A Sharper Ratio: A General Measure for Ranking Investment Risks

Kent Smetters[1] Xingtan Zhang[2]

[1]The Wharton School and NBER

[2]The Wharton School

March 2015

Disclaimer

No conflicts to report.

Size of active and passive indexing

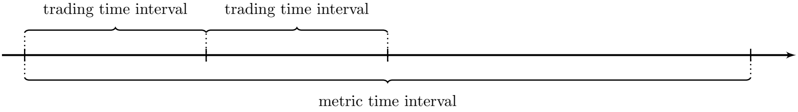
Assets under investment management: $34 trillion U.S. and $19 trillion EUR

"Alternative indexing" ("smart beta" / "multifactor") now 25% of all ETF flows

Hedge fund assets: $2.9 trillion

Passive index market cap growing but still a minority

Rebalancing occurs at a higher frequency than risk measurement



Rebalancing occurs daily (maybe weekly)

Hedge funds / active trade even more frequently

But Sharpe Ratios reported over life of the fund, at best over previous 3 years (GIPS).

Mismatch of trading - risk intervals under-explored

Even if the fundamental underlying risk (e.g., S&P1500) is Normally distributed, mismatch produces non-Normal investment returns

Alters the "higher" (3+) moments in critical ways

Our question: How to properly characterize the risk of a modern trading strategies.

Key question of:

- fund / manager / investment selection
- benchmarking on a truly risk-adjusted basis
- optimal portfolio construction and rebalancing
- calculating fair fees for non-market cap weighting trading strategies

Example Baseline: 50 / 50 portfolio [1]

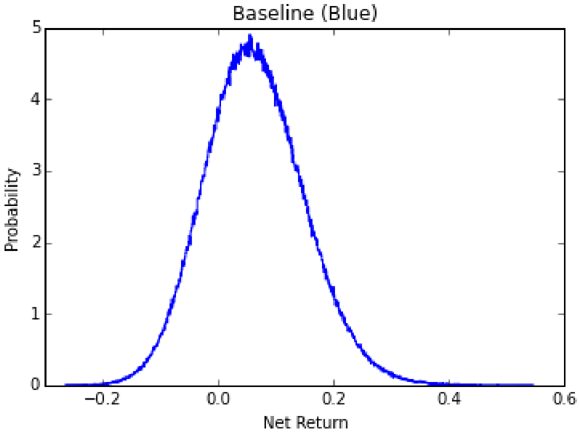
50% of wealth invested in risky asset, calibrated i.i.d. $\sim N(\mu, \sigma^2)$ using S&P1500 *weekly* data

50% in the risk-free asset paying annualized 1%

Returns reinvested 50/50 each week for one year

Sharpe Ratio and moments measured on final *annual* return distribution

Example Baseline: 50 / 50 portfolio [2]



Example Baseline: 50 / 50 portfolio [3]

$$\text{Annual Sharpe Ratio (SR)} = \frac{\mathbb{E}Y - r}{\sqrt{Var(Y)}} = 0.62$$

Index	**SR**	**M3**	**M4**	**M5**	**M6**	**M7**	**M8**	**M9**
Baseline	0.62	0.232	3.10	2.37	17.0	26	140	331

Andy Lo's "Capital Decimation Partners"

Baseline + leveraged selling 10% out-of-the-money puts

- Number of options set to produce income equal to 0.75% wealth, every three months (~ 3% additional return per year)

Index	SR	M3	M4	M5	M6	M7	M8	M9
Baseline	0.62	0.232	3.10	2.37	17.0	26	140	331
Baseline + Puts	0.96	0.227	3.18	1.18	31.7	-159	2436	-28241

- ***Odd*** higher moments (3,5,..) **decrease**, **even increase**
- Rational investor wants larger odd, smaller even (see paper)
- A lot of action shows up in moments 5+
- Chance of losing a fixed value grows rapidly

But, you don't invest in those strategies, right?

Even Lo hopes sophisticated people like you won't be suckered:

> *"Shorting deep out-of-the-money puts is a well-known artifice employed by unscrupulous hedge fund managers to build an impressive track record quickly, and most sophisticated investors are able to avoid such chicanery."*

But, as he notes, options are simply a delta hedge

... and delta hedge usually produced trading more frequently than measurement

Simple trading rule [1]

Baseline + a simple rebalancing rule/function (R)

Stock exposure = 50% + R($1 - $V), with $1 initial investment, V is value during the year

In words: stocks allocation varies away from 50% as V changes over the year

As before, weekly returns i.i.d.$\sim N\left(\mu, \sigma^2\right)$ using weekly S&P1500 parameters

I.e., a standard, rational CRRA agent would not rebalance
... but, we want a large Sharpe Ratio!

Simple trading rule [2]

Approximate R() as a quartic spline (similar results with cubic or higher-order polynomial, also for allowing another intercept)

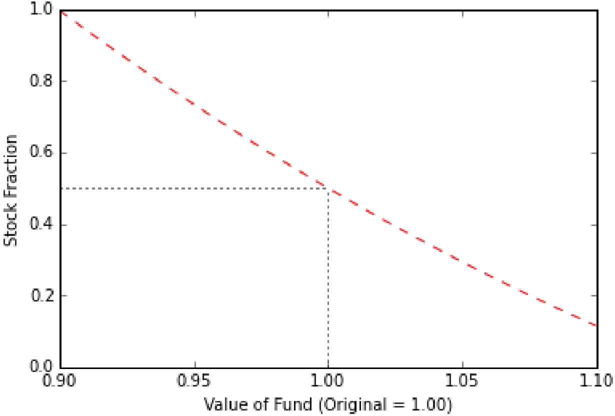
$$R(V) = 0.50 + \psi_1 (V-1) + \psi_2 (V-1)^2 + \psi_3 (V-1)^3 + \psi_4 (V-1)^4$$

Optimize (using BFGS) for coefficients ψ_1, ψ_2, ψ_3 and ψ_4 to maximize the Sharpe Ratio

$$SR = \frac{\mathbb{E}V - r_f}{\sigma_V}$$

Simple trading rule [3]

Optimal quartic:



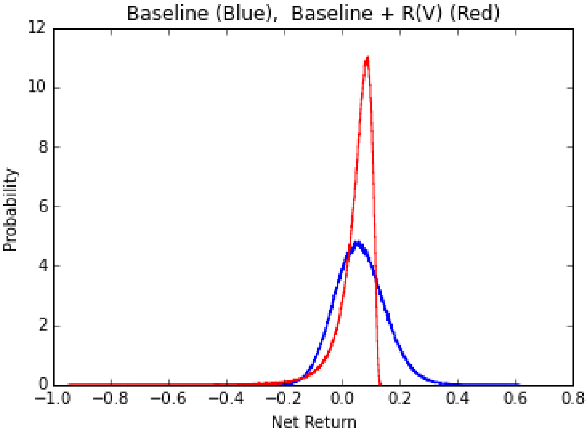
Simple trading rule [4]

Shockingly familiar trading rule?

- Value (or contrarian) investing: buy low, sell high
- Also, a crude form of selling deep out-of-money puts
- Similar to Lo's "Capital Decimation Partners II" where he used market data.
  - But, we are explicitly maximizing SR with i.i.d. draws

Simple trading rule [5]

Final (annual) return distribution (red) relative to baseline (blue):



Simple trading rule [6]

Annual Sharpe Ratio: 0.71 (14.5% increase from Baseline)

- But only weekly rebalancing, one year total, no excess vol, no margins in reasonable range.

Index	SR	M3	M4	M5	M6	M7	M8	M9
Baseline	0.62	0.232	3.10	2.37	17.0	26	140	331
Baseline + Puts	0.96	0.227	3.18	1.18	31.7	-159	2436	-28241
Baseline + R()	0.71	-2.319	13.14	-87.24	721.7	-6833	71230	-794985

- Again, raise SR by pushing higher moments in wrong direction
- Standard hedge fund moments (negative skew, fatter kurtosis, etc.) with simpler trading rule than previous clones
- *Because of i.i.d. draws, it is the trading rules, not autocorrelated market data, producing these results.*

Myth 1: "Max drawdown" is informative

Industry standard for measuring "tails" like these

Structural modeling ("parameterized model"):

- Max drawdown is lower support of net return distribution (i.e., -100% with L.L.)

Reduced-form modeling ("letting the data talk"):

- History is one sequence of draws from a distribution
- Lower support is again -100%
- Otherwise, likely presenting ex-post lucky strategies (data mining)

Myth 2: Multidimension Risk Management [1]

Indeed, common practice is to get a large SR with small tail risk measure, like:

- Maximum drawdown, Sortino, Treynor, Omega, Kappa, AVaR, or 100+ more measures

Multidimension RM sounds sophisticated, but reflects lack of a foundation

It is a myth for three reasons:

Myth 2: Multidimension Risk Management [2]

1. Ultimately, you must pick best portfolio:
    - What do you do if SR(risk Y_1) $>$ SR(risk Y_2) but Y_1 has a larger *AVaR* or max drawdown?
    - You need an aggregator producing an *ordinal* ranking
    - Normatively (i.e., first best), a *rational* or *valid* measure strictly increases in investor's Expected Utility (EU)
    - EU is still the most compelling way to measure investor's reward vs. risk

Myth 2: Multidimension Risk Management [3]

2. Most relevant investor question: how much should I be willing to pay over a market cap fund (efficient markets) for a non-cap fund?
   - Now, a *cardinal* ranking must be extracted from ordinal ranking

3. Even seemingly clear cases where investment strategy risk $D_s(Y_1) > D_s(Y_2)$, for all of your D_s risk screens, $EU(Y_2) > EU(Y_1)$ is still possible:
   - Put simply, seeming intuitive screens are often not very good at balancing reward v. risk

Myth 3: Higher moments are hard to estimate [1]

Structural modeling ("parameterized model"):

- Higher moments can be calculated precisely from trading model (last example)
  - Ex. Given distribution of underlying security in the *trading interval* (e.g., S&P500 $\sim N\left(\mu,\sigma^2\right)$), trading rules identify all higher moments in the *risk measurement interval*
- In theory, investment models should be structural, even passive
  - Ex. if you think dividend weighting is good, model the reason
- In practice, assumes you're the manager, or trading rules transparent

Myth 3: Higher moments are hard to estimate [2]

Reduced-form modeling ("black box" or "letting the data talk"):

- More common, easier to do, only choice with hidden IP
- Higher moments estimated with some noise, but:
  - With SR, you are already making specific higher-moment assumptions:
    - I.e., skew = 0, kurt = 3, ... (*without* estimation)
  - Alternative 1: histograms, but no better & costlier to compute
  - Alternative 2: force a distribution (e.g., G&H), but:
    - still using moments to fit, and very costly to compute
    - restricts the range of investments you can examine (no multi-asset optimization)

Toward a Summary Statistic

So, we need a valid ranking measure increasing in EU

Trivially, the EU calculation itself is a valid ranking measure, but complicated and costly to compute

So we want the valid ranking measure to also be a *summary statistic*

- Economics: Reduce the problem to focus on key driving variables (e.g., elasticity in optimal tax)
- Technical: Given the economics insight, provide the same solution with less work

Sharpe Ratio was the first valid summary statistic in finance, and it still dominates today

Sharpe Ratio [1]

Formally

$$\max_a \mathbb{E}u\left(w(1+r)+a(Y-r)\right)$$

where $u(\bullet)$ = investor preferences, w = initial wealth, r = risk-free rate, and Y = return to risky asset. For ranking purpose, we write

$(Y_1, r) \geq_u^w (Y_2, r)$ iff

$$\max_a \mathbb{E}u\left(w(1+r)+a(Y_1-r)\right) \geq \max_a \mathbb{E}u\left(w(1+r)+a(Y_2-r)\right).$$

If $(Y_1, r) \geq_u^w (Y_2, r)$ holds for all $w > 0$, then we write

$$(Y_1, r) \geq_u (Y_2, r).$$

Sharpe Ratio [2]

Lemma

If u is quadratic or Y is normally distributed, then $\max_a \mathbb{E}u(w(1+r)+a(Y-r))$ is an increasing function of Sharpe Ratio squared $\left(\frac{\mathbb{E}Y-r}{\sqrt{Var(Y)}}\right)^2$. If we further restrict random variables space such that $\mathbb{E}Y \geq r$ then $\max_a \mathbb{E}u(w(1+r)+a(Y-r))$ is an increasing function of Sharpe Ratio $\frac{\mathbb{E}Y-r}{\sqrt{Var(Y)}}$.

Quadratic u implies IARA (not appealing)

Paper derives a more general sufficient condition

Other metrics like Sortino, AVaR, etc., are not valid. Despite their seemingly intuitive appeal, they fail to maximize EU.

- Either penalize risk too much or too little relative to return

Sharpe Ratio [3]

Under Normality, SR is an elegant summary statistic that's easier to solve than original EU problem:

1. Independent of wealth
   - allows for direct comparison of risk Y_1 vs. Y_2, that is, without re-solving the nested bond-stock allocation problem for each Y
   - *Technical contribution over EU maximization:* Removes iterative integration associated with $max\, Eu(\bullet)$ problem
   - Technical advance relies on economic insights in 4. and 5. below
2. Valid over discrete time
   - i.e., not a pertubation that is only valid as time shrinks

Sharpe Ratio [4]

3. Indifferent to leverage, like EU problem
   - E.g., if $r = 0$ then $(2Y, 0) =_u^w (Y, 0)$
   - Otherwise, ranking is completely arbitrary
4. Risk can be fully described by its moments
5. Is independent of investor utility preferences
   - *Economic contribution over EU maximization:* Problem based on moments alone isolates key drivers
   - Technical + Economic contribution $\Rightarrow$ ease of use for industry $\Rightarrow$ quick adoption

Sharpe Ratio [5]

But when Sharpe Ratio is **not** valid, it often “breaks, not bends”

Sharpe Ratio [5]

But when Sharpe Ratio is **not** valid, it often "breaks, not bends"

Probability	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	Sharpe Ratio
Return of Asset A	-1%	1%	2%	1.0
Return of Asset B	-1%	1%	11%	0.8

$B >^{FOSD} A$, but $SR(A) > SR(B)$

- All EU's who like more money (nonsatiation) should want investment B, but A wins with SR

Examples like this one incorrectly called "puzzles" in the past.

- Not a puzzle: SR is simply not a valid ranker for this risk.

Past Attempts to Extend SR [1]

Earliest attempts by P. Samuelson and Tobin (to a lesser extent, Borch and Feldstein)

They hit a roadblock. Recall, the original investor problem:

$$\max_a \mathbb{E} u\left(w(1+r)+a(Y-r)\right)$$

First-order Condition:

$$\begin{aligned} 0 &= \mathbb{E} u'\left(w_r + a(Y-r)\right)(Y-r) \\ &= \mathbb{E}\left(\sum_{n=0}^{\infty} u^{(n+1)}(w_r)\frac{a^n(Y-r)^n}{n!}\right)(Y-r) \\ &= \sum_{n=1}^{\infty} \frac{u^{(n)}(w_r)\mathbb{E}(Y-r)^n}{(n-1)!} a^{n-1} \end{aligned}$$

which has one real root by construction (concavity).

Past Attempts to Extend SR [2]

To produce a ($N > 2$)-moment ranking measure, truncate the Taylor expansion at N

- But, you now have N roots, some real, some complex.
- Which one corresponds to the original real root?

Brief literature stopped here

Rise of ad-hoc measures (100+ by one count)

Other risk measures produced by math community (e.g., coherence)

These measures are generically not valid

Our approach: Solve the original truncated Taylor problem [1]

We revisit the original truncated Taylor expansion problem and provide a new general lemma for selecting the correct root:

Lemma

Suppose real function $f(x) = 0$ *has unique real solution* x_0*. Denote Maclaurin expansion of* f *to be* $\sum_{n=0}^{\infty} c_n x^n$*. Let* $f_N(x) = \sum_{n=0}^{N} c_n x^n$*. Then* $f_N = 0$ *has* N *solutions* S_N *on complex plane. Denote the convergent radius for the series as* λ*. If* $\lambda > |x_0|$*, then:*
(i) the smallest absolute real root in S_N *converges to* x_0 *as* $N \to \infty$
(ii) there is a finite value of N*, call it* $\bar{N}$*, such that there is only one real root for all* $N > \bar{N}$*.*

Our approach: Solve the original truncated Taylor problem [2]

Definition

We will say that the utility-risk pair (u, Y) satisfies the *Regularity Condition* if the corresponding series of satisfies the requirement $\lambda > |x_0|$ in above lemma. Denote $\mathscr{A}_{\mathscr{R}\mathscr{C}}$ as the admissible space of all the utility-risk pairs where the regularity condition holds.

- Regularity trivially holds for CARA, and across a wide spectrum of problems.

New Measure [1]

$$\mathscr{U}_h = \{u(\cdot): u(w) = \frac{\rho}{1-\rho}\left(\frac{\theta w}{\rho} + \phi\right)^{1-\rho}\}$$

Large range of preferences. Let $\mathscr{U}_h^\rho \subset \mathscr{U}_h$ be a subset, given ρ.

FOC:

$$0 = -\sum_{n=1}^{\infty} \frac{b_n t_n^Y}{(n-1)!} z^{n-1}$$

where $z = -\frac{\theta}{\rho}\frac{a}{\frac{\lambda w_r}{\rho}+\phi}$ and

$$b_n = \begin{cases} 1, & n = 1 \\ (\rho)\cdots(\rho+n-2), & n \geq 2 \end{cases},$$

and

$$t_n^Y \equiv \mathbb{E}(Y-r)^n$$

New Measure [2]

Definition

Denote $z_{N,Y}$ as the smallest absolute real root z that solves the truncated equation, $-\sum_{n=1}^{N} \frac{b_n t_n^Y}{(n-1)!} z^{n-1} = 0$. The N-th order HARA ranking measure is defined as:

$$q_N^H(t_\cdot^Y, b_\cdot) = -\sum_{n=1}^{N} \frac{b_n t_n^Y}{n!} z_{N,Y}^n.$$

Theorem

$q_H^N(t_n^Y, b_n)$ is a valid ranking measure sequence w.r.t. to the admissible space $\mathscr{A}_{\mathscr{H}} \equiv \left(\mathscr{U}_H^\rho \times \mathscr{Y}\right) \cap \mathscr{A}_{\mathscr{R}\mathscr{C}}$, where $\mathscr{Y}$ is the set of all random variables.

New Measure [3]

Under **any distribution** in $\mathscr{A}_{\mathscr{RC}}$, the ranking measure $q_N^H(t^Y, b)$ may **not** be as elegant as SR, but still easier to solve than max EU:

1. [Same] Independent of wealth
   - allows for direct comparison of risk Y_1 vs. Y_2, without re-solving the nested bond-stock allocation problem for each Y
   - *Technical contribution over EU maximization:* Removes the iterative integration associated with $max\, Eu(\bullet)$ problem, which is now very costly for non-Normal risk

2. [Same] Valid over discrete time
   - i.e., not a pertubation that is only valid as time shrinks

New Measure [4]

3. [Same] Indifferent to leverage
4. [Extension] Risk can be fully described by moments
    - [NEW] can rank risk Y_1 versus Y_2, even if each follows a different distribution (SR required the same distribution, i.e., Normal)
        - Y_1 (e.g., w/ option overlay) and Y_2 (e.g., w/o overlay)
        - allows for multi-asset portfolio construction
5. [More restricted] Ranking depends on ρ (but not on θ and ϕ)
    - Except for CARA ($\phi = 1, \rho \to \infty$), where only moments matter
    - *Economic contribution over EU maximization:* Problem based on moments and ρ alone isolates key drivers

“1. Independent of wealth” [1]

With non-Normal risk, wealth separation represents an even greater advantage relative to EU maximization.

Iterative integration associated with *max* EU problem even more costly with non-Normal risk

- No algebraic approx. for stock-bond allocation with non-Normal risk

So, calculation of $q_N^H(t^Y, b)$ is much cheaper

- ~ 100-500X faster than *max* EU problem
- Like Sharpe, quickly solve-able on, e.g., Excel
- Can use Excel “Solver” with initial guess of zero to find root
- As with Sharpe, ease of use is key for industry adoption

"5. Ranking depends on ρ" [1]

Not just a feature of our metric:

Theorem

There does not exist a moments-only ranking measure for HARA utility if portfolio risk Y can be any random variable.

Corollary

[Impossibility] *There does not exist a generic moments-only ranking measure independent of ρ if Y can be any random variable.*

“5. Ranking depends on ρ” [2]

Remark. But only ρ (“risk tolerance”) matters for valid ranking.
I.e., ϕ and θ do not matter.

Important because:

- Financial advisors already required (SEC and USA) to estimate client’s risk tolerance
- Same in Canada and many European countries
- In fact, lots of competing software for measuring risk tolerance
- In contrast, λ and ϕ are harder to measure

"5. Ranking depends on ρ" [3]

In practice, conditioning on ρ is not a big limitation:

- Again, by law, financial advisors must know this value anyway
- A fund then reports three ranking scores (say, corresponding to $\rho = 1, 3, 5$) rather than a single SR score
- Hasanhodzic (2015) shows plausible equity premium and small risk-free rate possible with $\rho = 2$ or 3.
- Advisor then picks fund conditional on client's ρ

Summary and Intuition [1]

EU selection between *risk-free and risky* asset depends on everything: wealth, preferences, and risk distribution

But, with Normal risk, Sharpe showed:

- Economics insight: EU selection between risk Y_1 and risk Y_2 only depends on risk distribution
- Technical contribution: Isolated the moments that characterize the risk distribution, avoiding iterative integration

Summary and Intuition [2]

With non-Normal risk, we show:

- Economics insight: EU selection between Y_1 and Y_2 only depends on risk distribution *and* ρ for broad HARA class
  - In fact, impossible to create a general ranker independent of ρ
- Technical contribution: Isolated the moments and ρ, avoiding iterative integration
  - Integration is especially costly with non-Normal distributions

Concern 1: Does $q_N^H(t^Y, b)$ work with little data? [1]

Just as accurate as EU with same data / distribution

Example: i.i.d. $N\left(\mu, \sigma^2\right)$ draws using S&P1500 weekly data

- Estimated market cap $q_N^H(t^{Y_1}, b)$ and EU with 1,000,000 weeks
- For each length of 30, 60, 120 or 240 weeks, did 5000 draws
- I.e., each draw only roughly N distributed, some draws fair away
- Computed fair fee f_q for *each* draw n using $q_N^H(t^Y, b)$ with 10 moments, and f_{EU} using longer EU
- Worst error of any draw, $\sup_n \| f_q^n - f_{EU}^n \| < 10^{-12}$, i.e., at machine numerical precision, even at 30 weeks

Robustness: i.i.d. $G\&H\left(\mu, \sigma^2, g, h\right)$ with $g < 0$ (- skew) and $h > 0$ (excess kurt.)

- Worst error still close to machine precision

Concern 2: Is $q_N^H(t^Y, b)$ score too complicated? [1]

Yes, but so is the Sharpe Ratio. The issue is a general one.

Investopedia says that a SR > 1 is a good investment

- But what does SR of 0.60 or 1.2 mean?
- In practice, you just know you want SR larger than market cap
- Suppose SR(Y_2) $= 1.2$ (smart beta) $>$ SR(Y_1) $= 0.6$ (market cap)
  - How much more should you pay for fund Y_2 than Y_1?
  - Twice as much (1.2 / 0.6) is not correct
- I.e., SR is an *ordinal* measure (ranker) w/o easy interpretation

Concern 2: Is $q_N^H(t^Y, b)$ score too complicated? [2]

Instead, construct a "fair fee" measure, f

Approach 1: $q_N^H(t^{Y_1}, b) = q_N^H(t^{Y_2 - f_{CV}}, b)$, where Y_1 is market cap *after* fee, Y_2 is alternative *before*

- "Compensating Variation"
- In words: "how many basis points over market cap should I pay (or be paid if $f < 0$) for alternative"
- However, f itself is not a ranking measure because two different distributions could have same EU (equivalently, same SZ score) but different f values. (Intuition: interplay of tails with MU).
- Still, gives the best answer for max. management fee I should pay for alternative

Concern 2: Is $q_N^H(t^Y, b)$ score too complicated? [3]

Approach 2: $q_N^H(t^{Y_1+f_{EV}}, b) = q_N^H(t^{Y_2}, b)$

- "Equivalent Variation"
- In words: "how much additional return must market cap yield to make me indifferent to alternative"
- Increasing in SZ (or EU), and so f is also a ranking measure

Application 1: Ranking Smart Betas

Let's rank various popular alternative passive indexing methods

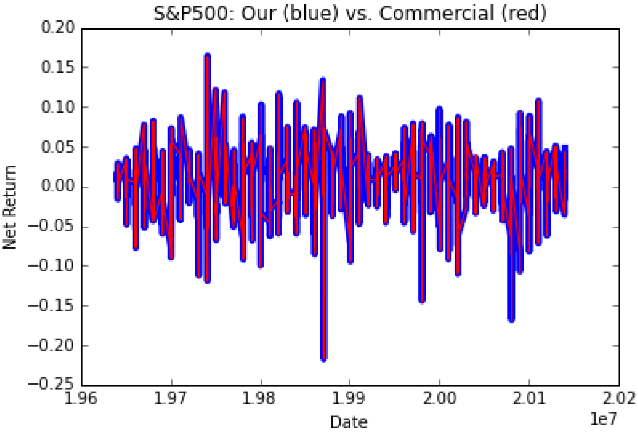
- But most indices don't push back very far
- So, we want to push back further

CRSP-COMPUSTAT match to construct four common passive indices:

- Market cap weighting, S&P500 and S&P1500 (to validate constituent match method)
- Equal weighting
- Dividend weighting
- Fundamental weighting

Monthly frequency

Market Cap Weighting



Corr(index we generated, commercial) > 0.9998

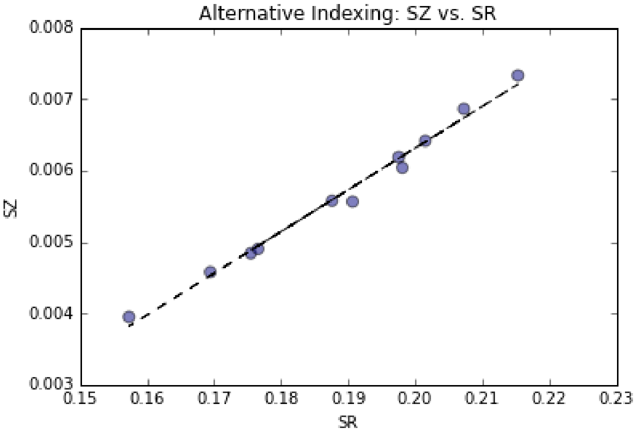
Market Cap vs. Equal, Div, and Fund

Index	SR	SZ	M3	M4	M5	M6	M7	M8	M9
S&P 1500	0.1755	0.0048	1.157	0.009	0.0020	-0.710	4.12	-8.70	36.5
S&P 500	0.1876	0.0056	1.883	0.009	0.0019	-0.417	4.87	-8.38	60.0
Fundamental	0.1976	0.0062	1.990	0.010	0.0019	-0.357	4.97	-7.04	55.6
Equal Weight	0.2015	0.0064	2.545	0.011	0.0025	-0.327	5.53	-7.79	79.6
Div. Weight	0.2154	0.0073	1.718	0.010	0.0018	-0.334	4.70	-6.44	49.0

Adding Commercial Indices [1]

INDEX	SR	SZ	M3	M4	M5	M6	M7	M8	M9
HFRX_America	0.1573	0.00395	-0.712	3.111	-4.946	16.0	-36.1	109.0	-291.8
RAFI_1000	0.1695	0.00458	-0.454	5.057	-5.213	54.7	-73.3	807.5	-1236.5
S&P 1500	0.1755	0.00484	-0.710	4.121	-8.696	36.5	-117.9	467.7	-1737.8
US_Value	0.1766	0.00490	-0.658	4.488	-8.519	41.2	-122.3	539.3	-1947.0
S&P 500	0.1876	0.00558	-0.417	4.867	-8.376	60.0	-198.8	1178.1	-5055.0
HFRX_Global	0.1907	**0.00556**	-0.644	8.234	-22.994	156.3	-682.0	3871.5	-19358.5
Fundamental	0.1976	0.00619	-0.357	4.973	-7.040	55.6	-148.3	913.0	-3256.4
US_Risk	0.1981	**0.00604**	-0.769	5.099	-13.489	68.5	-285.1	1406.4	-6665.0
Equal Cap	0.2015	0.00641	-0.327	5.527	-7.789	79.6	-211.1	1699.8	-5988.1
US_Quality	0.2073	0.00686	-0.352	3.808	-4.184	28.0	-54.1	293.3	-750.4

Adding Commercial Indices [2]



Intuition and Punchline

SZ produces some reversals relative to SR, but median error of SR is only 3.7 annual basis points

- Computed conditional on a reversal occurring
- Relative to last nearest neighbor of lower SR rank
- Computed as median of abs of "compensating variation" (f_{CV})

Intuition: SR works well for strategies close to market cap, which is already close to Normally distributed

But, still need to compute SZ values to know this

Application 2: Active Manager Selection [1]

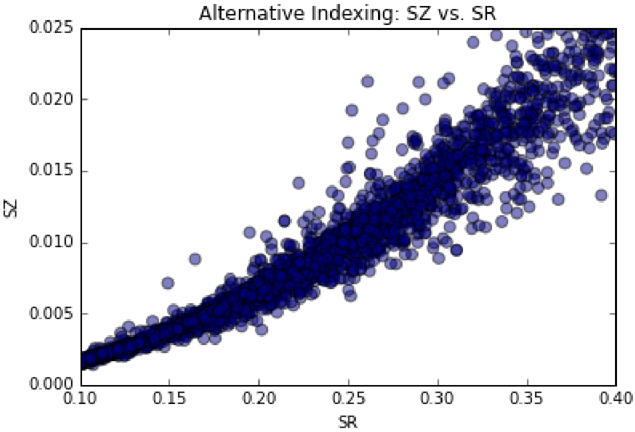
Entire HFR universe of hedge funds since 1993 with following screens:

- At least $100M at any point
- 36+ months of continuous reporting
- Includes non-survivors

6,463 funds pass these screens

- Results presented below magnified with smaller funds

Application 2: Active Manager Selection [2]



Application 2: Manager Selection [3]

$Corr(\text{SZ},\text{SR}) = 0.20$, i.e., positive but small

3,214 reversals relative to nearest neighbor (about half of data)

Median error (f_{CV}) is 174 bps per year

Sorting reversals from low to high, error at 75% is 469.8 bps

Bottom line: Using SR to rank active managers can now lead to large errors

Application 3: Multi-Asset Portfolio Optimization [1]

Let's do a really hard case of an "option overlay":

- today's price \$1
- price in 1 year is $S = \exp\left((\mu - \frac{1}{2}\sigma^2) + \sigma z\right)$, where z is the standard Normal random variable.
  $\mu = 0.10,\ r_f = 0.05,\ \sigma = 0.15$
- can also buy/sell a European put option with strike price \$0.88 and a European call option with strike price \$1.12, both mature in 1 year.
  - Put and call prices are \$0.0079 and \$0.0345, respectively.

Application 3: Multi-Asset Portfolio Optimization [2]

N	2 (=SR)	5	20	Simulation (verify)
CARA	(-1.374, -0.581)	(-0.402,-0.229)	(-0.390, -0.239)	(-0.390,-0.239)
CRRA(3)	(-1.374,-0.581)	(-0.482,0.529)	(-0.160,-0.077)	(-0.160,-0.076)
CRRA(4)	(-1.374,-0.581)	(-0.317,0.065)	(-0.214,-0.120)	(-0.214,-0.120)
CRRA(5)	(-1.374,-0.581)	(-0.310,-0.040)	(-0.247,-0.145)	(-0.247,-0.145)

Application 4: Is Piketty's Endowment Example Right?

[GETTING DATA NOW]

Works Cited in Talk I