Recovering Heterogeneous Beliefs and Preferences from Asset Prices

Anisha Ghosh* Arthur Korteweg◇ Qing Xu*

*McGill University

◇USC

Q-Group Seminar

Oct 25, 2021

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - ⇒ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - $\Rightarrow$ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - $\Rightarrow$ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - $\Rightarrow$ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - $\Rightarrow$ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - ⇒ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - $\Rightarrow$ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - ⇒ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...)

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - $\Rightarrow$ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...).

Motivation

- Asset prices reflect investors' risk preferences and beliefs about the future.
  - ⇒ Understanding preferences and beliefs of different investor types crucial in explaining and predicting the behavior of asset prices, design of optimal economic policies, ...
- Two paradigms in economics:
  1. Rational Expectations: agents use available data objectively to form beliefs about the future.
  2. Behavioral Models: agents have behavioral biases, distorting their beliefs relative to rationality.
- Models based on either paradigm make structural assumptions about:
  - the true data generating process (DGP),
  - the way investors' beliefs depart from the DGP.
- Most theories assume investors are homogeneous in terms of their risk preferences or beliefs, or both.
- Extant evidence of investor heterogeneity (see, e.g., Barberis & Thaler (2001), Calvet, Campbell & Sodini (2007), Malmendier & Nagel (2011), ...).

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

What We Do In This Paper

- A novel approach to identify heterogeneous investors' (subjective) beliefs and risk aversion levels from asset prices, bypassing the need for assumptions about:
  - the true data generating process,
  - investor rationality or lack thereof.
- Our approach recovers the *entire conditional distribution* of (or, beliefs about) macro and financial variables, as perceived by a given investor type, given:
  1. a pricing kernel
  2. a cross section of test assets
  3. the conditioning set
- The recovered beliefs are price-consistent – they satisfy the conditional Euler equations for the test assets for each investor type.
- Overcome shortcomings of existing survey data on investors' expectations – short time period, small cross section, scarce information on survey respondents.

Identification:

Investors with different risk preferences and/or beliefs choose different optimal portfolios.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Main Results

- Substantial heterogeneity in risk preferences and beliefs across different investor types.
  - Type I investors: allocate heavily to large market cap equities.
    - Risk preferences: $\gamma \approx 1.8 - 2.4$.
    - Beliefs: Strongly countercyclical expected stock market returns.
  - Type II investors: allocate heavily to small-growth stocks.
    - Risk preferences: Risk preferences: $\gamma \approx 0.2 - 0.8$.
    - Beliefs: Procyclical expected stock market returns

Reconciliation of Findings in Greenwood and Shleifer (2014)

Can reconcile the procyclical expected market returns found in survey data with the countercyclical expected returns implied by rational expectations representative agent models.

Reconciliation of Anomalous Small-Growth Portfolio Return

Very low historical average returns, producing large negative alphas with respect benchmark models (Fama and French (1993, 2012, 2015)), have proven difficult to explain thus far.

Outline

Appendix

Outline

Appendix

Related Literature

- **Entropy based inference**: Stutzer (JF1996), Owen (2001), Kitamura & Stutzer (ECMA1997, JE2002), Julliard & Ghosh (RFS2012), Almeida & Garcia (JE2012, MS2016), Ghosh and Rousellet (2019), Chen, Hansen, and Hansen (2020), GEL literature ...
- **Investor Heterogeneity**: e.g. Calvet & Campbell & Gomes & Sodini (2019), Meeuwis & Parker & Schoar & Simester (2019), Giglio & Maggiori & Stroebel & Utkus (2019), Wang (2021)...

Calvet & Campbell & Gomes & Sodini (2019)

"... to the extent that any heterogeneity in beliefs exists, it will be attributed to heterogeneous preferences by our estimation procedure."

Outline

Appendix

Conditional Euler Equations

- The absence of arbitrage opportunities implies the existence of a strictly positive **pricing kernel** M_{t+1} such that

$$\mathbb{E}^{\mathcal{P}}\left[M_{t+1}\mathbf{R}^{e}_{t+1}|\underline{\mathcal{F}_t}\right] = \mathbf{0_k}, \tag{1}$$

where $\underline{\mathcal{F}_t}$ denotes the investors' information set at time t.

  - If investors are fully rational, then $\mathcal{P}$ denotes the objective DGP.
  - If investors have behavioral biases, then $\mathcal{P}$ denotes their subjective measure.

- Allowing for different investor types with different risk preferences *and/or* beliefs:

$$\mathbb{E}^{\mathcal{P}^{(l)}}\left[M^{(l)}_{t+1}R^{e}_{i,t+1}|\underline{\mathcal{F}_t}\right] = 0. \tag{2}$$

$$M^{(l)}_{t+1} = M\left(F^{(l)}_{t+1};\gamma^{(l)}\right). \tag{3}$$

- Objective: *Recover* $\left(\gamma^{(l)}, \mathcal{P}^{(l)}\right)$ *from observed asset prices, for different investor types*

Conditional Euler Equations

- The absence of arbitrage opportunities implies the existence of a strictly positive **pricing kernel** M_{t+1} such that

$$\mathbb{E}^{\mathcal{P}}\left[M_{t+1}\mathbf{R}^{e}_{t+1}|\underline{\mathcal{F}_t}\right] = \mathbf{0}_{\mathbf{k}}, \tag{1}$$

where $\underline{\mathcal{F}_t}$ denotes the investors' information set at time t.

  - If investors are fully rational, then $\mathcal{P}$ denotes the objective DGP.
  - If investors have behavioral biases, then $\mathcal{P}$ denotes their subjective measure.

- Allowing for different investor types with different risk preferences *and/or* beliefs:

$$\mathbb{E}^{\mathcal{P}^{(l)}}\left[M^{(l)}_{t+1}R^{e}_{i,t+1}|\underline{\mathcal{F}_t}\right] = 0. \tag{2}$$

$$M^{(l)}_{t+1} = M\left(F^{(l)}_{t+1};\gamma^{(l)}\right). \tag{3}$$

- Objective: *Recover* $\left(\gamma^{(l)},\mathcal{P}^{(l)}\right)$ *from observed asset prices, for different investor types*

Conditional Euler Equations

- The absence of arbitrage opportunities implies the existence of a strictly positive **pricing kernel** M_{t+1} such that

$$\mathbb{E}^{\mathcal{P}}\left[M_{t+1}\mathbf{R}^{e}_{t+1}|\underline{\mathcal{F}_t}\right] = \mathbf{0_k}, \qquad (1)$$

where $\underline{\mathcal{F}_t}$ denotes the investors' information set at time t.
  - If investors are fully rational, then $\mathcal{P}$ denotes the objective DGP.
  - If investors have behavioral biases, then $\mathcal{P}$ denotes their subjective measure.
- Allowing for different investor types with different risk preferences *and/or* beliefs:

$$\mathbb{E}^{\mathcal{P}^{(l)}}\left[M^{(l)}_{t+1}R^{e}_{i,t+1}|\underline{\mathcal{F}_t}\right] = 0. \qquad (2)$$

$$M^{(l)}_{t+1} = M\left(F^{(l)}_{t+1};\gamma^{(l)}\right). \qquad (3)$$

- Objective: *Recover* $\left(\gamma^{(l)},\mathcal{P}^{(l)}\right)$ *from observed asset prices, for different investor types*

Conditional Euler Equations

- The absence of arbitrage opportunities implies the existence of a strictly positive **pricing kernel** M_{t+1} such that

$$\mathbb{E}^{\mathcal{P}}\left[M_{t+1}\mathbf{R}^{e}_{t+1}|\underline{\mathcal{F}_t}\right] = \mathbf{0_k}, \tag{1}$$

  where $\underline{\mathcal{F}_t}$ denotes the investors' information set at time t.
  - If investors are fully rational, then $\mathcal{P}$ denotes the objective DGP.
  - If investors have behavioral biases, then $\mathcal{P}$ denotes their subjective measure.
- Allowing for different investor types with different risk preferences *and/or* beliefs:

$$\mathbb{E}^{\mathcal{P}^{(l)}}\left[M^{(l)}_{t+1}R^{e}_{i,t+1}|\underline{\mathcal{F}_t}\right] = 0. \tag{2}$$

$$M^{(l)}_{t+1} = M\left(F^{(l)}_{t+1};\gamma^{(l)}\right). \tag{3}$$

- Objective: *Recover* $\left(\gamma^{(l)},\mathcal{P}^{(l)}\right)$ *from observed asset prices, for different investor types*

Conditional Euler Equations

- The absence of arbitrage opportunities implies the existence of a strictly positive **pricing kernel** M_{t+1} such that

$$\mathbb{E}^{\mathcal{P}}\left[M_{t+1}\mathbf{R}^e_{t+1}|\underline{\mathcal{F}_t}\right] = \mathbf{0_k}, \tag{1}$$

  where $\underline{\mathcal{F}_t}$ denotes the investors' information set at time t.
  - If investors are fully rational, then $\mathcal{P}$ denotes the objective DGP.
  - If investors have behavioral biases, then $\mathcal{P}$ denotes their subjective measure.
- Allowing for different investor types with different risk preferences *and/or* beliefs:

$$\mathbb{E}^{\mathcal{P}^{(l)}}\left[M^{(l)}_{t+1}R^e_{i,t+1}|\underline{\mathcal{F}_t}\right] = 0. \tag{2}$$

$$M^{(l)}_{t+1} = M\left(F^{(l)}_{t+1};\gamma^{(l)}\right). \tag{3}$$

- Objective: *Recover* $\left(\gamma^{(l)},\mathcal{P}^{(l)}\right)$ *from observed asset prices, for different investor types*

Smoothed Empirical Likelihood (Kitamura, Tripathi, & Ahn (2004))

- Approximate the physical DGP (objective beliefs) with a standard kernel density estimator: the transition probability from state j to state k is

$$\omega_{j,k} = \frac{\mathcal{K}\left(\frac{x_j - x_k}{b_T}\right)}{\sum_{k=1}^{T} \mathcal{K}\left(\frac{x_j - x_t}{b_T}\right)}$$

- Estimate subjective beliefs by the $(T \times T)$ transition matrix $\{p_{j,k}\}_{j,k=1}^{T}$ such that $\forall j \in \{1, \dots, T\}, \quad \forall \theta \in \Theta,$

$$\left\{\widehat{p_{j,\cdot}^{(l)}(\gamma^{(l)})}\right\} = \underset{(p_{j,\cdot}^{(l)}) \in \Delta_j}{\arg\min} \sum_{k=1}^{T} \omega_{j,k} \log\left(\frac{\omega_{j,k}}{p_{j,k}^{(l)}}\right) \quad \text{s.t.} \quad \sum_{k=1}^{T} p_{j,k}^{(l)} \cdot M(f_k^{(l)}; \gamma^{(l)}) \cdot r_{i,k}^{e} = 0, \tag{4}$$

- $\Rightarrow$ Recovered subjective beliefs are minimally distorted (minimizes KLIC divergence) with respect to the physical DGP, while also satisfying the conditional Euler restrictions – natural starting point in the analysis of subjective beliefs.

Smoothed Empirical Likelihood (Kitamura, Tripathi, & Ahn (2004))

- Approximate the physical DGP (objective beliefs) with a standard kernel density estimator: the transition probability from state j to state k is

$$\omega_{j,k} = \frac{\mathcal{K}\left(\frac{x_j - x_k}{b_T}\right)}{\sum_{k=1}^{T} \mathcal{K}\left(\frac{x_j - x_t}{b_T}\right)}$$

- Estimate subjective beliefs by the $(T \times T)$ transition matrix $\{p_{j,k}\}_{j,k=1}^{T}$ such that $\forall j \in \{1, \ldots, T\}, \quad \forall \theta \in \Theta,$

$$\left\{ \widehat{p_{j,\cdot}^{(l)}(\gamma^{(l)})} \right\} = \underset{(p_{j,\cdot}^{(l)}) \in \Delta_j}{\arg\min} \sum_{k=1}^{T} \omega_{j,k} \log\left(\frac{\omega_{j,k}}{p_{j,k}^{(l)}}\right) \quad \text{s.t.} \quad \sum_{k=1}^{T} p_{j,k}^{(l)} \cdot M(f_k^{(l)}; \gamma^{(l)}) \cdot r_{i,k}^{e} = 0, \tag{4}$$

- $\Rightarrow$ Recovered subjective beliefs are minimally distorted (minimizes KLIC divergence) with respect to the physical DGP, while also satisfying the conditional Euler restrictions – natural starting point in the analysis of subjective beliefs

Smoothed Empirical Likelihood (Kitamura, Tripathi, & Ahn (2004))

- Approximate the physical DGP (objective beliefs) with a standard kernel density estimator: the transition probability from state j to state k is

$$\omega_{j,k} = \frac{\mathcal{K}\left(\frac{x_j - x_k}{b_T}\right)}{\sum_{k=1}^{T} \mathcal{K}\left(\frac{x_j - x_t}{b_T}\right)}$$

- Estimate subjective beliefs by the $(T \times T)$ transition matrix $\{p_{j,k}\}_{j,k=1}^{T}$ such that $\forall j \in \{1, \ldots, T\}, \quad \forall \theta \in \Theta,$

$$\left\{\widehat{p_{j,\cdot}^{(l)}(\gamma^{(l)})}\right\} = \underset{(p_{j,\cdot}^{(l)}) \in \Delta_j}{\arg\min} \sum_{k=1}^{T} \omega_{j,k} \log\left(\frac{\omega_{j,k}}{p_{j,k}^{(l)}}\right) \quad \text{s.t.} \quad \sum_{k=1}^{T} p_{j,k}^{(l)} \cdot M(f_k^{(l)}; \gamma^{(l)}) \cdot r_{i,k}^{e} = 0, \tag{4}$$

- $\Rightarrow$ Recovered subjective beliefs are minimally distorted (minimizes KLIC divergence) with respect to the physical DGP, while also satisfying the conditional Euler restrictions – natural starting point in the analysis of subjective beliefs.

The SEL Estimator: Solution

- In the absence of Euler restrictions: $\widehat{p_{j,k}^{(l)}} = \omega_{j,k}$ – recovered beliefs coincide with physical DGP.
- In the presence of Euler restrictions: the recovered beliefs are distorted relative to the physical DGP, $\forall j,k \in \{1,\ldots,T\}$,

$$\widehat{p_{j,k}^{(l)}(\gamma^{(l)})} = \frac{\omega_{j,k}}{1+\widehat{\lambda_j(\gamma^{(l)})}\cdot M(f_k^{(l)};\gamma^{(l)})\cdot r_{i,k}^e}, \quad (5)$$

where $\widehat{\lambda_j(\gamma^{(l)})}$ are the Lagrange multipliers associated with the conditional Euler equation constraints, and solve:

$$\widehat{\lambda_j(\gamma^{(l)})} = \arg\max_{\lambda_j\in\mathbb{R}^l} \sum_{k=1}^{T} \omega_{j,k}\log\left[1+\lambda_j\cdot M(f_k^{(l)};\gamma^{(l)})\cdot r_{i,k}^e\right]. \quad (6)$$

⇒ For each investor type, the SEL procedure delivers a $(T\times T)$ transition matrix $\left\{\widehat{p_{j,k}^{(l)}}\left(\gamma^{(l)}\right)\right\}_{j,k=1}^{T}$, for each value of the parameter vector $\gamma^{(l)}$ (price-consistent beliefs)

The SEL Estimator: Solution

- In the absence of Euler restrictions: $\widehat{p_{j,k}^{(l)}} = \omega_{j,k}$ – recovered beliefs coincide with physical DGP.
- In the presence of Euler restrictions: the recovered beliefs are distorted relative to the physical DGP, $\forall j,k \in \{1,\ldots,T\}$,

$$\widehat{p_{j,k}^{(l)}(\gamma^{(l)})} = \frac{\omega_{j,k}}{1 + \widehat{\lambda_j(\gamma^{(l)})} \cdot M(f_k^{(l)};\gamma^{(l)}) \cdot \ r_{i,k}^{e}}, \tag{5}$$

where $\widehat{\lambda_j(\gamma^{(l)})}$ are the Lagrange multipliers associated with the conditional Euler equation constraints, and solve:

$$\widehat{\lambda_j(\gamma^{(l)})} = \underset{\lambda_j \in \mathbb{R}^l}{\arg\max} \sum_{k=1}^{T} \omega_{j,k} \log\left[1 + \lambda_j \cdot M(f_k^{(l)};\gamma^{(l)}) \cdot r_{i,k}^{e}\right]. \tag{6}$$

⇒ For each investor type, the SEL procedure delivers a $(T \times T)$ transition matrix $\left\{\widehat{p}_{j,k}^{(l)}\left(\gamma^{(l)}\right)\right\}_{j,k=1}^{T}$, for each value of the parameter vector $\gamma^{(l)}$ (price-consistent beliefs)

The SEL Estimator: Solution

- In the absence of Euler restrictions: $\widehat{p_{j,k}^{(l)}} = \omega_{j,k}$ – recovered beliefs coincide with physical DGP.
- In the presence of Euler restrictions: the recovered beliefs are distorted relative to the physical DGP, $\forall j, k \in \{1, \ldots, T\}$,

$$\widehat{p_{j,k}^{(l)}(\gamma^{(l)})} = \frac{\omega_{j,k}}{1 + \widehat{\lambda_j(\gamma^{(l)})} \cdot M(f_k^{(l)}; \gamma^{(l)}) \cdot \; r_{i,k}^e}, \tag{5}$$

where $\widehat{\lambda_j(\gamma^{(l)})}$ are the Lagrange multipliers associated with the conditional Euler equation constraints, and solve:

$$\widehat{\lambda_j(\gamma^{(l)})} = \underset{\lambda_j \in \mathbb{R}^l}{\arg\max} \sum_{k=1}^{T} \omega_{j,k} \log \left[1 + \lambda_j \cdot M(f_k^{(l)}; \gamma^{(l)}) \cdot r_{i,k}^e \right]. \tag{6}$$

⇒ For each investor type, the SEL procedure delivers a $(T \times T)$ transition matrix $\left\{ \widehat{p}_{j,k}^{(l)} \left(\gamma^{(l)} \right) \right\}_{j,k=1}^{T}$, for each value of the parameter vector $\gamma^{(l)}$ (price-consistent beliefs)

The SEL Estimator: Solution cont'd

- the SEL estimator of $\gamma^{(l)}$ is defined as:

$$\widehat{\gamma^{(l)}}^{SEL} = \underset{\gamma^{(l)} \in \Theta}{\text{argmin}} \underbrace{\sum_{j=1}^{T} \sum_{k=1}^{T} \omega_{j,k} \log\left(\frac{\omega_{j,k}}{\widehat{p_{j,k}^{(l)}(\gamma^{(l)})}}\right)}_{SEL\left(\gamma^{(l)}\right)} . \tag{7}$$

⇒ the approach searches for $\gamma^{(l)}$ that minimizes the profile relative entropy over the admissible parameter set.

The SEL Estimator: Solution cont'd

- the SEL estimator of $\gamma^{(l)}$ is defined as:

$$\widehat{\gamma^{(l)}}^{SEL} = \underset{\gamma^{(l)} \in \Theta}{\text{argmin}} \underbrace{\sum_{j=1}^{T} \sum_{k=1}^{T} \omega_{j,k} \log\left(\frac{\omega_{j,k}}{\widehat{p_{j,k}^{(l)}(\gamma^{(l)})}}\right)}_{SEL(\gamma^{(l)})} . \tag{7}$$

⇒ the approach searches for $\gamma^{(l)}$ that minimizes the profile relative entropy over the admissible parameter set.

Outline

Appendix

Implementation

- Inputs to SEL:
  - Pricing kernel: the SDF of a representative type-l investor is exponentially affine in the return on her total wealth portfolio:

$$M\left(F_{t+1}^{(l)};\gamma^{(l)}\right)=e^{-\gamma^{(l)}\log\left(R_{W,t+1}^{(l)}\right)},\tag{8}$$

where $R_{W,t+1}^{(l)}=x_{B,t}^{(l)}R_{B,t+1}+x_{SG,t}^{(l)}R_{SG,t+1}+(1-x_{B,t}^{(l)}-x_{SG,t}^{(l)})R_{F,t}$

  - Conditioning variables: exponentially smoothed past returns on these assets.
  - Test assets: Returns on these assets in excess of T-bill.
- Data: Quarterly U.S. data 1972:Q1–2018:Q4.
- Investor Types:
  - Type I: High $x_{B,t}^{(l)}$ (commonly studied in economic models).
  - Type II: High $x_{SG,t}^{(l)}$ (often viewed as public market analog of private venture backed companies).

Implementation

- Inputs to SEL:
  - Pricing kernel: the SDF of a representative type-l investor is exponentially affine in the return on her total wealth portfolio:

$$M\left(F_{t+1}^{(l)};\gamma^{(l)}\right)=e^{-\gamma^{(l)}\log\left(R_{W,t+1}^{(l)}\right)},\tag{8}$$

    where $R_{W,t+1}^{(l)}=x_{B,t}^{(l)}R_{B,t+1}+x_{SG,t}^{(l)}R_{SG,t+1}+(1-x_{B,t}^{(l)}-x_{SG,t}^{(l)})R_{F,t}$
  - Conditioning variables: exponentially smoothed past returns on these assets.
  - Test assets: Returns on these assets in excess of T-bill.
- Data: Quarterly U.S. data 1972:Q1–2018:Q4.
- Investor Types:
  - Type I: High $x_{B,t}^{(l)}$ (commonly studied in economic models).
  - Type II: High $x_{SG,t}^{(l)}$ (often viewed as public market analog of private venture backed companies).

Implementation

- Inputs to SEL:
  - Pricing kernel: the SDF of a representative type-I investor is exponentially affine in the return on her total wealth portfolio:

$$M\left(F^{(I)}_{t+1};\gamma^{(I)}\right) = e^{-\gamma^{(I)}\log\left(R^{(I)}_{W,t+1}\right)}, \tag{8}$$

    where $R^{(I)}_{W,t+1} = x^{(I)}_{B,t}R_{B,t+1} + x^{(I)}_{SG,t}R_{SG,t+1} + (1 - x^{(I)}_{B,t} - x^{(I)}_{SG,t})R_{F,t}$
  - Conditioning variables: exponentially smoothed past returns on these assets.
  - Test assets: Returns on these assets in excess of T-bill.
- Data: Quarterly U.S. data 1972:Q1–2018:Q4.
- Investor Types:
  - Type I: High $x^{(I)}_{B,t}$ (commonly studied in economic models).
  - Type II: High $x^{(I)}_{SG,t}$ (often viewed as public market analog of private venture backed companies).

Implementation

- Inputs to SEL:
  - Pricing kernel: the SDF of a representative type-I investor is exponentially affine in the return on her total wealth portfolio:

$$M\left(F_{t+1}^{(I)};\gamma^{(I)}\right)=e^{-\gamma^{(I)}\log\left(R_{W,t+1}^{(I)}\right)}, \tag{8}$$

    where $R_{W,t+1}^{(I)}=x_{B,t}^{(I)}R_{B,t+1}+x_{SG,t}^{(I)}R_{SG,t+1}+(1-x_{B,t}^{(I)}-x_{SG,t}^{(I)})R_{F,t}$
  - Conditioning variables: exponentially smoothed past returns on these assets.
  - Test assets: Returns on these assets in excess of T-bill.
- Data: Quarterly U.S. data 1972:Q1–2018:Q4.
- Investor Types:
  - Type I: High $x_{B,t}^{(I)}$ (commonly studied in economic models).
  - Type II: High $x_{SG,t}^{(I)}$ (often viewed as public market analog of private venture backed companies).

Implementation

- Inputs to SEL:
  - Pricing kernel: the SDF of a representative type-I investor is exponentially affine in the return on her total wealth portfolio:

$$M\left(F_{t+1}^{(I)};\gamma^{(I)}\right) = e^{-\gamma^{(I)}\log\left(R_{W,t+1}^{(I)}\right)}, \quad (8)$$

    where $R_{W,t+1}^{(I)} = x_{B,t}^{(I)} R_{B,t+1} + x_{SG,t}^{(I)} R_{SG,t+1} + (1 - x_{B,t}^{(I)} - x_{SG,t}^{(I)}) R_{F,t}$
  - Conditioning variables: exponentially smoothed past returns on these assets.
  - Test assets: Returns on these assets in excess of T-bill.
- Data: Quarterly U.S. data 1972:Q1–2018:Q4.
- Investor Types:
  - Type I: High $x_{B,t}^{(I)}$ (commonly studied in economic models).
  - Type II: High $x_{SG,t}^{(I)}$ (often viewed as public market analog of private venture backed companies).

Implementation

- Inputs to SEL:
  - Pricing kernel: the SDF of a representative type-I investor is exponentially affine in the return on her total wealth portfolio:

$$M\left(F_{t+1}^{(I)};\gamma^{(I)}\right)=e^{-\gamma^{(I)}\log\left(R_{W,t+1}^{(I)}\right)},\tag{8}$$

    where $R_{W,t+1}^{(I)}=x_{B,t}^{(I)}R_{B,t+1}+x_{SG,t}^{(I)}R_{SG,t+1}+(1-x_{B,t}^{(I)}-x_{SG,t}^{(I)})R_{F,t}$
  - Conditioning variables: exponentially smoothed past returns on these assets.
  - Test assets: Returns on these assets in excess of T-bill.
- Data: Quarterly U.S. data 1972:Q1–2018:Q4.
- Investor Types:
  - Type I: High $x_{B,t}^{(I)}$ (commonly studied in economic models).
  - Type II: High $x_{SG,t}^{(I)}$ (often viewed as public market analog of private venture backed companies).

Estimated Risk Preferences

Table: Estimates of Preference Parameter, 1972$Q1$ – 2018$Q4$

	$\gamma^{(i)}$		LR Test ($H_0 : \gamma^{(I)} = \gamma^{II}$)	
	Cond. Set 1	Cond. Set 2	Cond. Set 1	Cond. Set 2
Type-I: B^{FF25}, SG^{FF25}	1.8 (0.89)	2.1 (0.84)	3.59 [.058]	4.62 [.032]
Type-I: B^{FF6}, SG^{FF6}	2.0 (0.89)	2.4 (0.92)	2.55 [.110]	3.26 [.071]
Type-II: B^{FF25}, SG^{FF25}	0.2 (0.46)	0.3 (0.48)	-	-
Type-II: B^{FF6}, SG^{FF6}	0.7 (0.56)	0.8 (0.57)	-	-

- Type-II investors require a smaller price of risk compared to the Type-I investors – difference is statistically and economically significant.

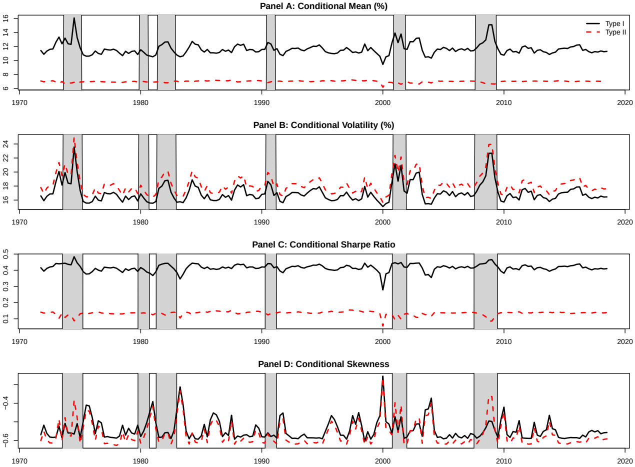
Estimated Risk Preferences

Table: Estimates of Preference Parameter, 1972$Q1$ – 2018$Q4$

	$\gamma^{(i)}$		LR Test ($H_0 : \gamma^{(I)} = \gamma^{II}$)	
	Cond. Set 1	Cond. Set 2	Cond. Set 1	Cond. Set 2
Type-I: B^{FF25}, SG^{FF25}	1.8 (0.89)	2.1 (0.84)	3.59 [.058]	4.62 [.032]
Type-I: B^{FF6}, SG^{FF6}	2.0 (0.89)	2.4 (0.92)	2.55 [.110]	3.26 [.071]
Type-II: B^{FF25}, SG^{FF25}	0.2 (0.46)	0.3 (0.48)	-	-
Type-II: B^{FF6}, SG^{FF6}	0.7 (0.56)	0.8 (0.57)	-	-

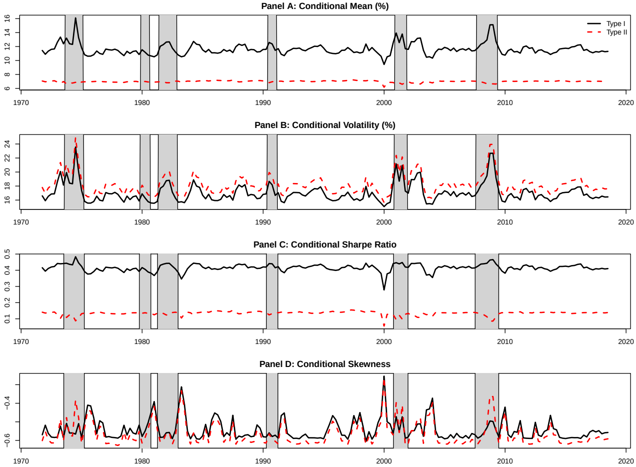
- Type-II investors require a smaller price of risk compared to the Type-I investors – difference is statistically and economically significant.

Beliefs About Aggregate Market return



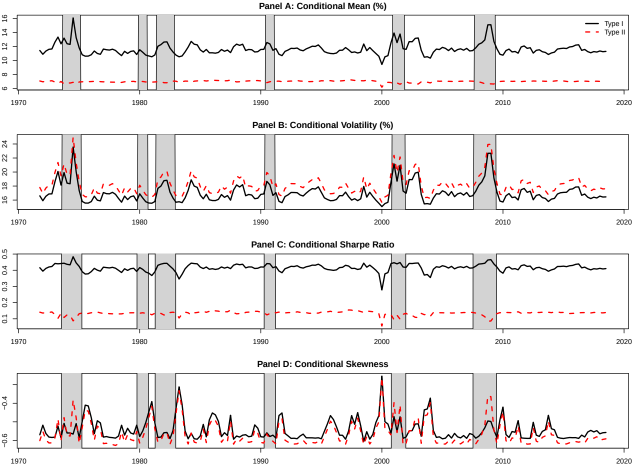
- Type-I investors: countercyclical expected market returns and Sharpe ratio – correlations 51.6% and 37.9%, respectively, with recession dummy.
- Type-II investors: procyclical expected market returns and Sharpe ratio – correlation -44.8% and -44.5%, respectively, with recession dummy.
- Disagreement in expected returns, but not volatility.

Beliefs About Aggregate Market return



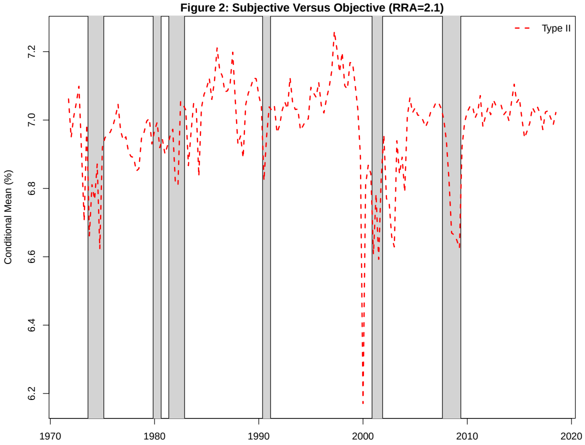
- Type-I investors: countercyclical expected market returns and Sharpe ratio – correlations 51.6% and 37.9%, respectively, with recession dummy.
- Type-II investors: procyclical expected market returns and Sharpe ratio – correlation -44.8% and -44.5%, respectively, with recession dummy.
- Disagreement in expected returns, but not volatility.

Beliefs About Aggregate Market return



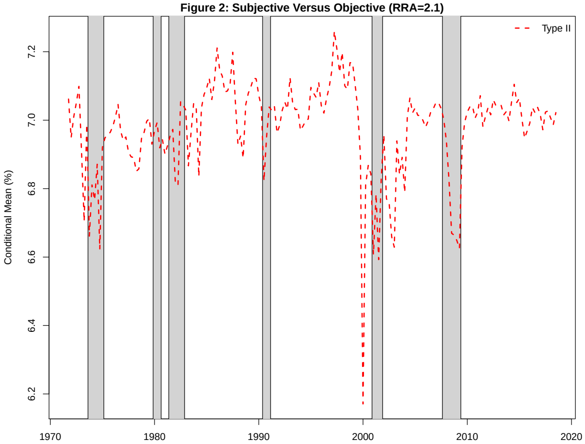
- Type-I investors: countercyclical expected market returns and Sharpe ratio – correlations 51.6% and 37.9%, respectively, with recession dummy.
- Type-II investors: procyclical expected market returns and Sharpe ratio – correlation -44.8% and -44.5%, respectively, with recession dummy.
- Disagreement in expected returns, but not volatility.

Beliefs of Type II Investors



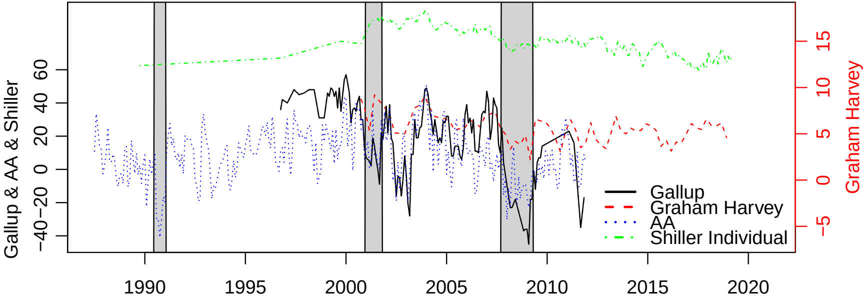
- The beliefs of the Type II investor appear flat because both the level and the variability of the expected market returns are a lot higher for the Type I investor compared to the Type II investor.

Beliefs of Type II Investors



- The beliefs of the Type II investor appear flat because both the level and the variability of the expected market returns are a lot higher for the Type I investor compared to the Type II investor.

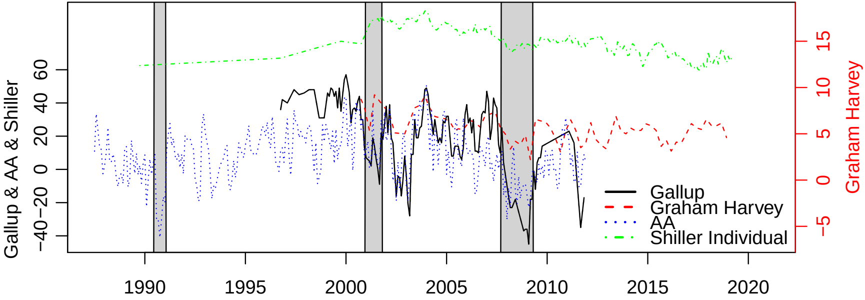
Surveys with Procyclical Expectations About the Stock Market: Greenwood and Shleifer (2014)



⇒ Point to procyclical beliefs about expected market return: correlation with recession dummy

- Gallup: -53.6%
- AA: -30.7%
- Graham-Harvey (GH): -17.3%
- *Shiller individual: 1.1%*

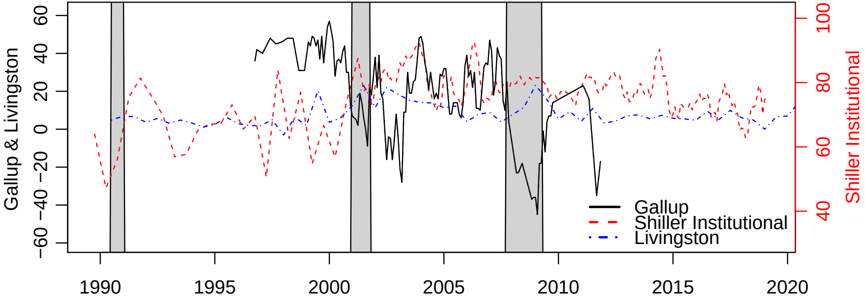
Surveys with Procyclical Expectations About the Stock Market: Greenwood and Shleifer (2014)



⇒ Point to procyclical beliefs about expected market return: correlation with recession dummy

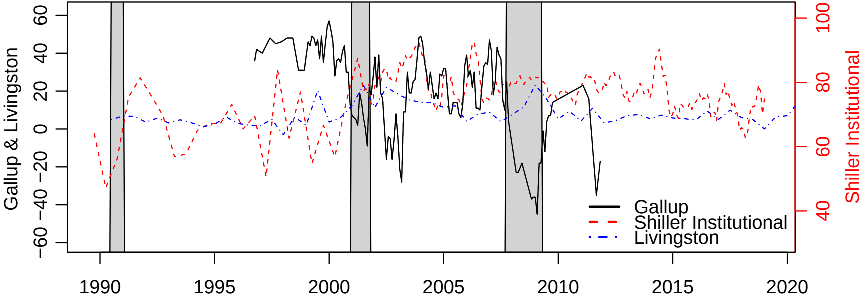
- Gallup: -53.6%
- AA: -30.7%
- Graham-Harvey (GH): -17.3%
- *Shiller individual: 1.1%*

Surveys with Countercyclical Expectations



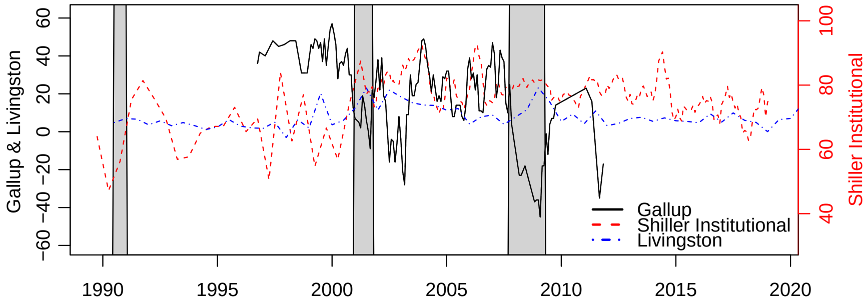
- Shiller's Institutional investor survey points to countercyclical beliefs about expected market return – correlation 15% with recession dummy and -11.5% with Gallup survey.
- Livingston survey also points to countercyclical beliefs – correlation 24.4% with recession dummy.

Surveys with Countercyclical Expectations



- Shiller's Institutional investor survey points to countercyclical beliefs about expected market return – correlation 15% with recession dummy and -11.5% with Gallup survey.
- Livingston survey also points to countercyclical beliefs – correlation 24.4% with recession dummy.

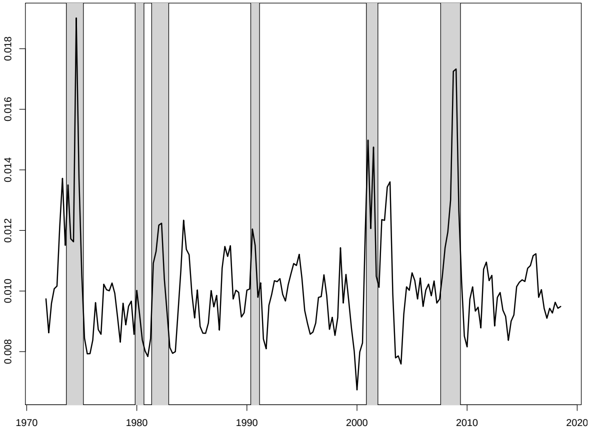
Surveys with Countercyclical Expectations



- Shiller's Institutional investor survey points to countercyclical beliefs about expected market return – correlation 15% with recession dummy and -11.5% with Gallup survey.
- Livingston survey also points to countercyclical beliefs – correlation 24.4% with recession dummy.

Beliefs Disagreement

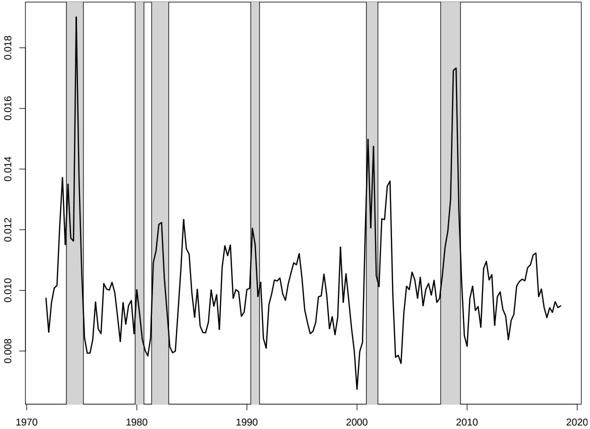
KLIC Divergence Between Type I and II Investors



- ⇒ the two investor types disagree more with each other during economic downturns – correlation between the KLIC divergence and the recession dummy is 52.6%.

Beliefs Disagreement

KLIC Divergence Between Type I and II Investors



- ⇒ the two investor types disagree more with each other during economic downturns – correlation between the KLIC divergence and the recession dummy is 52.6%.

Outline

Appendix

Conclusion

- A novel methodology for the recovery of heterogeneous beliefs and risk preferences of different investor types from observed asset prices, without relying on specific assumptions about beliefs or rationality.
- Main Results:
  - I. The presence of at least two types of investors, differing both in terms of risk preferences and beliefs.
  - II. Investors who primarily invest in large cap equities are more risk averse and believe that the expected stock market return is countercyclical.
  - III Investors who invest heavily in small-growth equities are substantially less risk averse, with procyclical beliefs about the expected market return
  - ⇒ reconciliation of the seemingly contradictory evidence of procyclical beliefs about the market return in survey data versus the countercyclical expected returns implied by rational expectations representative agent models.
- Methodology is quite general and may be applied to encompass other asset classes, such as currencies, commodities, private equity, venture capital, and derivative securities.

Conclusion

- A novel methodology for the recovery of heterogeneous beliefs and risk preferences of different investor types from observed asset prices, without relying on specific assumptions about beliefs or rationality.
- Main Results:
    I. The presence of at least two types of investors, differing both in terms of risk preferences and beliefs.
    II. Investors who primarily invest in large cap equities are more risk averse and believe that the expected stock market return is countercyclical.
    III Investors who invest heavily in small-growth equities are substantially less risk averse, with procyclical beliefs about the expected market return
    ⇒ reconciliation of the seemingly contradictory evidence of procyclical beliefs about the market return in survey data versus the countercyclical expected returns implied by rational expectations representative agent models.
- Methodology is quite general and may be applied to encompass other asset classes, such as currencies, commodities, private equity, venture capital, and derivative securities.

Conclusion

- A novel methodology for the recovery of heterogeneous beliefs and risk preferences of different investor types from observed asset prices, without relying on specific assumptions about beliefs or rationality.
- Main Results:
  - I. The presence of at least two types of investors, differing both in terms of risk preferences and beliefs.
  - II. Investors who primarily invest in large cap equities are more risk averse and believe that the expected stock market return is countercyclical.
  - III Investors who invest heavily in small-growth equities are substantially less risk averse, with procyclical beliefs about the expected market return
  - ⇒ reconciliation of the seemingly contradictory evidence of procyclical beliefs about the market return in survey data versus the countercyclical expected returns implied by rational expectations representative agent models.
- Methodology is quite general and may be applied to encompass other asset classes, such as currencies, commodities, private equity, venture capital, and derivative securities.

Conclusion

- A novel methodology for the recovery of heterogeneous beliefs and risk preferences of different investor types from observed asset prices, without relying on specific assumptions about beliefs or rationality.
- Main Results:
  - I. The presence of at least two types of investors, differing both in terms of risk preferences and beliefs.
  - II. Investors who primarily invest in large cap equities are more risk averse and believe that the expected stock market return is countercyclical.
  - III Investors who invest heavily in small-growth equities are substantially less risk averse, with procyclical beliefs about the expected market return
  - ⇒ reconciliation of the seemingly contradictory evidence of procyclical beliefs about the market return in survey data versus the countercyclical expected returns implied by rational expectations representative agent models.
- Methodology is quite general and may be applied to encompass other asset classes, such as currencies, commodities, private equity, venture capital, and derivative securities.

Conclusion

- A novel methodology for the recovery of heterogeneous beliefs and risk preferences of different investor types from observed asset prices, without relying on specific assumptions about beliefs or rationality.
- Main Results:
  - I. The presence of at least two types of investors, differing both in terms of risk preferences and beliefs.
  - II. Investors who primarily invest in large cap equities are more risk averse and believe that the expected stock market return is countercyclical.
  - III Investors who invest heavily in small-growth equities are substantially less risk averse, with procyclical beliefs about the expected market return
  - ⇒ reconciliation of the seemingly contradictory evidence of procyclical beliefs about the market return in survey data versus the countercyclical expected returns implied by rational expectations representative agent models.
- Methodology is quite general and may be applied to encompass other asset classes, such as currencies, commodities, private equity, venture capital, and derivative securities.

Conclusion

- A novel methodology for the recovery of heterogeneous beliefs and risk preferences of different investor types from observed asset prices, without relying on specific assumptions about beliefs or rationality.
- Main Results:
  - I. The presence of at least two types of investors, differing both in terms of risk preferences and beliefs.
  - II. Investors who primarily invest in large cap equities are more risk averse and believe that the expected stock market return is countercyclical.
  - III Investors who invest heavily in small-growth equities are substantially less risk averse, with procyclical beliefs about the expected market return
  - ⇒ reconciliation of the seemingly contradictory evidence of procyclical beliefs about the market return in survey data versus the countercyclical expected returns implied by rational expectations representative agent models.
- Methodology is quite general and may be applied to encompass other asset classes, such as currencies, commodities, private equity, venture capital, and derivative securities.

Outline

5 Appendix

▸ Appendix

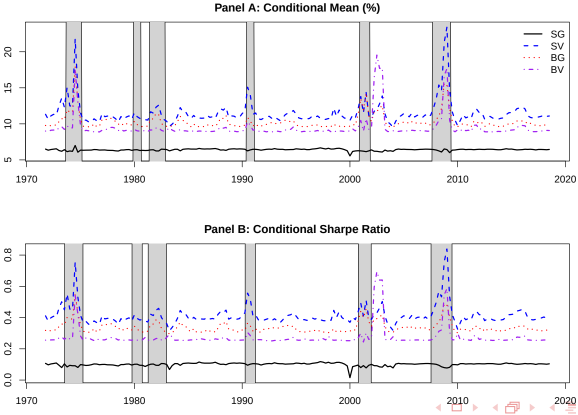
Robustness Checks

Table: Beliefs and Risk Preferences Across Investors with Different Optimal Portfolios

	$\gamma^{(l)}$	$\rho\left(E^{\mathbb{P}^{(l)}}(R_m), I_{rec}\right)$ %	$\rho\left(Vol^{\mathbb{P}^{(l)}}(R_m), I_{rec}\right)$ %	$\rho\left(SR^{\mathbb{P}^{(l)}}(R_m), I_{rec}\right)$ %
Panel A: $R_W = R_n$				
$n = SG$	0.3	−25.9	53.0	−29.0
$n = SV$	2.3	56.0	59.8	56.3
$n = BG$	2.1	56.9	56.8	58.4
$n = BV$	2.0	26.3	40.2	20.8
Panel B: $R_W = xR_f + (1-x)R_{SG}$				
$x = 0.0$	0.30	−25.9	53.0	−29.0
$x = 0.2$	0.35	−30.9	53.0	−29.8
$x = 0.4$	0.45	−32.8	53.0	−29.8
$x = 0.6$	0.60	−38.9	53.0	−31.0
$x = 0.9$	0.95	−54.0	52.8	−37.3
Panel C: $R_W = 0.2R_f + xR_{SG} + (1-0.2-x)R_m$				
$x = 0.8$	0.35	−30.9	53.0	−29.8
$x = 0.6$	0.60	15.3	53.2	−14.4
$x = 0.4$	1.00	40.3	53.5	11.0
$x = 0.2$	1.70	48.7	54.0	34.2
$x = 0.0$	2.90	51.9	54.6	44.6

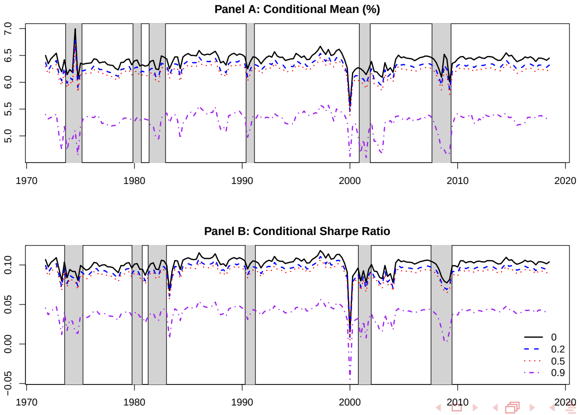
Robustness Checks cont'd

Beliefs About the Stock Market Across Investors with Different Optimal Portfolios: $R_W = R_n, n = SG, SV, BG, BV$



Robustness Checks cont'd

Beliefs About the Stock Market Across Investors with Different Optimal Portfolios: $R_W = xR_f + (1 - x)R_{SG}$



Robustness Checks cont'd

Beliefs About the Stock Market Across Investors with Different Optimal Portfolios: $R_W = 0.2R_f + xR_{SG} + (1 - 0.2 - x)R_m$