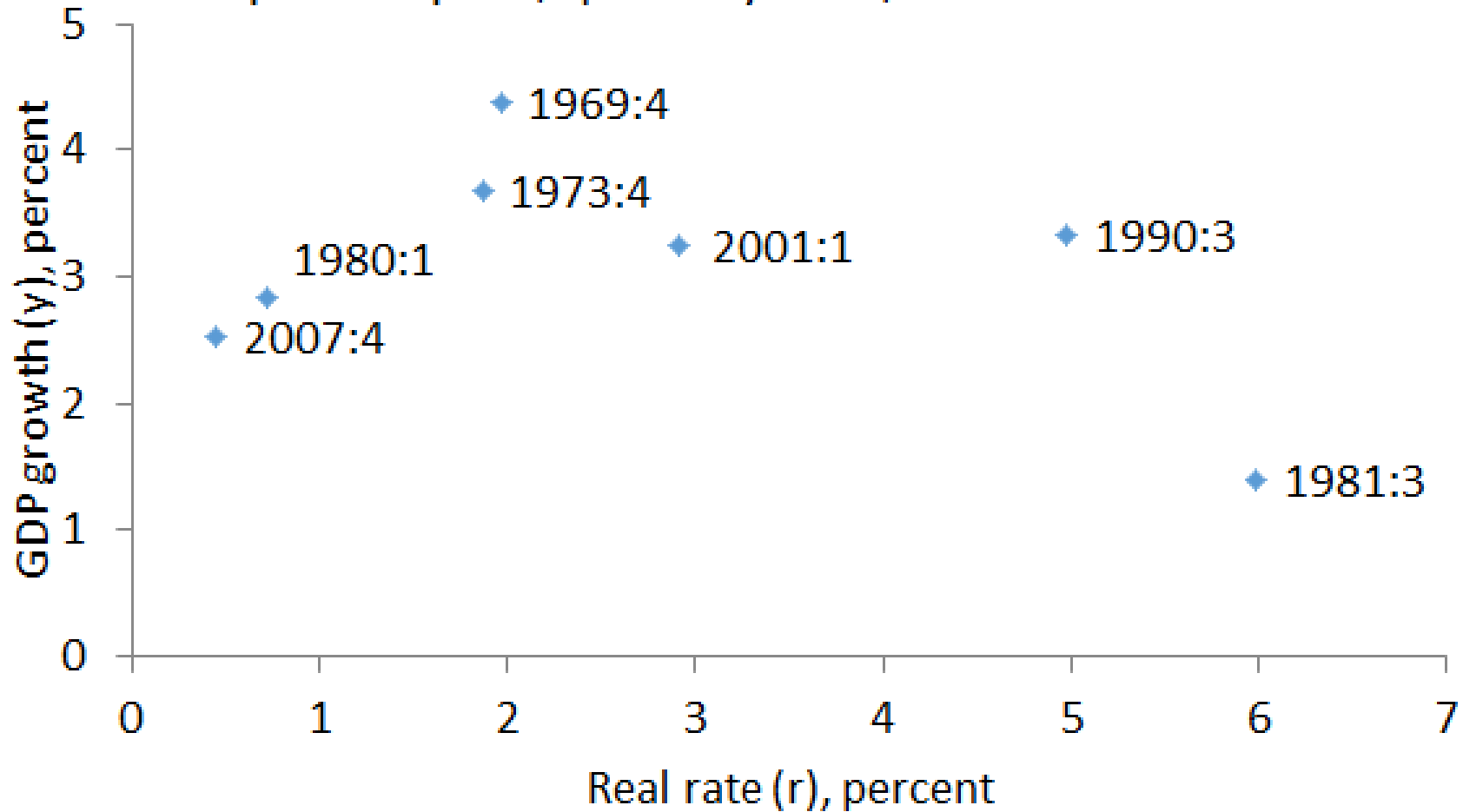
### 2. Annual

29 data points,

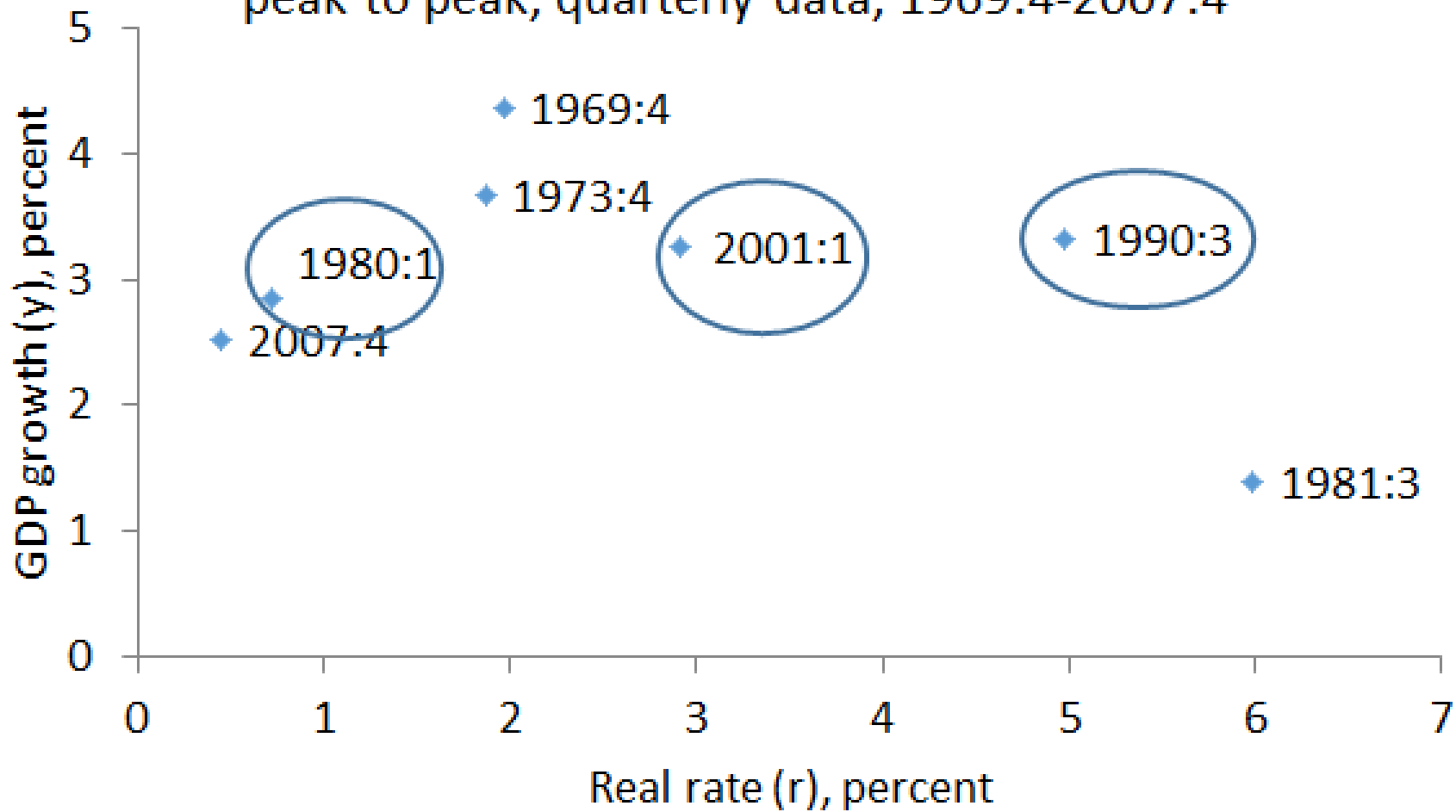
1869-1873 delivers first observation,

2001-2007 delivers last observation.

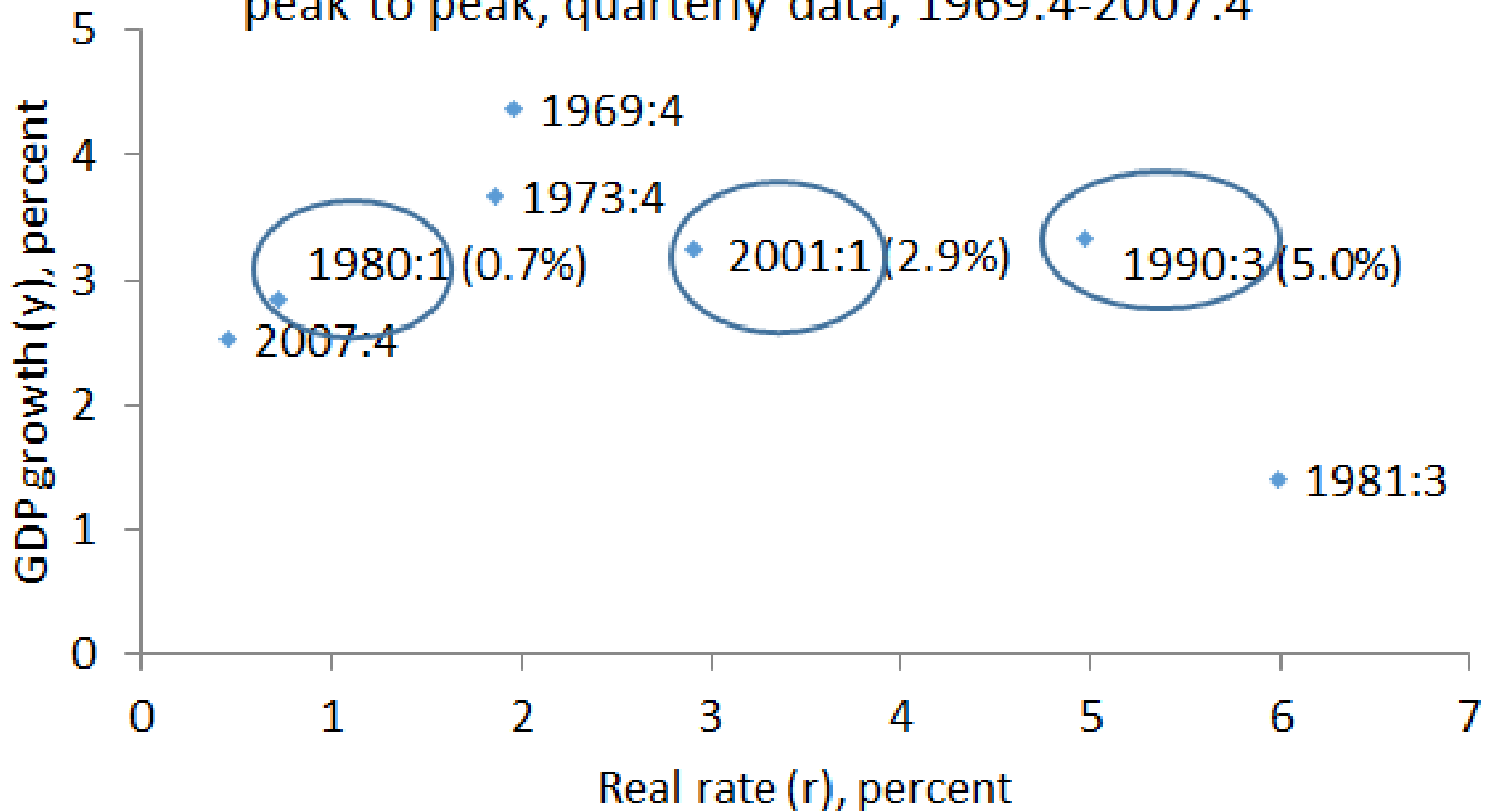
Average GDP growth vs. average real rate:  
peak to peak, quarterly data, 1969:4-2007:4



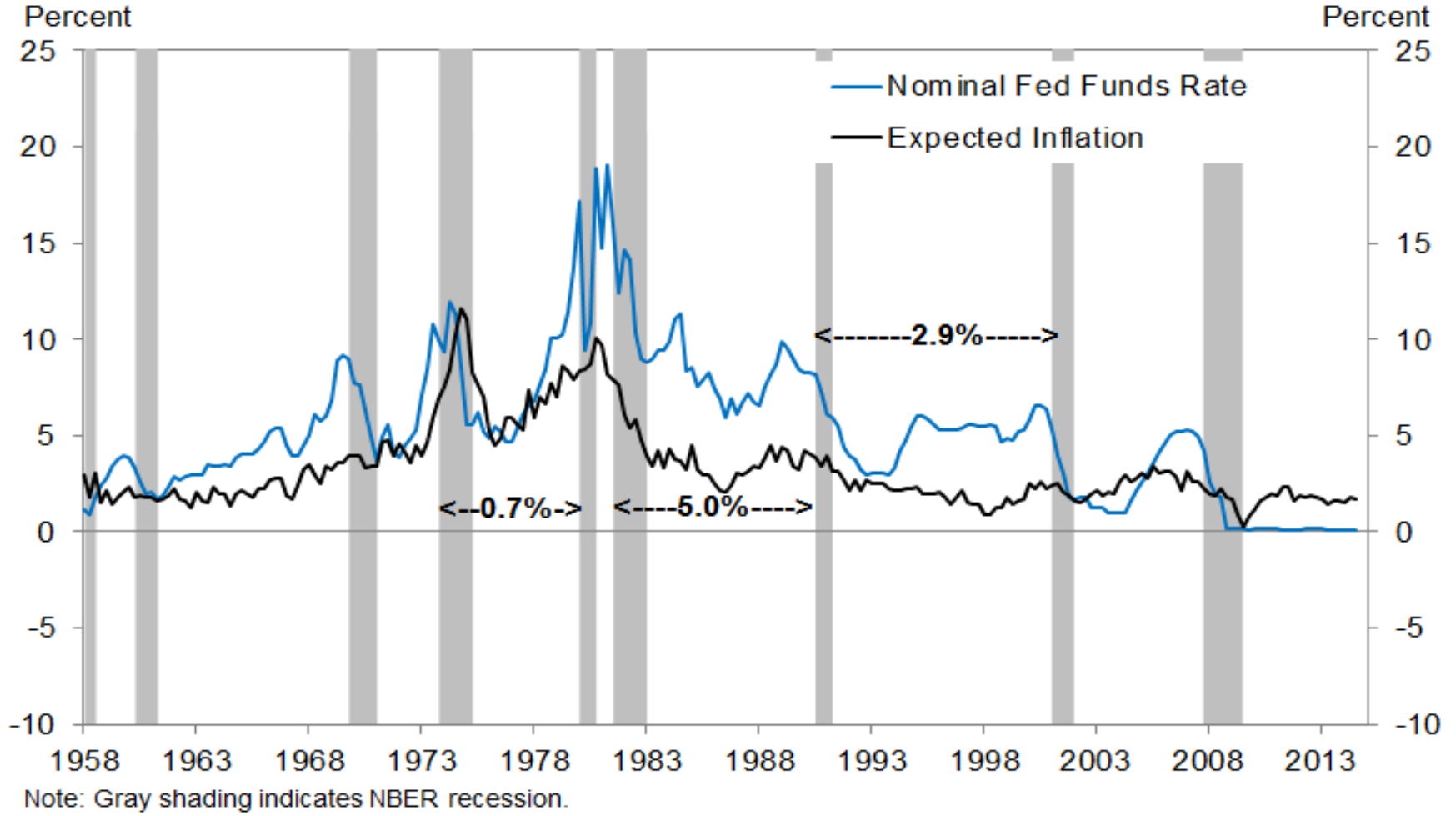
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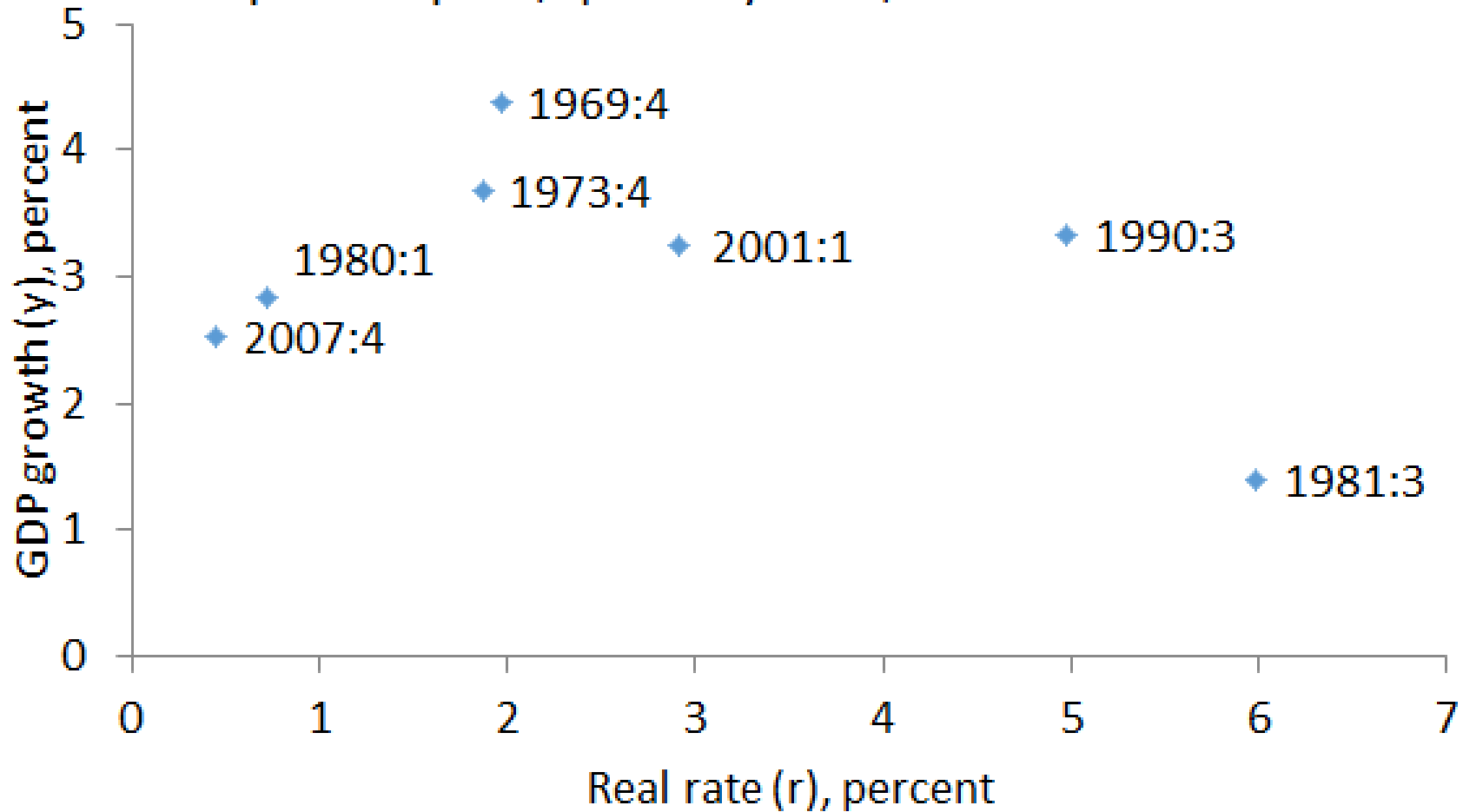
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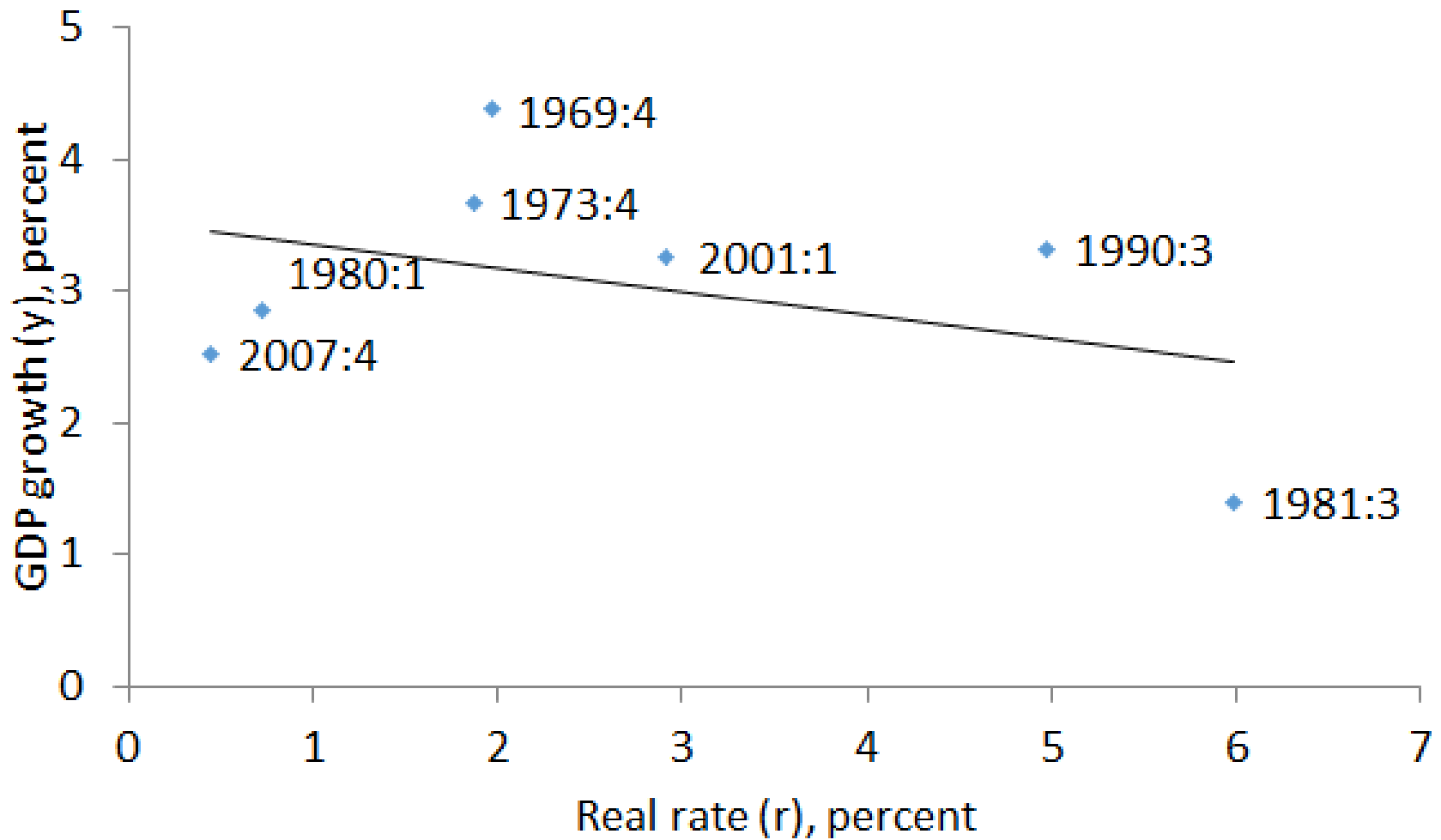




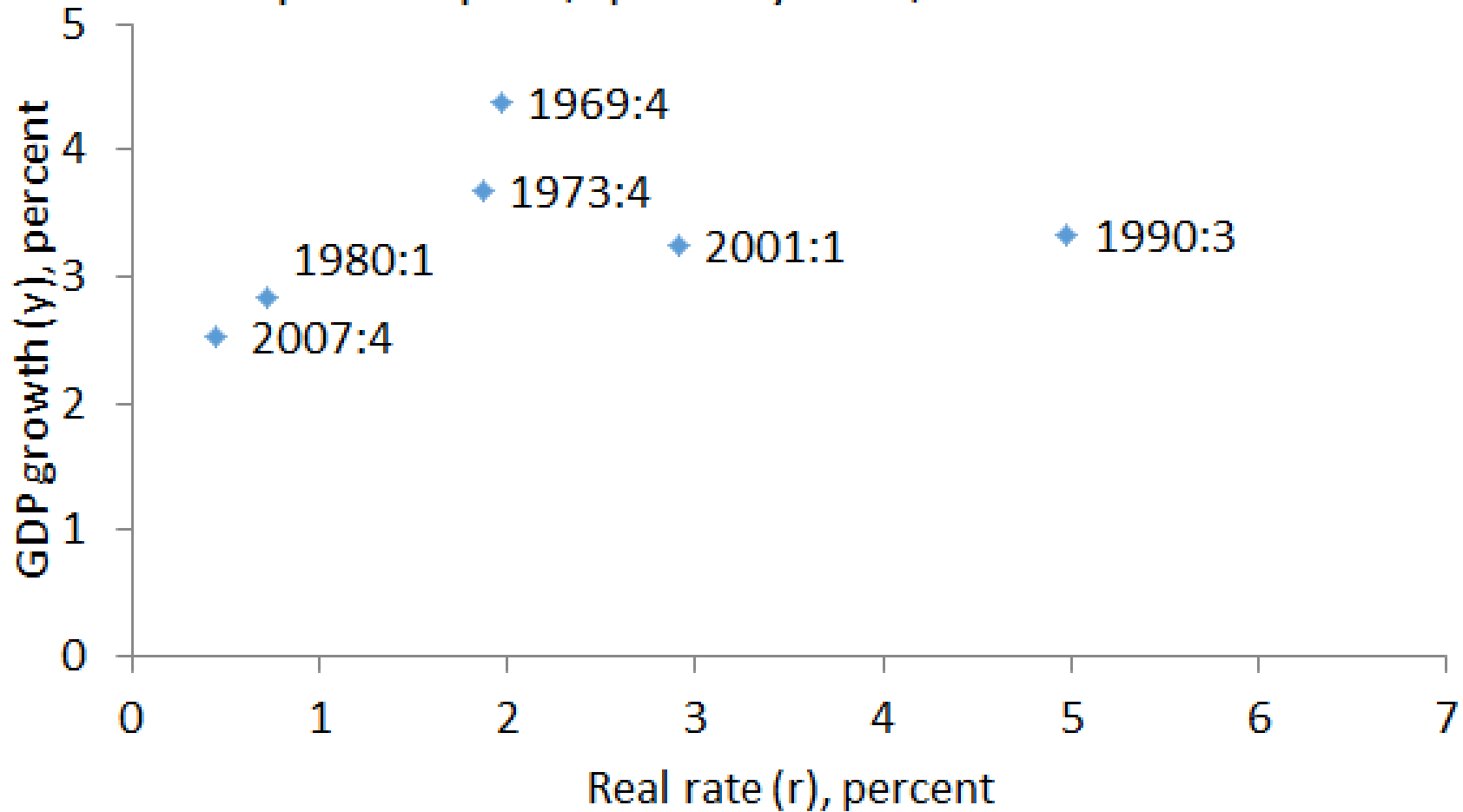
Average GDP growth vs. average real rate:  
peak to peak, quarterly data, 1969:4-2007:4



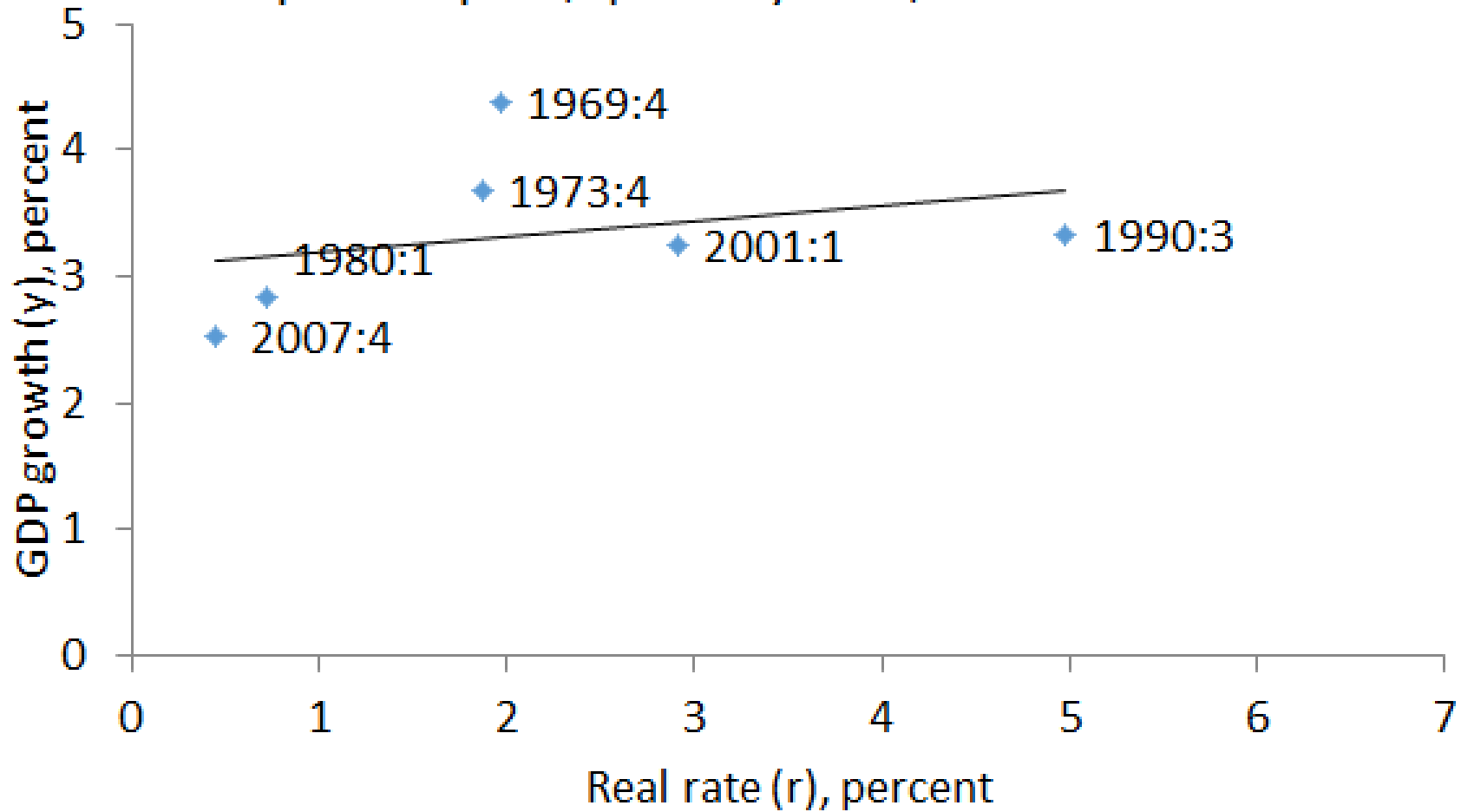
Average GDP growth vs. average real rate:  
peak to peak, quarterly data, 1969:4-2007:4



Average GDP growth vs. average real rate:  
peak to peak, quarterly data, omit 1981:3



Average GDP growth vs. average real rate:  
peak to peak, quarterly data, omit 1981:3



## Numerical values of correlations, peak to peak calculations

Quarterly ( $N=7$ )	-0.40
Quarterly, omit 1980:1-1981:3 ( $N=6$ )	0.32
Annual ( $N=29$ )	0.23
Annual, omit 1918-1920, 1944-1948 ( $N=27$ )	-0.23

Correlations for other samples and data measures are reported in the paper (Exhibit 3.4). The numbers above are representative:

- (a) The absolute value of the correlation is small.
- (b) The sign of the correlation is sensitive to minor changes in sample.

## B. Average GDP growth $y$ vs average $r_t$ : 10 year averages (U.S. data)

Numerical values of correlations:

Quarterly, 1968:1-2014:3 ( $N=187$ )	0.39
Quarterly, 1968:1-2007:4 ( $N=160$ )	-0.19
Annual, 1879-2014 ( $N=136$ )	-0.25
Annual, 1879-2014, omit 1930-1950 ( $N=115$ )	0.31

Correlations for other samples and data measures are reported in the paper (Exhibit 3.5). The numbers above are representative:

- (a) The absolute value of the correlation is small.
- (b) The sign of the correlation is sensitive to minor changes in sample.

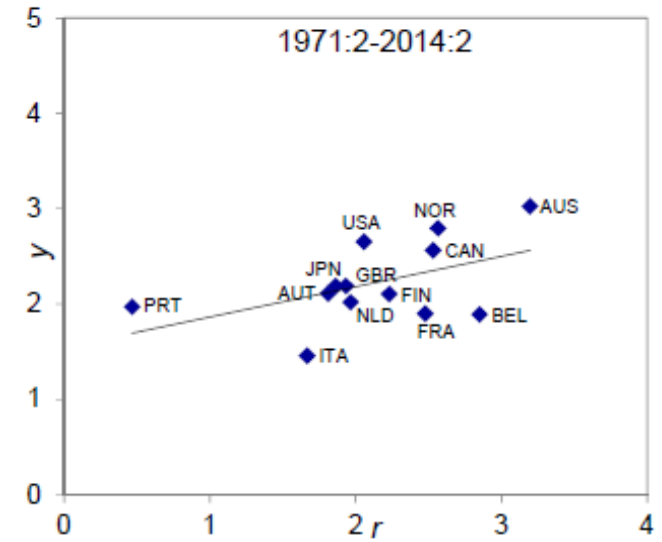
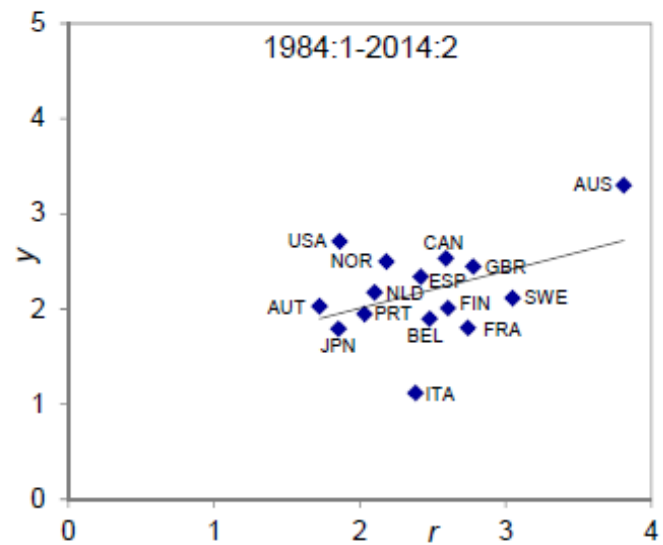
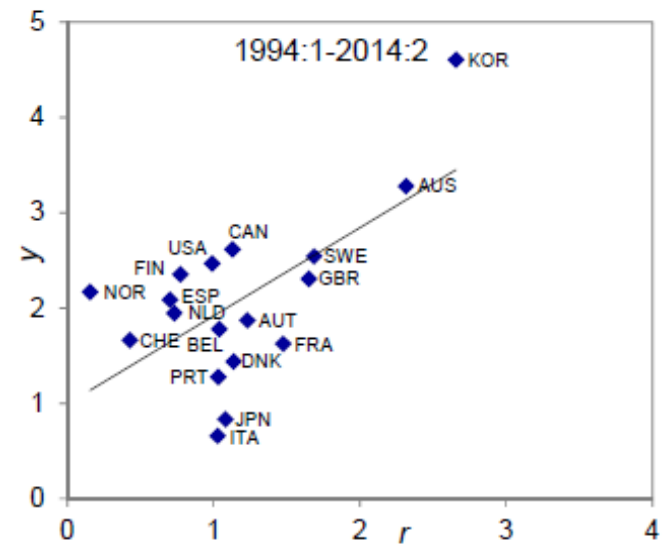
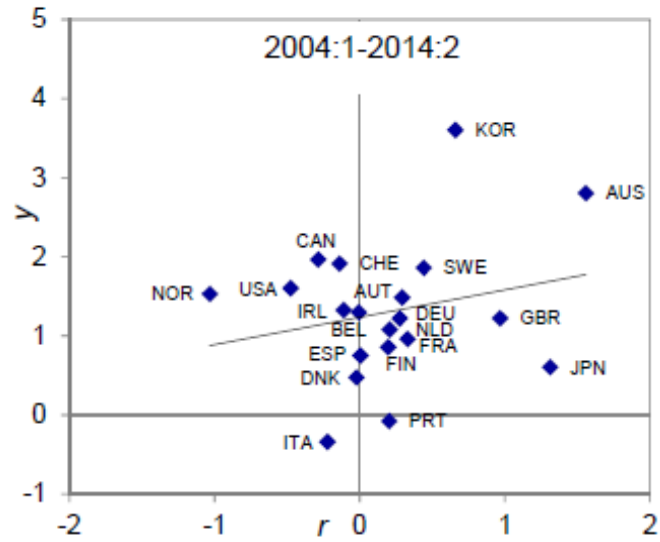
## C. Average GDP growth $y$ vs average $r_t$ : cross-country data

- Quarterly data

- Unit of observation is (average GDP growth, average  $r$ ) for a given country, computed over four samples

- 2004:1-2014:2,  $N=20$  countries;  $\text{corr}(y, r) = 0.23$
- 1994:1-2014:2,  $N=18$  countries;  $\text{corr}(y, r) = 0.63$
- 1984:1-2014:2,  $N=15$  countries;  $\text{corr}(y, r) = 0.42$
- 1971:2-2014:2,  $N=13$  countries;  $\text{corr}(y, r) = 0.36$





## Summary: average GDP growth $y$ vs average $r_t$

- Wide range of average ex-ante real interest rates associated with a given average output growth rate.
- Weak correlation between average ex-ante real rate and average growth rate, with the sign of the correlation sensitive to inclusion or exclusion of a small number of observations.

## Summary: average GDP growth $y$ vs average $r_t$ , cont'd

Even if one puts more weight on samples with a positive correlation, we think there are two implications:

- If, indeed, we are headed for stagnation for supply side reasons (Gordon (2012, 2014)), any such slowdown should not be counted on to translate to a lower equilibrium rate over periods as short as a cycle or two or a decade.
- The relation between average output growth and average real rates is so noisy we are forced to conclude that other factors play a large, indeed dominant, role in determination of average real rates. In the next section we take a narrative approach to sorting out some of these factors.

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## V. Narrative evidence on real rates in the U.S.

- Lots of shifting or hard to model variables
  - Trend growth
  - Time varying volatility
  - Shape of utility function
  - Financial frictions
  - Incomplete markets
  - Heterogeneous agents
  - Monetary transmission mechanism
- Historical narrative allows intuition to roam where formal analysis might flounder

## Narrative evidence: overview

- Since our focus is on the equilibrium rate we continue to look at averages over various time periods (not necessarily peak to peak or exactly 10 years)
- Goal: to understand what forces are influencing the equilibrium rate today.
- Bottom line: the record supports a wide range of plausible values for the equilibrium rate, up to 2% or so.
- This presentation: just a couple of points.

A. Real rates in the 1991-2007 cycles

B. Outlook for the current cycle

## A. Real rates in the 1991-2007 cycles

- 1991:1 (trough) -2001:1 (peak) cycle:
  - $r_t$  at or below 1.05% for nearly 2 years (1992:2-1993:4)
  - Subsequent peak: 4.7% (1998:2)
- 2001:4-2007:4 cycle:
  - $r_t$  at or below 0.3% for nearly four years (2001:4-2005:3), below zero for over 2 years (2002:4-2005:1)
  - Subsequent peak: 3.1% (2006:4)



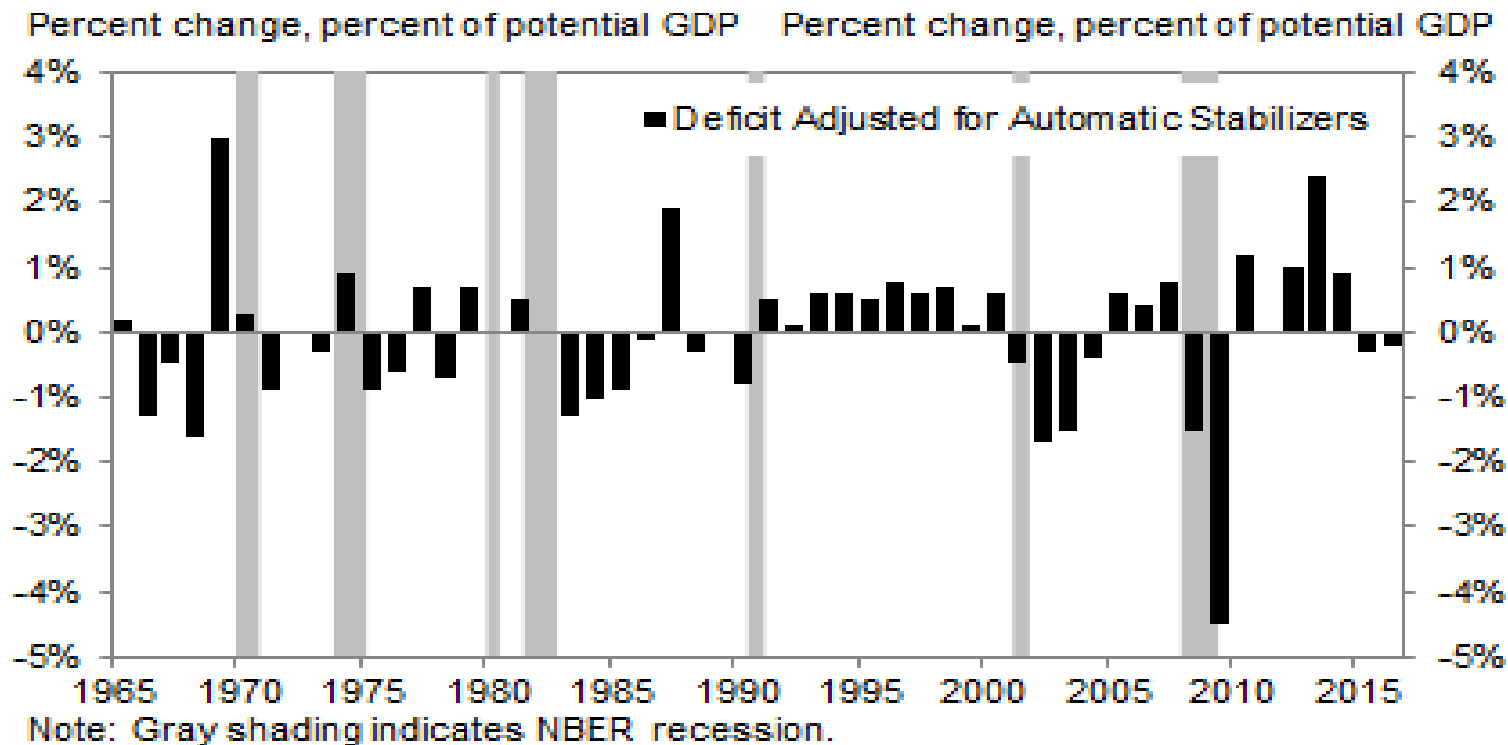
## B. Outlook for the current cycle

- Not a normal business cycle. Reinhart and Rogoff data on systemic banking crises:

	Peak to trough GDP (%)	Years to recovery	Severity index
100 crises	-10.3	8.4	19.6
37 EM crisis	-14.2	9.9	24.2
63 DM crisis	-9.6	7.4	17.0
US 1907	-12.5	9.0	21.5
US 1929/33	-28.6	10.0	38.6
memo:			
US today	-3.1	4.0	7.1
EA today	-4.5	6.0+	10.5+
Japan today	-6.5	6.0	12.5
DM today	-3.7	4.0	7.7

Today: Is It Secular?

- Not a normal business cycle
- Fiscal tightening: 2012-14: record 5% of GDP (see graph of deficit adjusted for automatic stabilizers as % of GDP)
  - Special bonus: lots of brinkmanship!



## History lessons

1. Equilibrium rate sensitive to: changing policy transmission, regulatory headwinds, inflation cycles and delayed recoveries.
2. Post-WWII data allows a wide range of estimates for the equilibrium real rate, even as high as 2%. That is a good deal higher than the near-zero number priced into the market.
3. Given how hard it is to distinguish cycle and secular stagnation, we can't think of a tougher time to pin down the equilibrium rate.

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## VI. Long run tendencies of the real rate

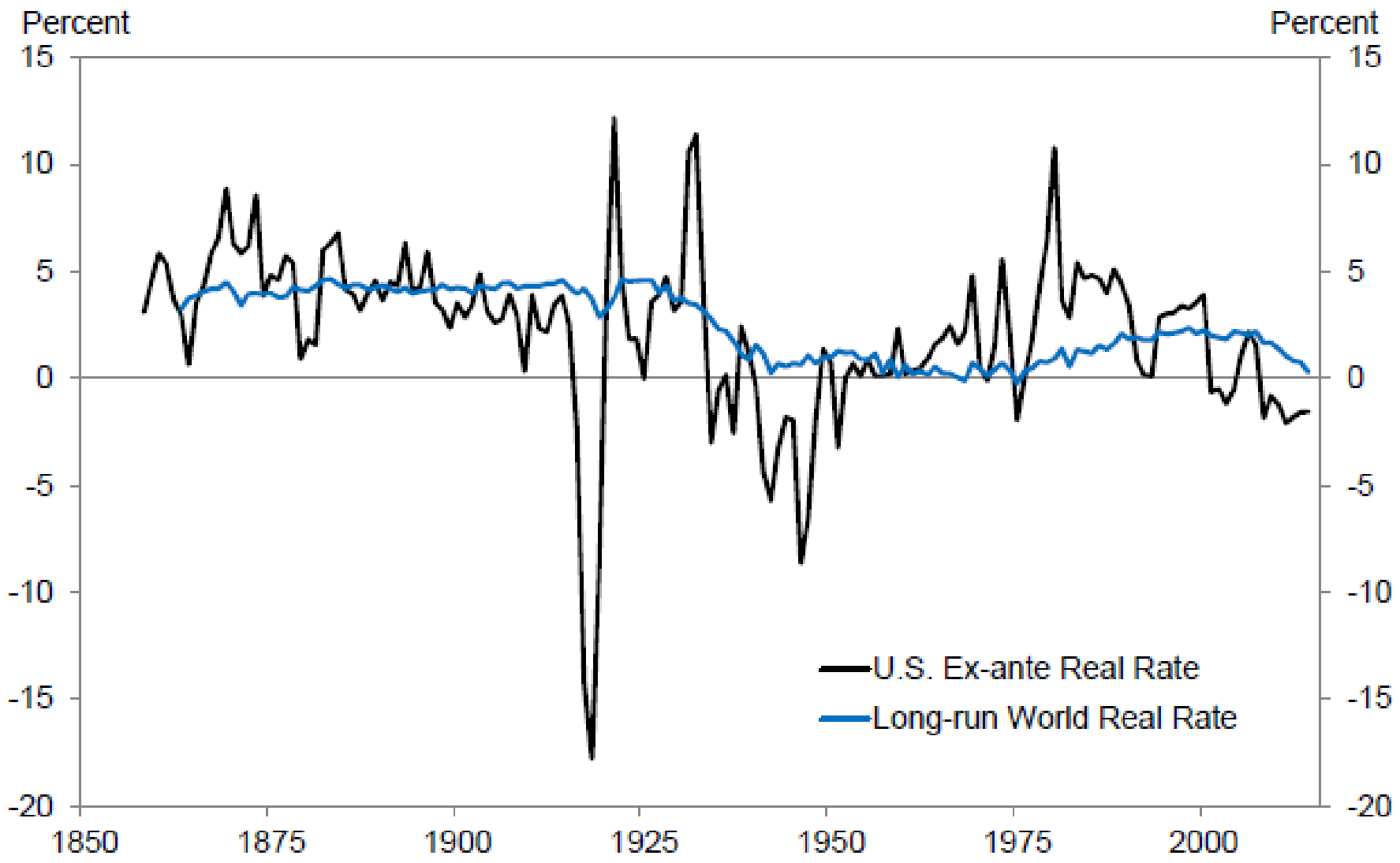
- Goal: develop and estimate time series model for annual data that can be used to forecast the U.S. real rate
- End product: first order bivariate vector error correction model in U.S. real rate and the “world rate”
  - “error correction”: we treat real rates as nonstationary
  - “world rate”: median over our 17 countries of country-specific average real rates, computed in each country using 30 year rolling samples

## Nonstationary real rates?

- Regime shifts / structural breaks / unit roots commonly found in time series on real rates, in the U.S. and other countries
- We test for stability of the U.S. real rate, decisively rejecting the joint null of stationarity and stability.
- We elect to model the data in differences, testing for stability of the VECM. The VECM does not reject the null of stability. It also does not reject the null that the constant terms are zero.

## The long run world rate

- For country  $n$  ( $n=1, \dots, 17$ ) let  $r_{nt}$  be the real rate.
- In country  $n$ , compute the average real rate using the previous 30 years of data on  $r_{nt}$  (i.e., roll through the sample using 30 year windows). Call this  $\ell_{nt}$ .
- The long run world rate  $\ell_t$  is the median over  $n=1, \dots, 17$  of  $\ell_{nt}$ .



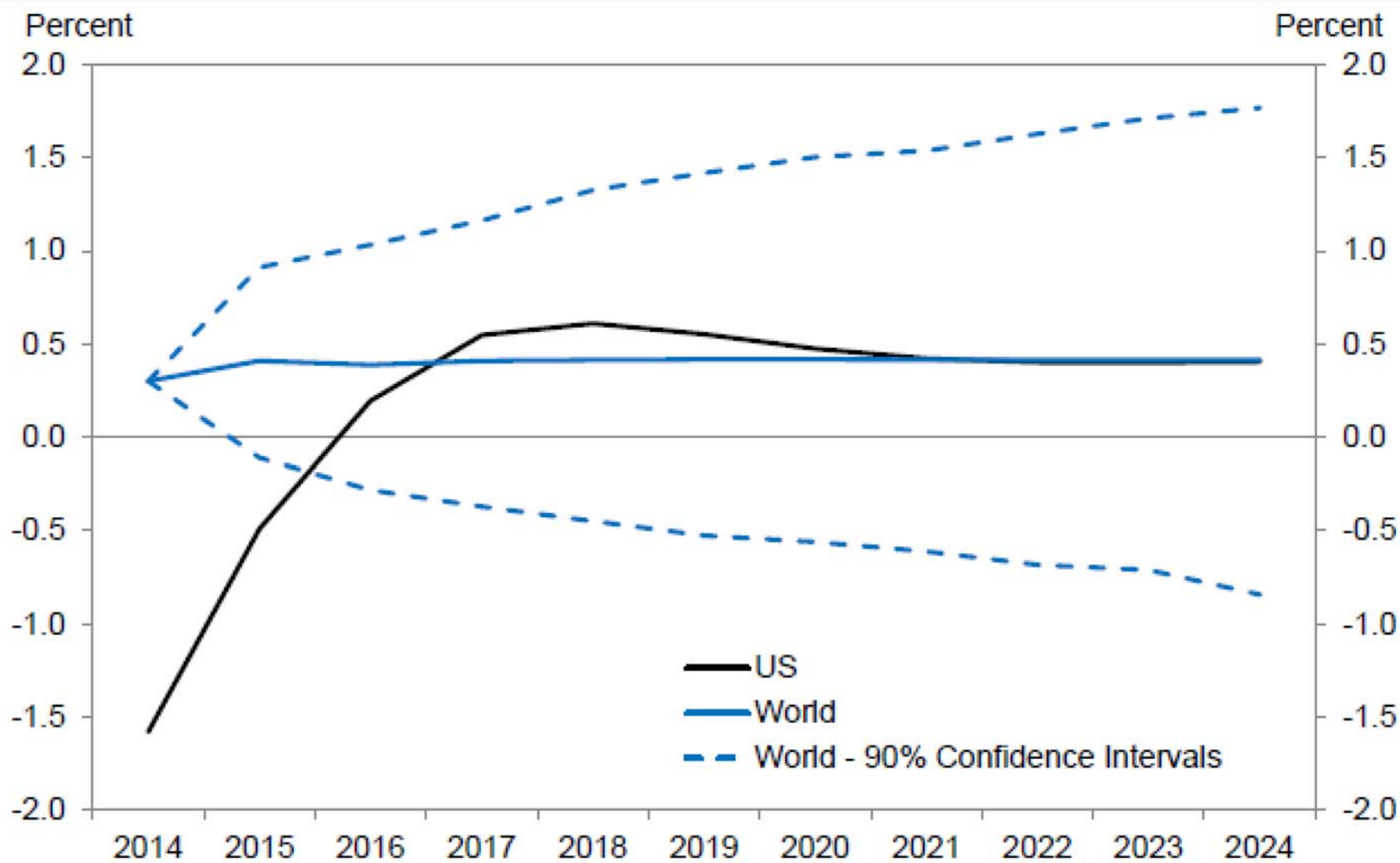


## VECM Estimates

$$\Delta r_{US,t} = 0.4\Delta r_{US,t-1} - 0.8\Delta \ell_{t-1} - 0.4(r_{US,t-1} - \ell_{t-1}) + e_{US,t}, \hat{\sigma}_{US}=2.6,$$

$$\Delta \ell_t = 0.03\Delta r_{US,t-1} - 0.3\Delta \ell_{t-1} + 0.02(r_{US,t-1} - \ell_{t-1}) + e_{\ell,t}, \hat{\sigma}_{\ell}=0.3.$$

- World rate  $\Delta \ell_t$ : feedback from  $r_{US}$  is small
- U.S. rate  $\Delta r_{US,t}$ :
  - Feedback from  $\ell_t$  is substantial. If the U.S. rate is 1% below the world rate, then all else equal we expect the U.S. rate to move 40 basis points closer to the world rate in the next year.
  - Std dev of residual = 260 basis points: despite cointegration, in any given year, substantial divergence between U.S. and world rate is possible.

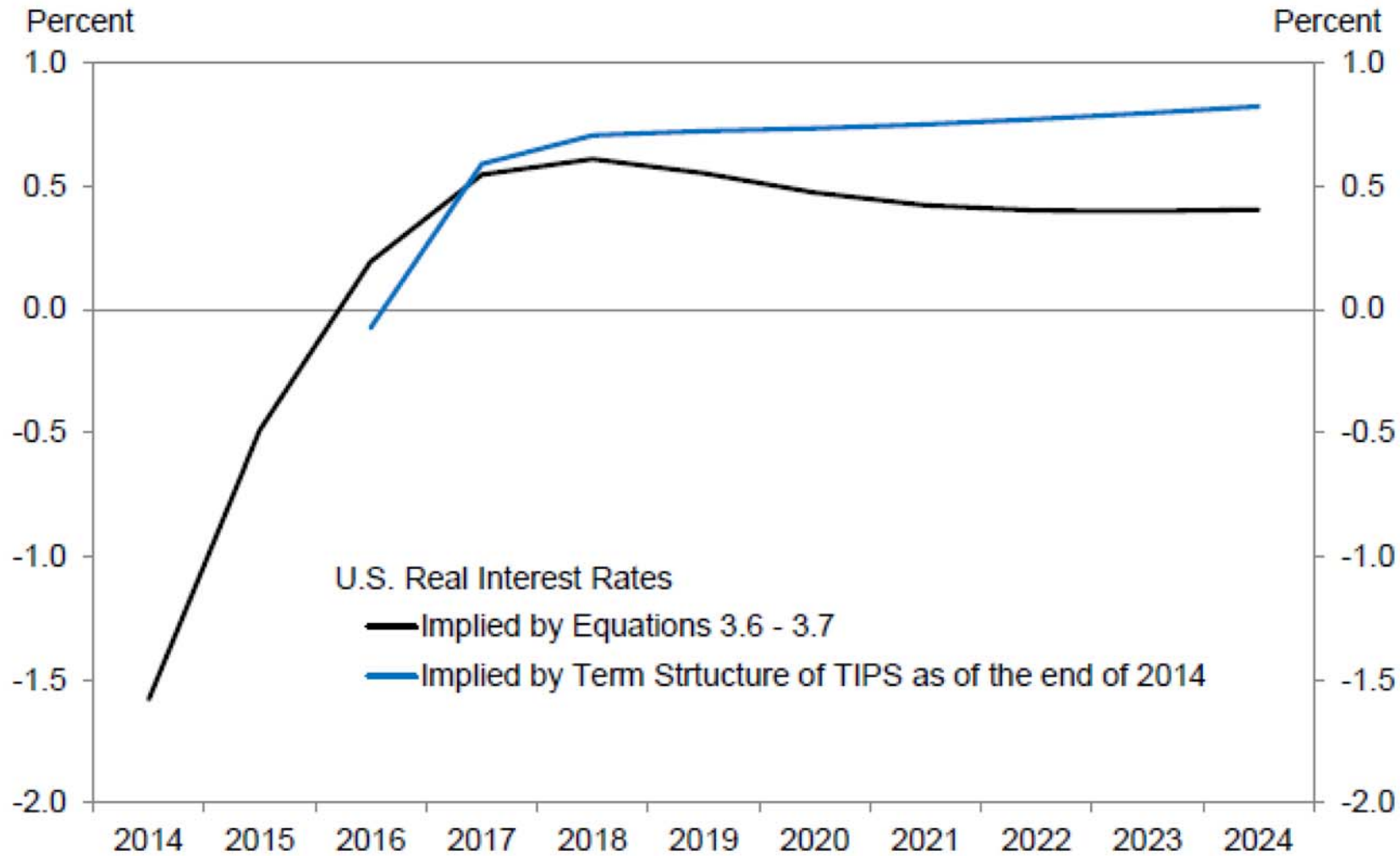


## Variability and Uncertainty of Estimates in Some Other Studies

	sample period	range (%)	max discrep (bp)
Barsky et al. (2014), Fig. 1	1990-2012	-6 to +11	n.a.
Curdia et al. (2014), Fig. 1	1987-2009	-9 to +4	150bp
Clark and Kozicki (2005), Fig. 1b	1962-2004	0 to +7	200bp
Laubach and Williams (2002), Fig. 3	1961-2002	2 to 4	100bp

Note: “Range” presents the lowest and highest value in the indicated sample, using the authors’s preferred specification.

“Max discrep” is the maximum point in time discrepancy (i.e., maximum difference) in two estimates of the equilibrium rate at a given quarter, with the two estimates computed from seemingly similar specifications.



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## VII. Monetary policy implications of uncertainty

- What does uncertainty about the equilibrium rate imply for monetary policy?
- Building on intuition and analysis of Orphanides and Williams (2002), we use FRB/US to quantify effects of uncertainty about  $r^*$ 
  - Notation:  $r^*$  = equilibrium rate.

## Orphanides and Williams (2002)

- Orphanides and Williams (“OW”, 2002) consider optimal monetary policy rules using a small stylized model of the US economy.
- They show that uncertainty around  $r^*$  implies that the optimal monetary policy rule should be more “inertial” than a standard Taylor rule.
- An inertial rule puts more weight on the lagged funds rate and less weight on the estimated value of  $r^*$ .
- An extreme inertial rule is a “difference rule.” In a difference rule, the change in interest rates depends on the level of inflation and the employment gap.

## Our analysis

- We revisit the OW analysis in a richer and more realistic setting, namely the board staff model FRB/US benchmarked to the FOMC's current economic outlook.
- We augment FRB/US by introducing errors into the FOMC's perception of  $r^*$ , which feed back into the path for the federal funds rate.
- We then compare the behavior of the economy under a standard Taylor (1999) rule with the behavior of the economy under an alternative “inertial” Taylor 1999 rule that responds only partially to changes in the estimate of  $r^*$ .



## Some details

- We use the unemployment rate  $u_t$  as the real activity variable, in both the Taylor rule and the Fed's loss function.
- The loss function is expected discounted sum of per period losses, with baseline

$$\text{per period loss} = (u_t - u_t^*)^2 + (\pi_t - \pi^*)^2 + 0.5(\Delta i_t)^2$$

and robustness checks wrt weight on  $(\Delta i_t)^2$

## Taylor rule

- Taylor rule is maximum of 0 and

$$i_t = \alpha_0 i_{t-1} + (1-\alpha_0)(r_t^* + \pi_t) + \alpha_1(\pi_t - \pi^*) + \alpha_2(u_t - u_t^*)$$

- $\alpha_1 = 0.5$ ,  $\alpha_2 = -2.0$ ,
- $\alpha_0$  the parameter that indexes inertia;  $\alpha_0 = 1 \Rightarrow$  difference rule,
- $r_t^*$  follows one of the three paths described on the next page.

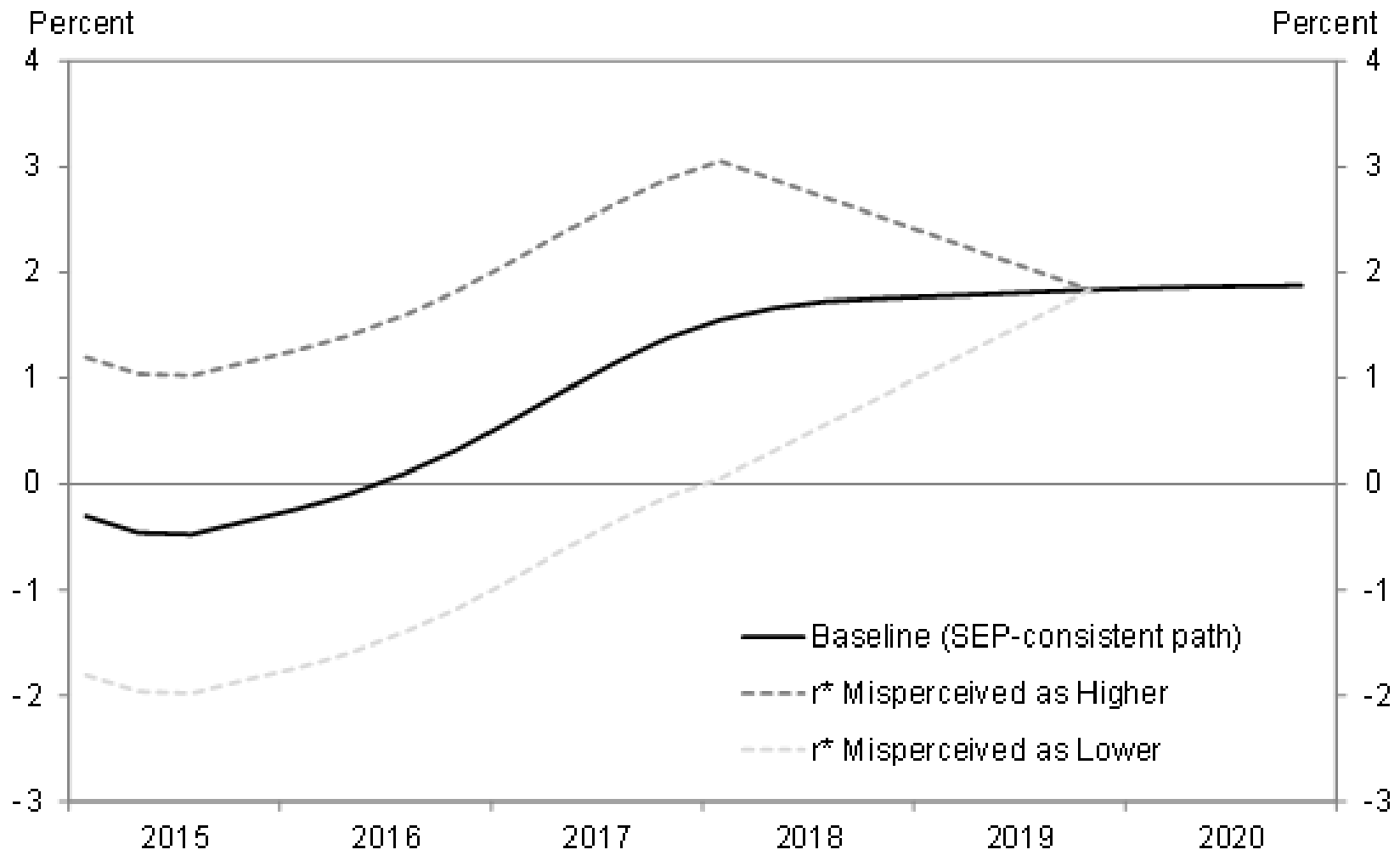
## Modeling uncertainty about $r^*$

- Uncertainty about  $r^*$  means:

- One possible path for  $r_t^*$  going forward is one consistent with the Dec. 2014 SEP projections for  $i$ ,  $\pi$  and  $u$ , along with  $\pi^*=2.0$  and the FRB/US path for  $u_t^*$ . This the SEP-consistent or baseline path.

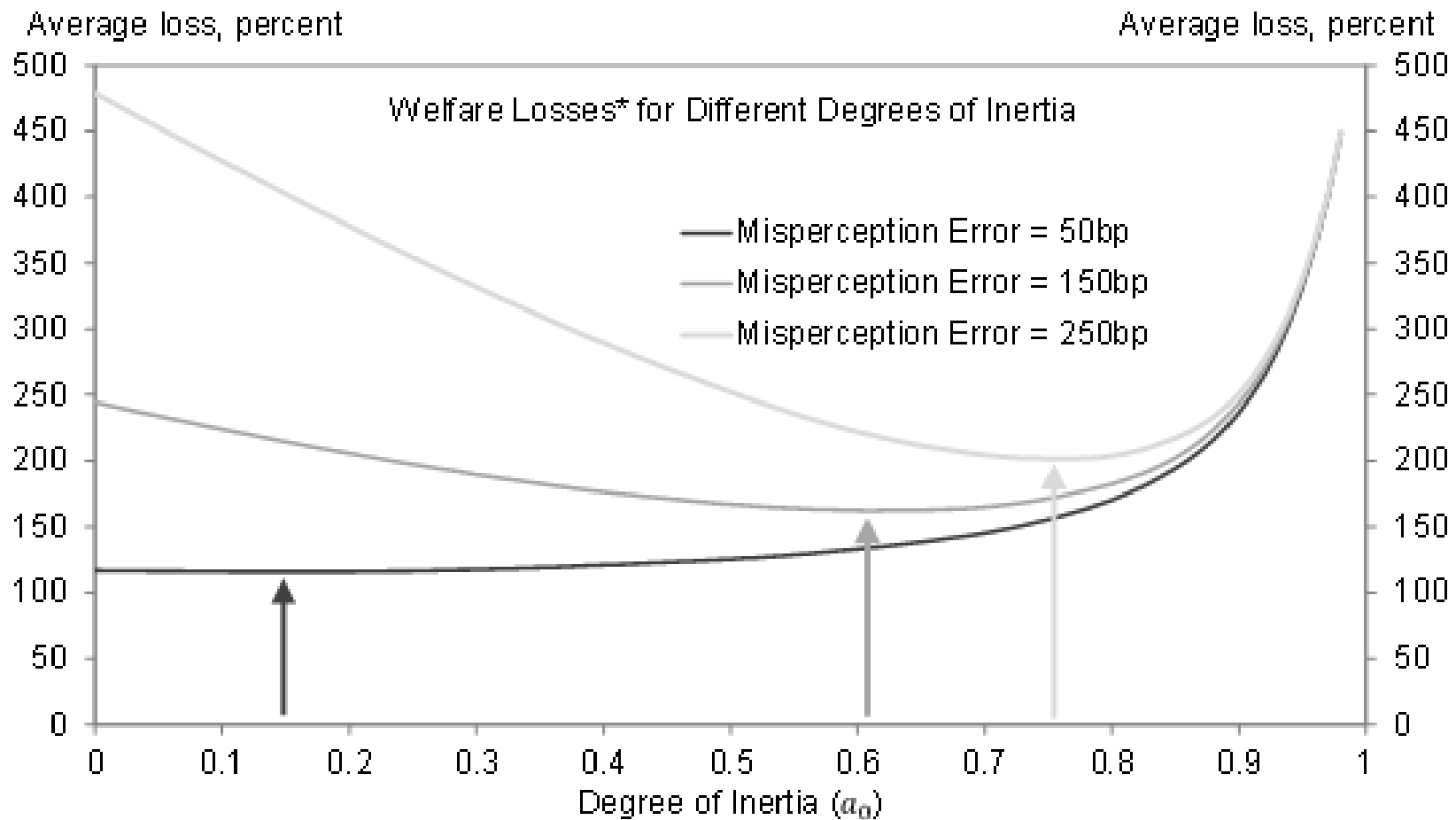
- Two other possible paths begin as  $\pm 150$  bp from the medium path, converging to the baseline path in 2020 (picture on next overhead). These are the high and low paths. (With robustness checks for  $\pm 50$  and  $\pm 250$ .)

- In computing expected losses, each of the three paths are equally likely (each have probability  $1/3$ ).



## Our results

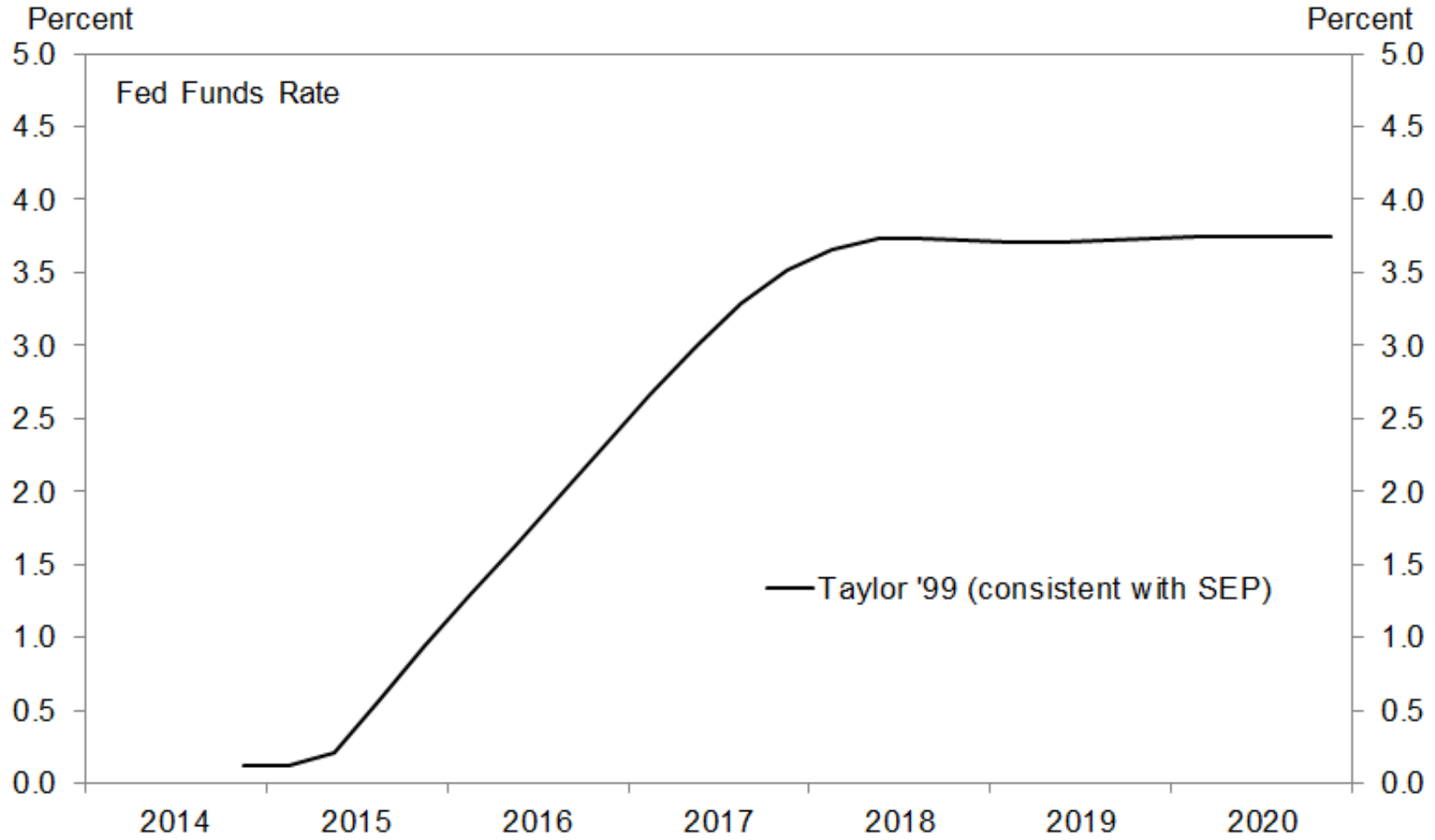
- We assume backward looking expectations, with a quick check for model consistent expectations.
- Consistent with OW, we find that a more “inertial” policy rule leads to better economic performance if there is more uncertainty around  $r^*$  (next overhead).
- We show that when the starting point for the funds rate is zero, an “inertial” policy rule yields a later but steeper normalization.



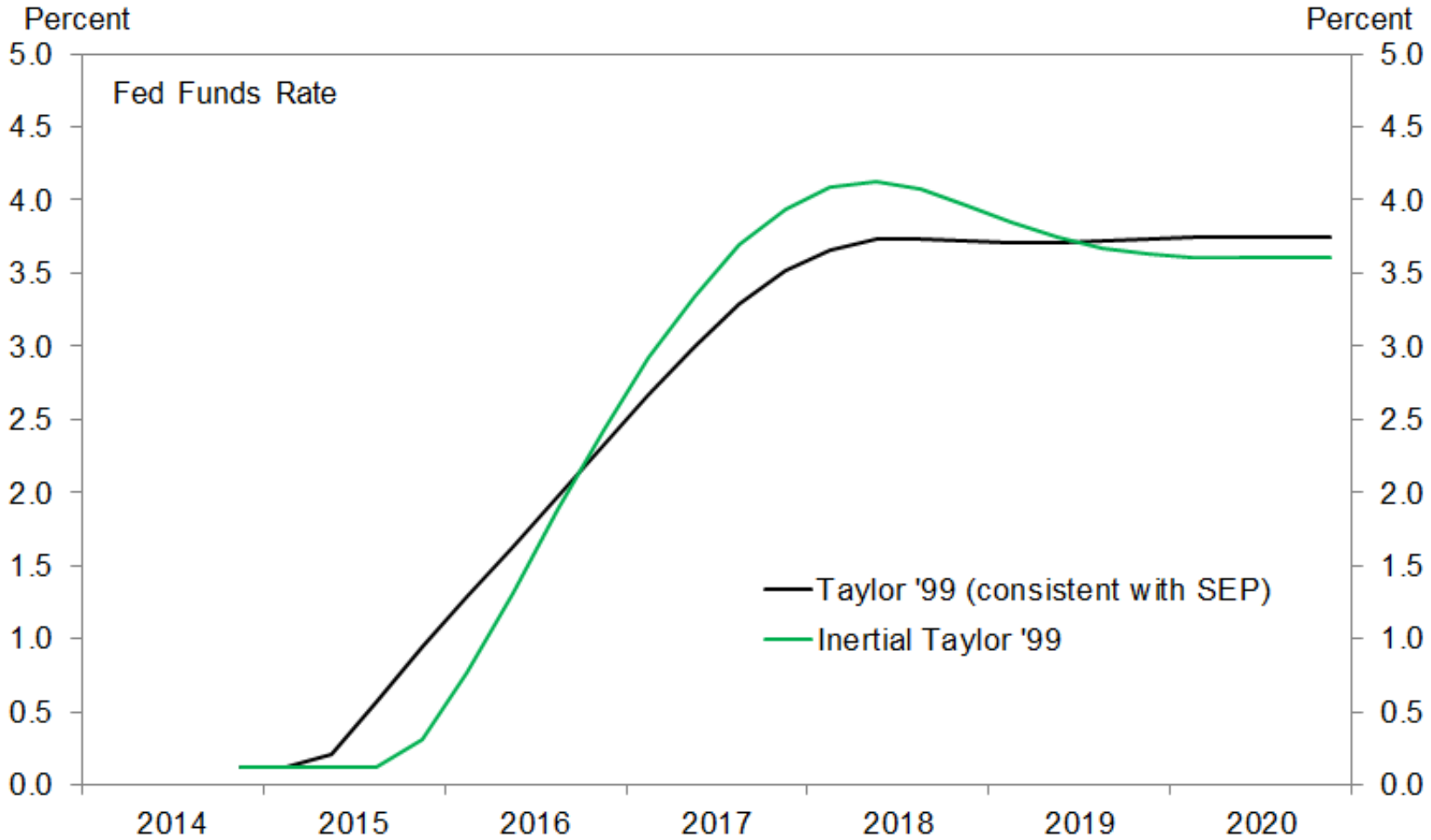
Note: Arrows indicate optimal inertia coefficients.

\* Relative to no-misperception optimum.

# Implications for the Policy Path

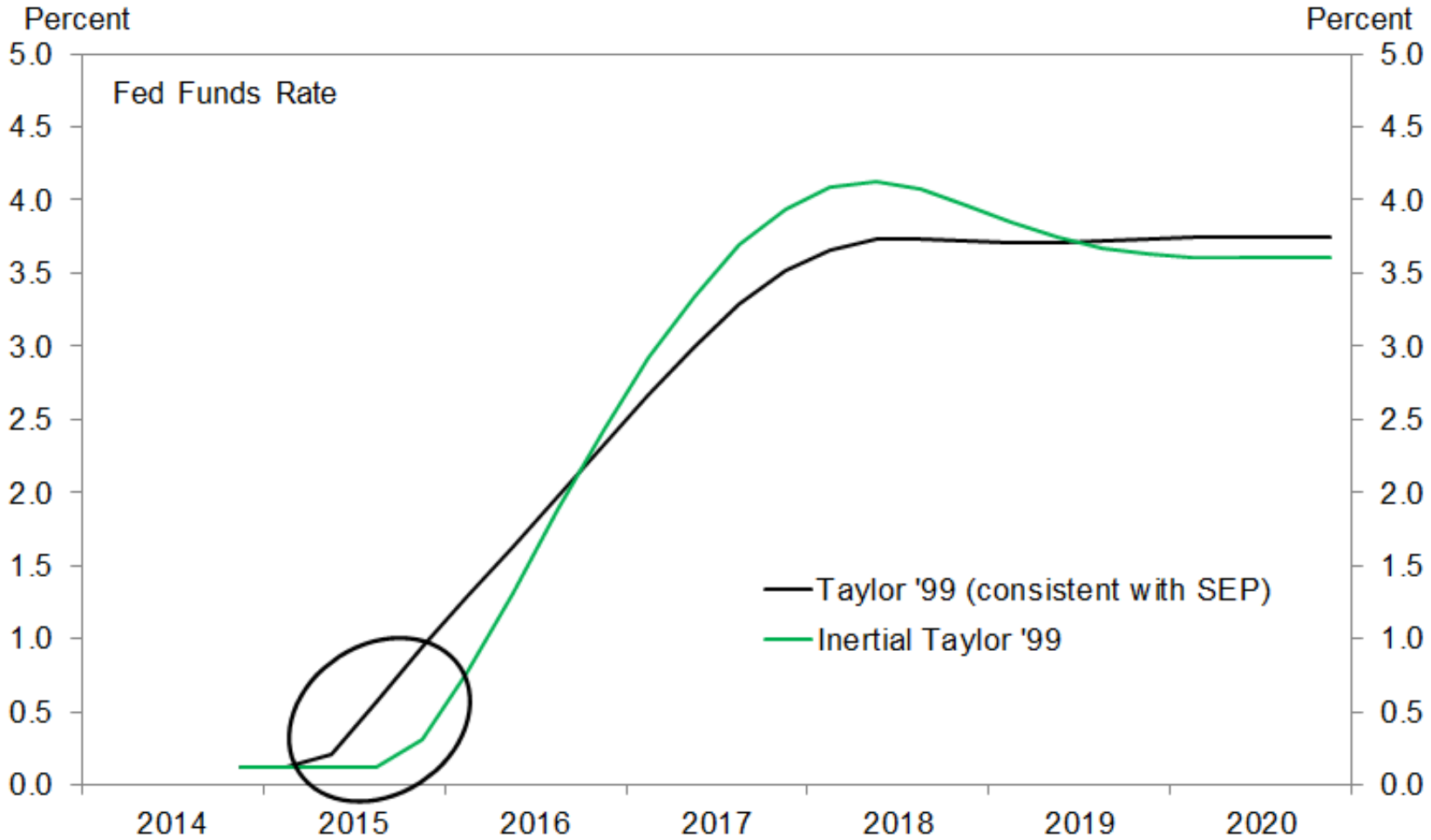


# Implications for the Policy Path-2

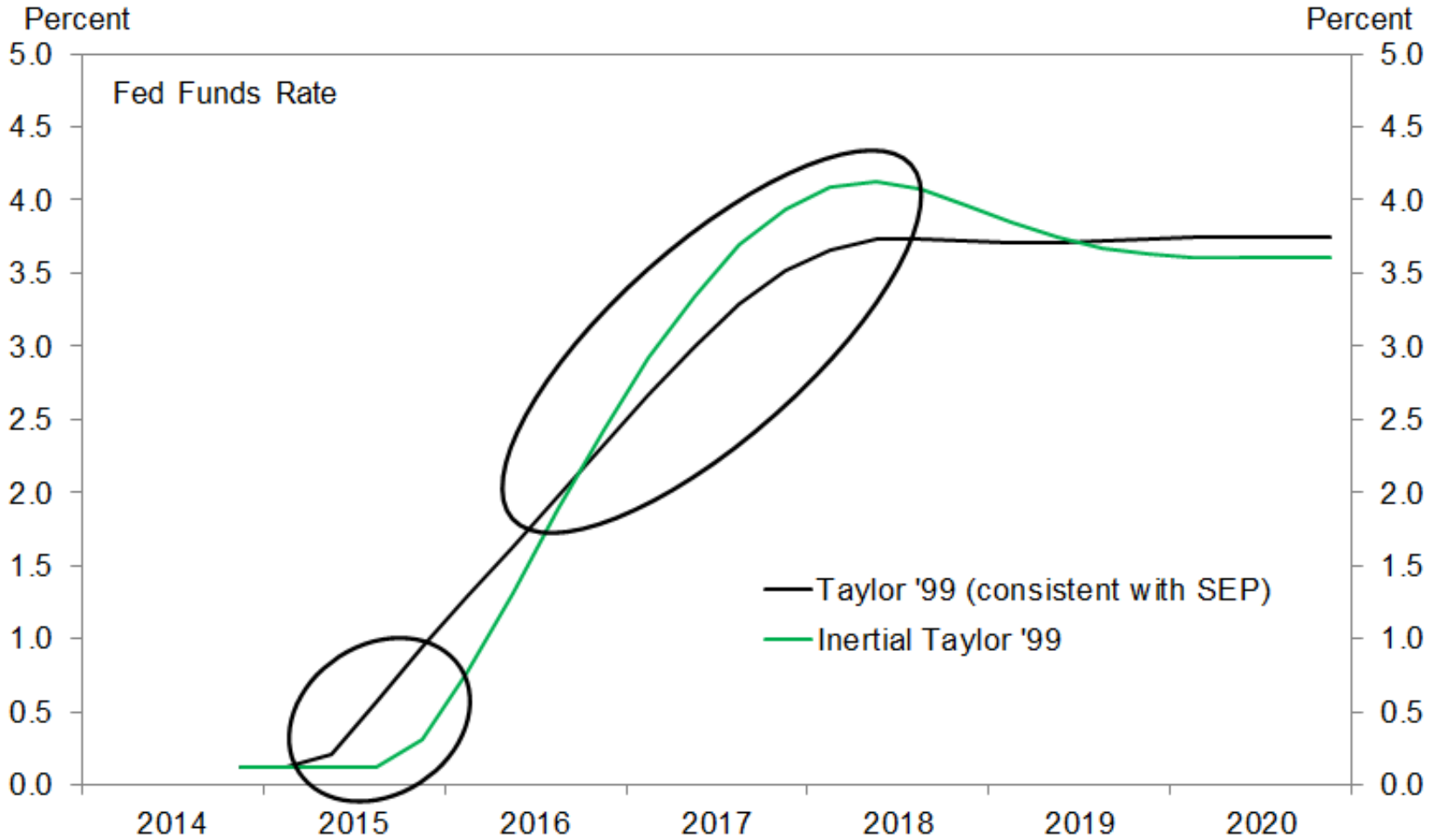




# Implications for the Policy Path-3



# Implications for the Policy Path-4



## Broader context

- Our analysis complements other arguments for a later liftoff:
- While some job market measures such as job openings and headline unemployment have tightened a lot, broad measures such as U6 and E/P still show substantial slack.
- The continued weakness of nominal wage growth supports a focus on broad as opposed to narrow slack measures.
- Core inflation remains well below the 2% target, and only some of this is explained by oil and dollar pass-through.
- The risks to global growth and inflation remain on the downside. At the ZLB, hiking too early is riskier than hiking too late.

I. Introduction

II. Construction of ex-ante real rates

III. The real rate, consumption growth and aggregate growth

IV. The real rate and aggregate growth: empirical analysis

V. Narrative evidence on real rates in the U.S.

VI. Long run tendencies of the real rate

VII. Monetary policy implications of uncertainty

**VIII. Conclusion**

## VIII. Conclusion

- There is much uncertainty about the equilibrium rate, which varies considerably over time, and arguably is well modeled as having a unit root.
- The determinants of the equilibrium rate are manifold and time varying, with the effects of trend output growth generally dominated by those of other factors.
- A vector error correction model that looks only to U.S. and world real rates well captures the behavior of U.S. real rates.
- Looking forward, a plausible range for the equilibrium rate is wide, perhaps ranging from a little above 0 up to 2%.