Abstract

This paper presents new evidence that international investors are compensated for bearing currency risk. We present a new three-factor international capital asset pricing model, comprising a global equity factor denominated in local currencies, and two currency factors, dollar and carry. The model is able to explain a wide cross-section of equity returns from 46 developed and emerging countries from 1976 to the present, is also useful at explaining the risks of international mutual funds and hedge funds, and outperforms standard models proposed in the international asset pricing literature. We rationalize our findings with a simple model of endogenous exchange rate risk in complete markets, and identify the importance of correctly identifying the dynamics of quantities and market prices of risk.
1 Introduction

Are international equity investors compensated for bearing exchange rate risk? The question of whether currency risk is priced is increasingly relevant, especially in light of the rapid acceleration of international financial integration over the past three decades. Figure 1 shows that aggregate foreign equity holdings as a percentage of global gross domestic product have increased steadily from roughly 3% in the 1980s to approximately 30% in 2011, alongside a steady dismantling of de-jure cross-border capital flow restrictions. The magnitudes are large, highlighting the importance of this issue – in the United States, for example, foreign equity holdings are currently worth roughly US$ 6 trillion.

Academic work on the subject has faced the usual challenge in international finance, namely, that empirical work has struggled to provide evidence to support the theory. Theoretically, exchange rate risk should matter. In a world with real rigidities, frictions in the goods market prevent perfect risk-sharing, implying deviations from purchasing power parity. International investors invest abroad, but consume at home. As a result, investors should require compensation for bearing the risk of low returns on their foreign investments once these returns are expressed in real domestic terms. Reasonable though this rationale is, empirical work in international asset pricing, with a few notable exceptions, has not been able to provide convincing evidence that currency risk is priced in international equity markets.

In this paper, we present a new three-factor model to capture the risks in international equity portfolios. Using the model, we show that international equity investors are rewarded for the currency risk that they take. The three factors that are necessary to price the cross-section of global equity returns are the return on a world market portfolio denominated in local currency terms, and two currency factors which effectively summarize variation in a broad cross-section of bilateral exchange rates, namely, the
Figure 1
Portfolios of Foreign Equity Assets and Financial Openness

This figure shows the portfolio equity assets (bar plots) for 40 developed and emerging countries along with a de jure financial openness index. In each year, portfolio assets of all countries are summed up and divided by the corresponding world GDP. The financial openness index is based on the restrictions in capital flows listed by the IMF over time; a world index is obtained by GDP-weighting country indices. Estimates of foreign assets come from Lane and Milesi-Ferretti (2007). The financial openness index comes from Chinn-Ito (2008). The set of developed countries comprises Australia, Austria, Belgium, Canada, Hong Kong, Czech Republic, Denmark, Euro Area, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Saudi Arabia, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Arab Emirates, United Kingdom, and United States. The data are annual and the sample period is 1971 to 2011.
dollar factor, and the carry factor.

The model is an important departure from the usual approaches followed by the literature on international equity returns. The key idea that we bring to the table is that it is possible to approximate the foreign exchange component of the return on the world market portfolio denominated in a common currency, as well as all additional currency risk premia required by the International CAPM with dollar and carry. This insight builds on recent work showing that a substantial fraction of the variation in bilateral exchange rates can be captured by these two factors. The dollar factor is the average excess return earned by an investor that borrows in the U.S. and invests in a broad portfolio of foreign currencies. The carry factor is the average excess return earned by an investor that goes short (long) in a portfolio of low (high) interest rate currencies.

To test the model, we compile a comprehensive dataset of market returns, value- and growth-indexes, and small- and large-market capitalization indices from 25 developed markets and 21 emerging markets over the sample period between February 1976 and April 2013. Using this dataset, we provide evidence that in conditional asset pricing tests, our model outperforms the single-global factor World CAPM as well as the International CAPM estimated by Dumas and Solnik (1995). Our model also has comparable or better performance to the global versions of the popular Fama and French models described in Fama and French (2012). We also show that our model is relevant to those interested in delegated portfolios of international assets – we are able to explain a significant fraction of the variation in international mutual and hedge fund returns using the same set of risk factors.

We explore the reasons for the success of our empirical model using a simple reduced form model of equity and currency returns in complete markets in which we endogenize exchange rates. In this model, the world aggregate equity return expressed in a common currency actually does contain all relevant information necessary for pricing
international assets. But the presence of time-variation in the prices of global shocks – necessary to account for time-varying currency risk premia – confounds estimation using a single-factor model, especially when the relevant state variables are unknown to the econometrician. We show that the three-factor model that we propose arises as a natural solution to the challenges of empirical estimation in this framework.

Our three-factor model is related to, but distinct from earlier empirical approaches to international asset pricing. In the World CAPM of Sharpe (1964) and Lintner (1965), the world aggregate return in a common currency (usually the U.S. dollar) is the unique risk factor. In the International CAPM of Adler and Dumas (1983), all bilateral exchange rates constitute additional factors, but exchange rate shocks are driven by exogenous factors. The literature review provides a more expansive summary of the large body of related work.

The paper is organized as follows. Section 2 briefly reviews the major approaches proposed in the asset pricing literature to understand the risk-return trade-off faced by international investors. Section 3 uses a simple reduced form model which guides our choice of empirical specification. Section 4 describes the data on which we conduct international asset pricing tests. The results are discussed in Section 5, including our work on international mutual and hedge funds. Section 6 concludes. All robustness checks and additional results that are mentioned but not reported in the paper are provided in the Online Appendix.

## 2 Literature Review

Early theoretical work in international asset pricing generally assumed that consumption and investment opportunity sets do not differ across countries. This restrictive assumption was relaxed in later work, which eliminated the assumption of investor in-
difference between domestic and foreign markets. \(^1\) With country-specific investor heterogeneity in preferences, or homogenous preferences but differences in relative prices faced by investors in different countries, currency risk matters for asset pricing. Such relative price differences naturally arise in a world in which there are deviations from purchasing power parity (PPP); moreover in such a world, investing in foreign currencies is risky, since adverse exchange rate movements imply low income from foreign investment expressed in domestic currency terms. \(^2\) In this context, Solnik (1974), Sercu (1980), Stulz (1981), and Adler and Dumas (1983) incorporated currency risk into theoretical asset pricing models, allowing for differences across countries in consumption opportunity sets.

Specifically, Solnik (1974) modeled exchange rates as cross-country relative prices of consumption baskets, and Sercu (1980) generalized the approach, allowing stock returns expressed in local currencies to be correlated with exchange rates (we refer to this henceforth as the Solnik-Sercu model). Stulz (1981) and Adler and Dumas (1983) assumed stochastic country-specific inflation in addition to deviations from PPP.

Early empirical work in international asset pricing generally extended the standard Capital Asset Pricing Model (CAPM) (Sharpe, 1964 and Lintner, 1965) to global markets, or simply augmented the domestic CAPM with the addition of a few international factors. In this early literature, unconditional tests of these models generally proved inconclusive (see, for example, Stehle, 1977, Solnik, 1974 and Korajczyk and Viallet, 1989).

\(^1\)See Glen and Jorion (1993) and Stulz (1995) for a comprehensive survey of the theoretical and empirical literature on international portfolio choice and asset pricing. More recently, Campbell, de Medeiros, and Viceira (2010) consider optimal currency hedging in international equity investment, while Kroencke, Schindler and Schrimpf (2013) study the optimal portfolio allocation across equity and exchange rate investment styles.

\(^2\)Empirical work suggests that PPP holds only in the long run. See Froot and Rogoff (1995) for a comprehensive survey of the early literature on PPP.
Later conditional studies yielded more promising results, for example, Bekaert and Harvey (1995) provided evidence that countries’ capital markets become increasingly globally integrated over time, Chan, Karolyi and Stulz (1992) identified a time-varying global market premium in U.S. equity markets, and Karolyi and Stulz (1996) studied co-movement in Japanese and U.S. stock markets.\(^3\) Ferson and Harvey (1993) studied the predictability of foreign equity returns and showed that most of it is related to time-variation in global risk premia, and Bekaert and Hodrick (1992), and Bekaert (1995, 1996) studied the predictability of equity and currency returns.

Harvey (1991) introduced a novel approach to the literature, modelling time-variation in both the exposure and the price of risk by conditioning on common and country-specific set of instruments, and finding that time-variation reveals differences across countries, but fails to fully predict conditional expected returns. Implementing a variation on this methodology, and incorporating currency risk explicitly, Dumas and Solnik (1995) found that an exchange rate risk model outperforms a simple “World CAPM” with global market equity returns but no explicit role for currency risk. In their empirical work, Dumas and Solnik (1995) assumed that currency risk for a U.S. investor is well captured by three major currencies: the Japanese Yen, the British Pound, and the German Mark. Providing evidence that currency risk is priced in global capital markets, they attributed the widespread failure of the World CAPM to a misspecification problem. These findings were subsequently corroborated by De Santis and Gerard (1998), who extended the analysis to account for volatility dynamics. Harvey, Solnik and Zhou (2002) also provided support for currency risk in international equity returns. Using latent factors, they find that their first factor premium resembles the expected

\(^3\)A large related literature studies the integration of emerging and developed equity markets [see, e.g., Bekaert and Harvey (2000), Bekaert, Harvey and Lundblad (2003), Bekaert, Harvey, Lundblad and Siegel (2011), Bekaert, Harvey, Lundblad and Siegel (2007b), Bekaert, Hodrick and Zhang (2009), and Bekaert and Harvey (2014)].
return on the world market portfolio, while their second factor premium is related to foreign exchange risk. These studies provided strong empirical support for the predictions of international asset pricing theory, but were generally implemented on a small sample of assets from developed countries, over relatively short sample periods.\(^4\)

More recently, a variety of alternate explanations have been proposed for differences in international average equity returns (see Lewis (2011) for the most complete and recent survey of international asset pricing). These explanations include global economic risks (Ferson and Harvey, 1994); inflation risk (Chaieb and Errunza, 2007); liquidity risk (Karolyi, Lee, and van Dijk, 2012, Bekaert, Harvey, and Lundblad, 2007a, Malkhazov, Mueller, Vedolin, and Venter, 2014) factors including momentum and a global cash-flow-to-price factor (Hou, Karolyi, and Kho, 2011); and investability restrictions (Karolyi and Wu, 2014). These specifications do not account for currency risk, nevertheless they significantly outperform the World CAPM. This is also a feature of the work of Fama and French (2012), who present international versions of both the Fama and French (1993) three-factor model, and the Carhart (1997) four-factor model.

Our paper has close links with recent advances in research on currencies. It is well-known (see Meese and Rogoff, 1983) that economic models of exchange rate determination generally lack empirical support in the short-run, with few identifiable links between nominal exchange rates and economic fundamentals.\(^5\) However, a related literature has attempted to explain returns on portfolios of currencies as compensation for risk. Lustig, Roussanov, and Verdelhan (2011) show that the cross-section of currency portfolios sorted by interest rates can be well-explained by a “slope” factor – which


\(^5\)At high frequencies, a number of papers, including Evans and Lyons (2002), and Froot and Ramadorai (2005) show that order flow in exchange rate markets is helpful at predicting and explaining exchange rate movements, but there is considerable debate about the source of this explanatory power, with both rational and behavioral explanations rationalizing these findings.
corresponds to the “carry” factor that we employ as one of the factors in our model. Lustig, Roussanov, and Verdelhan (2011), Menkhoff, Sarno, Schmeling and Schrmpf (2012), Maggiori (2012), and Lettau, Maggiori and Weber (2013) link the currency factors to measures of volatility and downside risk in equity and currency markets. Verdelhan (2014) shows that the carry factor and the “dollar” factor together account for a large share of the variation in bilateral exchange rates, and provides evidence that dollar risk is priced in the cross-section of currencies. These two currency risk factors are nearly orthogonal to one other, and capture two distinct sources of currency risk relevant for explaining variation in currency returns. Our use of carry and dollar as risk-factors in our international asset pricing model is a product of these applications of risk-based arbitrage pricing (Ross, 1976) to currency markets.

The next section describes our approach more formally, as well as describing the international asset pricing models that we use as benchmarks for our empirical tests.

3 Theoretical Framework

In this section, we begin by rapidly reviewing the key result of Adler and Dumas (1983), which forms the basis of the empirical work of Dumas and Solnik (1995), the leading international asset pricing model. In this setting, while exchange rate shocks are priced, they are considered as exogenous to equity returns. We then propose a simple reduced-form model in complete markets that endogenizes the exchange rate, and links together equity risk, currency risk, and interest rates. This model rationalizes our empirical implementation and helps to explain the success of our empirical approach.

3.1 CAPM, World CAPM, and International CAPM

The International CAPM and the World CAPM build on the CAPM, with which they share many assumptions. The CAPM assumes that all investors maximize expected
returns and minimize the expected variance of their portfolios (i.e., investors have homogenous preferences). All investors have access to the same list of assets (i.e., homogenous opportunities) and have equal perceptions about the return characteristics of all assets (i.e., homogenous expectations). All agents thus agree on the composition of the tangency portfolio and hold risky assets in the same proportions. Since all agents hold risky assets in the same proportion, the tangency portfolio must be the market portfolio, and a simple perturbation argument implies that any expected excess return depends on the beta of that asset return and the market portfolio return.

The World CAPM is a simple extension of the standard CAPM to global markets, which assumes that PPP holds instantaneously and continuously, thus rendering currency risk irrelevant. In the World CAPM, global market risk is the single source of systematic risk driving asset prices, and international investors should only earn a premium for exposure to this source of risk. Empirically, the measure of global market risk has generally been the excess return on the world market portfolio, denominated in a common currency, e.g. in U.S. dollars. The use of this proxy invites Roll’s (1977) critique as this proxy for the return on world wealth is clearly subject to substantial measurement error.

The International CAPM differs from the World CAPM, in that it assumes that all investors in the world share the same preferences and opportunities, but not the same expectations, since deviations from PPP imply that investors in different countries pay different prices for their consumption goods. On the one hand, the domestic (e.g., U.S.) investor’s portfolio choice is based on how each asset contributes to the variance and expected excess return on a portfolio measured in U.S. dollars. On the other hand, the foreign (e.g., Japanese) investor’s portfolio choice is based on the assets’ contributions to a portfolio whose risk and return are expressed in foreign currency (here, Japanese Yen).

The International CAPM is best expressed in the work of Adler and Dumas (1983).
In their model, there are \( L + 1 \) countries, and a set of \( m = n + L + 1 \) assets – other than the base-currency deposit – comprised of \( n \) portfolios of equities, \( L \) foreign currency deposits, and the world market portfolio. The returns are listed in the following order: first, the \( n + 1 \) country-specific and world equity returns, and then the currency deposit returns, which are driven by exchange rate changes. In that model, the expected return on asset \( j \) is:

\[
E_t(r_{j,t+1} - r_{f,t}) = \sum_{i=1}^{L} \lambda_{i,t} \text{cov}_t(r_{j,t+1}, r_{n+i,t+1}) + \lambda_{m,t} \text{cov}_t(r_{j,t+1}, r_{m,t+1})
\]  

(1)

where \( r_{j,t+1} - r_{f,t} \) is the nominal return on the equity portfolio \( j \) in excess of the risk-free rate (denoted \( r_{f,t} \)) of the currency in which returns are measured, and \( r_{m,t+1} \) is the excess return on the world market portfolio. The covariance terms \( \text{cov}_t(r_{j,t+1}, r_{n+i,t+1}) \) measure the quantity of exchange rate risk. The time-varying coefficients \( \lambda_{i,t} \) are the world prices of exchange rate risk. The time-varying coefficient \( \lambda_{m,t} \) is the world price of market risk. Equation (1) above derives from Equation (14) in Adler and Dumas (1983), and forms the basis of the empirical work of Dumas and Solnik (1995).

### 3.2 Reduced-Form Model with Endogenous Exchange Rates

In the International CAPM described above, there is no assumption on the set of assets or goods traded; financial markets may be complete or incomplete. The only assumption needed is that PPP does not hold. The model is therefore very general, but this strength comes with a price. The risk factor is the return on world wealth, which does not have to correspond to the world equity market return, and the exchange rate shocks are left unspecified, despite potentially being linked to the market returns. The model taken literally recommends the use of all bilateral exchange rates as additional risk factors, which is somewhat cumbersome empirically; as described above, recent research suggests that a large set of bilateral exchange rates can be summarized using
carry and dollar factors.

To address those shortcomings, we describe a simple reduced-form model in which exchange rates, currency risk factors, and equity market returns are all precisely defined. In our model, we specify the law of motion of the lognormal stochastic discount factors (SDFs) in all countries, and we assume that financial markets are complete.

**Intuition** When the law of one price holds and investors can form portfolios freely, there exists a SDF $M_{t+1}$ that prices any return $R_{t+1}^i$ such that $E_t (M_{t+1}R_{t+1}^i) = 1$. The same condition holds for the risk-free rate $R_f$. Assuming that the returns and SDF are lognormal, the Euler equation implies:

$$E_t \left( r_{t+1}^i - r_{f,t} + \frac{1}{2} var_t(r_{t+1}^i) \right) = -\frac{cov_t(m_{t+1}, r_{t+1}^i)}{var_t(m_{t+1})} \frac{var_t(m_{t+1})}{\Lambda_t}$$

where lower letters denote logs. Expected excess returns are the product of the quantity of risk, $\beta_t^i$, which is asset-specific, and the market price of risk, $\Lambda_t$. The quantities of risk can be thought as simple regression coefficients of returns on the risk factors.

Two features of currency markets imply that estimating the risk-return relationship in international asset pricing is delicate. First, it is well-known since Bekaert (1996) and Bansal (1997) that in a lognormal model, the log currency risk premium equals half the difference between the conditional volatilities of the log domestic and foreign SDFs. Since currency risk premia are time-varying (as shown by the large literature on uncovered interest rate parity and the forward premium puzzle), log SDFs must be heteroskedastic.$^6$ That is, empirical estimation must account for time-varying market prices of risk ($\Lambda_t$).

$^6$When SDFs and returns are not lognormal, a similar result implies that the higher moments of the SDF must be time-varying.
Second, we know that at least two risk factors, dollar and carry, are necessary to describe systematic variation in bilateral exchange rates. International equity returns comprise much more than just exchange-rate variation, which strongly suggests the need for additional risk factors over and above dollar and carry.

Taken together, it appears that identifying the risk-return tradeoff in international asset pricing will require at least three factors. Moreover, we will need to account for time-variation in the prices of risk, and possibly in the quantity of risk \( \beta_t \) as well. Intuitively, a model of international asset pricing that explains variation in excess returns on risky assets \( (r_{t+1}^i - r_{f,t}) \) will likely have the general form:

\[
E_t \left( r_{t+1}^i - r_{f,t} + \frac{1}{2} \text{var}_{t}(r_{t+1}^i) \right) = \beta_{t,1}^i \Lambda_1 + \beta_{t,2}^i \Lambda_2 + \beta_{t,3}^i \Lambda_3.
\]

Consider a single-factor model such as the World CAPM. Such a model would only accurately describe returns in the unlikely case in which all quantities and prices of risk move in lockstep, and the empirical proxy correctly captured variation in the underlying source of risk. This seems a priori unlikely. We now formalize this intuition. In our simple reduced-form model, the World CAPM holds in theory, but our three-factor model will fit the data far better.

**SDFs** In the tradition of Backus, Foresi, and Telmer (2001), we assume that pricing kernels \( m_{t+1}^i \) are exponentially affine:

\[
-m_{t+1}^i = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i u_{t+1}^i} + \tau z_t^w + \sqrt{\delta z_t^w u_{t+1}^w} + \sqrt{\kappa z_t^i u_{t+1}^i} + \sqrt{\omega z_t^w u_{t+1}^w},
\]

\[
z_{t+1}^i = (1 - \phi) \theta + \phi z_t^i - \sigma \sqrt{z_t^i u_{t+1}^i},
\]

\[
z_{t+1}^w = (1 - \phi^w) \theta^w + \phi^w z_t^w - \sigma^w \sqrt{z_t^w u_{t+1}^w}
\]
where \( u_{i,t+1}, u_{w,t+1}, u_{g,t+1}, u_{c,t+1} \) are i.i.d, mean-zero, variance-one Gaussian shocks, \( m_{i,t+1} \) is the log SDF of country \( i \), and \( z_{t}^{i} \) and \( z_{t}^{w} \) are the state variables that govern the conditional volatility of the SDF. Each SDF is heteroskedastic because currency risk premia are driven by the conditional variances of the SDFs. Inflation is not a priced risk in this model – the SDFs can be interpreted as nominal SDFs. We use the U.S. dollar as the base currency and drop the superscript \( i \) to describe any U.S. variable.

A similar model is studied in Lustig, Roussanov, and Verdelhan (2011, 2014) and in Verdelhan (2014).\(^7\) The key distinguishing feature of our model is the introduction of equity-specific shocks \( u_{c,t+1} \). As we shall see, these shocks affect both dividends and SDFs but not exchange rates, and drive the world equity return factor.

The SDFs depend on country-specific shocks, \( u_{i,t+1} \), and three global shocks, \( u_{w,t+1}, u_{g,t+1}, u_{c,t+1} \). We refer to the volatilities of the SDF related to those three shocks (namely, \( \sqrt{\delta_{i}z_{t}^{w}}, \sqrt{\kappa z_{t}^{i}}, \) and \( \sqrt{\omega z_{t}^{w}} \)) as the market prices of risk of those shocks.

The first shock \( u_{w,t+1} \) is priced similarly in each country up to a scaling factor, denoted \( \delta_{i} \). Examples of such a shock might be a global financial crisis which affects prices in all countries in a perfectly correlated fashion, but with differential intensity. To be parsimonious, we model differences in exposure \( \delta_{i} \) as the only source of heterogeneity in countries’ SDFs, and fix all the other parameters of the SDFs to be the same across countries. Countries also differ in their aggregate dividend growth rates.

The second shock \( u_{g,t+1} \) is priced differently across countries, even if countries share the same exposure \( \kappa \). An example of this might be a productivity shock that affects some economies more than others.

The third shock \( u_{c,t+1} \) is priced in exactly the same way in all countries.

\(^7\)In those papers, the prices of risk (i.e., the square roots in the law of motion of the SDFs), depend on both the country-specific and global state variables in order to differentiate between unconditional and conditional currency risk premia. For the sake of clarity, we leave this difference aside here, but the model can be easily extended in this direction. Earlier examples of affine models in international finance include Frachot (1996) and Brennan and Xia (2006).
Exchange Rates  When markets are complete, log changes in exchange rates correspond to the differences between domestic and foreign log pricing kernels (Bekaert, 1996, Bansal, 1997):  

\[
\Delta s^i_{t+1} = m_{t+1} - m^i_{t+1},
\]

\[
= \chi(z^i_t - z_t) + \sqrt{\gamma z^i_t} u^i_{t+1} - \sqrt{\gamma z_t} u_{t+1} \\
+ (\sqrt{\delta^i} - \sqrt{\delta}) \sqrt{z^w_t} u^w_{t+1} + \sqrt{\kappa} (\sqrt{z^i_t} - \sqrt{z_t}) u^g_{t+1},
\]

where the exchange rate is defined in foreign currency per U.S. dollar. Therefore, an increase in the exchange rate corresponds to an appreciation of the U.S. dollar. Although financial markets are complete, real exchange rates are not necessarily constant as soon as some frictions exist in the goods markets (e.g., non-traded goods, or trading costs). The exchange rate between country \(i\) and the domestic economy depends on the country-specific shocks \(u^i\) and \(u\), as well as on the global shocks \(u^w\) and \(u^g\), but not on the global shocks \(u^c\) since their prices of risk are the same across countries.

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\(8\)This result derives from the Euler equations of the domestic and foreign investors buying any asset \(R^i\) that pays off in foreign currency: \(E_t[M_{t+1} R^i S^i_t/S^i_{t+1}] = 1\) and \(E_t[M^i_{t+1} R^i] = 1\). When markets are complete, the pricing kernel is unique and thus exchange rates are defined as \(S^i_{t+1}/S^i_t = M_{t+1}/M^i_{t+1}\), or in logs \(\Delta s^i_{t+1} = m_{t+1} - m^i_{t+1}\).
**Equity Returns**  In order to define equity returns, the model posits a dividend growth process in each country $i$:

$$\Delta d_i^{t+1} = \mu_D + \psi z_i^t + \psi w z_w^t + \sigma_D \sqrt{z_i^{t+1}} u_i^{t+1} + \sigma^w_D \sqrt{z_i^{t+1}} u_w^{t+1} + \sigma_g^D \sqrt{z_i^{t+1}} u_g^{t+1} + \sigma_{c,i}^D \sqrt{z_i^{t+1}} u_c^{t+1},$$

where the innovations are the same as those described above. Dividend growth rates respond to both country-specific and global shocks. The only systematic difference across countries comes from the impact of global shocks $u_{c,t+1}$ on dividend growth, governed by the parameters $\sigma_{c,i}^D$. This source of heterogeneity drives the differences in global equity market betas. Innovations to the log gross equity excess return are then:\[9\]

$$r_{i,t+1}^{e,i} - E_t(r_{i,t+1}^{e,i}) = (\sigma_D - k_1 B_{pd}^i)\sqrt{z_i^{t+1}} u_i^{t+1} + (\sigma^w_D - k_1 C_{pd}^i)\sqrt{z_w^{t+1}} u_w^{t+1} + \sigma_g^D \sqrt{z_i^{t+1}} u_g^{t+1} + \sigma_{c,i}^D \sqrt{z_w^{t+1}} u_c^{t+1},$$

where $r_{i,t+1}^{e,i}$ is the logarithmic gross rate of return on each country's stock market index denominated in that country’s currency.

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\[9\] In this model, the log price-dividend ratio is affine in the state variables, $z_i^t$ and $z_w^t$:

$$pd_i^t = A_{pd}^i + B_{pd}^i z_i^t + C_{pd}^i z_w^t,$$

where the constants $A_{pd}^i$, $B_{pd}^i$, and $C_{pd}^i$ are defined as function of the SDF and dividend growth parameters. The model is solved in closed form using the standard log-linear approximation for the log gross return on the aggregate dividend claim:

$$r_{i,t+1}^{e,i} \approx k_0 + k_1 pd_i^{t+1} - pd_i^t + \Delta d_i^{t+1},$$

where $k_0$ and $k_1$ are defined by the Taylor approximation of the log price-dividend ratio $pd_i^t$ around its mean. More precisely, the Euler equation applied to the stock market return implies that the coefficients $A_{pd}^i$, $B_{pd}^i$ and $C_{pd}^i$ are defined by:

$$A_{pd}^i = -\alpha + k_0 + k_1 A_{pd}^i + k_1 B_{pd}^i (1 - \phi) \theta + k_1 C_{pd}^i (1 - \phi w) \theta w + \mu_D,$$

$$B_{pd}^i = k_1 B_{pd}^i \phi + \psi - \chi + \frac{1}{2}(\sqrt{\gamma} + k_1 B_{pd}^i \sigma - \sigma_D)^2 + \frac{1}{2}(\sigma_D^2 - \sqrt{\kappa})^2,$$

$$C_{pd}^i = k_1 C_{pd}^i \phi w + \psi w - \tau + \frac{1}{2}(\sqrt{\delta} + k_1 C_{pd}^i \sigma w - \sigma_D^2)^2 + \frac{1}{2}(\sqrt{\omega} - \sigma_D^2)^2.$$
International CAPM Redux  The equity return of country $i$ expressed in U.S. dollars, denoted $r_{i,t+1}$, is simply derived from the equity return of country $i$ in local currency and the change in the exchange rate:

$$r_{i,t+1} - E_t(r_{i,t+1}) = \sqrt{\gamma z_t} u_{t+1} + (\sigma_D - k_1 B_{pd}\sigma - \sqrt{\gamma}) \sqrt{\gamma z_t} u_{t+1}$$

$$+ \left(\sigma^w_D - k_1 C_{pd}\sigma^w - \sqrt{\gamma^w} + \sqrt{\delta^w}\right) \sqrt{\gamma^w} u_{t+1} + \sigma^c_i \sqrt{\gamma^c} u_{t+1}$$

$$+ \left(\sigma_D^g - \sqrt{\kappa}\right) \sqrt{z_t^g} + \sqrt{\kappa z_t^g} u_{t+1}.$$

The foreign equity return expressed in U.S. dollars therefore depends on the foreign and U.S. specific shocks ($u^i$ and $u$), as well as the global shock captured by the carry and dollar factors ($u^w$) and ($u^g$) and the world equity shock ($u^c$).

The expected equity excess return from the perspective of a U.S. investor is:

$$E_t[r^e_{i,t+1} - r_{f,t+1}] = \frac{1}{2} Var_t[r_{i,t+1}] = Cov_t[M_{i,t+1}, r_{i,t+1}]$$

$$= \gamma z_t + \left(\sigma^w_D - k_1 C_{pd}\sigma^w - \sqrt{\gamma^w} + \sqrt{\delta^w}\right) \sqrt{\gamma^w} z_t^w + \sigma^c_i \sqrt{\gamma^c} z_t^w$$

$$- \left(\sigma_D^g - \sqrt{\kappa}\right) \sqrt{z_t^g} + \sqrt{\kappa z_t^g}.$$

(2)

Note that our assumption of complete markets implies that we need only verify the Euler condition for one country’s investor. As soon as the Euler equation is satisfied for the U.S. investor, for example, it implies that the Euler condition for any foreign investor is also satisfied.

The Euler condition for the U.S. investor is $E_t[M_{i,t+1} R_{i,t+1} S_i / S_i^{t+1}] = 1$, which implies $E_t[M_{i,t+1} R_{i,t+1}] = 1$ — the Euler condition of the representative investor in country $i$ — as well as $E_t[M_{i,t+1} R_{i,t+1}(S_i / S_i^{t+1})(S_i^{t+1} / S_i^i)] = 1$, the Euler condition of the representative investor in country $j$.

What do we learn from this reduced-form model? In order to understand our empirical approach, we first express various factors in the language of the model, namely,
the world equity return in U.S. dollars $WMKT_{t+1}$, the world equity return in local currencies $LWMKT_{t+1}$, and the carry and dollar factors.

**Equity Factors** The innovations to the average world equity market return in local currency terms, which we define for ease of exposition as the simple average of local equity returns, are:

$$
LWMKT_{t+1} = r_{t+1}^{\bar{e}_i} - E_t(r_{t+1}^{\bar{e}_i}) = (\sigma^w_D - k_1 C_{pd}^w \sigma^w) \sqrt{z^w_t u^w_{t+1}} + \sigma^g_D \sqrt{z^g_t u^g_{t+1}} + \sigma^c_i D \sqrt{z^w_t u^c_{t+1}},
$$

where $\overline{x}$ denotes the cross-country average of a variable $x$.

Country-specific shocks average out, and the world equity market return is only driven by the global shocks $u^w_{t+1}$, $u^g_{t+1}$, and $u^c_{t+1}$. If the law of large numbers applies, the cross-sectional mean of $z^i_t$ is constant (and equal to $\theta$). The world equity return in U.S. dollars is:

$$
WMKT_{t+1} = r_{t+1}^{\bar{e}_i,d} - E_t(r_{t+1}^{\bar{e}_i,d}) = \sqrt{\gamma z^i_t u^i_{t+1}} + \left(\sigma^w_D - k_1 C_{pd}^w \sigma^w - \sqrt{\delta'} + \sqrt{\delta}\right) \sqrt{z^w_t u^w_{t+1}} + \sigma^c_i D \sqrt{z^c_t u^c_{t+1}}.
$$

**Currency Factors** We first express currency excess returns, which are returns on the following strategy: the investor borrows at the domestic risk-free rate, converts the amount into foreign currency and lends at the foreign risk-free rate, converting back the proceeds at the end of the investment period and paying back the debt. The log currency excess return is thus:

$$
r^{i,t}_{x,t+1} = r^{i,t}_f - r^{i,t}_r - \Delta s^{i,t}_{t+1}.
$$

---

$^{10}$The risk-free rate in country $i$, denoted $r^{i,t}_f$, is given by:

$$
r^{i,t}_f = -E_t [m^{i,t+1}_t] - \frac{1}{2} Var_t [m^{i,t+1}_t] = \alpha + \left(\chi - \frac{1}{2}(\gamma + \kappa)\right) z^i_t + \left(\tau - \frac{1}{2}(\delta' + \omega)\right) z^w_t.
$$
The systematic components of currency excess returns are driven by at least two risk factors (Lustig, Roussanov, and Verdelhan, 2011, 2014, and Verdelhan, 2014), i.e., carry and dollar factors.

The carry factor is the excess returns of a strategy that invests in high- and borrows in low-interest rate currencies:

$$ Carry_{t+1} = \frac{1}{N_H} \sum_{i \in H} r_{x_{t+1}}^i - \frac{1}{N_L} \sum_{i \in L} r_{x_{t+1}}^i, $$

where $N_H$ ($N_L$) denotes the number of high (low) interest rate currencies in the sample.

For the sake of exposition, assume that most of the cross-country difference in interest rates is due to their exposure ($\delta^i$) to the world state variable. In this case, baskets of high and low interest rate currencies will exhibit the same level of country-specific volatility, assuming that the law of large numbers holds.\footnote{In practice of course, the number of currencies is small (the dataset used in this paper contains at most 39 currencies). As a result, the law of large numbers is only an approximation used here to provide intuition.} Under this assumption, innovations to the carry factor only depend on shocks $u^w_t$, not on shocks $u^\delta$.

$$ Carry_{t+1} - E_t(Carry_{t+1}) = \left( \sqrt{\delta^L_i} - \sqrt{\delta^H_i} \right) \sqrt{z^w_t} u^w_{t+1}, $$

The interest rate difference, or forward discount, between country $i$ and the U.S is therefore equal to:

$$ r_{f,t}^i - r_{f,t} = \left( \chi - \frac{1}{2}(\gamma + \kappa) \right) (z^i_t - z_t) - \frac{1}{2}(\delta^i - \delta) z^w_t. $$

The expected currency excess return is therefore:

$$ E_t(rx_{t+1}^i) = r_{f,t}^i - r_{f,t} - E_t(\Delta s_{t+1}^i) = \frac{1}{2} Var_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1}^i) $$

$$ = \frac{1}{2}(\gamma + \kappa)(z_t - z^i_t) + \frac{1}{2}(\delta - \delta^i) z^w_t. $$

If the SDFs were not heteroscedastic, the expected currency excess returns would be constant and the uncovered interest rate parity, which is strongly rejected in the data, would be satisfied in the model.
where $\bar{x}^i_H$ and $\bar{x}^i_L$ denote the cross-sectional average of the variable $x$ across countries in the high- and low-interest rate portfolios: $\bar{x}^i_H = \frac{1}{N_H} \sum_{i \in H} x^i$, $\bar{x}^i_L = \frac{1}{N_L} \sum_{i \in L} x^i$.

The dollar risk factor is the average of all currency excess returns defined in U.S. dollars:

$$Dollar_{t+1} = \frac{1}{N} \sum_i r x^i_{t+1},$$

where $N$ denotes the number of currencies in the sample. In large baskets of currencies, foreign country-specific shocks average out (again assuming that there are enough currencies in the baskets for the law of large numbers to apply). As a result, innovations to the dollar risk factor depend on both U.S.-specific and world shocks, but not on country-specific shocks:

$$Dollar_{t+1} - E_t(Dollar_{t+1}) = \sqrt{\gamma z_t u_{t+1}} + \left( \sqrt{\delta} - \sqrt{\delta_i} \right) \sqrt{z_t^w} u^w_{t+1} + \sqrt{\kappa} \left( \sqrt{z_t^c} - \sqrt{z^c_t} \right) u^g_{t+1},$$

If the U.S. SDF exhibits the average exposure to shocks $u^w$ (i.e., $\bar{\delta} = \delta$), the dollar factor is orthogonal to the carry factor. Under that assumption, the dollar factor captures shocks $u^g$, while the carry factor captures shocks $u^w$.

**Discussion of the Model** The world equity return, $LWMKT_{t+1}$, built from returns in local currencies, does not span all systematic shocks: it depends on shocks $u^w$, $u^g$, and $u^c$, but not on the U.S. shock $u$. These shocks are systematic from the perspective of the U.S. investor, although not from the perspective of other investors. They appear in the cross-section of equity excess returns because the returns need to be expressed in one common currency (e.g., the U.S. dollar).

In contrast, the world equity return in U.S. dollars $WMKT_{t+1}$ contains all the systematic shocks that drive foreign equity returns. At first sight, it appears as a sufficient tool to measure aggregate risk, without the need to add any bilateral exchange.
rates. This result contrasts with the findings of Adler and Dumas (1983) because the exchange rate is endogenous in our setup. In practice, however, even in this simple model, the world equity return in U.S. dollars is an imperfect measure of risk in the (obviously general) case when the econometrician does not know the country-specific and global state variables.

To see this point, consider the time-variation in the quantity and market price of aggregate risk, starting with the betas. The betas on the global shocks $u^w$ and $u^c$ are constant, while the betas on the global shocks $u^g$ are not. The total beta on the world equity return in U.S. dollars $W MKT_{t+1}$ is time-varying, following the dynamics of the country-specific state variables $z$ and $z^i$:

$$\beta_{W MKT,t}^i = 1 + \frac{\sigma_w^D - k_1 C_{pd} \sigma^w - \sqrt{\delta^i} + \sqrt{\delta}}{\sigma^D - k_1 C_{pd} \sigma^w - \sqrt{\delta^i} + \sqrt{\delta}} + \frac{\sigma_{g}^D}{\sigma^D} \left( \sigma_{g}^D - \sqrt{\kappa} \right) \sqrt{z^i_t + \sqrt{\kappa z_t}} \tag{3}$$

In our simple setup, all country-specific state variables $z^i$ are characterized by the same persistence and volatility. However, in actual data, country-specific state variables seem very likely to evolve at different frequencies, confounding estimation.

Market prices of risk are also time-varying in our simple model. Expected excess returns are the product of betas and market prices of risk, i.e., the market price of world equity risk is the ratio of the expected excess return, defined in Equation (2), to the beta in Equation (3). The price of risk thus also varies with the country-specific and global state variables ($z$, $z^i$, and $z^w$). If those state variables were known to the econometrician, a conditional asset pricing experiment could recover the time-variation in the quantities and prices of risk. In practice however, the state variables are not known, forcing reliance on rolling windows or other such empirical approaches.

The issue becomes even more acute if the heteroskedasticity of the $u^w$ or $u^c$ shocks
were to depend on both global and country-specific state variables. In that case, the
aggregate market beta $\beta_{W_{MKT},t}^i$ also depends on the global state variable $z^w$. The criti-
cal issue here is that if the local and global state variables evolve at different frequencies,
it becomes impossible to use a single beta to perfectly summarize the time-variation in
two state variables.

Taken together, the world equity return expressed in U.S. dollars bundles variables
with different time-dynamics together. This makes it difficult to use this single factor
to uncover risk exposures in international asset pricing. While our simple model with
endogenous exchange rates suggests that we should go back to the World CAPM, the
heteroskedasticity of the SDF implies that this single-factor model would struggle to
accurately capture the risk-return tradeoff in practice.

This motivates our choice of a multiple-factor model. With $LW_{MKT}$, $Carry$, and
$Dollar$, we can summarize all the shocks in the system, and we can allow all quantities
and prices of risk to have their own time-dynamics. When we empirically implement
the model in the data, the only difference lies in the approach to aggregation, i.e., we
weight countries and currencies by stock market capitalizations, instead of the equal
weights that we employ in the model for ease of exposition.

The model has three additional implications that we check in the data. First, the
carry and dollar factors should matter even for equity excess returns expressed in local
currency. Likewise, the equity factor $LW_{MKT_{t+1}}$, built without introducing exchange
rates, should be correlated with the carry and dollar factors. Second, despite exchange

\[\begin{align*}
-\alpha + \chi z^i_t + \sqrt{\gamma z^i_t u^i_{t+1}} + \tau z^w_t + \sqrt{\delta z^w_t u^w_{t+1}} + \lambda z^i_t u^i_{t+1} + \sqrt{\omega z^w_t u^w_{t+1}}
\end{align*}\]

We do not consider that variation here as it does not admit a closed-form solution for equity returns
and betas.

---

12Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2014) consider this variation in
order to reproduce some features of the currency markets. In the context of our model, the law of
motion of the log SDF would be:

\[\begin{align*}
-\alpha + \chi z^i_t + \sqrt{\gamma z^i_t u^i_{t+1}} + \tau z^w_t + \sqrt{\delta z^w_t u^w_{t+1}} + \lambda z^i_t u^i_{t+1} + \sqrt{\omega z^w_t u^w_{t+1}}
\end{align*}\]
rates being endogenous, exchange rate shocks in this model do not span equity returns. Shocks $u^c$ affect equity returns but do not affect exchange rates, because their impact is exactly the same across countries. Equity and currency markets therefore appear segmented to an extent. Third, while using bilateral exchange rates as risk factors directly is consistent with the model, they are also driven by country-specific shocks that are irrelevant to asset pricing and thus weaken the identification of aggregate risk.

3.3 Model simulation

We simulate the model in order to check its implications of the model and to compare it to the World CAPM. The calibration is presented in Table 1; all parameters governing the state variable dynamics and the stochastic discount factor, except for $\omega$ and $\alpha$, are from Lustig, Roussanov and Verdelhan (2014). The parameter $\alpha$ is calibrated to match an average annualized risk-free rate of 1.31% given the other parameter values. The value of $\omega$ as well as the parameters of the dividend growth process are calibrated to match an annualized volatility of equity returns in local currency of 16% and a price-dividend ratio (level) of about 30. The model delivers reasonable interest rates, exchange rates, equity and currency excess returns.

Figure 2 reports the simple exercise that we conduct on simulated data from the model, and later reproduce on actual data. The vertical axis corresponds to average excess returns from simulated data. The horizontal axis corresponds to predicted excess returns from the World CAPM (left panel) and our three-factor model (right panel). Both models are estimated using rolling-windows in order to capture time-variation in quantities and prices of risk. The figure presents a striking demonstration of the difference: the World CAPM fits the data poorly, while our model accurately describes the cross-section of returns.
Table 1
Parameter Values

This table reports the parameter values used to simulate the model. All parameters governing the state variable dynamics and the stochastic discount factor, except for $\omega$ and $\alpha$, are from Lustig, Roussanov and Verdelhan (2014). The parameter $\alpha$ is calibrated to match an average annualized risk-free rate of 1.31% given the parameter values for $\chi$, $\gamma$, $\kappa$, $\tau$, $\omega$ and the average $\delta$. The parameters of the dividend growth process are calibrated to match an annualized volatility of equity returns in local currency of 16% and a price-dividend ratio of about 30 (in level). The law of motion of the stochastic discount factor (Panel A), the state variable dynamics (Panel B) and the dividend growth process (Panel C) are reported at the top of each panel.

<table>
<thead>
<tr>
<th>Panel A: Stochastic discount factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-m_{t+1} = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i u_{t+1}^i} + \tau z_t^w + \sqrt{\delta z_t^w u_{t+1}^w} + \sqrt{\kappa z_t^u i_{t+1}^i} + \sqrt{\omega z_t^w u_{t+1}^c}$</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>SDF</th>
<th>Heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ (%)</td>
<td>1.80</td>
<td>0.36</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.89</td>
<td>0.22</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.04</td>
<td>0.49</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2.78</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: State variable dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{t+1}^i = (1 - \phi)\theta_t + \phi z_t^i - \sigma z_t^i u_{t+1}^i$</td>
</tr>
<tr>
<td>$z_{t+1}^w = (1 - \phi^w)\theta_t + \phi^w z_t^w - \sigma^w z_t^w u_{t+1}^w$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\theta$ (%)</th>
<th>$\sigma$ (%)</th>
<th>$\phi^w$</th>
<th>$\theta^w$ (%)</th>
<th>$\sigma^w$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.91</td>
<td>0.77</td>
<td>0.68</td>
<td>0.99</td>
<td>2.09</td>
<td>0.28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Dividend growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta d_{t+1} = \mu_D + \psi z_t^i + \psi_w z_t^w + \sigma_D z_t^i u_{t+1}^i + \sigma^D z_t^w u_{t+1}^w + \sigma_D c_{t+1}^i + \sqrt{z_t^w u_{t+1}^c}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dividends</th>
<th>Heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_D$ (%)</td>
<td>$\psi$</td>
</tr>
<tr>
<td>2.50</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 2  
Realized vs Predicted Average Excess Returns: Simulated Data

This figure plots annualized realized average excess returns against those predicted by the World CAPM and the CAPM Redux using simulated series. The model is simulated for 45 countries. Empirical asset pricing models are estimated conditionally. For each country the predicted average excess return is computed as follows: a) Time-varying factor betas are estimated using 60-month rolling windows; b) At each time t estimated conditional betas are multiplied by the corresponding factor means computed over the same time-window (from time t-59 to time t); c) Predicted values are averaged, expressed in percentage terms and multiplied by 12. The straight line is the 45-degree line through the origin. The sample size is 10,000 periods.
3.4 Fama-French Global Factor Model

We end this overview of international asset pricing with a note on the Fama-French factors. Since their discovery, the Fama and French (1993) three factors and the Carhart (1997) four factors, although not based on a particular theoretical model, have become standard benchmarks in empirical asset pricing studies. These models ignore exchange rate risk, and offer an alternate risk-based explanation to patterns in international average stock returns based on size, value, and momentum premia. Fama and French (2012) find that these global models perform reasonably well in unconditional time-series tests on global size and book-to-market sorted, and size and momentum sorted portfolio returns (apart from micro-cap stocks), but perform very poorly when the same portfolios are constructed at a regional level (i.e., North America, Japan, Asia Pacific, and Europe). We also use these models as useful, albeit atheoretical competitors for our model.

3.5 Empirical Approach

We compare the performance of our three-factor model above against a range of alternative asset pricing models, notably the World and International CAPM, but also the Fama-French-Carhart factor models. For the readers’ convenience, all those models are
summarized as follows:

\[ r_{i,t+1} - r_{f,t} = \alpha_i^{WCAPM} + \beta_i^{WMKT} [WMKT_{t+1} - r_{f,t}] + \epsilon_{t+1} \]  

(4)

\[ = \alpha_i^{ICAPM} + \beta_i^{WMKT} [WMKT_{t+1} - r_{f,t}] 
+ \beta_{GBP}^i r_{GBP,t+1} + \beta_{JPY}^i r_{JPY,t+1} + \beta_{GBP}^i r_{GBP,t+1} + \epsilon_{t+1} \]  

(5)

\[ = \alpha_i^{CAPMredux} + \beta_{LWMKT}^i [LWMKT_{t+1} - r_{f,t}] 
+ \beta_{Dollar}^i r_{Dollar,t+1} + \beta_{Carry}^i r_{Carry,t+1} + \epsilon_{t+1} \]  

(6)

\[ = \alpha_i^{FF} + \beta_{WMKT}^i [WMKT_{t+1} - r_{f,t+1}] 
+ \beta_{SMB}^i SMB_{t+1} + \beta_{HML}^i HML_{t+1} + \beta_{WML}^i WML_{t+1} + \epsilon_{t+1} \]  

(7)

where, again, \( WMKT_{t+1} \) denotes the return on the world market portfolio denominated in US dollars, while \( LWMKT_{t+1} \) denotes the return on the world market portfolio obtained by aggregating country returns in local currencies. Equation (4) describes the World CAPM, Equation (5) describes the International CAPM, while Equation (6) describes our model. Equation (7) corresponds to the Fama and French (2012) and Carhart (1997) model, motivated by international evidence on four anomalies extensively documented in the asset pricing literature.\(^{13}\) All these models are estimated using rolling window betas. Before reporting time-series tests, as well as the two-pass cross-sectional asset pricing tests of Fama and MacBeth (1973), we describe succinctly our dataset.

\(^{13}\)These anomalies are the “size” effect (Banz, 1981), the “value” effect (Chan, Hamao, and Lakonishok, 1991, and Asness, Moskowitz, and Pedersen, 2013), as well as the “momentum” effect (see, for example, Griffin, Ji, and Martin, 2003, and Chui, Titman, and Wei, 2010 among others).
4 Data

Our dataset comprises aggregate equity return indices as well as mutual and hedge fund individual returns.

4.1 Test Assets

The test assets span 46 countries, comprising 225 different indices, over the period from January 1976 to April 2013. The coverage of countries follows the constituents of the 2011 Morgan Stanley Capital International (MSCI) Global Investable Market Indices. Following MSCI’s approach, countries are classified into two categories: 25 “developed markets” and 21 “emerging markets”. For each of these countries, Datastream reports daily total return series denominated in U.S. dollars for five different MSCI indices, namely, (i) the aggregate market, (ii) an index of growth stocks, (iii) an index of value stocks, (iv) an index of large market-capitalization stocks, and (v) an index of small market-capitalization stocks. Monthly returns are obtained from end-month to end-month. The risk-free rate is the U.S. 30-day Treasury bill rate, obtained from Kenneth French’s website. Countries and asset types enter the equity data set at different points in time, depending on data availability. There are 54 test assets at the beginning of the sample in 1976, covering 18 developed markets and three asset types, namely, aggregate market, value, and growth. The size of the cross-section progressively increases from 1986 onwards.

The MSCI portfolios offer a challenging cross-section of returns to explain. Tables 2 and 3 provide evidence that these test assets exhibit large cross-sectional variation in average aggregate equity excess returns, across both developed and emerging countries, and across the different types of indices. Table 2 focuses on developed markets, while Table 3 focuses on emerging markets. For aggregate market excess returns and the full sample period, the annualized average spread is approximately 10% for developed
markets and 22% for emerging markets. Moreover, Tables 2 and 3 confirm the existence of both a size and a value effect in international equity markets. On average, small cap-firms outperform large-cap firms in 15 of 21 emerging markets, and value firms pay higher average excess returns than growth firms in three-fourths of the countries in our sample. The “value premium” is absent in only five developed markets and seven emerging markets. The time-series standard deviations also show that these excess returns vary considerably over time, and not merely in the cross-section. Figure 3 provides a pictorial description of the same facts – in the figure, for each asset type, countries are sorted according to their average market excess returns.

We expand our set of test assets beyond equity markets by building two sets of currency cross-sections, both stemming from recent research in currency markets. Specifically, we build six portfolios sorting all currencies in our data set on their forward discounts and six other portfolios sorting the same set of currencies on their dollar exposures. In constructing these portfolios we follow Lustig and Verdelhan (2005, 2007) and Verdelhan (2014), respectively. All portfolios are rebalanced monthly. Their properties are similar to those documented in the literature.
Table 2
Descriptive Statistics: Country Equity Excess Returns in Developed Markets

This table reports the annualized sample mean (Mean) and the annualized standard deviation (Std) of monthly equity excess returns in developed countries. For each country, equity excess returns are defined as the monthly simple return on five types of MSCI equity indices (i.e., aggregate market (Aggr. Market), value-stocks (Value), growth-stocks (Growth), small-stocks (Small) and big-stocks (Big)) quoted in U.S. dollars and computed in excess of the U.S. one-month Treasury bill rate. All values are in percentage terms. Country daily total return indices are obtained from Datastream; returns are from end-of-month series. The sample period is February 1976 to April 2013.

<table>
<thead>
<tr>
<th>Country</th>
<th>Aggr. Market</th>
<th>Value</th>
<th>Growth</th>
<th>Small</th>
<th>Big</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std</td>
<td>Mean</td>
<td>Std</td>
<td>Mean</td>
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<td>10.86</td>
<td>23.07</td>
<td>7.83</td>
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<td>2.74</td>
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<td>7.52</td>
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</table>
Table 3
Descriptive Statistics: Country Equity Excess Returns in Emerging Markets

This table reports the annualized sample mean (Mean) and the annualized standard deviation (Std) of monthly equity excess returns in emerging countries. For each country, equity excess returns are defined as the monthly return on five types of MSCI equity indices (i.e. aggregate market (Aggr. Market), value-stocks (Value), growth-stocks (Growth), small-stocks (Small) and big-stocks (Big)) quoted in U.S. dollars and computed in excess of the U.S. one-month Treasury bill rate. All values are in percentage terms. Country daily total return indices are obtained from Datastream; returns are from end-of-month series. The sample period is February 1976 to April 2013.

<table>
<thead>
<tr>
<th>Country</th>
<th>Aggr. Market</th>
<th>Value</th>
<th>Growth</th>
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<th>Big</th>
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<td>56.66</td>
<td>27.91</td>
<td>55.94</td>
<td>19.46</td>
</tr>
</tbody>
</table>
4.2 Mutual Fund and Hedge Fund Returns

We also evaluate the exposure of international mutual funds and hedge funds to currency risk using our model.

For mutual funds, we acquire monthly data from CRSP. The sample includes all funds classified as “Foreign Equity Funds” according to the CRSP fund style code. The sample period is 11/1990–4/2013, that is the horizon over which the Fama-French global factors are constructed. This choice ensures a large cross-section of mutual fund returns. The dataset also includes the total net asset value (NAV) managed by each fund: when NAV are annual or quarterly, we linearly interpolate monthly values. We adopt the same procedure for single missing observations, which we interpolate using the two adjacent values; note that we do not interpolate returns, only NAV. In order to compare equally-weighted and value-weighted statistics, only the returns with corresponding NAV are kept in the data set. There are 74 funds with 5 years of past returns as of October 1995, but up to 1148 funds at the end of the sample. Those funds as a group currently manage about US$ 800 billion.

We acquire monthly hedge fund data from the updated version of the consolidated hedge fund database built by Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013). The sample period is 1/1994–4/2013. From this universe, we select only funds that are classified as “Macro” or “Emerging” according to their strategy code, and do not select any funds-of-funds. The database includes fund returns net of management and incentive fees, and fund assets under management (AUM). All series are denominated in U.S. dollars. In order to compare equally-weighted and value-weighted statistics, only returns associated with corresponding AUM are kept in the data set. Single missing observations of the AUM are linearly interpolated, but we do not use

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14 Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013) consolidate data from the TASS, HFR, CISDM, Morningstar, and BarclayHedge databases. The database used in these papers include both hedge funds and funds-of-funds.
Figure 3
Annualized Average Excess Returns by Country Group and Asset Type

This figure shows annualized average excess returns (in percentage) for Developed Markets (left plot) and Emerging Markets (right plot). Monthly returns are computed on five types of MSCI country equity indices (aggregate market, growth stock, value stock, big cap and small cap indices) and reported in excess of the U.S. one month Treasury bill rate. In both graphs, sorting is based on aggregate market excess returns: countries are ranked from the highest to the lowest value of those returns. Average excess returns are computed over different samples due to data availability. For data coverage refer to Table A.1. Daily indices are from Datastream; monthly returns are computed from end-of-month series; annualized returns are obtained multiplying monthly sample average excess returns by 12. The sample is February 1976 to April 2013.
interpolation for returns. The number of hedge funds varies over time – with 85 funds at the beginning of the sample and 362 towards the end. These funds as a group managed approximately US$ 149 billion in April 2013.

4.3 Risk Factors

Our risk factors are built from exchange rate and equity return series.

4.3.1 Exchange Rates

Daily spot and one-month forward exchange rate series (midpoint quotes) quoted in British pounds for the same set of countries as above are obtained from Datastream. To maximize data coverage, different providers (Reuters, Barclays, and additional sources) are merged. The Online Appendix describes the series in details. Assuming that the covered interest parity condition (CIP) holds, the difference between the (log) forward and spot exchange rates (i.e., the forward discount) is equal to the interest rate differential (in log-form) between the foreign and domestic nominal one-month risk-free rates.\footnote{The one-month forward rate series for Japan does not span the full sample period. Before June 1978, the Japanese interest rate differential is obtained as the difference between the (monthly) three-month Japanese Treasury bill rate obtained from Global Financial Data and the (monthly) U.S. 30-day Treasury bill rate. For five emerging markets (i.e., Brazil, Chile, Colombia, Egypt and Peru), forward rate series are not available. For this reason, the country coverage of the international equity and currency data sets does not fully overlap.} Countries enter the currency data set at different points in time according to the availability of their forward rate series. There are 15 currencies at the beginning of the sample in 1976 and 28 at the end. The maximum monthly coverage is 34, as the Euro replaces national Euro area currencies from January 1999 onwards.

Following Lustig and Verdelhan (2005, 2007), at each time \( t \), we create six currency portfolios by sorting all available currencies in our data set by their forward discounts. These portfolios are rebalanced at the end of each month. properties as in the literature.
The carry is constructed as the return on the top portfolio minus the return on the bottom portfolio, and the average excess return earned by a U.S. investor on the carry trade strategy is 7.65%. To construct the dollar return, we assume that in each period an investor borrows in the U.S. and invests in all currencies in our data set. The correlation between the dollar factor and the carry factor has historically been low – in our data set, it is 0.18 over the full sample period, but rises to 0.40 in the post-1990 period, driven primarily by the incidence of currency and financial crises in this latter period.

4.3.2 World Equity Market Returns

The global equity factor $LWMKT_{t+1}$ in local currencies is constructed as the weighted average excess return on a portfolio of MSCI aggregate market returns denominated in local currencies. Weights are derived from lagged market capitalization in U.S. dollars. Market capitalization series for all countries in our sample are collected from Datastream; countries enter the world portfolio according to their data coverage. To build the global equity factor $WMKT_{t+1}$ in U.S. dollars, we collect the daily total return series for the MSCI World Index denominated in U.S. dollars from Datastream. Monthly returns are obtained from end-month to end-month. All excess returns are computed over the U.S. 30-day Treasury bill rate. The size, value, and momentum international Fama and French (2012) factors and the U.S. size, value and momentum Fama and French factors are obtained from Kenneth French’s website. All these series are denominated in U.S. dollars. International series are available from July 1990, except for the momentum series that start in November 1990.

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16The weights indirectly introduce some second-order exchange rate variation in the equity factor. Using (monthly interpolated) annual P.P.P exchange rates in order to compute those weights lead to very similar results.
5 Time-series and Cross-sectional Asset Pricing Tests

We begin with simple unconditional time-series regressions, in which we regress the aggregate excess returns of the countries in our sample on the factors specified by the five different models over the full sample period.

5.1 Unconditional Time-series Tests

Table 4 reports OLS estimates of the intercepts $\hat{\alpha}$ and adjusted R-squared statistics ($\bar{R}^2$) in percentage points. To conserve space, the associated beta estimates from the models are provided in the Online Appendix to the paper. The table shows these statistics for the set of developed markets in our sample first, followed by the statistics for the emerging markets.

The table shows that these models exhibit similar explanatory power in the set of developed countries, with one notable exception, namely, the International CAPM performs exceptionally well in Japan and the U.K., with $\bar{R}^2$ statistics of 61% and 70%, respectively. This is not particularly surprising, considering that the empirical implementation of the model uses the Japanese Yen and U.K. pound exchange rates as two of the three currency factors, with the German mark/Euro as the third. In the set of emerging markets, however, our model generally delivers larger $\bar{R}^2$ statistics than the competition; this is particularly evident in markets such as China, Mexico, and South Africa.

The estimated intercepts show that deviations from all three models are often large and statistically significant in these unconditional tests. When a statistically significant intercept is estimated, this is often correlated across models for the same country – this is clearly evident in the cases of Netherlands, Chile, Brazil, and Mexico.

These statistically significant intercepts may be a result of sampling error: even if factor returns are ex-ante mean-variance efficient, they may be interior to the ex-post
Table 4
OLS Intercepts and Explanatory Power

This table reports OLS estimates of intercepts and associated adjusted R-squared ($R^2$, in percentage) for country aggregate market excess returns. Estimates are from: the World CAPM, the International CAPM (Int. CAPM), the CAPM Redux and the three- and four-Fama-French factor model (3FF and 4FF). Standard errors are Newey-West (1987) computed with the optimal number of lags according to Andrews (1991). *, **, *** denote statistical significance at the 10%, 5% and 1% level, respectively. The GRS test statistics whether all intercepts in a set of 46 regressions are zero. This statistic (under the assumption of normality) is F-distributed and defined as $\frac{T-N-K}{N} \left(1 + E_T(f)\hat{\Sigma}^{-1}E_T(f)\right)^{-1/2} \hat{\alpha} \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{(N,T-N-K)}$, where $T$, $N$ and $K$ denote the sample size, the number of test assets, and the number of factors, respectively, $\hat{\Sigma}$ is the variance-covariance matrix of the factors $f$ and $\hat{\Sigma}$ is the variance-covariance matrix of estimated residuals. A $p$-value lower than 0.05 means that we can reject the null hypothesis at the 5% confidence level. MAPE and RMSE denote the Mean Absolute Pricing Error and the Root of Mean Square pricing Errors. Obs. denotes the country sample size. All variables are monthly and in percentage points. Fama-French factors are obtained by combining U.S. factors with their global counterparts. The sample period is February 1976 to April 2013.

<table>
<thead>
<tr>
<th>Country</th>
<th>World CAPM</th>
<th>Int. CAPM</th>
<th>3FF-Combined</th>
<th>4FF-Combined</th>
<th>CAPM Redux</th>
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<td>$\hat{\alpha}$</td>
<td>$R^2$</td>
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<td>4FF-Combined</td>
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<td>Malaysia</td>
<td>0.428</td>
<td>19.39</td>
<td>0.423</td>
<td>19.50</td>
<td>0.287</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.230***</td>
<td>31.19</td>
<td>1.147***</td>
<td>37.29</td>
<td>1.207***</td>
</tr>
<tr>
<td>Morocco</td>
<td>0.591</td>
<td>4.54</td>
<td>0.637*</td>
<td>17.33</td>
<td>0.418</td>
</tr>
<tr>
<td>Peru</td>
<td>1.064**</td>
<td>20.26</td>
<td>1.095**</td>
<td>20.58</td>
<td>0.903*</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.486</td>
<td>19.11</td>
<td>0.469</td>
<td>19.21</td>
<td>0.305</td>
</tr>
<tr>
<td>Poland</td>
<td>0.872</td>
<td>27.19</td>
<td>0.917</td>
<td>26.88</td>
<td>0.511</td>
</tr>
<tr>
<td>Russia</td>
<td>1.282</td>
<td>26.86</td>
<td>1.167</td>
<td>28.29</td>
<td>1.147</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.441</td>
<td>42.44</td>
<td>0.497</td>
<td>43.57</td>
<td>0.237</td>
</tr>
<tr>
<td>South Korea</td>
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<td>27.65</td>
<td>0.396</td>
<td>29.58</td>
<td>0.250</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.344</td>
<td>16.43</td>
<td>0.265</td>
<td>19.41</td>
<td>0.113</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.560</td>
<td>24.59</td>
<td>0.597</td>
<td>26.28</td>
<td>0.502</td>
</tr>
<tr>
<td>Turkey</td>
<td>1.379</td>
<td>12.69</td>
<td>1.296</td>
<td>12.57</td>
<td>1.606*</td>
</tr>
</tbody>
</table>

| GRS test | 1.43 | 1.46 | 1.45 | 1.60 | 1.76 |
| p-value   | (0.04) | (0.03) | (0.03) | (0.01) | (0.00) |
| MAPE      | 0.457 | 0.453 | 0.406 | 0.454 | 0.364 |
| RMSE      | 0.625 | 0.608 | 0.560 | 0.637 | 0.475 |
frontier as a result of luck. We therefore run the “GRS” $F$-test of Gibbons, Ross and Shanken (1989) on each model to test whether all 46 country-intercepts are jointly zero. For all models, we reject the null hypothesis that the intercepts are jointly zero at the 5% level of confidence. Unconditionally, it seems that none of the international asset pricing models that we consider is able to fully explain the expected risk premia on country aggregate market excess returns.

One important issue here is that these unconditional tests are restricted to the cross-section of country excess returns, which may not exhibit sufficient cross-sectional variation to allow us to identify these models particularly effectively. We therefore conduct the remainder of our tests on the broader cross-section of asset returns including size and value sorted portfolios across countries.

A second clue as to the poor unconditional performance of these models is obtained by inspecting the factor exposures. In the Online Appendix, we see that while the loading on aggregate equity market risk is generally positive, close to one, and statistically significant at the one-percent level across countries and models, the unconditional loading on currency risk varies across countries and models. It may be that the unconditional versions of the models perform poorly because time-variation in currency factor loadings average out over the entire sample period. We now turn to investigating variation in these factor loadings more fully, and to testing conditional versions of these models.

5.2 Conditional Time-Series Tests

Harvey (1991) and Dumas and Solnik (1995) capture time-variation in factor risk premia by conditioning on a set of instruments. As Cochrane (2001) notes, since the econometrician does not know the true set of state variables, conditional asset pricing tests are joint tests of the set of variables employed as conditioning information and
whether the asset pricing model minimizes pricing errors.

Our approach to this issue is to estimate time-variation in factor loadings using a simple rolling window approach in the spirit of Lewellen and Nagel (2006). Following their implementation, we use 60-month rolling windows for our regressions. This choice means that the maximum number of rolling regressions we run for a single country is 388, and the minimum is 161 – this variation is a result of country-specific data availability. In the Online Appendix, we verify that our results are robust to the use of other window sizes (namely, 48- and 72-month windows).

Figures 4 and 5 show, using our model, that there is substantial time-variation in factor betas. For each country in the data set and each risk factor in the model, the figures report the average rolling factor loading (the central dot in each figure), as well as the range between the minimum and the maximum estimated rolling factor loadings (the two ends of the line in each figure).\(^\text{17}\)

The figure shows that for all countries, there is substantial time-variation in factor loadings, especially for the currency factors. Across countries, there are significant differences both in the magnitude of the risk exposures, and the degree to which they vary over time. While these features are evident in both sets of markets, they are more pronounced in emerging markets.

With few exceptions (primarily in developed countries), estimated carry loadings switch sign over time, and dollar betas are more volatile than carry betas. Extreme examples of time variation include the dollar loadings of Indonesia, and the carry loadings of Turkey. The Netherlands and the United States show the most stable exposures

\(^{17}\text{Country-by-country time-series of the betas, along with Newey and West (1987) standard error bands computed with the optimal number of lags according to Andrews (1991), are reported in the Online Appendix. They confirm that time-variation in these risk-exposures is not driven by outliers, and is usually statistically significant. The Online Appendix also reports these time-varying factor loadings scaled by the cross-sectional standard deviation of the unconditional factor loadings for ease of interpretation.}\)
Figure 4
Time-varying Factor Betas: Developed Markets

This figure shows, for each country, the interval between the minimum and the maximum value (black arrow) and the average (white dot) of a time-series of 60-month rolling factor betas. These loadings are estimated at each time $t$ regressing aggregate market excess returns of country $i$ on a constant, the excess return on a world portfolio denominated in local currencies (LWMKT), the average excess return earned by borrowing in the U.S and investing in all other currencies (Dollar) and the average excess return earned by going long in a basket of high interest rate currencies and short in a basket of low interest rate currencies (Carry) using a 60-month rolling window. The sample is February 1976 to April 2013.
Figure 5
Time-varying Factor Betas: Emerging Markets

This figure shows, for each country, the interval between the minimum and the maximum value (black arrow) and the average (white dot) of a time-series of 60-month rolling factor betas. These loadings are estimated at each time $t$ regressing aggregate market excess returns of country $i$ on a constant, the excess return on a world portfolio denominated in local currencies (LWMKT), the average excess return earned by borrowing in the U.S and investing in all other currencies (Dollar) and the average excess return earned by going long in a basket of high interest rate currencies and short in a basket of low interest rate currencies (Carry) using a 60-month rolling window. The sample is February 1976 to April 2013.
to currency risk. The heterogeneity across countries is so pronounced that it is difficult to identify common patterns. For example, while Japan and Switzerland are typical carry-trade funding countries, the carry factor loadings of their equity returns often move in opposite directions. Allowing betas to vary implies clear differences across models.

Figure 6 provides a pictorial representation of the relative performance of the models. The vertical axis in these plots is common to all, and reports the realized average excess returns (in percentage) of our widest cross-section of test assets over the full sample period. The horizontal axis reports the average excess returns of the same test assets as predicted by each model. To compute predicted returns, we estimate time-varying factor loadings for each asset using 60-month rolling windows, and multiply these estimated factor loadings by factor means computed over the same time window. We then average these conditional predictions across all periods to obtain the values that we plot. As usual, better performing models will generate points which lie close to the 45\(^\circ\) line, and deviations from this line indicate pricing errors.

The figure shows that the World CAPM and the International CAPM underestimate realized average excess returns: the large cross-sectional variation observed in the data is not matched because there is little cross-sectional variation in the predictions of these models, leading to a more vertical line. From the figure it is apparent that this poor performance is not driven by a particular type of equity asset; the differences between predictions and realizations are similar for the various different types of assets. The Fama-French-Carhart four-factor model does do substantially better than the more theoretically grounded models, but there is still significant deviation from the 45\(^\circ\) line.

Our model significantly improves the visual relationship between predicted and realized average returns. These improvements come from two sources. First, we explicitly model the currency component embedded in foreign equity market returns. Second, we reduce the noise in measured currency risks by relying on two global currency risk
This figure plots realized average excess returns against those predicted by the World CAPM, the International CAPM, the four-factor model (4FM, where U.S. and global factors are combined) and the CAPM Redux. All models are estimated conditionally. For each country the predicted average excess returns are computed as follows: a) Time-varying factor betas are estimated using 60-month rolling windows; b) At each time $t$ estimated conditional betas are multiplied by the corresponding factor means computed over the same time-window (from time $t-59$ to time $t$); c) Predicted values are averaged. Test assets are equity excess returns for all asset types and all countries, six currency portfolios sorted on forward discounts (Carry portfolios) and six currency portfolios sorted on dollar exposure (Dollar portfolios). Average returns are in percentage. The straight line is the 45-degree line through the origin. The sample is February 1981 to April 2013.
factors rather than a selected few currency excess returns.

We check the robustness of our result by inspecting the pricing errors obtained for each asset and each rolling window. Panel A of Figure 7 reports the kernel estimates of all the alphas for the World CAPM, the Fama-French Four Factor model and the CAPM Redux. The latter tends to have comparable or lower pricing errors than the former across the whole distribution.

Panel B of Figure 7 compares the cross-sectional average absolute alphas computed in each rolling window generated by the same three models. The figure shows that apart from a longer period from 1997 until the NASDAQ crash in 2000, and a few shorter episodes, our model generally appears to deliver lower average absolute alphas. This is especially the case towards the end of the sample period.

We confirm this result in a number of ways in the Online Appendix. First, we find that the cross-sectional average $\bar{R}^2$ from these time-series regressions is generally higher for our model than for the competition. At each date in the sample, the global equity, dollar, and carry factors explain a larger share of the time-series variation in international equity returns than the world CAPM. We also find that our model outperforms the International CAPM as we move from the distant past towards the recent past. This latter finding suggests that the increasing integration of global markets might account for the increasing explanatory power of global currency factors.

To summarize our results from these time-series tests, international equity returns have heterogeneous and time-varying exposures to global currency factors. Accounting for this variation increases the explanatory power of all international asset pricing models, including our model. A number of standard tests conducted using time-series regressions reveal that our model appears to outperform the others. We now turn to standard cross-sectional asset pricing tests to verify these findings, and to estimate the market prices of global equity and currency risk.
This figure shows the kernel densities of rolling alphas (Panel A) and the time-series of rolling mean absolute alphas (Panel B) obtained from the World CAPM, the Fama-French Four Factor model and the CAPM Redux. Rolling alphas are obtained by regressing country-i equity excess returns on a constant and the appropriate set of factors over rolling windows of 60 months. In Panel B alpha estimates (in absolute value) are averaged across countries at each point in time. U.S. Fama-French factors are combined with their global counterparts. Test assets are equity excess returns of all types and for all countries. The sample period is February 1976 to April 2013.
5.3 Cross-sectional Tests

We now relax the no-arbitrage condition that pins down the market prices of risk and estimate them using the cross-section of excess returns. We conduct our cross-sectional tests using the standard two-pass cross-sectional approach of Fama and MacBeth (1973, henceforth FMB). For each of the models in our comparisons, we run FMB tests in three different ways. In the first variant of these FMB tests (which we denote as FMB$^1$), we estimate the factor loadings using unconditional time-series regressions over the full sample as in Table 4. We then compute market prices of risk ($\lambda$) via a cross-sectional regression of average excess returns of the test assets on these unconditional factor loadings. In the second variant (which we denote FMB$^2$), first-stage betas continue to be obtained using unconditional time-series regressions over the full sample as in Table 4, and as in FMB$^1$. However, in the second stage, we run $T$ cross-sectional regressions, one for each time period, of country excess returns on these estimated factor loadings. The average market prices of risk are then computed as simple averages of the slope coefficients obtained from these $T$ cross-sectional regressions. In the third variant (denoted FMB$^{TV}$), we obtain time-varying factor loadings using rolling regressions over 60-month windows. In each period $t + 1$, we estimate market prices of risk $\lambda_{t+1}$ using cross-sectional regressions of test asset returns on these time-varying factor loadings estimated using windows ending in period $t$. Average market prices of risk are once again simple averages of $\lambda_{t+1}$ over all periods $T$. In all three variants of these tests, we omit constants in the second stage of the FMB procedure. To ensure that the three tests are comparable, the FMB$^1$ and FMB$^2$ tests are carried out on the second-stage estimation sample of the FMB$^{TV}$ procedure.

Table 5 reports the estimates of the average market prices of risk along with Shanken (1992)-corrected standard errors (in parentheses) from these tests across models which are in blocks of rows. The columns identify the set of test assets on which we run these
tests. The first set of test assets includes aggregate market excess returns, and value
and size-sorted portfolios for the developed markets in the sample (120 assets), as well
as 12 currency portfolios. The second set of test assets expands the equity cross-section
to include assets from emerging markets, leading to a total of 225 equity assets plus 12
currency portfolios.

The table shows that across all model specifications, cross-sections of test assets,
and testing approaches, the world equity factor is priced, whether it is measured in local
currency or U.S. dollar terms. The statistical significance of this result holds at the five
percent level or better. This finding supports the early evidence in the international
finance literature that international investors are compensated for taking on risk that
is correlated with returns on the world equity market portfolio.

In contrast, there is little evidence to support the pricing of currency risk in models
other than our own. Looking across the three currencies included in the International
CAPM, only the British pound appears to carry a significant currency premium in our
sample. Moreover, this result holds only when time-variation is taken into account
(FMB\(^{TV}\)). The importance of using a conditional model is consistent with the findings
of Dumas and Solnik (1995), however even allowing for time-variation in factor loadings
and risk premia, the German mark/Euro and Japanese Yen are not priced over the
sample period for the wider cross-section of assets. This result is potentially attributable
to the longer sample period, the larger cross-section of test assets, and our use of a
methodology based on rolling windows instead of instrumental variables to model time-
variation.

The table also shows that there is no statistical evidence for a value or momentum
premium in the cross-section of test assets, either conditionally or unconditionally.
However there is some evidence to support the existence of a size premium, especially
when we evaluate the Fama-French model allowing for time-variation in factor loadings.

In contrast with these results, we find evidence to support the pricing of currency
Table 5
Asset Pricing Fama-MacBeth Tests

This table reports results from the Fama-MacBeth asset pricing procedure for the World CAPM, the International CAPM, the Fama-French four-factor model (4FF, U.S. and global factors are combined) and the CAPM Redux. In FMB\(^1\), the average market prices of risks (\(\lambda\)) are obtained via a cross-sectional regression of average excess returns on the (unconditional) first-step betas. In FMB\(^2\), the \(\lambda\)s correspond to the average across \(T\) cross-sectional regressions of excess returns on the same unconditional betas. In FMB\(^{TV}\), in the first step of the procedure time-varying (TV) betas are estimated over 60-month rolling windows ending at time \(t\). In the second-step, the market prices of risk are estimated at each time \(t+1\) via a cross-sectional regression of country excess returns on the first-step conditional betas. The \(\lambda\)s are obtained as average of these second-stage estimates. In all cases, the second stage of the procedure does not include a constant. MAPE (RMSE) denotes the Mean Absolute Pricing Error (Root of Mean Square pricing Errors). Shanken (1992)-corrected standard errors are in brackets. *, **, *** denote statistical significance of the average prices of risk at the 10%, 5% and 1% level, respectively. Test assets are country equity excess returns for developed markets (DM, left panel) or all markets (right panel) and 12 currency portfolios, namely six currency portfolios sorted on forward discounts (Carry portfolios) and six currency portfolios sorted on dollar exposure (Dollar portfolios). The sample period is February 1981 to April 2013. Unconditional tests (factor means) are carried out (computed) over the second-stage estimation sample of the FMB\(^{TV}\) procedure.

<table>
<thead>
<tr>
<th>Model</th>
<th>DM Equity + FX Portfolios</th>
<th>All Equity + FX Portfolios</th>
<th>Factor Means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FMB(^1)</td>
<td>FMB(^2)</td>
<td>FMB(^{TV})</td>
</tr>
<tr>
<td>World (\lambda_{WMT})</td>
<td>0.623**</td>
<td>0.680***</td>
<td>0.686***</td>
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<tr>
<td></td>
<td>(0.258)</td>
<td>(0.263)</td>
<td>(0.264)</td>
</tr>
<tr>
<td>CAPM (\lambda_{WMT})</td>
<td>0.252</td>
<td>0.260</td>
<td>0.309</td>
</tr>
<tr>
<td></td>
<td>0.334</td>
<td>0.353</td>
<td>0.409</td>
</tr>
<tr>
<td>Int. CAPM (\lambda_{WMT})</td>
<td>0.666**</td>
<td>0.703***</td>
<td>0.590**</td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td>(0.262)</td>
<td>(0.254)</td>
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<tr>
<td>(\lambda_{GBP})</td>
<td>0.199</td>
<td>0.158</td>
<td>0.411**</td>
</tr>
<tr>
<td></td>
<td>(0.209)</td>
<td>(0.246)</td>
<td>(0.180)</td>
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<tr>
<td>(\lambda_{JPY})</td>
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<td>0.044</td>
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</tr>
<tr>
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<td>(0.271)</td>
<td>(0.334)</td>
<td>(0.232)</td>
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<td>(\lambda_{EUR})</td>
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<td>(0.266)</td>
<td>(0.196)</td>
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<td>MAPE</td>
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<td>0.241</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td>0.330</td>
<td>0.334</td>
<td>0.414</td>
</tr>
<tr>
<td>(\lambda_{FFMKT})</td>
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<td>0.676**</td>
<td>0.754***</td>
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<tr>
<td></td>
<td>(0.264)</td>
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<td>(0.289)</td>
</tr>
<tr>
<td>(\lambda_{SMB})</td>
<td>0.173</td>
<td>0.018</td>
<td>0.348**</td>
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<td>(0.158)</td>
<td>(0.256)</td>
<td>(0.158)</td>
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<td>(\lambda_{HML})</td>
<td>-0.195</td>
<td>-0.157</td>
<td>-0.244</td>
</tr>
<tr>
<td></td>
<td>(0.184)</td>
<td>(0.211)</td>
<td>(0.169)</td>
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<td>(\lambda_{MOM})</td>
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<td>(0.340)</td>
<td>(0.435)</td>
<td>(0.311)</td>
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<td>MAPE</td>
<td>0.246</td>
<td>0.224</td>
<td>0.281</td>
</tr>
<tr>
<td></td>
<td>0.328</td>
<td>0.309</td>
<td>0.377</td>
</tr>
<tr>
<td>(\lambda_{LWMT})</td>
<td>0.570**</td>
<td>0.623**</td>
<td>0.552**</td>
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<td></td>
<td>(0.253)</td>
<td>(0.262)</td>
<td>(0.250)</td>
</tr>
<tr>
<td>CAPM (\lambda_{Dollar})</td>
<td>0.175</td>
<td>0.176</td>
<td>0.234*</td>
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<tr>
<td></td>
<td>(0.145)</td>
<td>(0.163)</td>
<td>(0.134)</td>
</tr>
<tr>
<td>(\lambda_{Carry})</td>
<td>0.403*</td>
<td>0.461</td>
<td>0.445**</td>
</tr>
<tr>
<td></td>
<td>(0.225)</td>
<td>(0.300)</td>
<td>(0.181)</td>
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<tr>
<td>MAPE</td>
<td>0.240</td>
<td>0.241</td>
<td>0.296</td>
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<tr>
<td></td>
<td>0.331</td>
<td>0.339</td>
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risk when we measure this risk using the dollar and carry factors. This is even true to some extent unconditionally, in the sense that the carry factor is priced using FMB\(^1\) across developed and developed plus emerging cross-sections, and using FMB\(^2\) in the broader developed plus emerging cross-section.

The results supporting our model become substantially stronger when we account for time-variation in factor loadings as well as in risk premia, using FMB\(^{TV}\). The prices of dollar and carry risk are all significantly different from zero at the 5\% level or better, in both cross-sections.

Additional evidence on the models is provided when we inspect the prices of risk of the equity, dollar, and carry factors and compare them with the average excess returns of those factors which are provided in the final column of Table 5. The no-arbitrage condition implies, since the beta of each factor on itself is obviously one, that the market price of risk of each factor should be equal to the average of the factor. While the sample is short, leading to difficulties in estimating this relationship precisely, we do see that these factor means are relatively close to the estimated factor risk premia. For the other models, the price of equity risk is much higher, further removed from its sample mean. As noted, however, the sample is indeed short, leading to substantial imprecision, and susceptibility to Daniel and Titman’s (1997) critique. We cannot of course rule out a characteristics-based behavioral explanation of the cross-sectional variation in test asset returns.

For each model, Table 5 also reports the cross-sectional Mean Absolute Pricing Error (MAPE) and the cross-sectional Root Mean Square Error (RMSE). Consistent with the evidence discussed above, our factor model delivers relatively smaller RMSEs than the
competition, although these differences are not substantial.\footnote{Bootstrap tests help assess whether RMSE differences ($\Delta$RMSE) between two models are statistically different from zero. The Online Appendix reports results from a bootstrap exercise, and shows that our model is able to beat the competition in all cases that use the largest possible cross-section of test assets and allow for time-variation in factor loadings.}

The Online Appendix reports a number of additional robustness checks including re-estimating the model on samples of expanding sizes, either moving forwards in time, beginning in 2/1976 or moving backwards in time, beginning in 12/2013. The dollar and carry factors appear priced even in samples that exclude the recent financial crisis (i.e., before 2007). Small perturbations in the estimation sample do not imply abrupt changes in the estimates. The magnitude of these currency premia vary smoothly over time suggesting that the price of risk (not merely risk exposure) is time-varying. We also find that the significance of the pricing results for the model increases over time. Clearly, part of the explanation for this pattern lies in the increase in power arising from a broader cross-section of asset returns available to test the model; we also attribute some part of this to the effects of increasing global capital market integration over time.

One obvious question is whether our strong results are simply a consequence of the currency return component embedded in dollar-denominated test asset returns, and the pricing of these currency components by carry and dollar factors. Figure 8 shows that a simple story of this nature would be insufficient to explain the somewhat involved dynamics of equity and currency risk.

The figure reports the cross-sectional average correlation coefficient between country aggregate market equity returns expressed in local currency and the dollar factor (left panel) and the carry factor (right panel). Clearly, equity and currency risk are not orthogonal to one another, and demonstrate significant time-variation in their relationship. Particularly over the second half of the sample period that we consider, these
Figure 8
Cross-Sectional Average Time-Varying Correlation Between Equity Returns in Local Currency and Each Currency Factor

This figure shows, for each 60-month rolling window, the average cross-sectional correlation between country aggregate market returns expressed in local currency and each currency factor (dollar/carry factor in the left/right graph, respectively). Specifically, at each time $t$ we compute pairwise correlations between the equity return of each country-$i$ and a given currency factor and report the average correlation across countries. In each rolling window, a 95% confidence interval (dash-dotted line) is obtained via pairwise bootstrapping (1000 bootstrap samples). The lower/upper bound corresponds to the 2.5$^{th}$ and 97.5$^{th}$ percentile of the bootstrap distribution of the cross-sectional average correlation in that time-window. The sample is February 1976 to April 2013.
correlations are significant and increasing, sometimes non-linearly. These correlations reach 60% (50%) at the end of our sample period for the dollar (carry) factor.

The next section documents this issue in more detail, using our reduced-form model to elucidate why exposures to equity and currency risk cannot be assessed independently.

5.4 Comparison with the reduced-form model

The model is parsimonious and can be solved in closed form. But its simplicity entails some shortcomings when compared to the data. As we have shown in the data global equity and currency betas are clearly time-varying. In the model, a univariate regression of equity excess returns in U.S. dollars on the carry factor delivers a constant beta; the data suggest otherwise. Time-variation in all betas can be obtained by assuming that the market prices of risk (i.e., the square roots in the SDFs) depend not only on one but on two state variables. In this case, however, the model does not admit a closed-form.

The model features two sources of heterogeneity: one in the SDF and one in the dividend growth rates. The first source of heterogeneity is necessary to account for the cross-section of interest rates and currency excess returns. As Lustig, Roussanov, and Verdelhan (2011) show, high interest rate countries must be characterized by low exposure ($\delta^i$) to world shocks for the high interest rate currency to depreciate in bad times — the key mechanism of any risk-based explanation of carry trade profits. This first source of heterogeneity entails differences in global equity betas, but if it were not for the differences in dividend growth rates, the equity betas would line up with the carry betas. In the data, they do not, consistent with our modelling choice of two sources of cross-country differences.

The model exhibits a role for a pure equity factor, even though financial markets are complete and SDFs reflect all priced shocks. In the data, the global equity factor
appears key; currency factors clearly are not enough to describe foreign equity returns. The reduced-form model suggests that an equity factor expressed in U.S. dollars reflects all the priced shocks. In practice, however, disentangling the pure equity factor from the currency factors relies on estimating different betas separately.

In the data, the global equity, carry, and dollar betas are not highly correlated. The time-series correlations (obtained country-by-country) range from $-0.7$ to $0.7$ for the $LWMKT – Dollar$, $LWMKT – Carry$, and $Dollar – Carry$ pairs. Across countries, these correlations are close to zero. Our estimates of a three-factor model allow for these factor-specific dynamics in the loadings and market prices of factors to be accounted for. Estimating a single world market beta, as the World CAPM does, simply misses this heterogeneity in dynamics, as it constrains all factors to affect risky asset returns in the same way.

5.5 Global Risk in Mutual and Hedge Fund Returns

Our empirical analysis has concentrated on the usual cross-section of equity assets around the world, and we show, using our model, that currency risk is priced in this cross-section. This section shows that a large proportion of mutual and hedge funds investing internationally are also exposed to currency risk, which we are well able to detect using our model.

5.5.1 Mutual Fund Currency Exposure

Mutual fund managers can choose whether or not to hedge the currency exposure in their foreign equity investments.$^{19}$ While many mutual fund brochures remain vague

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$^{19}$According to the Wall Street Journal (August 4, 2013), for example, Fidelity Overseas manages $2.3$ billion without hedging its foreign currency risk, while Oakmark International, which has $19.5$ billion in assets, “hedges up to $80\%$ of its exposure to a given currency when the currency’s exchange rate against the dollar is more than $20\%$ above what the management team considers fair value.” It is not clear whether this fair value is known to outside investors or not.
about currency risk, we find that their realized returns reveal a clear and economically significant exposure to exchange rate fluctuations.

As an initial exercise, we simply document the explanatory power of our model for these returns over the entire sample period. A clear pattern emerges in the time-series, as shown in Figure 9: over time, the explanatory power of the local currency, dollar, and carry factors increases steadily, along with steadily increasing exposure to the global equity, dollar, and carry factors in the data. The average cross-sectional $R^2$ at each date increases from around 60% in the early 1990s to more than 90% at the end of our sample, the percentage of mutual funds significantly exposed to dollar risk increased from around 75% in the early 1990s to close to 100% at the end of the sample, and the percentage of mutual fund returns loading on carry risk is on average 30%, but varies considerably over time between 10% and more than 60%. Clearly, international mutual fund returns are exposed to currency risk, and as a consequence, their investors bear significant currency risk as well.\footnote{We check the robustness of our findings on longer rolling windows of 120 months, instead of 60 months. The number of funds with available data decreases. There are in this case only 568 funds at the end of the sample, but those funds as a group still manage more than $620 billions. The results are broadly similar as above. On average over the whole period, 80% of the mutual funds are significantly exposed to currency risk, representing 91% of the total funds under management in the dataset.}

In the Online Appendix, we report summary statistics on the share of mutual funds with significant exposure to risk factors. The results are clear: on average over the whole period, close to 80% of the mutual funds are significantly exposed to currency risk, representing approximately the same share of the total dollars under management in the dataset.

Figure 10 presents a simple comparison across the models that we consider, regressing monthly mutual fund returns on the competing models, and plotting CDFs of the $t$-statistics of alphas (using bootstrap standard errors) obtained from these different models. The figure reveals that the performance of our model is better than that of
Figure 9
Mutual Funds’ Significant Exposure to Global Factors — Equally-Weighted Funds

The figure plots the time-series of the share of mutual funds with significant exposure to currency factors. Monthly mutual fund returns are regressed on a constant and the equity, carry and dollar factors over rolling-windows of 60 months. The first panel reports the number of funds with available data. The second panel reports the cross-sectional average $R^2$ at each date. The shaded area corresponds, at each date, to the interval obtained as the cross-sectional average $R^2$ plus/minus two cross-sectional standard deviations of the funds’ specific $R^2$s. The third (respectively, fourth, and fifth) panels reports the percentage of funds with absolute $t$-stats on the equity (respectively, dollar, and carry) factor above 1.96. The last panel reports the percentage of funds with $F$-test probabilities below 5% (FX $F$-test). The null hypothesis of the $F$-test is that both loadings on the carry and dollar factors are zero. A probability below 5% means that the null hypothesis can be rejected at the 95% confidence level. Data are monthly, from CRSP. The sample includes all funds classified as “Foreign Equity Funds” according to the CRSP fund style code. The sample period is 11/1990–4/2013.
the World CAPM and the International CAPM, and comparable to that of the Carhart four-factor model.
Figure 10
Statistical Significance of Mutual Funds’ Alphas

This figure shows the empirical cumulative distribution function (ECDF) of t-statistics of all the mutual funds’ rolling alphas that are obtained from the World CAPM, the International CAPM, the Fama-French four-global factor model (4GFM) and the CAPM Redux. The left (right) graph examines negative (positive) alphas, only. Monthly mutual fund returns are regressed on a constant and the appropriate set of factors over rolling-windows of 60 months. Standard errors are obtained via bootstrapping (1000 bootstrap samples). The sample includes all mutual funds that are classified as 'Foreign Equity Funds' according to the CRSP fund style code. The sample period is November 1990 to April 2013.
5.5.2 Hedge Fund Currency Exposure

Figure 11 assesses the growth in international hedge funds’ exposures to the factors in our model. The figure shows that while exposure to the global equity factor is stable throughout the period, the combined exposure to the dollar and carry factors has increased notably: less than a fifth of hedge funds load significantly on the currency factors in the 1990s but close to half the funds do so by the end of the sample.

Overall, while hedge fund returns appear to be less exposed to the factors in our model than mutual funds, a large number of hedge fund returns load significantly on our risk factors. This means that their returns can potentially be reproduced at a much lower cost than the substantial fees incurred by investing in a hedge fund.

Once again, the Online Appendix reports summary statistics on the share of hedge funds with significant exposure to risk factors. On average across the sample, one can reject the null joint hypothesis that their returns do not load on the equity and currency factors for close to 60% of international hedge funds, representing a similar share of the money invested in the “Macro” and “Emerging” hedge fund sectors.

Finally, Figure 12 presents a simple comparison across the models that we consider, regressing monthly hedge fund returns on the competing models, and plotting CDFs of the $t$-statistics of alphas (using bootstrap standard errors) obtained from these different models. The figure reveals that the performance of our model is better even than that of the Carhart four-factor model, delivering a slightly lower number of funds with statistically significant alphas.
Figure 11
“Macro” and “Emerging” Hedge Funds’ Significant Exposure to Global Factors — Equally-Weighted Funds

The figure plots the time-series of the share of hedge funds with significant exposure to global equity and currency factors. Monthly hedge fund returns are regressed on a constant and the equity, carry and dollar factors over rolling-windows of 60 months. The first panel reports the number of funds with available data. The second panel reports the cross-sectional average $R^2$ at each date. The shaded area corresponds, at each date, to the interval obtained as the cross-sectional average $R^2$ plus/minus two cross-sectional standard deviations of the funds’ specific $R^2$s. The third (respectively, fourth, and fifth) panels reports the percentage of funds with absolutes $t$-stats on the equity (respectively, dollar, and carry) factor above 1.96. The last panel reports the percentage of funds with $F$-test probabilities below 5%. The null hypothesis of the $F$-test is that both loadings on the carry and dollar factors are zero (FX $F$-test). A probability below 5% means that the null hypothesis can be rejected at the 95% confidence level. Data are monthly, from the updated version of the consolidated hedge fund database by Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013). The sample includes all funds classified as “Macro” or “Emerging” according to the strategy code. The sample period is 1/1994–4/2013.
Figure 12
Statistical Significance of Hedge Funds’ Alphas

This figure shows the empirical cumulative distribution function (ECDF) of t-statistics of all the hedge funds’ rolling alphas that are obtained from the World CAPM, the International CAPM, the Fama-French four-global factor model (4GFM) and the CAPM Redux. The left (right) graph examines negative (positive) alphas, only. Monthly hedge fund returns are regressed on a constant and the appropriate set of factors over rolling-windows of 60 months. For each alpha, standard errors are obtained via bootstrapping (1000 bootstrap samples). Hedge fund returns are from the updated version of the consolidated hedge fund database by Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013). The sample includes all funds classified as “Macro” or “Emerging” according to the strategy code. The sample period is January 1994 to April 2013.
6 Conclusion

This paper presents a new international asset pricing model, which we use to test whether currency risk is priced in international equity portfolios. Our model comprises three factors, namely, a global equity factor denominated in local currencies, and two currency factors, dollar and carry, which are constructed from the space of global currency excess returns. In a comprehensive set of equities from 46 developed and emerging countries spanning value, growth, and country index returns from 1976 to the present we find that currency risk is priced in the cross-section of international equity returns. The global equity, dollar, and carry factors outperform the World and International CAPM as well as the Fama-French three and four factor models in our sample. We also find evidence of substantial exposure to currency risk in the universe of global mutual funds and hedge funds, suggesting that whether exposure to international markets is direct or indirect, currency risk is a significant factor influencing investor performance.

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