The Not-so-well-known Three-and-one-half Factor Model

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Abstract

Equity analysts conceptualize the Fama-French framework as a tool for studying the size and value characteristics of equity portfolios along with the market return. But the market return is not the return to market beta. In fact, commercial providers of equity risk models typically include both a market factor and a beta factor, along with variations of the size and value factors. In other words, in equity risk modeling practice, the basic Fama-French framework includes four factors not just three. Unlike the other three factors, the intercept term (i.e., market factor) does not have a coefficient that varies across securities so can be described as just half a factor. We clarify the nature and role of the "first" factor in equity return models and explain that the distinction between the market portfolio return and the return to the cross-sectional variation in security beta also applies to portfolio performance measurement. Specifically, the realized alphas of low (high) beta portfolios are reduced (increased) when a beta factor is included. The problem of ignoring the beta factor in performance measurement pertains to fully invested portfolios that have a low or high beta based on security selection, not to changes in portfolio beta induced by cash or leverage.

The Not-so-well-known Three-and-one-half Factor Model

"I know what you're thinking. You're thinking, did he fire six shots or only five? Now to tell you the truth, I've forgotten myself in all this excitement. Do you feel lucky, punk?"

Famous misquote of Harry Callahan (1971)

What are the three factors in the well-known Fama-French model? Some analysts will answer the market, size, and value, while others answer beta, size, and value. So is it the return on the capitalization-weighted market portfolio or the return on CAPM beta? Consider the definition of the "Fama and French Three Factor Model" in the popular online educational site *Investopedia*: "A factor model that expands on the capital asset pricing model by adding size and value factors in addition to the market risk factor in the CAPM." Ask someone studying for the CFA exams what the phrase "market risk factor in the CAPM" refers to and they will probably answer beta. But if there's just three factors, with the first one being beta, where is the market?

To illustrate this issue we compare equations from two Fama-French papers published in the *Journal of Finance* in the 1990's. The empirical results in this study are based on the first equation, estimated by regression analysis and econometric techniques now used in industry. Specifically, we report the intercept and coefficient estimates from 600 monthly cross-sectional regressions using data on essentially all U.S. common stocks for the past half-century. That methodology, without the econometric enhancements, was used to create the last table in "The Cross-Section of Expected Stock Returns" by Fama and French (1992). With minor notational changes, their table is based on the equation

$$R_i = \mathrm{A} + \mathrm{B1}\ beta_i + \mathrm{B2}\ size_i + \mathrm{B3}\ value_i + \varepsilon_i \qquad \text{(FF 1992)}$$

where $beta_i$, $size_i$ and $value_i$ are security-specific parameters that allow the estimation of the returns to those factors each month. Fama and French (1992) then report the returns to *four* factors, not just three. The first factor, estimated by the regression intercept term, is described as the return on a "standard portfolio in which the weighted-average of the explanatory variables are zero." Fama and French (1992) then go on to explain that the other three factors are the return to individual stock betas (trailing 60-month historical regression estimates), size (beginning-of-month log market capitalization), and value (log book-to-market ratio).

The second equation, which we contrast with FF 1992 above, is prominently displayed in the introduction to "Multifactor Explanations of Asset Pricing Anomalies" by Fama and French (1996). With minor notational changes the now popular Fama-French three-factor equation is

$$R_i = a_i + b_i \text{ MRF} + s_i \text{ SMB} + h_i \text{ HML} + \varepsilon_i \qquad \text{(FF 1996)}$$

Interpretations of FF 1996 generally ignore the intercept term and then go on to explain that the first factor is the return on a capitalization-weighted market portfolio minus the risk-free rate (MRF), and the other two factors are the differential returns to Small-cap Minus Big-cap (SMB) portfolios and High Minus Low (HML) book-to-market portfolios. We emphasize that FF 1996 does not include a separate beta factor, while FF 1992 does, although there are other important distinctions. FF 1992 has the security-specific subscript "*i*" on the stock characteristics while FF 1996 uses the "*i*" subscript for factor return coefficients. FF 1992 is a cross-sectional regression used to calculate monthly factor returns. FF 1996 is a time-series regression used to estimate an individual stock or portfolio's alpha and sensitivity to a set of previously determined set of factors. Before using FF 1996, the SMB and HML returns are calculated each period by sorting stocks along the variable of interest, assigning those that fall above and below the 70th and 30th percentile into portfolios, and then taking the difference in portfolio returns.

Alternatively, FF 1992 and the main empirical contribution of this study employ what are commonly called Fama-MacBeth (1973) regressions. Cross-sectional regressions on security characteristics are at the core of equity risk-factor models provided by firms like Axioma and Barra, among others. Unlike the original Fama-MacBeth (1973) regressions, current risk modeling practice standardizes the characteristics across stocks to have zero mean and unit variance (i.e., calculate z-scores) and use various techniques to tilt the results towards larger capitalization stocks. But one common aspect of most factor-based risk models is that the cross-sectional regression equation contains both a market factor, without a coefficient, and a separate beta factor.[1]

The methodological choice characterized by the cross-sectional regression in FF 1992 and the time-series regression in FF 1996 often depends on the intended application. Cross-sectional regressions are clearly the methodology of choice in applied risk-factor models, while time-series regressions against MRF, SMB, HML, and sometimes UMD (for Up Minus Down momentum portfolios), are usually associated with the measurement of a fund's alpha. On the other hand, Back, Kapadia, and Ostdiek (2013) argue that Fama-Macbeth cross-sectional regressions yield purer factor returns than portfolio sorts, so do a better job at measuring alpha. They note that multivariate cross-sectional regression factor returns are less correlated with each other, use returns for all stocks, not just those above or below the 70th and 30th percentile, and contain all the information in stock characteristics, not just their order.

In this paper, we first discuss the conceptual distinction between the market return and the return to market beta and how the two can be separated in multivariate cross-sectional regressions. We explain that the slope coefficients in these regressions can also be described as the returns to specific long/short portfolios. We then document the highly divergent average returns to these two factors over the last half century in the U.S. equity market. Most equity market observers appreciate that the traditional CAPM prediction about the payoff to beta has not held up empirically, but may not appreciate how dramatic that divergence has been. Finally, we explore how time-series based alpha measurement can be adapted to accommodate separate market and beta factors and show that funds with market betas that are materially different than one may have misstated alphas when the beta factor is ignored.

Stock Return Models and Regression Specifications

Asset pricing theory in financial economics has developed well beyond Sharpe's (1964) original CAPM. Since the 1960s, the identification of additional risk factors has been primarily driven by observed patterns in stock returns, rather than basic theory derived from a set of assumed investor preferences and constraints. One empirical result relevant to this study is that

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[1] See, for example, the Axioma risk model US3AxiomaMH, where the separate beta factor is labeled "market sensitivity" or the MSCI Barra risk model USE4 which has separate beta and non-linear beta factors.

back tests of risk-based portfolio strategies like minimum variance have been shown to reduce risk compared to the market portfolio without lowering the average realized return (see Clarke, de Silva, and Thorley (2006)). The primary driver of this result is the now well-documented failure of a key prediction in the traditional CAPM. Market beta does not appear to be priced in that higher beta stocks do not in fact have higher realized returns.

To provide some intuition, consider the classic "market model" for stock returns

$$\mathrm{R}_i = \alpha_i + \beta_i \, \mathrm{R}_M + \varepsilon_i \qquad (1)$$

In this simple statistical model, returns in excess of the risk-free rate for a given stock, R_i , are assumed to be the sum of a stock-specific alpha, α_i , the beta of the stock times the excess market return, $\beta_i \, \mathrm{R}_M$, and a zero-mean random error term, ε_i . The formal CAPM prediction in the context of the market model in Equation 1 is that $\alpha_i = 0$ for all stocks. But even if the alphas for individual stocks are not zero, the expected return still appears to increase with its market beta. Specifically, taking the expected values of both sides of Equation 1 gives $\mathrm{E}(\mathrm{R}_i) = \alpha_i + \beta_i \, \mathrm{E}(\mathrm{R}_M)$. The only way for expected returns across stocks to *not* increase with market beta is if the individual security alphas just happen to linearly decline with beta, a rather odd concept. Thus, the statistical model itself, not just equilibrium CAPM theory, suggests that higher beta stocks have higher expected returns.

To avoid this problem, we develop a variation of Equation 1 which is more like the framework used by equity risk model providers. First, we simply split the middle term into a market constant and a relative beta factor,

$$\mathrm{R}_i = \alpha_i + \mathrm{R}_M + (\beta_i - 1) \, \mathrm{R}_M + \varepsilon_i \, . \qquad (2)$$

Next, we standardize the cross-sectional variation in relative betas, $(\beta_i - 1)$, to have unit variance, in other words calculate z–scores, $z_{\beta,i}$. Finally, we allow the pure beta factor return to be potentially different than R_M , renaming it R_B ,

$$\mathrm{R}_i = \alpha_i + \mathrm{R}_M + z_{\beta,i} \, \mathrm{R}_B + \varepsilon_i \, . \qquad (3)$$

Generically, the factors R_M and R_B in the statistical model in Equation 3 could be any two sources for co-variation in stock returns, but based on a careful specification of the regression process, the estimated value of R_M will be equal to the capitalization-weighted market return, leaving R_B as a pure return to the cross-sectional variation in security betas. If the expected value of the pure beta factor in Equation 3 is $E(R_B) = 0$, then taking the expectation of both sides gives $E(R_i) = \alpha_i + E(R_M)$ so that the expected security return does not necessarily increase with beta. Alternatively, the empirical prediction of the CAPM is an upward sloping Security Market Line (SML) of average realized returns (as a proxy for expected returns) against security betas. In fact, according to the traditional CAPM, the expected return on unscaled betas (i.e., before standardization) should be equal to the equity market risk premium, $E(R_B) = E(R_M)$.

Empirical research has found that while the realized equity market risk premium is in fact positive over any sufficiently long time period of time, the observed SML is either too flat or downward sloping. The empirical failure of the traditional CAPM is not news; even early tests in the academic finance journals were not promising, for example Black, Jensen, and Scholes (1972). But recently, there have been a number of researchers, for example Frazzini and Pedersen (2013), who have proposed explanations for *why* the original CAPM prediction doesn't hold. In this study, we do not weigh in on the specific reasons why there has not been a large positive payoff to market beta, just document how dramatic that effect has been using long-term historical data on essentially all U.S. stocks.

The empirical sections that follow employ two forms of Equation 3, one for factor return estimation using Fama-MacBeth cross-sectional regressions, and a second for time-series estimation of fund alphas. The cross-sectional factor return regression does not include security specific alphas, which can be subsumed in the error term *across* stocks, and accommodates several other security characteristics,

$$R_i = R_M + R_B z_{\beta,i} + \ldots + \varepsilon_i \tag{4}$$

where $+ \ldots +$ is a place-holder for a list of additional terms in the form of a factor return times a security-specific characteristic. As with security beta (i.e., 60-month historical beta) we standardize the other security-specific characteristics (e.g., market-to-book ratio) to z-scores in order to facilitate a comparison between the factors' returns. The observations in the regression specified by Equation 4 are the returns to the characteristics of individual stocks in a single time period, so we do not include "*t*" subscripts.

The second type of regression in the empirical section uses monthly time-series observations of a managed portfolio return, $\mathrm{R}_{P,t}$ and the factor returns estimated in Equation 4,

$$\mathrm{R}_{P,t} = \alpha_P + \mathrm{R}_{M,t} + c_B\, \mathrm{R}_{B,t} + ... + \varepsilon_{P,t} \quad (5)$$

where c_B is a fund-specific coefficient for the beta factor return, and $+ \dots +$ is a place-holder for other terms in the form of a fund-specific coefficient times a factor return. The market factor return does not have a coefficient, in accordance with the way the factor returns are estimated in Equation 4. Depending on the statistical package, one can either restrict the $\mathrm{R}_{M,t}$ coefficient to be one, or simply subtract $\mathrm{R}_{M,t}$ from the fund return on the left-hand side. The observations in Equation 5 are sequential time-series returns for a single fund so we include "*t*" subscripts.

One key implication of our study is that returns-based portfolio performance measurement should include a beta factor to allow for the possibility that the payoff to cross-sectional variation in security betas during a given period is different than the value predicted by the traditional CAPM. In other words, performance measurement using the well-known three-factor Fama-French framework, or four-factor Fama-French-Carhart framework, may be missing an important factor. We also make a distinction between market betas of fully invested portfolios that differ from one due to security selection, and changes in portfolio beta induced by holding cash or using leverage. In the language of the traditional CAPM, we examine the slope of the Security Market Line (SML), rather than the Capital Market Line (CML) from basic portfolio theory. Separation of the cross-sectional beta from the return on the market in Equation 4 allows for a calculation of the realized slope of the SML. To the extent that the estimated payoff to beta is not as high as predicted by the traditional CAPM, portfolios of low beta stocks will have higher average Sharpe ratios than portfolios of high beta stocks. Alternatively, cash holdings or leverage will proportionally shift both the portfolio return and portfolio risk as measured by standard deviation, preserving the portfolio's Sharpe ratio in accordance basic portfolio theory.

For example, the performance measurement process in Equation 5 should only be applied to a fully invested portfolio, not to a portfolio that has a beta of 0.7 because it includes 30 percent cash.

The Return to Pure Beta and Other Factors

We use Equation 4 to run 600 monthly cross-sectional regressions from January 1963 to December 2012 on all but the smallest quintile of stocks in the CRSP database. Depending on the month, each regression has between 1600 and 5800 realized common stock return observations in excess of the contemporaneous risk-free rate on the left-hand side, and four (including momentum) beginning-of-month observable characteristics on the right-hand side. To be admitted as an observation in any cross-sectional regression, we require that the security have a CRSP share-code of 10 or 11 (domestic common stock, excluding ETFs and REITs), a non-missing realized return for that month, a non-missing market capitalization at the end of the prior month, and to be above the 20^{th} percentile by size of all such securities in CRSP. The four observable stock characteristics are:

1) *Beta*: The 60-month historical market beta from a time-series regression of the excess stock return on the cap-weighted excess market return. The market return and the risk free rate are from the Morningstar SBBI dataset. If 60 months of prior return data are not available in CRSP for a given stock in a given month, then at least 24 are required or the historical beta characteristic is declared missing. Historical betas below -1.0 or above 3.0 are Winsorized to those limits.

2) *Small*: Negative one times the natural log of market capitalization, a characteristic which is never missing based on the previously mentioned admission criteria for inclusion of a stock for that month. Because of the negative one multiplier, size returns in this study are to smallness, not largeness.

3) *Value*: Book-to-market ratio based on the combination of CRSP and Compustat databases. We use the prior month-end market value, and the three-month lagged book value, similar to Asness and Frazzini (2013), rather than the timing conventions used by Fama and French (1996) to create HML. The book-to-market characteristic has a fair

number of missing values in early years but becomes more populated as time goes on. The book-to-market ratio is Winsorized to plus or minus 5.0.

4) *Momentum*: The Carhart (1997) 11-month stock return, lagged by one month, which on rare occasions is Winsorized to a maximum value of 3.0, a quadrupling of stock price over 11 months. All 11 monthly returns are required or the momentum characteristic is declared missing.

The selection and definitions of these four stock characteristics are fairly well established, starting with Fama and French (1992) then adding momentum first documented by Jegadeesh and Titman (1993). With a simple negative sign, “small” has replaced “size” in many studies, and “value” could refer to earnings yield (inverse P/E ratio) but remains book-to-market in this study. While the CRSP database goes back further, the 1963 start-date is based on the availability of accounting book values from Compustat. The methodological enhancements to the classic Fama-MacBeth regression specification that are informed by current risk-modeling practice are:

1) Observations in the monthly cross-sectional regressions are weighted by market capitalization. Thus, including increasingly smaller stocks has little impact on the regression results, except that they introduce more missing or outlier right-hand side values.

2) Each of the four characteristics is shifted each month to have a cross-sectional capitalization-weighted mean of zero, and then missing values are set to zero. Together with step number 1, this makes the estimated regression intercept term exactly equal to the return on a capitalization-weighted portfolio of all admitted stocks. Non-zero values for the other four factors then measure exposures that are relative to the market portfolio.

3) Each characteristic is scaled each month to have a cross-sectional standard deviation of one. Calculating “z-scores” for stock characteristics, including beta, is a common risk-factor modeling practice and allows for a more direct comparison of the magnitude of the

return between factors.[2] We label the factor returns from the our regressions as z-Beta, z-Size, z-Value and z-Mom to distinguish them from factor returns based on raw characteristics

As an aside, we note that the cross-sectional regression coefficients in Equation 4 can be described as the returns to factor-mimicking long/short portfolios, as well as the return to standardized stock characteristics. Formulas for the security weights in such portfolios can be derived from the matrix algebra expression for weighted regressions.[3] To reinforce this point, Table 1 provides statistics on the weight structure of all five factor portfolios at one point in time, the beginning of January 2012.

Table 1: Factor-Mimicking Portfolio Weights for January 2012

	Market	z-Beta	z-Small	z-Value	z-Mom
Maximum Weight	2.90%	3.84%	0.11%	3.71%	2.11%
Minimum Weight	0.00%	-2.29%	-4.65%	-2.06%	-2.23%
Sum of Long Weights	100.00%	51.35%	41.78%	51.06%	58.86%
Sum of Short Weights	0.00%	-51.35%	-41.78%	-51.06%	-58.86%
Long Security Count	3,008	1,150	2,886	1,443	1,227
Short Security Count	0	1,858	122	1,565	1,781

As shown in Table 1, the Market portfolio is long-only, with 3,008 security weights that are all positive and sum to one, while the weights for the other four portfolios contain long and short positions that sum to zero. For example, the highest individual stock weight in the z-Beta

[2] In statistics, the term z-score generally refers to the scaling of observations to unit standard deviation (and variance) by dividing raw values by their cross-sectional standard deviation. z-scores are also typically shifted to have a mean value of zero, but in our application we shift to a cap-weighted (not equal-weighted) mean value of zero. In addition, the underlying characteristics do not necessarily follow a normal probability distribution either before or after standardization.

[3] The direct calculation for the *N*-by-5 matrix of factor portfolio weights is $\mathbf{W} = (\mathbf{X} \circ \mathbf{M})\left((\mathbf{X} \circ \mathbf{M})' \,\mathbf{X}\right)^{-1}$, where $\mathbf{X}$ is an *N*-by-5 matrix of right-hand-side values (including a leading column of ones), $\mathbf{M}$ is an *N*-by-5 matrix composed of the market portfolio weight vector, repeated in five columns, $\circ$ designates the matrix dot product (i.e., element-wise multiplication) function, $'$ designates the matrix transpose function, and $^{-1}$ designates the matrix inverse function.

portfolio is 3.84 percent, the lowest weight is -2.29 percent, and the weights form a generally symmetric distribution around zero. Alternatively, the highest weight in the z-Small portfolio is 0.11 percent, the lowest weight is -4.65 percent, and the cross-sectional distribution of the weights in the z-Small portfolio is negatively skewed. Because of capitalization-weighting in the cross-sectional regressions, the largest weights in absolute magnitude for the various factor portfolios are associated with the largest stocks.

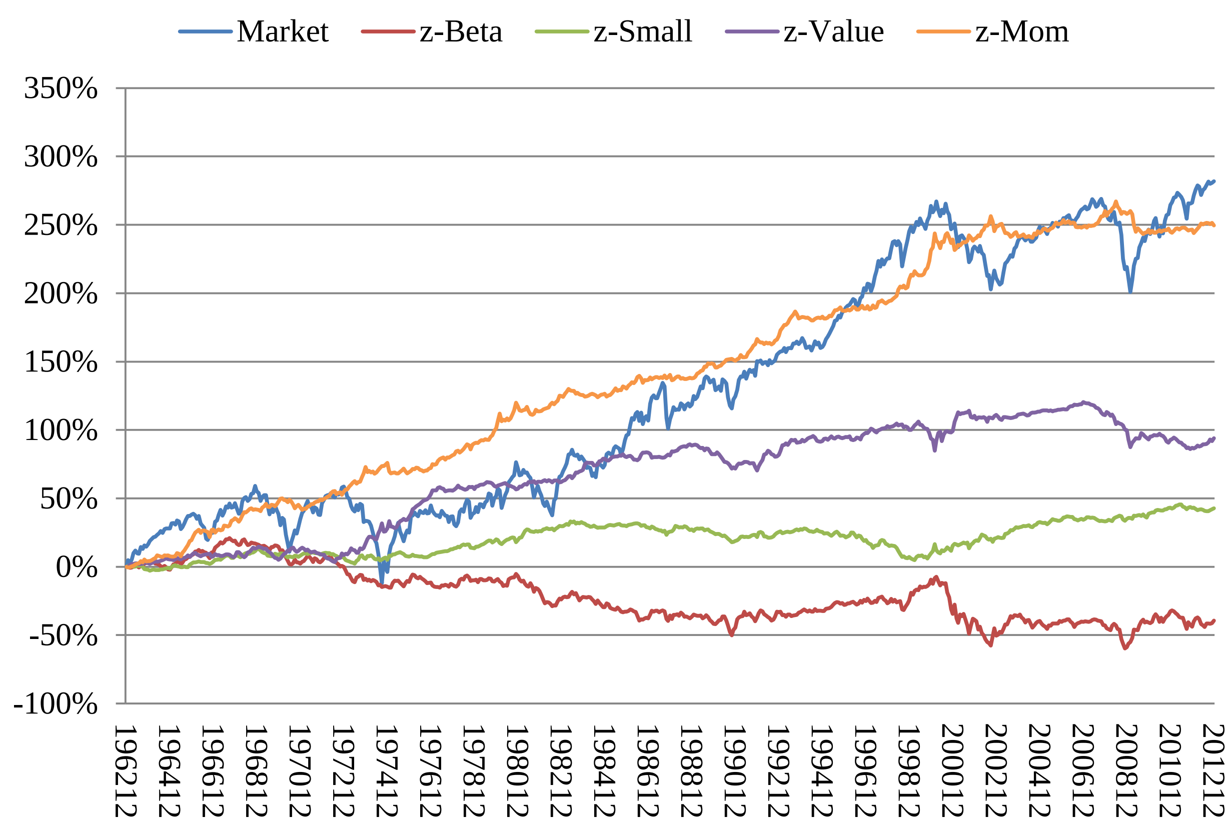
Table 2 reports summary statistics of the returns on the market portfolio and the four additional factor-mimicking portfolios over the last half century. To annualize, monthly averages are multiplied by 12 and standard deviations by the square-root of 12. Table 2 reports the familiar long-term average return (in excess of the risk-free rate) for the Market portfolio of 5.64 percent as well as the Market volatility of 15.54 percent, leading to a Sharpe ratio of 5.64/15.54 = 0.363. Because the four long/short portfolios all have the same amount of leverage (approximately 50 percent long and 50 percent short) the average return and volatility magnitudes can be compared to each other. On the other hand, Sharpe ratios are required for a direct comparison to the excess return for the Market portfolio, which is 100 percent long the market and short 100 percent cash.

Table 2: Summary Statistics of Factor Returns from 1963 to 2012

	Market	z-Beta	z-Small	z-Value	z-Mom
Average Return	5.64%	-0.79%	0.86%	1.88%	4.99%
Standard Deviation	15.54%	7.02%	3.61%	5.30%	6.45%
Sharpe Ratio	0.363	-0.112	0.237	0.354	0.774
Correlation to:	Market	z-Beta	z-Small	z-Value	z-Mom
Market	1.000	0.684	0.226	-0.049	0.002
z-Beta	0.684	1.000	0.257	-0.131	-0.043
z-Small	0.226	0.257	1.000	-0.131	0.058
z-Value	-0.049	-0.131	-0.131	1.000	-0.264
z-Mom	0.002	-0.043	0.058	-0.264	1.000
	Market	z-Beta	z-Small	z-Value	z-Mom
Market Beta	1.000	0.309	0.052	-0.017	0.001
Market Alpha	0.00%	-2.53%	0.56%	1.97%	4.99%
Active Risk		5.12%	3.52%	5.30%	6.45%
Information Ratio		-0.494	0.160	0.372	0.773

Time-series statistics for an entire half century can mask the evolving nature of the factor returns over time. Whatever the true parameter values are for these factors (if indeed they are the correct factors and the world is linear) they are probably not the same today as they were forty years ago. Figure 1 plots the cumulative factor return since year-end 1962 for the five portfolios in Table 2. We do not compound the returns since the portfolios are not fully funded and represent incremental returns that would be added to a fully funded base. As is common practice, the graph of the cumulative (summed) incremental returns is only meant to show trends over time and add insight beyond the summary statistics.

Figure 1: Cumulative Factor Returns from 1963 to 2012



The long-term average return for the z-Beta factor, as reported in the second column of Table 2, is -79 basis points. On examination of Figure 1, one can find multi-year periods where the average return to the z-Beta factor is positive, like the early 1960's, but they are the exception not the rule. Similarly, one can tweak the specifications in this study to make the returns to beta look slightly better, but there's really no practical sense in which the traditional CAPM is empirically valid. The SML is not only "too flat" but downward sloping as manifest by the negative drift for the z-Beta factor in Figure 1. Although not reported in this paper, cross-sectional regressions on CRSP data from earlier years, 1928 to 1962, without a value factor (which requires book-values from Compustat) are not materially different from Table 2. Specifically, for the earlier 420 months, the Market portfolio has a healthy realized risk premium, the average return to z-Beta is about zero, z-Small has a relatively small positive payoff, and z-Mom has a relatively large positive payoff.

The portfolio return standard deviations, reported in the second row of Table 2, are also informative. While beta exposure greater than the market does not pay off in the long run, inclusion as a separate factor is important from a risk modeling perspective, as explained in Chan, Karceski, and Lakonishok (1998). Their argument for "importance" is that an irrelevant stock characteristic (e.g., alphabetical order) would yield factor returns that are small in absolute value from month to month leading to a small portfolio return standard deviation. With a standard deviation of 7.02 percent in Table 2, z-Beta is the most important non-market risk factor. In other words, stocks with similar historical betas co-vary more in their realized cross-sectional returns than stocks with similar market capitalizations or similar book-to-market ratios. Beta is certainly not "dead" from the perspective of being an important risk factor; it's just not priced according to the traditional CAPM.

The middle panel of Table 2 reports the correlation coefficients between the five portfolio returns, which are generally low except for the z-Beta to Market correlation of 0.684. The lower panel of Table 2 provides market-relative performance statistics from five separate time-series regressions of each factor return on the Market. Market beta in Table 2 is the coefficient from those time-series regressions and market alpha is the intercept term. For example, the market beta of the Market factor is exactly equal to one, while the estimated market betas of the z-Small, z-Value, and z-Mom factors are all relatively close to zero. On the other hand, the estimated market beta of 0.309 for the z-Beta factor is substantially greater than zero which verifies that "historical security beta" is, in fact, a somewhat successful predictor of realized betas. Note that the market beta of the z-Beta factor is not approximately 1.0, as it would be for an unscaled beta characteristic. Specifically, the cross-sectional standard deviation of unscaled historical betas is about 0.4 over time. Scaling betas into z-scores increases the cross-sectional standard deviation to 1.0 every month, proportionally decreasing the magnitude of factor returns in the Fama-MacBeth regressions.

One can calculate the CAPM predicted return to the z-Beta factor using its realized market beta and the average return to the Market, 0.309 * 5.64 percent = 1.74 percent. The CAPM predicted return is 253 basis points higher than the actual average return of -0.79 percent, as shown by the -2.53 percent market alpha for the z-Beta factor. Note that if one swapped "Low Beta" for Beta as the factor definition, like swapping small for large size, the z-Beta factor alpha would be *positive* 2.53 percent. While the magnitude of the z-Beta factor average return is smaller than the other factors, the alpha of the z-Beta factor is second only to z-Mom. The final two rows in Table 2 report active risk, defined as the standard deviation of alpha, and the Information Ratio, the quotient of alpha to active risk. The Information Ratio is the relative-return equivalent to the more familiar absolute-return Sharpe Ratio measure, and by this measure z-Beta remains in second place to z-Mom.

Although not reported here, we tried several variations on the set of stock characteristics, the set of investable securities, and the weighting scheme in the cross-sectional regressions.[4] Equal-weighted Fama-Macbeth cross-sectional regressions as employed in some studies made the average return to the z-Beta portfolio slightly positive, although still well below the value predicted by the CAPM. However, we believe the equal-weighted results are less indicative of investable strategies because so much collective weight is placed on literally thousands of smaller-cap stocks. Commercial risk-factor models often use square-root-of-market-capitalization-weighted cross-sectional regressions, which produce results similar to this study.

In Table 3, we compare our factor returns to the corresponding Fama-French factor (i.e., SMB, HML, and UMD) returns from the Ken French website. For a comparison to the z-Beta factor, we also include VMS described in Clarke, de Silva and Thorley (2010), which uses the Fama-French methodology with historical beta as the sorting variable. As shown in Table 1, our

[4] Among other perturbations, we investigated an investable set of only the largest 1000 stocks each month, rather than the much greater and variable number of U.S. common stocks included in CRSP. The average return on the z-Beta portfolio was even more negative than in Table 2, while the average returns to the other three long-short portfolios, z-Small, z-Value, and z-Mom, were less positive. In other words, the "beta anomaly" is even more acute within the set of large-cap U.S. stocks.

regression-based portfolios are generally about 50 percent long and short so our factor returns have about half the magnitude of the Fama-French portfolios, which are 100 percent long and short. But the Sharpe ratios for the Fama-French factors are similar to our factors, and as shown at the bottom of Table 3 the correlations are generally high, although far from perfect. The Fama-French sorted portfolios exclude the middle 40 percent of CRSP securities, while the cross-sectional regressions include essentially all stocks. The constituent long and short portfolios are capitalization-weighted in the Fama-French methodology, but the constituent small-cap and large-cap portfolios are given equal weight so our factors tend to be less dependent on small-cap effects. Our portfolios are based on multivariate regressions that include all the other factors while the Fama-French methodology only takes into account the size dimension in portfolio construction.

Table 3: Summary Statistics for Fama/French Factors from 1963 to 2012

	MRF	VMS	SMB	HML	UMD
Average Return	5.63%	0.33%	3.00%	4.74%	8.42%
Standard Deviation	15.57%	15.36%	10.80%	10.01%	14.81%
Sharpe Ratio	0.361	0.021	0.277	0.473	0.568
Correlation to:	Market	z-Beta	z-Small	z-Value	z-Mom
	1.000	0.913	0.811	0.728	0.850

Performance Measurement with a Beta Factor

Understanding that there are essentially four factors in the "three factor" Fama-French framework may be important in performance measurement as well as risk modeling. In this section, we review examples of returns-based performance measurement that include an explicit beta factor to account for the realized slope of the SML during the measurement period. Note that returns-based performance measurement is less precise than standard holdings-based performance attribution. The parameters from the time-series regressions reflect the average ex-post exposure of the portfolio to the factors during the measurement period. A returns-based approach provides insight about the source of relative performance when the underlying factor exposures are relatively constant and the portfolio is fully invested. However, the returns-based approach will tend to miscalculate the effect of cash in the portfolio. In other words, when the slope of the SML is not equal to the excess return on the market, a portfolio that is fully invested in stocks with an average beta less than one will perform differently than a portfolio that has a low beta because it contains cash.

As our first example, we use the last ten years (2003 to 2012) of returns on the nine economic Select Sector SPDR ETFs, sponsored by State Street Global Advisors. As a second and more direct example of the low beta anomaly, we examine the returns to the MSCI USA Minimum Volatility Index. Data for the Sector SPDRs are monthly returns on market-traded ETFs. Monthly returns for the Minimum Volatility Index are based on back-tests from MSCI because the ETF that tracks this index did not trade until October 2011. Table 4 reports the annualized average monthly returns to each portfolio in excess of the risk-free rate (annualized average of 1.64 percent) for that decade. The Energy SPDR has the highest average excess return, at 14.09 percent, while the Financial SPDR has the lowest average excess return at 0.57 percent.

Table 4: ETF Excess Returns from 2003 to 2012

C. Discretionary SPDR	C. Staples SPDR	Energy SPDR	Financial SPDR	Healthcare SPDR	Industrial SPDR	Materials SPDR	Technology SPDR	Utilities SPDR	MSCI Minimum Volatility
XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	USMV
8.39%	6.71%	14.09%	0.57%	4.80%	8.04%	9.23%	7.73%	8.72%	7.29%

Table 5: Summary Statistics for Factor Returns from 2003 to 2012

	Market	z-Beta	z-Small	z-Value	z-Mom
Average Return	7.10%	1.11%	2.18%	-1.73%	0.11%
Standard Deviation	15.14%	7.12%	3.06%	5.01%	6.21%
Sharpe Ratio	0.469	0.156	0.713	-0.344	0.018
Market Beta	1.000	0.336	0.074	0.123	-0.104

For reference purposes, Table 5 reports the excess return statistics during the 2003 to 2012 decade for the factor-mimicking portfolios, a subset of the half-century of data in Table 2.

An examination of Chart 1 indicates that the cumulative Market return has been relatively high for the last ten years while the cumulative return to z-Beta has been relatively flat. As reported in Table 5, the annualized Market factor return was 7.10 percent and the annualized z-Beta factor return was 1.11 percent. While positive, the z-Beta factor return for this decade is not as high as predicted by the CAPM. Specifically, the realized market beta of the z-Beta factor for these ten years was 0.336, similar to the value of 0.309 for the entire half century shown in Table 2. The alpha of the z-Beta factor was thus 1.11 – 0.336*7.10 = -1.28 percent, not as large as the -2.53 percent alpha over the last half century, but negative enough. As a result, the inclusion of the z-Beta factor has a material impact on the ETF alphas, depending on how close their betas are to the overall market.

Table 6 reports the coefficient estimates, *t*-statistics, and *R*-squared for two specifications of ten multivariate time-series regressions. Specification "A" is the traditional formulation, with the excess portfolio return on the left-hand side and returns for the Market and other factors on the right-hand side. Depending on the statistical package, one can either restrict the coefficient of the z-Beta factor to be exactly *0.000*, shown in italics, or simply leave the z-Beta factor returns out of the regression. Specification "B" is our alternative formulation, with a Market factor coefficient of one, consistent with each stock's exposure to that factor in the cross-sectional regressions used to estimate the factor returns.[5] Depending on the statistical package, one can either restrict the coefficient of the Market factor to be exactly *1.000*, shown in italics, or simply subtract the Market factor returns from the fund returns before running the time-series regression. The *t*-statistics for the Market factor test the null hypothesis that the coefficient is different than one, while the *t*-statistics for the other four factors test the null hypothesis that the coefficients are different than zero, meaning that the factor exposures are different than the market. The *R*-squared values in Table 6 for specification B are from regressions with restricted coefficients, not with the Market return subtracted from the left-hand side, so that the *R*-squared can be compared between specifications.

[5] A third regression specification with unrestricted coefficients on all five factors, Market, z-Beta, z-Small, z-Value, and z-Mom, are not reported. Besides the potential misspecification of allowing the Market factor coefficient to vary, the Market and z-Beta factor returns are quite correlated, as shown in the middle panel of Table 2. This third regression specification has enough multicollinearity that the coefficient estimates on Market and z-Beta factors are difficult to disentangle.

The coefficient estimates in Table 6 for the z-Small, z-Value, and z-Mom factors are generally similar between specifications A and B for any given ETF, and confirm some well-known aspects of the various industrial sectors. Because the SPDR ETFs are composed of large stocks and are capitalization weighted, the z-Small coefficients for the sector ETFs in Table 6 are either statistically insignificant or negative, with the exception of the Consumer Discretionary SPDR. The Financial SPDR has the largest z-Value coefficients and the Technology SPDR has the lowest (i.e., most negative) z-Value coefficients. On the other hand, the Energy SPDR has the highest z-Mom coefficients for the 10-year period from 2003 to 2012.

Table 6: Returns-Based Performance Measurement of Ten Funds from 2003 to 2012

C. Discretionary	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	R^2
A Coefficients	1.015	*0.000*	0.718	0.030	-0.377	-0.29%	0.867
(*t*-statistics)	(0.3)		(3.2)	(0.2)	(3.5)	(0.1)	
B Coefficients	*1.000*	0.106	0.643	0.040	-0.361	-0.13%	0.868
(*t*-statistics)		(1.1)	(2.9)	(0.3)	(3.3)	(0.1)	
C. Staples	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	0.576	*0.000*	-0.436	-0.156	-0.113	3.31%	0.631
(*t*-statistics)	(9.1)		(2.1)	(1.2)	(1.1)	(1.6)	
B Coefficients	*1.000*	-1.051	-0.222	-0.551	-0.192	0.33%	0.774
(*t*-statistics)		(14.4)	(1.3)	(5.6)	(2.4)	(0.2)	
Energy	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	1.149	*0.000*	0.069	-0.264	1.069	5.20%	0.546
(*t*-statistics)	(1.3)		(0.1)	(0.8)	(4.3)	(1.1)	
B Coefficients	*1.000*	0.249	0.110	-0.119	1.074	6.14%	0.544
(*t*-statistics)		(1.1)	(0.2)	(0.4)	(4.3)	(1.2)	
Financial	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	1.137	*0.000*	-0.530	1.719	-0.639	-3.32%	0.932
(*t*-statistics)	(3.0)		(2.5)	(12.8)	(6.3)	(1.6)	
B Coefficients	*1.000*	0.270	-0.533	1.850	-0.627	-2.41%	0.932
(*t*-statistics)		(2.9)	(2.5)	(14.8)	(6.1)	(1.2)	
Healthcare	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	0.674	*0.000*	-0.693	-0.098	-0.083	1.36%	0.570
(*t*-statistics)	(5.3)		(2.5)	(0.5)	(0.6)	(0.5)	
B Coefficients	*1.000*	-0.896	-0.443	-0.398	-0.160	-1.01%	0.664

(*t*-statistics)		(8.3)	(1.8)	(2.7)	(1.3)	(0.4)	
Industrial	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	1.118	*0.000*	0.116	0.145	-0.260	0.12%	0.884
(*t*-statistics)	(2.5)		(0.5)	(1.1)	(2.5)	(0.1)	
B Coefficients	*1.000*	0.217	0.129	0.260	-0.252	0.89%	0.883
(*t*-statistics)		(2.3)	(0.6)	(2.0)	(2.4)	(0.4)	
Materials	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	1.290	*0.000*	0.374	-0.211	0.136	-1.12%	0.812
(*t*-statistics)	(4.2)		(1.2)	(1.1)	(0.9)	(0.4)	
B Coefficients	*1.000*	0.542	0.396	0.068	0.157	0.76%	0.809
(*t*-statistics)		(3.9)	(1.2)	(0.4)	(1.0)	(0.2)	
Technology	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	1.200	*0.000*	-0.337	-0.863	0.106	-1.56%	0.882
(*t*-statistics)	(4.5)		(1.7)	(6.7)	(1.1)	(0.8)	
B Coefficients	*1.000*	0.571	-0.511	-0.680	0.157	-0.09%	0.904
(*t*-statistics)		(7.1)	(2.8)	(6.2)	(1.8)	(0.0)	
Utilities	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	0.530	*0.000*	-0.200	-0.013	0.305	5.34%	0.317
(*t*-statistics)	(5.8)		(0.5)	(0.1)	(1.7)	(1.4)	
B Coefficients	*1.000*	-1.097	-0.027	-0.455	0.231	2.09%	0.388
(*t*-statistics)		(7.1)	(0.1)	(2.2)	(1.4)	(0.6)	
MSCI Min Vol	Market	z-Beta	z-Small	z-Value	z-Mom	Alpha	
A Coefficients	0.692	*0.000*	-0.204	0.202	-0.026	3.16%	0.874
(*t*-statistics)	(10.3)		(1.5)	(2.3)	(0.4)	(2.3)	
B Coefficients	*1.000*	-0.765	-0.046	-0.085	-0.083	0.99%	0.937
(*t*-statistics)		(18.0)	(0.5)	(1.5)	(1.8)	(1.9)	

The z-Beta factor has an impact on the estimated alphas of the Sector SPDRs in Table 6 depending on how close their betas are to the overall market. As a neutral example, the Consumer Discretionary SPDR in the first panel has a specification A Market factor coefficient estimate of 1.015. Because the Market factor coefficient is close to one, the specification B coefficient estimate of z-Beta is not statistically different from zero (t-statistic of 1.1) and the specification B alpha of -0.13 percent is similar to the specification A alpha of -0.29 percent. In contrast, the Consumer Staples SPDR has a Market factor coefficient estimate of 0.576, well below one, and the z-Beta coefficient in specification B is negative and highly significant (t-statistic of 14.4). Once the negative alpha of the z-Beta factor is explicitly acknowledged, the 3.31 percent Alpha for Consumer Staples is reduced to just 0.33 percent. Similar reductions in Alpha are shown in Table 6 for two other low-beta SPDRs, Healthcare and Utilities. Alternatively, the Technology SPDR has one of the highest Market factor coefficient estimates at 1.200, and the highest z-Beta factor coefficient of 0.571. Once we explicitly acknowledge the effect of a positive loading on the z-Beta factor, the Technology SPDR shows a large increase in Alpha from -1.56 percent to -0.09 percent in specification B. The *R*-squared increases between specifications A and B to the extent that the z-Beta coefficients are significantly different than zero. For example, the *R*-squared for Consumer Staples goes from 0.631 to 0.774.

The returns-based regressions for the MSCI Minimum Volatility Index at the bottom of Table 6 are particularly informative. The Market factor coefficient of 0.692 is indicative of a low beta strategy, confirmed by the significant coefficient of -0.765 for the z-Beta factor in specification B. The Alpha of 3.16 percent is reduced to just 0.99 percent in specification B, where the impact of its exposure to the z-Beta factor is properly acknowledged. The comparison of specifications A and B for the Minimum Volatility Index confirms that most of the value-added in minimum variance strategies is the selection of low beta stocks. In summary, much of the variation in alphas across the ETFs is explained by the z-Beta factor. Specifically, the specification B alphas in Table 4 are generally closer to zero than the specification A alphas.

Summary and Conclusions

Following the practice of equity risk model providers, as well as early tests of the CAPM, we split the market's influence on individual security returns into a capitalization-weighted market portfolio and a pure beta factor. Beta and other volatility-based measures are among the most important factors in equity risk models, explaining more of the realized cross-sectional variation in individual security returns than size, value or momentum. However, the long-term average return to beta is negative in stark contrast to the traditional CAPM prediction that greater exposure to systematic risk should be rewarded over time. In fact, as measured by the Information Ratio, the beta anomaly in the U.S. equity market over the last half century is larger than either the size or value anomalies, second only to momentum. Sensitivity analysis indicates that the beta anomaly is even more evident in larger capitalization stocks.

The beta anomaly presents challenges in portfolio performance measurement as well as opportunities in portfolio construction. Returns-based performance measurement for the MSCI Minimum Volatility Index indicates that selecting low beta stocks provides most of the value added in low volatility strategies. In general, a separate beta factor should be incorporated in portfolio performance calculations to avoid misperceptions about the source and magnitude of the alpha when the market beta is materially different from one. Specifically, we show that much of the variation in the performance of economic sector portfolios is explained by their differential exposures to a separate beta factor.

Our observations about the beta anomaly are with respect to security selection in fully invested portfolios. Reducing portfolio beta by adding cash to an equity portfolio, or borrowing to increase beta by leverage, proportionally decreases or increases expected return and risk. The Capital Market Line (CML) is by definition a positively sloped line given a positive realized market risk premium. On the other hand, the average slope of the Security Market Line (SML) is dependent on the realized return to beta, which is generally lower than the positive expected value predicted by the traditional CAPM, and is often negative. Understanding of the distinction between the market and beta factors in the not-so-well-known three-and-one-half factor model should help avoid misperceptions about the sources of portfolio performance.

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