The International CAPM Redux

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Aggregate Foreign Equity Holdings
(as % of global GDP)
Motivation

- Large increase in financial integration over the past 30 years.
  - Aggregate foreign equity holdings, as a % of global GDP: from 3% in the 1980s to approximately 30% in 2011.
  - Foreign equity holdings of the U.S.: $6 trillion.

- What are the expected returns on those foreign equity holdings?
  - Finance logic:
    - If an asset’s return tends to be low in bad times, the asset is risky. Its expected return should be above the risk-free rate.
  - How to measure bad times?
    - Capital Asset Pricing Model (CAPM): Aggregate market return
  - How to compare returns?
    - Foreign equity returns are in foreign currencies
Currency Risk

- Exchange rate risk should matter...
  - When investors invest abroad, but consume at home, and domestic and foreign purchasing powers differ.

- Yet, empirical evidence is difficult to obtain.

- Recent findings: Two risk factors account for a large share of systematic changes in bilateral exchange rates
  - **Idea:** Instead of using raw bilateral exchange rates, use currency risk factors that summarize their systematic variation.
This Paper

1. Three factors describe the time-series of foreign equity benchmark returns
   ▶ Smaller difference between expected and realized returns than in the competitors’ models (World CAPM, International CAPM, and Fama-French factors)

2. Those three factors account for a large share of foreign equity mutual funds’ and macro/emerging hedge funds’ returns

3. A simple reduced-form model to study the interaction between currency and equity risk
Realized vs Predicted Excess Returns

- World CAPM
- International CAPM
- Fama–French Four Factors
- CAPM Redux

Y-axis: Actual mean excess return (%)
X-axis: Predicted excess return (%)

Legend:
- Aggr. Market
- Value
- Growth
- Small
- Big
- Carry Portfolios
- Dollar Portfolios
Literature

- **World CAPM:**
  - Sharpe (1964) and Lintner (1965) to Stehle (1977), Solnik (1974), …

- **International CAPM:**

- **New equity factors and frictions:**

- **Currency drivers:**
Road map

- Empirical evidence:
  - Equity benchmark indices
  - Hedge funds and mutual funds

- Theoretical framework:
  - Intuition
    - Replication in a reduced-form model

- Conclusion
Empirical Asset Pricing
Asset Pricing

- When the law of one price holds and investors can form portfolios freely, there exists a SDF $M_{t+1}$ that prices any return $R_{t+1}^i$:
  \[ E_t (M_{t+1}R_{t+1}^i) = 1 \]

- The same condition holds for the risk-free rate $R_f$.

- Assuming that the returns and SDF are lognormal, the Euler equation implies:
  \[
  E_t \left( r_{t+1}^i - r_{f,t} + \frac{1}{2} \text{var}_t(r_{t+1}^i) \right) = - \frac{\text{cov}_t(m_{t+1}, r_{t+1}^i)}{\text{var}_t(m_{t+1})} \text{var}_t(m_{t+1}) \beta_t^i \Lambda_t
  \]
  where lower letters denote logs.

- What is the SDF $M_{t+1}$?

- If $M$ is an excess return, its market price of risk $\Lambda$ should be equal to its mean.
Equity Literature Summary

- World CAPM: \( WMT_{t+1} \) = world stock market return in U.S. dollars

\[
r_{i,t+1} - r_{f,t} = \alpha^i + \beta^i_{WMT}[WMT_{t+1} - r_{f,t}] + \epsilon_{t+1}
\]

- International CAPM: + 3 bilateral exchange rates

\[
r_{i,t+1} - r_{f,t} = \alpha^i + \beta^i_{WMT}[WMT_{t+1} - r_{f,t}] + \beta^i_{GBP}r_{GBP,t+1} + \beta^i_{JPY}r_{JPY,t+1} + \beta^i_{EUR}r_{EUR,t+1} + \epsilon_{t+1}
\]

- Three/four Fama-French factor models: size, value, and momentum anomalies

\[
r_{i,t+1} - r_{f,t} = \alpha^i + \beta^i_{WMT}[WMT_{t+1} - r_{f,t+1}] + \beta^i_{SMB}SMB_{t+1} + \beta^i_{HML}HML_{t+1} + \beta^i_{WML}WML_{t+1} + \epsilon_{t+1}
\]
Currency Literature Summary

- Carry trade risk: sort currencies by their short-term interest rates

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(R^e)$</td>
<td>-3.17</td>
<td>-0.46</td>
<td>1.26</td>
<td>1.80</td>
<td>2.29</td>
<td>4.48</td>
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</table>

Source: Lustig, Roussanov, and Verdelhan, 2011.

- “Dollar” risk: sort currencies by their exchange rate betas, long if average interest rate > U.S. interest rate, short otherwise

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</tr>
</thead>
<tbody>
<tr>
<td>$E(R^e)$</td>
<td><strong>1.31</strong></td>
<td>2.86</td>
<td>3.87</td>
<td>5.13</td>
<td>4.99</td>
<td>7.14</td>
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</tbody>
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Intuition

- Three risk factors: Equity, Dollar, and Carry.

\[
E_t \left( r_{t+1}^i - r_{f,t} + \frac{1}{2} \text{var}_t(r_{t+1}^i) \right) = \beta_t^{i,1} \Lambda_t^1 + \beta_t^{i,2} \Lambda_t^2 + \beta_t^{i,3} \Lambda_t^3.
\]

- One could ignore currency risk and focus on equity risk only if all the quantities and prices of risk were in sync.

\[
E_t \left( r_{t+1}^i - r_{f,t} + \frac{1}{2} \text{var}_t(r_{t+1}^i) \right) = \tilde{\beta}_t^{i} \tilde{\Lambda}_t.
\]

- No reason to expect so.
This Paper: International CAPM Redux

- Three-factor model:

\[ r_{i,t+1}^{e,i,\$,} - r_{f,t} = \alpha^i + \beta^i_{LWMKT}[LWMKT_{t+1} - r_{f,t}] + \beta^i_{Dollar} r_{t+1}^{Dollar} + \beta^i_{Carry} r_{t+1}^{Carry} + \epsilon_{t+1} \]

- Factors:

3. Carry: average excess return of a U.S. investor going long high interest rate currencies and short low interest rate currencies.

- Estimation: time-varying betas obtained on 60-month rolling windows.
Data and Estimation

- Test assets: from 46 developed and emerging countries, spanning value, growth, and country index returns from 1/1976 to 12/2013.

- Risk factors: global equity factor (built from local currency equity returns) and two currency factors: Dollar and Carry

- Estimation: time-varying betas obtained on rolling windows
Expected vs Realized Equity Excess Returns

▶ Since factors are excess returns, the market prices of risk should be the means of the risk factors (no-arbitrage condition)

\[
\begin{align*}
    r_{t+1}^{e,i} - r_{f,t} &= \alpha^i + \beta^i F_{t+1} + \epsilon_{t+1} \\
    E_T \left( r_{t+1}^{e,i} - r_{f,t} \right) &= \alpha^i + \beta^i E_T (F_{t+1})
\end{align*}
\]

▶ For each asset and each rolling window:

▶ Estimate the beta on rolling windows, from \( t - 59 \) to \( t \):

\[ \beta_t \]

▶ Estimate the price of risk as the mean of the risk factor over the same period, from \( t - 59 \) to \( t \):

\[ \lambda_t = \frac{1}{60} \sum_{i=0}^{59} F_{t-i} \]

▶ Form expected equity excess returns as the product of the quantities and prices of risk:

\[ E_t \left( r_{t+1}^{e,i} - r_{f,t} \right) = \beta_t \lambda_t \]

▶ Average across time and compare expected to realized excess returns
Time-varying Factor Betas: Developed Markets

Exposure to LWMKT

- Australia
- Austria
- Belgium
- Canada
- Denmark
- Finland
- France
- Germany
- Greece
- Hong Kong
- Ireland
- Israel
- Italy
- Japan
- Netherlands
- New Zealand
- Norway
- Portugal
- Singapore
- Spain
- Sweden
- Switzerland
- United Kingdom
- United States

Exposure to Dollar

- Average

Exposure to Carry

- Min/Max
Time-varying Factor Betas: Emerging Markets

- Exposure to LWMKT
  - Brazil
  - Chile
  - China
  - Colombia
  - Czech Republic
  - Egypt
  - Hungary
  - India
  - Indonesia
  - Malaysia
  - Mexico
  - Morocco
  - Peru
  - Philippines
  - Poland
  - Russia
  - South Africa
  - South Korea
  - Taiwan
  - Thailand
  - Turkey

- Exposure to Dollar

- Exposure to Carry

○ Average

Min/Max
Realized vs Predicted Excess Returns

- World CAPM
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The graphs show scatter plots comparing actual mean excess returns to predicted excess returns for different models and factors. The models include World CAPM, International CAPM, Fama–French Four Factors, and CAPM Redux. The factors are categorized by Aggr. Market, Value, Growth, Small, Big, Carry Portfolios, and Dollar Portfolios.
Robustness Checks

- Other comparisons of the pricing errors:
  - Histograms and kernel density estimates of rolling alphas
  - Rolling mean absolute alphas

- Even if the market prices of risk are estimated (Fama-McBeth procedure), the CAPM Redux still compares favorably:
  - the market prices of risk are positive and significant for our three factors
  - equity risk is priced for the CAPM, International CAPM, and FF models, but not the other factors (except GBP); the prices of risk appear further removed from the mean of the risk factors.

- Subsamples:
  - time-windows (increasing sizes starting from 1976 or backwards from 2013)
  - test assets (country aggregates, developed vs emerging markets)
  - length of the rolling windows
Kernel Density of Rolling Alphas

Rolling alphas

Kernel density

World CAPM  Fama–French Four Factors  CAPM Redux
Mutual and Hedge Fund Returns
Hedge Funds and Mutual Funds

- Fund’s exposure to global systematic factors:

\[ R_{t+1}^i = \alpha^i + \beta^i \text{Carry}_{t+1} + \gamma^i \text{Dollar}_{t+1} + \delta^i \text{LWMKT}_{t+1} + \epsilon^i_{t+1}, \]

where \( R_{t+1}^i \) denotes the monthly realized return of fund \( i \).

- Data:
  - MFs: U.S. “Foreign Equity Funds” from CRSP.
  - HFs: International “Macro” and “Emerging” Funds from Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013).

- Estimation: time-varying betas obtained on rolling windows.

- Sample period: November 1990 (MF)/January 1994 (HF) to April 2013.
“Macro/Emerging” Hedge Funds’ Significant Exposure to Currency Factors

![Graph showing the number of funds and exposure to currency factors from 2000 to 2012.](image-url)
Statistical Significance of Positive HF and MF’s Alphas

Mutual Funds

Hedge Funds

ECDF

World CAPM - Int. CAPM - Fama–French Four Factors - CAPM Redux
Theoretical Framework
Recall that:

\[ E_t \left( r_{t+1}^i - r_{f,t} + \frac{1}{2} \text{var}(r_{t+1}^i) \right) = -\frac{\text{cov}_t(m_{t+1}, r_{t+1}^i)}{\text{var}(m_{t+1})} \frac{\text{var}(m_{t+1})}{\Lambda_t} \beta_t \]

Expected currency excess returns when markets are complete and shocks are gaussian (Bekaert, 1996, Bansal, 1997, Backus, Foresi, and Telmer, 2001):

\[ E_t \left( r_{x_{t+1}}^i \right) = r_{f,t}^i - r_{f,t} - E_t \left( \Delta s_{t+1}^i \right) \]
\[ = \frac{1}{2} \text{Var}(m_{t+1}) - \frac{1}{2} \text{Var}(m_{t+1}^i) \]

Failure of the uncovered interest rate parity (U.I.P.) implies that expected currency excess returns are time-varying.

SDF must be heteroscedastic, and thus market prices of risk must be time-varying.
Framework

- Assume that financial markets are complete (thus log changes in exchange rates are differences in log SDF)

- Start from the law of motion of each SDF, which depends on country-specific and global shocks

- Assume that each country-aggregate dividend growth rate depends on the same shocks as the SDF

- Two key assumptions:
  - Prices of risk are time-varying (time-varying currency risk premia)
  - One global shock affects all SDF in the same way (role for a pure equity risk factor)
Reduced-Form Model with Endogenous Exchange Rates

- In the tradition of Frachot (1996), Backus, Foresi and Telmer (2001), Brennan and Xia (2006)
- Exponentially affine SDFs in complete financial markets

\[-m_{it+1} = \alpha + \chi z_{it} + \sqrt{\gamma z_{it}} u_{it+1} \]
\[+ \tau z_{wt} + \sqrt{\delta i z_{wt}} u_{wt+1} + \sqrt{\kappa z_{it}} u_{gt+1} + \sqrt{\omega z_{wt}} u_{ct+1},\]
\[z_{it+1} = (1 - \phi)\theta + \phi z_{it} - \sigma \sqrt{z_{it}} u_{it+1},\]
\[z_{wt+1} = (1 - \phi^w)\theta^w + \phi^w z_{wt} - \sigma^w \sqrt{z_{wt}} u_{wt+1}\]
\[\Delta d_{iit+1} = \mu_D + \psi z_{it} + \psi w z_{wt} + \sigma_D \sqrt{z_{it}} u_{it+1} \]
\[+ \sigma^w_D \sqrt{z_{wt}} u_{wt+1} + \sigma^g_D \sqrt{z_{it}} u_{gt+1} + \sigma^c_{D,i} \sqrt{z_{wt}} u_{ct+1}\]

where $u_{it+1}, u_{wt+1}, u_{gt+1}, u_{ct+1}$ are i.i.d, mean-zero, variance-one Gaussian shocks.

- Similar model studied in Lustig, Roussanov, and Verdelhan (2011, 2014) and in Verdelhan (2014) but without the global “equity” shocks $u_{ct+1}$
Model Solution

- Simple and tractable reduced-form model:
  - Interest rates, exchange rates, and price-dividend ratios in closed-forms, as well as realized and expected equity and currency excess returns, and equity and currency risk factors.

- Implications:
  - The world stock market return in U.S. dollars depends on all global shocks.
  - No role for bilateral exchange rates in theory
  - But in the model the market price of global equity risk depends on several state variables.
    - Those state variables are unknown to the econometrician.
    - Quantities and market prices of risk evolve at different frequencies.
  - Role for currency factors in practice, even in simulated data.
Realized vs Predicted Excess Returns: Simulated Data

World CAPM

CAPM Redux
Conclusion

- Three factors (Global Equity, Dollar, and Carry) describe the time-series of foreign equity benchmark returns.
  - Predicted and realized equity excess returns are closer than in the competitors’ models.

- Currency risk matters for foreign equity mutual fund and “macro/emerging” hedge fund returns.

- Interaction between currency and equity risk in a simple reduced-form model:
  - Time-variation in quantities and prices of risk is key.