Mispriced Index Option Portfolios

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Introduction

- We consider 2 generic traders:
  - the <u>**I**</u>ndex <u>**T**</u>rader (IT) holds the S&P 500 index and T-bills and maximizes expected utility
  - the <u>**O**</u>ption <u>**T**</u>rader (OT) with same utility function who holds the IT's portfolio plus a zero-net-cost portfolio of index options and T-bills
- We identify a zero-net-cost options portfolio such that the OT's portfolio stochastically dominates the IT's portfolio, irrespective of their particular common utility function
- That is, we identify an options portfolio such that the OT attains higher expected utility than the IT irrespective of their particular common utility function

Results

- Sample period 1990-2013
- European S&P 500 options with 28, 14, or 7 days to maturity
- Options are bought at their ask price and written at their bid price
- We find dominating options portfolios in almost every month in our sample
- This works because the dominating options portfolio shifts the probability mass to the left part of the support
- Similar results obtain with options on the CAC and DAX indices
- The results are not explained away with risk adjustments using the FF factors, option-specific factors, or a U-shaped stochastic discount factor

Results (cont.)

- Dominance is prevalent when
  - ➢ATM-IV is high
  - ➢right skew is low
  - ➢option maturity is short
- The dominating portfolios include mostly calls
- Positions in the dominating portfolios are overwhelmingly short

Finding a dominating options portfolio

- Let S_T denote the index price at maturity
- Let $A(S_T)$ denote the value of the options portfolio at maturity
- The following 2 conditions are sufficient for the OT's portfolio to dominate the IT's portfolio

(a) $E_t\,[\,A(S_T)\,] \geq 0$

(b) there exists a number $\hat{S}$ such that

$$A(S_T) > 0 \quad \text{for} \quad S_T \leq \hat{S}$$

and

$$A(S_T) < 0 \quad \text{otherwise}$$

Finding a dominating options portfolio (cont.)

- Each month we find several options portfolios that satisfy conditions (a) and (b) using linear programming
- Of these we pick the options portfolio that maximizes the Sharpe ratio
- We obtain similar results when we replace the Sharpe ratio with the Sortino ratio or the gain-loss criterion

Empirical procedure

- The test is strictly out-of-sample
- We generate the time series of the realized returns of the IT and OT portfolios at the option maturity
- 1^{st}: we test for the significance of the difference in the mean returns of the OT and IT portfolios
- 2^{nd}: we apply the Davidson-Duclos (DD, 2013) test for restricted second-order stochastic dominance of the OT over the IT returns
- This test is based on the null hypothesis of non-dominance
- Rejection of the null is a powerful statement about OT dominating IT, much stronger than non-rejection of the null of dominance

General description of the tables

- μ is the mean difference of the annualized percentage return between the OT and IT portfolios
- The *p*-values for the difference in means are derived via bootstrap with 10,000 draws
- For the DD test, 10% trimming in the left tail is uniformly performed
- Trimming in the right tail is as shown
- We find option portfolios in 270, 272 and 272 months out of the 278 months for 28-, 14-, and 7-day options, respectively
- Therefore almost all months contain mispriced options

Table 2: 28-day options

Portfolio selection criterion	μ	p-value for μ≤0	OT vol.	IT-OT vol.	DD test p-value	
					5% trim	10% trim
28-Day Options, IT vol. 16.48%						
Sharpe ratio	0.50	0.112	15.89	1.97	0.039	0
Gain/loss ratio	0.92	0.029	15.81	2.19	0.008	0
Sortino ratio	0.45	0.128	15.90	1.97	0.045	0
Max S hat	0.66	0.057	15.92	1.89	0.008	0

Table 2: 14-day options

Portfolio selection criterion	μ	p-value for μ≤0	OT vol.	IT-OT vol.	DD test p-value	
					5% trim	10% trim
14-Day Options, IT vol. 17.15%						
Sharpe ratio	2.07	0.062	15.68	3.99	0	0
Gain/loss ratio	2.55	0.026	15.64	3.79	0	0
Sortino ratio	1.82	0.041	15.71	3.88	0	0
Max S hat	2.15	0.051	15.76	3.73	0	0

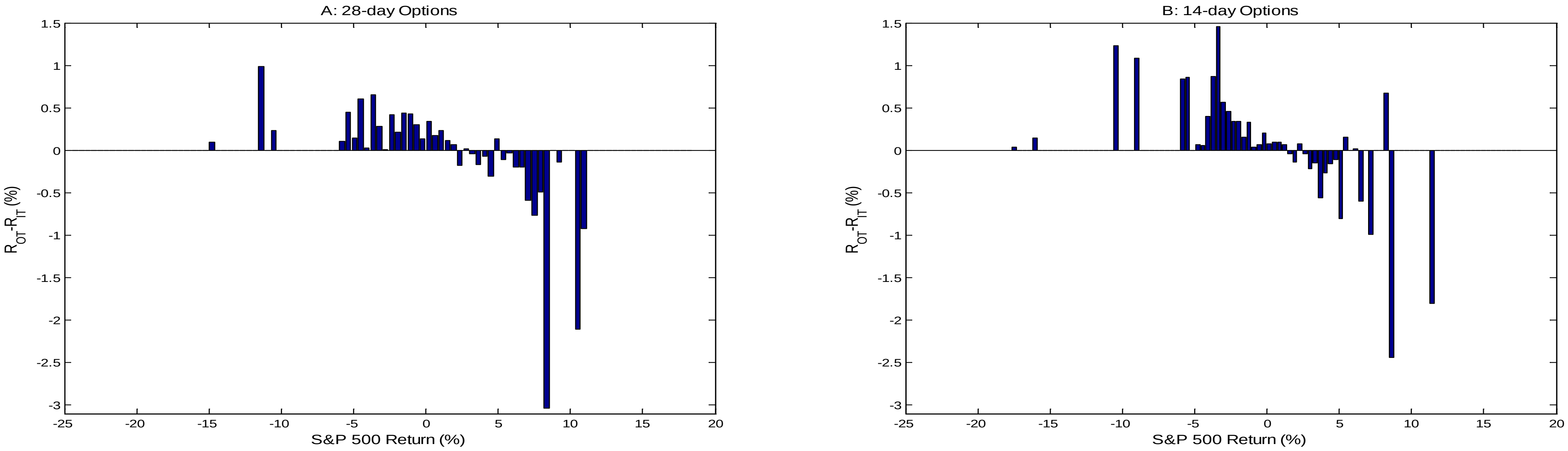
Table 2: 7-day options

Portfolio selection criterion	μ	p-value for μ≤0	OT vol.	IT-OT vol.	DD test p-value	
					5% trim	10% trim
7-Day Options, IT vol. 18.12%						
Sharpe ratio	1.90	0.062	17.43	2.79	0	0
Gain/loss ratio	2.72	0.008	17.36	2.49	0	0
Sortino ratio	2.24	0.038	17.45	2.82	0	0
Max S hat	2.22	0.007	17.61	2.06	0	0

Table 2 discussion

- The DD tests strongly reject the null of non-dominance for all maturities and all selection criteria
- The annualized means of the OT portfolio exceed those of IT by 50 bps for 28-day options and by well over 100 bps for 14- and 7-day options
- The excess returns are not statistically significant from 0 for 28-day options where dominance comes from lower risk
- The excess returns are strongly significant for 14- and 7-day options
- The IT portfolio is more volatile for all 3 option maturities
- The results pass robustness tests

Figure 2: Difference in realized returns between the OT and IT portfolios as a function of the S&P 500 index return for 28- and 14-day options



Stochastic dominance and the IV smile

- Tables 4-6 show the stochastic dominance tests for the low and high tercile of a given smile characteristic (ATM IV, left skew, and right skew) for 28-, 14-, and 7-day options, respectively
- The power of the tests is diminished by the fact that we have only 92 observations in each tercile instead of 278 observations for the main results in Table 2
- For all maturity options, the mean difference of the returns between the OT and IT portfolios is significantly higher when the ATM IV is in the high tercile of the ATM IV but not in the low tercile
- In the high IV tercile the DD tests strongly reject the null of non-dominance

Stochastic dominance and the IV smile (cont.)

- For all maturity options the mean difference of the returns between the OT and IT portfolios is significantly higher when the right skew is low, for all maturities and all selection criteria
- By contrast, no firm conclusions can be extracted about the role of the left skew in stochastic dominance
- The “flatness” of the right skew is significantly associated with
  - high ATM IV
  - OTM calls
- Hence, we expect our portfolios to have more OTM calls than OTM puts, which are related to the left skew

Table 4: Relation between stochastic dominance and the smile for 28-day option portfolios

	Lowest Tercile				Highest Tercile		
Portfolio selection criterion	μ	p-value $\mu \le 0$	DD test p-value		μ	p-value $\mu \le 0$	DD test p-value
ATM IV							
Sharpe ratio	-0.22	0.666	1		0.91	0.149	0
Gain/loss ratio	0.26	0.246	0.347		1.65	0.059	0
Sortino ratio	-0.41	0.806	1		1.02	0.134	0
Max S hat	0.01	0.486	0.587		1.17	0.109	0
Left Skew							
Sharpe ratio	0.99	0.119	0.027		-0.47	0.725	1
Gain/loss ratio	1.62	0.036	0.009		-0.29	0.649	1
Sortino ratio	1.04	0.109	0.028		-0.57	0.766	1
Max S hat	1.24	0.070	0.016		-0.48	0.748	1
Right Skew							
Sharpe ratio	0.58	0.192	0.043		-0.41	0.730	1
Gain/loss ratio	1.18	0.095	0.057		0.41	0.264	0.360
Sortino ratio	0.68	0.148	0.032		-0.59	0.794	1
Max S hat	0.84	0.100	0.006		0.05	0.430	0.444

Table 5: Relation between stochastic dominance and the smile for 14-day option portfolios

	Lowest Tercile				Highest Tercile		
Portfolio selection criterion	μ	p-value μ ≤ 0	DD test p-value		μ	p-value μ ≤ 0	DD test p-value
ATM IV							
Sharpe ratio	0.01	0.486	0.356		1.17	0.109	0.003
Gain/loss ratio	0.46	0.324	0.208		4.19	0.081	0.001
Sortino ratio	0.89	0.096	0.107		4.55	0.069	0
Max S hat	0.03	0.453	0.327		4.06	0.085	0
Left Skew							
Sharpe ratio	1.24	0.070	0.016		-0.48	0.748	1
Gain/loss ratio	2.38	0.195	0.054		3.76	0.035	0.024
Sortino ratio	3.22	0.125	0.040		4.57	0.013	0.032
Max S hat	1.94	0.221	0.052		3.80	0.032	0.026
Right Skew							
Sharpe ratio	0.84	0.100	0.006		0.05	0.430	0.144
Gain/loss ratio	4.79	0.007	0		-0.21	0.544	1
Sortino ratio	5.34	0.004	0.006		-0.01	0.465	1
Max S hat	4.65	0.010	0		-0.61	0.666	1

Table 6: Relation between stochastic dominance and the smile for 7-day option portfolios

	Lowest Tercile				Highest Tercile		
Portfolio selection criterion	μ	p-value μ ≤ 0	DD test p-value		μ	p-value μ ≤ 0	DD test p-value
ATM IV							
Sharpe ratio	-1.33	0.715	1		3.97	0.033	0.001
Gain/loss ratio	0.79	0.176	0.154		4.77	0.033	0
Sortino ratio	-0.87	0.627	1		4.27	0.027	0.001
Max S hat	0.61	0.194	0.075		4.31	0.026	0.001
Left Skew							
Sharpe ratio	1.68	0.239	0.296		1.79	0.233	0.184
Gain/loss ratio	2.85	0.117	0.063		3.43	0.013	0
Sortino ratio	2.00	0.208	0.308		2.29	0.172	0.134
Max S hat	2.03	0.137	0.300		3.13	0.009	0
Right Skew							
Sharpe ratio	5.13	0.007	0		1.33	0.219	0.354
Gain/loss ratio	6.01	0.008	0		2.07	0.008	0.001
Sortino ratio	5.41	0.006	0		1.41	0.215	0.343
Max S hat	5.21	0.005	0		1.55	0.020	0.107

Composition of dominating portfolios

- Table 7 shows the composition of the dominating portfolios for all maturities for the Sharpe ratio selection criterion, with the other criteria yielding similar results
- We distinguish three sub-periods: pre-, post- and during the crisis
- For the 28-, 14-, and 7-day options, over the whole sample period and in the sub-period before the financial crisis, the total number of call contracts is more than double the number of put contracts
- This is also mostly true after the financial crisis
- In all cases the call positions are overwhelmingly short positions

Table 7: Composition of option portfolios

Option Maturity (days)	Total # of call contracts	# short call contracts	# long call contracts		Total # of put contracts	# short put contracts	# long put contracts
1990.01-2013.02 (N = 278)							
28	0.72	0.60	0.12		0.27	0.17	0.10
14	0.79	0.69	0.10		0.18	0.10	0.08
7	0.86	0.72	0.13		0.12	0.06	0.06
1990.01-2008.10 (N = 220)							
28	0.79	0.65	0.14		0.21	0.11	0.10
14	0.81	0.70	0.11		0.17	0.09	0.08
7	0.87	0.73	0.14		0.12	0.06	0.06
2008.11-2009.10 (N = 12)							
28	0.28	0.25	0.03		0.72	0.67	0.05
14	0.47	0.41	0.06		0.44	0.36	0.09
7	0.73	0.65	0.09		0.17	0.12	0.05
2009.11-2013.02 (N = 46)							
28	0.50	0.46	0.04		0.44	0.32	0.12
14	0.78	0.69	0.09		0.15	0.08	0.08
7	0.83	0.71	0.12		0.08	0.04	0.04

Dominating portfolios composition (cont.)

- Calls are more overpriced than puts, consistent with the earlier findings in Constantinides *et al.* (2009, 2011)
- The period 2008.11-2009.10 of the financial crisis is different
  - ➢the number of put positions is double the number of call positions
  - ➢the put positions are overwhelmingly short positions: during the crisis put prices overreacted to the prospect of a financial disaster and the slope of the skew steepened to the point that it became attractive to write overpriced puts rather than calls
  - ➢this is true for the 28-day options; there is a gradual decrease in put trading as the maturity gets shorter and the crash risk less likely

Dominating portfolios composition (cont.)

- Over the whole sample period and the sub-periods before and after the financial crisis, the OT investor primarily transfers payoffs from the high market return states to the low market return states by writing OTM calls
- In most months, the OT investor writes only one and at most two types of OTM calls
- During the crisis the OT investor primarily writes OTM puts
- The option portfolios are parsimonious, with at most one or two options of each category entering the portfolios

Are the options in the dominating portfolio outliers of the skew?

- In each cross section we regress the spread midpoint of the IV of all options that pass all our filters except for the moneyness filter on each option's moneyness, K/S, and its squared value

$$IV_i = a + b\left(K_i / S_t\right) + c\left(K_i / S_t\right)^2 + e_i$$

- We run separate regressions for calls and puts
- The long option positions in the dominating portfolio do not have negative average regression residuals
- The short option positions in the dominating portfolio do not have positive average regression residuals
- Therefore, options in the dominating portfolio are not skew outliers

Is dominance driven by a small number of "mispriced" options?

- We remove from each cross-section the options that are included in the optimal portfolio and repeat the search with the remaining options
- We still find dominating option portfolios (Table 11)
- The results are weaker than in Table 2 but still highly significant for 14- and 7-day options, but lesser so for 28-day options

Is the excess returns of the OT portfolios reward for risk?

- The 3-factor Fama-French model does not explain the excess returns (Table 13)
- The Constantinides, Jackwerth, and Savov (2013) option-based factors (price jumps, volatility jumps, liquidity) do not explain the excess returns (Table 14)
- The Christoffersen, Heston, and Jacobs (2013) U-shaped stochastic discount factor does not explain the excess returns (Table 15)

Why does the option mispricing persist?

- Index funds and ETFs:
  - ➢minimize tracking error and the inclusion of options in their portfolios likely increases tracking error
  - ➢may find it difficult to explain to their investors the benefits of stochastic dominance
- Active mutual funds and hedge funds may not hold the market portfolio because of different strategies
- Option traders’ and intermediaries’ credit constraints and funding liquidity may distort the prices of index options

Conclusion

- We start with the optimal portfolio of an investor who trades the S&P index and a riskless asset and maximizes expected utility
- In almost every month over 1990-2013 we identify a zero-net-cost portfolio of S&P 500 options that added to the investor's portfolio results in a stochastically dominating portfolio—the investor's expected utility increases
- We verify this claim in out-of-sample empirical tests
- The results hold for any risk-averse investor
- We allow for bid/ask spread and trading costs
- The option mispricing may persist because of institutional reasons

Thank you!