

Fund Tradeoffs

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- Mutual funds manage some \$50 trillion dollars worldwide (2017)
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- Traditional focus: Fund performance
 - Berk and Green (2004) critique
- Our focus: Fund behavior
 - Tradeoffs between fund characteristics?

What we do

- Build an equilibrium model connecting key **fund characteristics**
 - Fund size
 - Expense ratio
 - Turnover
 - Portfolio liquidity

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 - Relate it to portfolio diversification
 - Derive simple measures of portfolio liquidity and diversification

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- Introduce the concept of **portfolio liquidity**
 - Relate it to portfolio diversification
 - Derive simple measures of portfolio liquidity and diversification
- Test the model's predictions for U.S. equity mutual funds
 - Find strong empirical support
- Use the model to identify misallocation to mutual funds

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 - Holding fund size and turnover constant

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- Simple trading cost function \Rightarrow

$$\text{Portfolio Liquidity} = \text{Stock Liquidity} \times \text{Diversification}$$

- Diversification also has two components:

$$\text{Diversification} = \text{Coverage} \times \text{Balance}$$

- Greater coverage \Leftrightarrow More stocks in portfolio
- Greater balance \Leftrightarrow Portfolio weights closer to market-cap weights

Main empirical results

- Mutual funds with less **liquid** portfolios have
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 - More liquid holdings
- Larger funds are cheaper
- Funds that trade less are larger and cheaper
- Funds that are too big perform worse
- All these findings are consistent with our model, and with decreasing returns to scale in active management

- Decreasing returns to scale in active management
 - Several papers relate fund size to performance
 - We relate fund size to portfolio liquidity and turnover
 - Berk and Green (2004), Chen et al. (2004), Pollet and Wilson (2008), Yan (2008), Pástor, Stambaugh, and Taylor (2015), Reuter and Zitzewitz (2015), Busse et al. (2017), Harvey and Liu (2017)

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- Portfolio diversification
 - Common measures: Number of stocks, Herfindahl index of weights
 - Our measure blends both ideas, has strong theoretical motivation
 - We explore what kinds of funds are more likely to diversify
 - Blume and Friend (1975), Kacperczyk, Sialm, and Zheng (2005), Goetzmann and Kumar (2008), Cremers and Petajisto (2009)

- Introduce portfolio liquidity
 - Theory
 - Empirics

- Tradeoffs among fund characteristics
 - Theory
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- Predict fund returns
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Introducing portfolio liquidity

$$L = \left(\sum_{i=1}^N \frac{w_i^2}{m_i} \right)^{-1}$$

N = number of stocks in portfolio

w_i = portfolio's weight on stock i

m_i = weight on stock i in a value-weighted benchmark

Introducing portfolio liquidity

- Fund's dollar trading costs: $C = \sum_i^N D_i C_i$
 - D_i = dollar amount traded of stock i
 - C_i = cost per dollar traded of stock i

- **Assumption 1:** Larger trades have higher proportional costs

$$C_i = \tilde{c} \frac{D_i}{M_i}, \quad M_i = \text{stock } i\text{'s market cap.}$$

- **Assumption 2:** Fund expects to trade portfolio proportionally

$$E(D_i) = \underbrace{A}_{\text{AUM}} \times \underbrace{T}_{\text{turnover}} \times \underbrace{w_i}_{\text{portfolio weight}}$$

Introducing portfolio liquidity

These assumptions \Rightarrow

$$E(C) = \left(\frac{c}{M}\right) (AT)^2 \underbrace{\left(\sum_{i=1}^N \frac{w_i^2}{m_i}\right)}_{L^{-1}}$$

Definition:

If two funds have equal size and equal turnover, the fund with lower trading costs has greater portfolio liquidity.

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Introducing portfolio liquidity

Expected cost is quadratic in V :

$$E(C) = \left(\frac{c}{M}\right) \underbrace{(AT)^2 L^{-1}}_{V^2}$$

where $V = ATL^{-\frac{1}{2}}$ is liquidity-adjusted dollar trading volume

Properties of portfolio liquidity

- $L \in (0, 1]$
- Least liquid portfolio: Single, smallest stock in benchmark
- Most liquid portfolio: Benchmark portfolio ($L = 1$)

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- Decomposition:

$$L = \underbrace{\frac{1}{N} \sum_{i=1}^N \mathcal{L}_i}_{\text{Stock Liquidity}} \times \underbrace{\left(\frac{N}{N_M} \right) \left[1 + \text{Var}^* \left(\frac{w_i}{m_i^*} \right) \right]^{-1}}_{\text{Diversification}}$$

$$\mathcal{L}_i = M_i / \bar{M}$$

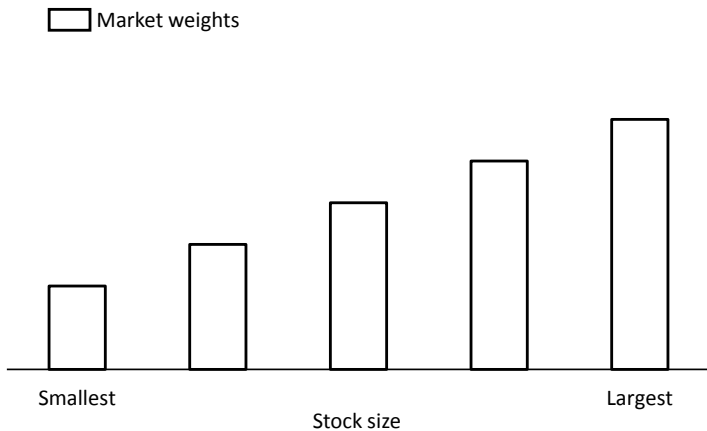
\bar{M} = average market cap of stocks in benchmark

Properties of portfolio diversification

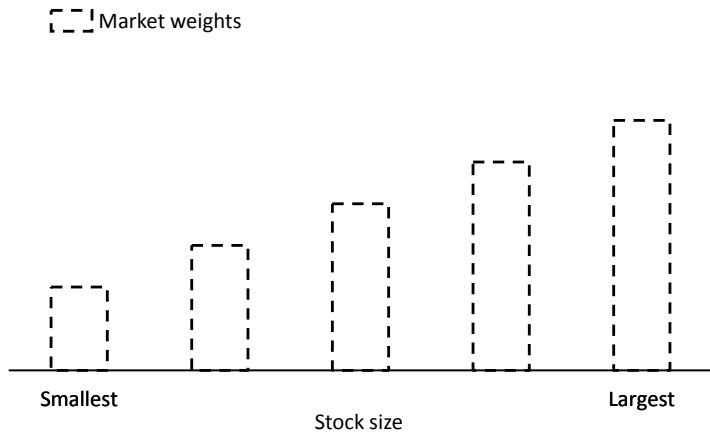
$$\text{Diversification} = \underbrace{\left(\frac{N}{N_M}\right)}_{\text{Coverage}} \times \underbrace{\left[1 + \text{Var}^*\left(\frac{w_i}{m_i^*}\right)\right]^{-1}}_{\text{Balance}}$$

- **Coverage:** What fraction of available stocks do you hold?
- **Balance:** How close are your weights to market-cap weights among stocks you hold?
- Diversification, Coverage, Balance are all $\in (0, 1]$

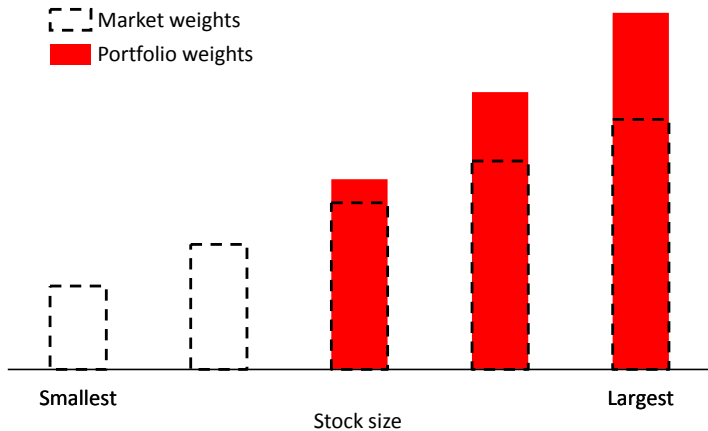
Example



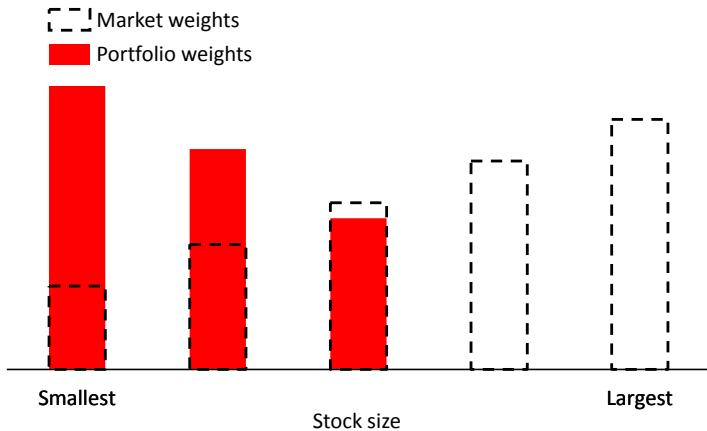
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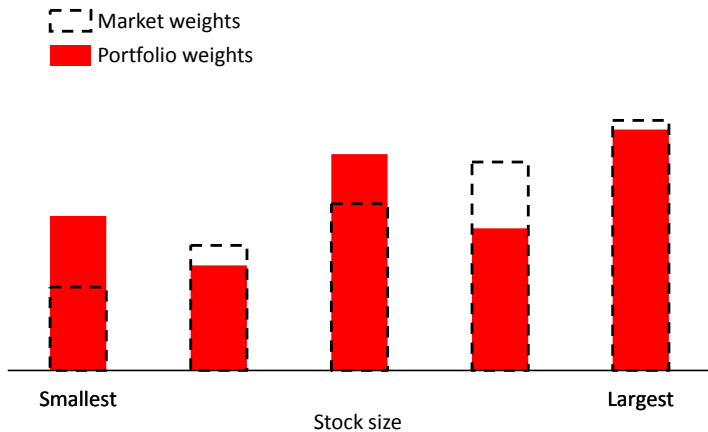
Example: High Stock Liquidity



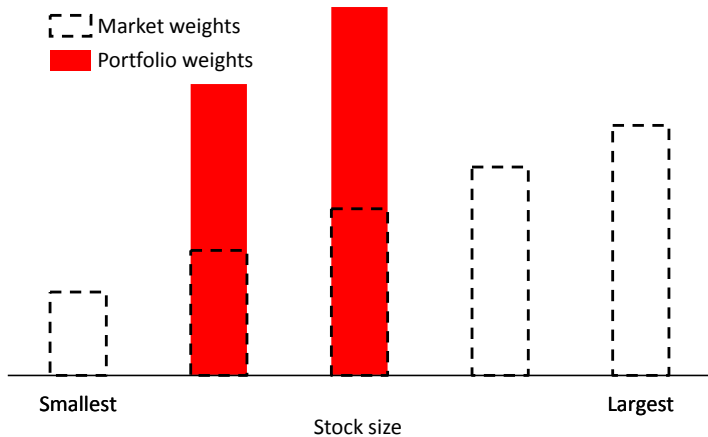
Example: Low Stock Liquidity



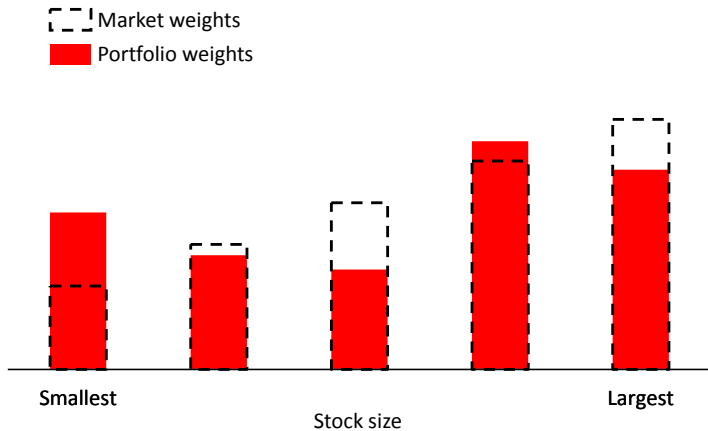
Example: High Diversification



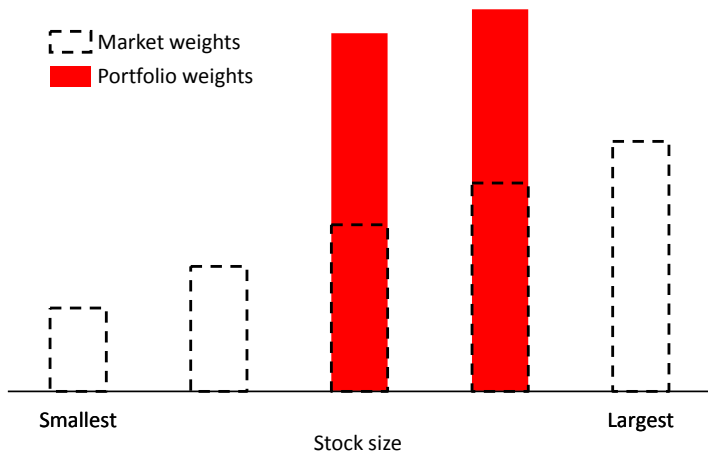
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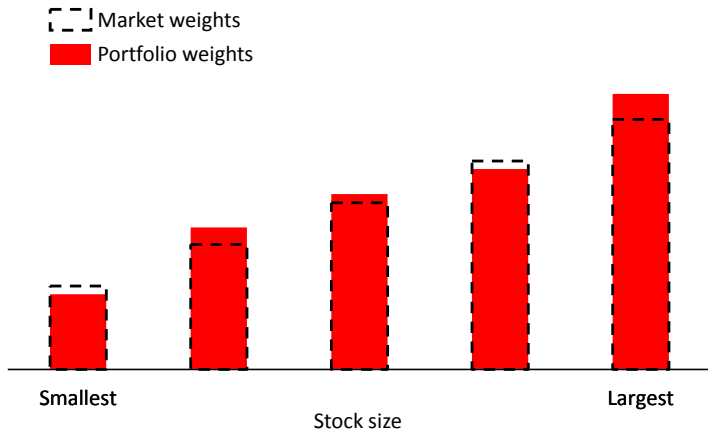
Example: High Coverage



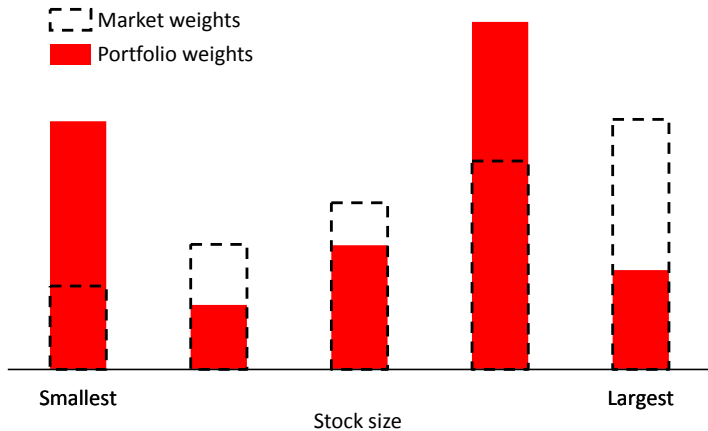
Example: Low Coverage



Example: High Balance

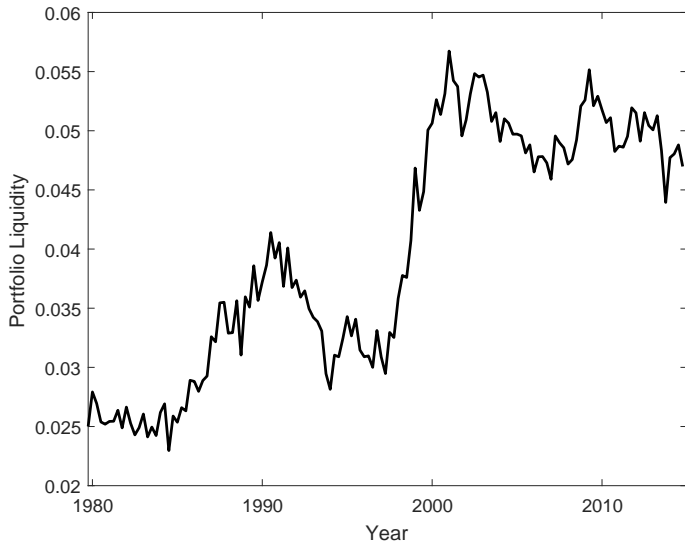


Example: Low Balance

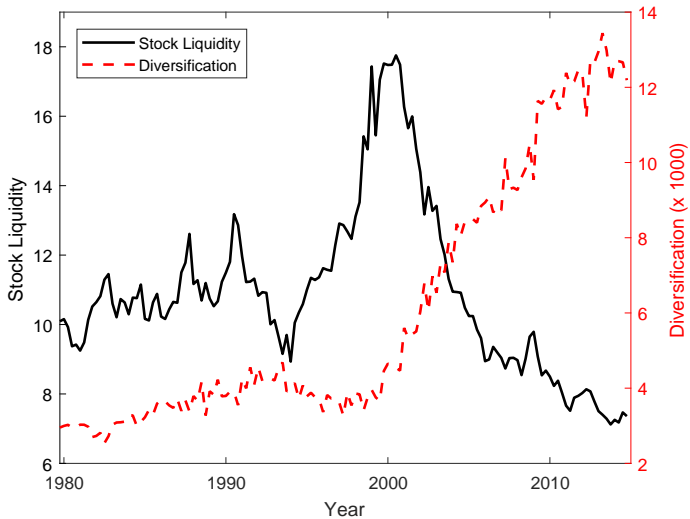


- 2,789 active U.S. domestic equity mutual funds, 1979–2014
- **Data:** Combine CRSP, Morningstar, Thomson Reuters
 - Check accuracy across databases
 - Exclude index funds, non-equity funds, international funds, industry funds, target-date funds, funds of funds, funds with size $<$ \$15 million

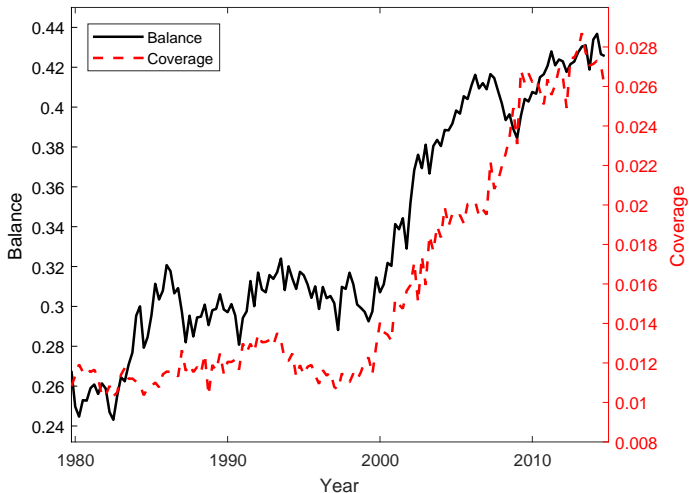
Portfolio liquidity has doubled...



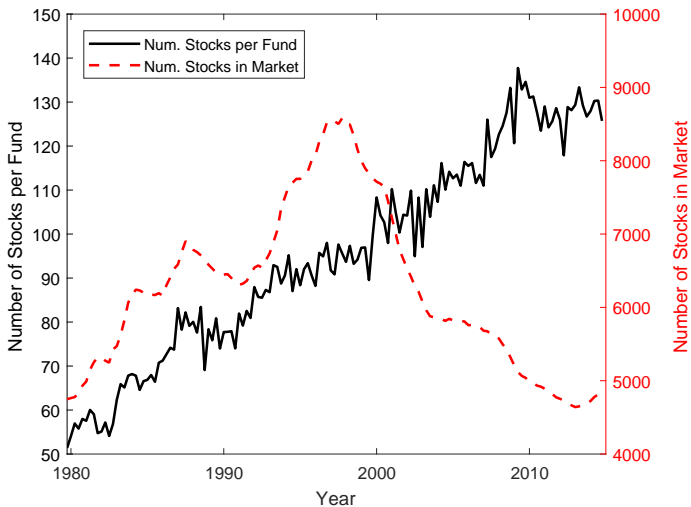
...because diversification has tripled



Both components of diversification have trended up

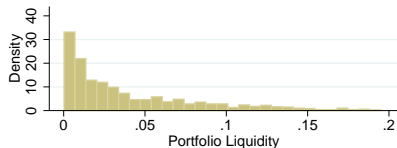


Funds are holding more and more stocks

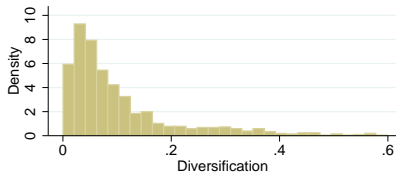
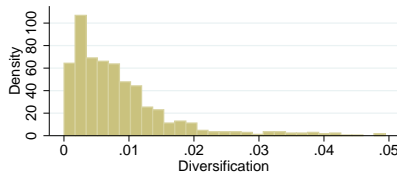
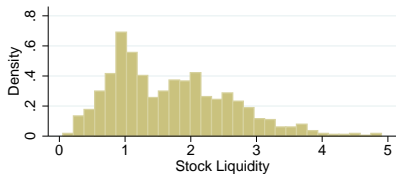
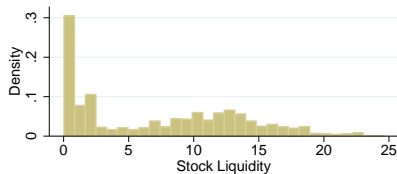
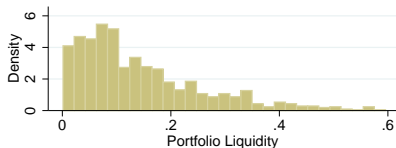


Portfolio liquidity is low, due to low diversification

Benchmark: Market

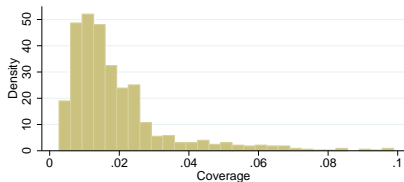


Benchmark: Sector

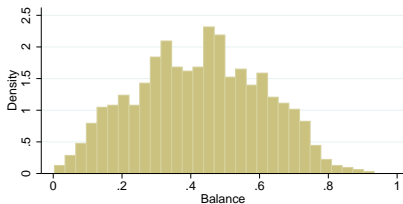
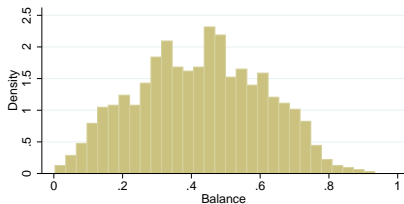
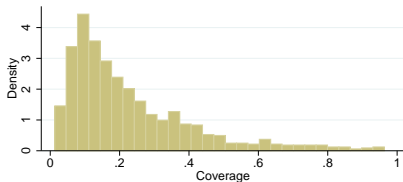


Low diversification is driven mostly by low coverage

Benchmark: Market



Benchmark: Sector



- Introduce portfolio liquidity
 - Theory
 - Empirics
- Tradeoffs among fund characteristics
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- Predict fund returns
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Equilibrium model

- Fund's expected benchmark-adjusted return, before costs & fees:

$$a = \underbrace{\mu}_{\text{skill}} \times \underbrace{TL^{-\frac{1}{2}}}_{\text{activeness}}$$

- Increasing in μ and T
- Decreasing in L

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Why do illiquid portfolios generate higher expected returns?

$$L = \underbrace{\text{Stock Liquidity}}_{\text{more mispricing}} \times \underbrace{\text{Coverage}}_{\text{best ideas}} \times \underbrace{\text{Balance}}_{\text{larger bets}}$$

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- Fund's expected trading profit, gross of costs:

$$aA = \mu \underbrace{ATL^{-\frac{1}{2}}}_V$$

Equilibrium model

- Fund's expected trading cost (from before):

$$E(C) = \left(\frac{c}{M}\right) \underbrace{(AT)^2 L^{-1}}_{V^2}$$

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- Fund's expected trading profit, net of costs:

$$\Pi = \mu V - (c/M)V^2$$

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- Fund chooses L , T , and f to **maximize Π**
 - Choice of f determines A but does not affect Π
 - Choice of L and T determines V , given A

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- Fund chooses L , T , and f to **maximize Π**
 - Choice of f determines A but does not affect Π
 - Choice of L and T determines V , given A
- Investors compete $\Rightarrow \alpha = 0$ (à la Berk and Green, 2004)
 - This implies $\Pi = Af$

- Fund's first-order condition:

$$\underbrace{ATL^{-\frac{1}{2}}}_V = \frac{\mu M}{2c}$$

- Impose $\alpha = 0 \Rightarrow$

$$A = \frac{\mu^2 M}{4cf}$$

- Combine previous equations \Rightarrow

$$\mathbf{\log(L) = \log(A) - \log(f) + 2 \log(T) + \text{constant}}$$

Testing the model

- Interpret our main prediction as a panel regression:

$$\log(L) = \beta_0 + \beta_1 \log(A) + \beta_2 \log(f) + \beta_3 \log(T) + \epsilon$$

- 76,928 fund/quarter observations
- Sector \times quarter fixed effects, cluster by fund

Testing the model

$$\log(L) = \log(A) - \log(f) + 2 \log(T) + \text{constant}$$

	(1)	(2)	(3)	(4)	(5)
	Portfolio Liquidity				
Fund Size	0.124 (13.76)				
Expense Ratio	-0.608 (-11.26)				
Turnover	0.101 (4.93)				
R^2	0.652				
R^2 (FEs only)	0.598				

Testing the model

$$\log(\text{Portfolio Liquidity}) = \log(\text{Diversification}) + \log(\text{Stock Liquidity})$$

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Fund Size	0.124 (13.76)	0.134 (15.00)			
Expense Ratio	-0.608 (-11.26)	-0.622 (-11.00)			
Turnover	0.101 (4.93)	0.122 (5.96)			
Stock Liquidity		-0.621 (-21.61)			
R^2	0.652	0.465			
R^2 (FEs only)	0.598	0.240			

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Turnover	0.101 (4.93)	0.122 (5.96)	0.102 (6.37)		
Stock Liquidity		-0.621 (-21.61)	-0.337 (-14.21)		
Balance			-0.0447 (-2.08)		
R^2	0.652	0.465	0.336		
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	Portfolio Liquidity	Diversification	Coverage	Balance	
Fund Size	0.124 (13.76)	0.134 (15.00)	0.0940 (12.08)	0.0452 (7.54)	
Expense Ratio	-0.608 (-11.26)	-0.622 (-11.00)	-0.408 (-9.33)	-0.238 (-6.95)	
Turnover	0.101 (4.93)	0.122 (5.96)	0.102 (6.37)	0.0247 (1.92)	
Stock Liquidity		-0.621 (-21.61)	-0.337 (-14.21)	-0.308 (-14.90)	
Balance			-0.0447 (-2.08)		
Coverage				-0.0343 (-2.09)	
R^2	0.652	0.465	0.336	0.286	
R^2 (FEs only)	0.598	0.240	0.163	0.172	

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Expense Ratio	-0.608 (-11.26)	-0.622 (-11.00)	-0.408 (-9.33)	-0.238 (-6.95)	-0.132 (-5.26)
Turnover	0.101 (4.93)	0.122 (5.96)	0.102 (6.37)	0.0247 (1.92)	-0.0146 (-1.32)
Stock Liquidity		-0.621 (-21.61)	-0.337 (-14.21)	-0.308 (-14.90)	
Balance			-0.0447 (-2.08)		
Coverage				-0.0343 (-2.09)	
Diversification					-0.264 (-24.49)
R^2	0.652	0.465	0.336	0.286	0.882
R^2 (FEs only)	0.598	0.240	0.163	0.172	0.857

Correlations among fund characteristics

- Model also makes predictions about simple correlations

Assumptions

Correlations among fund characteristics

- Model also makes predictions about simple correlations

Assumptions

Prediction	Variables (logs)	Correlations	
		X-sectional	T-series
1. Larger funds are cheaper	Fund Size, Expense Ratio	-0.315 (-15.3)	-0.251 (-17.6)

Correlations among fund characteristics

- Model also makes predictions about simple correlations

Assumptions

Prediction	Variables (logs)	Correlations	
		X-sectional	T-series
1. Larger funds are cheaper	Fund Size,	-0.315	-0.251
	Expense Ratio	(-15.3)	(-17.6)
2. Funds that trade less are larger and cheaper	Turnover,	-0.105	-0.147
	Fund Size	(-6.0)	(-12.2)
	Turnover,	0.130	0.105
	Expense Ratio	(6.4)	(7.6)

Correlations among fund characteristics

- Model also makes predictions about simple correlations

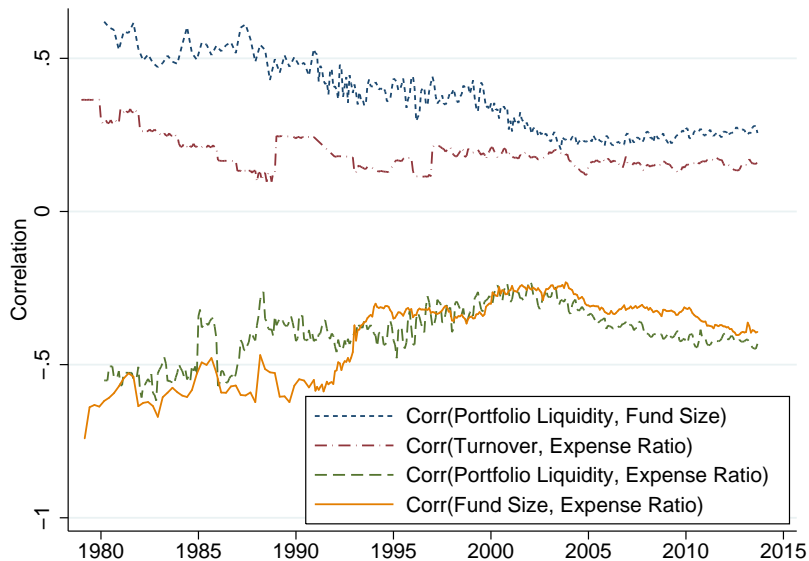
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	Expense Ratio	(-15.3)	(-17.6)
2. Funds that trade less are larger and cheaper	Turnover,	-0.105	-0.147
	Fund Size	(-6.0)	(-12.2)
3. Funds with more liquid portfolios are larger and cheaper	Turnover,	0.130	0.105
	Expense Ratio	(6.4)	(7.6)
3. Funds with more liquid portfolios are larger and cheaper	Port. Liquidity,	0.285	0.308
	Fund Size	(17.9)	(18.2)
3. Funds with more liquid portfolios are larger and cheaper	Port. Liquidity,	-0.291	-0.118
	Expense Ratio	(-13.4)	(-6.9)

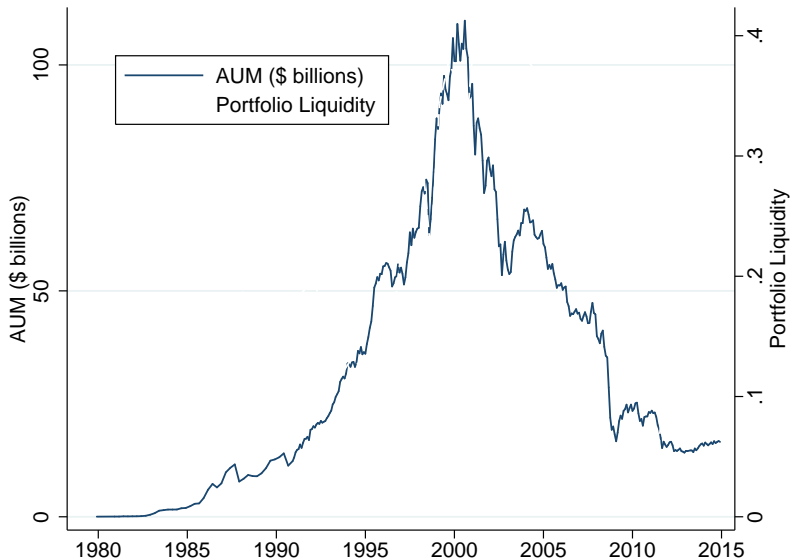
Correlations among fund characteristics

Prediction	Variables (logs)	Correlations	
		X-sectional	T-series
4. Larger funds are less active	Fund Size, Activeness	-0.231 (-13.3)	-0.268 (-17.6)
5. Cheaper funds are less active	Expense Ratio, Activeness	0.255 (13.58)	0.136 (9.20)

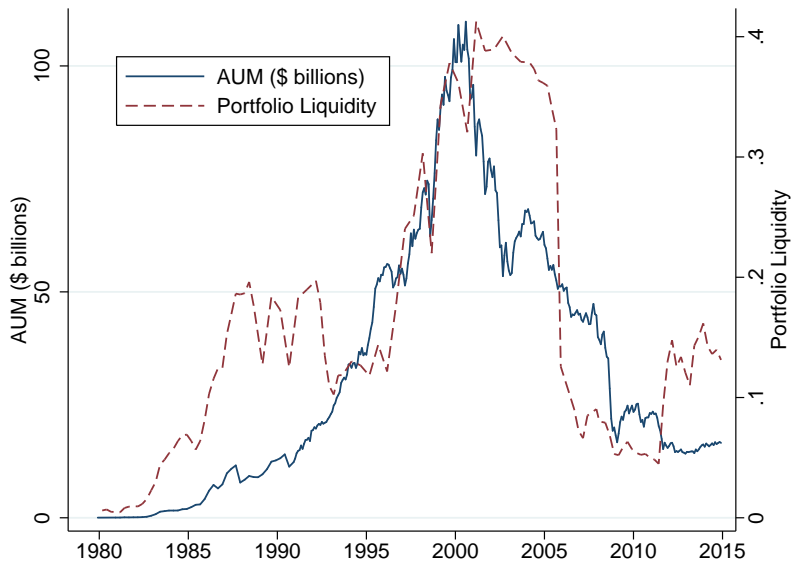
Cross-sectional correlations over time



Example: Fidelity's Magellan fund



Example: Fidelity's Magellan fund



- Introduce portfolio liquidity
 - Theory
 - Empirics

- Tradeoffs among fund characteristics
 - Theory
 - Empirics

- Predict fund returns
 - Theory
 - Empirics

Predicting fund returns: Theory

- So far: Investors allocate capital perfectly to funds
- Extend model to allow capital misallocation:

$$\underbrace{A}_{\text{actual}} = \underbrace{\bar{A}}_{\text{equilibrium}} \times \left(1 + \underbrace{\delta}_{\text{excess}}\right)$$

Predicting fund returns: Theory

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- Fund observes δ , adjusts its activeness optimally:

$$\underbrace{TL^{-\frac{1}{2}}}_{\text{actual}} = \underbrace{\bar{T}\bar{L}^{-\frac{1}{2}}}_{\text{equilibrium}} / (1 + \delta)$$

Fund gets too much capital \Rightarrow Either $T \downarrow$ or $L \uparrow$

Predicting fund returns: Theory

- Fund's net alpha:

$$\alpha = - \left(\frac{\delta}{1 + \delta} \right) f$$

- Funds that are too big have $\alpha < 0$. Especially for high- f funds.

Predicting fund returns: Theory

- Fund's net alpha:

$$\alpha = - \left(\frac{\delta}{1 + \delta} \right) f$$

- Funds that are too big have $\alpha < 0$. Especially for high- f funds.
- Challenge: How to measure excess fund size (δ)
- Model predicts

$$\log(L) \approx \log(A) - \log(f) + 2 \log(T) + \log(c) + \delta$$

- We estimate excess fund size as $\hat{\delta}_{i,t}$ from a panel regression

Predicting fund returns: Panel regressions

Dependent variable: Benchmark-adjusted fund return in month $t + \tau$

	Forecasting Horizon (τ , in months)				
	1	6	12	24	48
Excess size	-0.0360 (-4.00)	-0.0282 (-3.31)	-0.0283 (-3.51)	-0.0236 (-2.97)	-0.0118 (-1.57)
Observations	199,311	190,612	180,763	161,449	127,440

* Regressions include sector \times month FEs, cluster by sector \times month

Predicting fund returns: Portfolio sorts

Each month t , sort funds into 5 EW portfolios based on $\hat{\delta}_{i,t-12}$
Portfolios' average benchmark-adjusted returns (% per month):

	Excess Fund Size					
	Low	2	3	4	High	High-Low
All Funds	-0.028 (-0.78)	-0.055 (-1.91)	-0.062 (-2.26)	-0.071 (-2.73)	-0.082 (-3.02)	-0.053 (-1.32)
High Expense Ratio	-0.006 (-0.14)	-0.078 (-2.29)	-0.094 (-3.26)	-0.081 (-2.77)	-0.115 (-4.18)	-0.109 (-2.47)
Low Expense Ratio	-0.040 (-0.98)	-0.038 (-1.23)	-0.037 (-1.16)	-0.052 (-1.72)	-0.046 (-1.48)	-0.006 (-0.13)
High-Low	0.034 (0.91)	-0.040 (-1.26)	-0.057 (-2.08)	-0.029 (-1.08)	-0.069 (-3.23)	-0.103 (-2.59)

- ① Model & document strong relations among **fund characteristics**
 - Funds with more liquid portfolios are larger, cheaper, and trade more
 - Larger funds are cheaper
 - Funds that trade less are larger and cheaper
- ② Introduce concept of **portfolio liquidity**
 - Portfolio Liquidity = Stock Liquidity \times Diversification
 - Derive simple measures of portfolio liquidity and diversification
- ③ Analyze capital misallocation to mutual funds
 - Funds that are too big underperform

ADDITIONAL SLIDES

Assumptions about μ

1 Larger funds are cheaper

- $\beta_{\mu,f}$ = slope of $\log(\mu)$ on $\log(f)$
- Assumption: $\beta_{\mu,f} < 1/2$

2 Funds that trade less are larger and cheaper

- $\beta_{\mu,A}$ = slope of $\log(\mu)$ on $\log(A)$
- Assumption: $\beta_{\mu,A} < 1$
- Assumption: $\beta_{\mu,f} < 1$

3 Funds with more-liquid portfolios are larger and cheaper

- Assumption: $\beta_{\mu,A} < 1$
- Assumption: $\beta_{\mu,f} < 1$