Neglected Information: What is in the Shape of the Return Distribution?

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Introduction

what is a proxy for investor information in the return distribution?

- how to measure the proxy?
- is the investor information priced?
- what does it tell us about the future returns?

This Paper: a New Entropy-Based Measure

- **event-based analysis:** information publicly released
- **normal-divergence**
  - distance of two distributions in “entropy-space"
  - between the **true** return distribution before an event, and normal distribution with the same standard deviation
- significant (negative) explanatory power for price jump at the event
- robust to many controls
  - higher moments of return distribution, earning surprise, option-implied-vol
- normal-divergence $\uparrow\rightarrow$ information $\uparrow\rightarrow$ jump at event $\downarrow$

Probability versus Entropy

- both measures of **information**
  - *probability* is the measure to capture the randomness of one data point
  - *entropy* is the measure to capture the randomness of a distribution
- information $\leftrightarrow$ probability

Normal-Divergence. an Entropy-Based Distance

- normal distribution: minimum information $\rightarrow$ highest entropy
- private information reflected in the pre-event return distribution
  $\Rightarrow$ high deviation from normal distribution
- **high normal divergence** $\equiv$ a lot of information in the distribution
- the information about the event is priced in $\Rightarrow$ smaller jump at event

Normal-Divergence. an Entropy-Based Distance

- normal distribution: minimum information $\rightarrow$ highest entropy
- private information reflected in the pre-event return distribution
  $\Rightarrow$ high deviation from normal distribution
- **high normal divergence** $\equiv$ a lot of information in the distribution
- the information about the event is priced in $\Rightarrow$ smaller jump at event
- consider **earning announcements**
  - return **distribution** prior to announcement reflects investor information
  - not captured by the commonly used moments of the return distribution
  - not fully priced by options either!
  - **but**, internalized by the investors $\rightarrow$ affects the price jump at announcement

Event Risk

- relevant events for stock market
  - firm earning announcements
  - FED announcements
  - clinical trial outcomes
  - SEC announcements: first crypto ETF (10/18/2021!)
- **event-driven strategies**
- **risk management**
  - tail risk: level vs *uncertainty*

Example
How is Information Related to the Price Jump?

- two periods, $t = 0, 1$
- single asset
  - $t = 0$ price p_0
  - $t = 1$ price distribution $F(p_1)$

$$p_1 = p_0 + \begin{cases} x_1 & \text{with probability } p \\ -x_2 & \text{with probability } 1 - p \end{cases}$$

- asset price martingale: $px_1 - (1 - p)x_2 = 0$ $(x_1, x_2 > 0)$

Example
How is Information Related to the Price Jump?

- expected absolute price difference: $EJ = \mathbb{E}[|p_1 - p_0|] = 2px_1$
- standard deviation of price difference: $\sigma = \sqrt{\frac{p}{1-p}}x_1$
- **normalized expected absolute price difference**

$$NEJ = \frac{EJ}{\sigma} = 2\sqrt{p(1-p)}$$

- when $p = 0.5$ (no information): $NEJ = 1$
- when $p = 0$ or $p = 1$ (a lot of information): $NEJ = 0$

Example.
How is Information Related to the Price Jump?

- $D_{KL}(.||.)$ = KL divergence of probability distributions $(\frac{1}{2}, \frac{1}{2})$ and $(p, 1-p)$

$$D_{KL}(0.5||p) = -\left(1 + \log\left(\sqrt{p(1-p)}\right)\right)$$

$$NEJ = 2 \times 2^{-(1+D_{KL}(0.5||p))}$$

$$\log(NEJ) \equiv -\underbrace{D_{KL}(0.5||p)}_{\text{information}}$$

the more information, the smaller the jump!

Outline

Normal-Divergence, χ_h

Definition

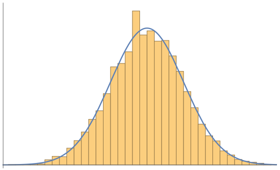
Let $X \sim F(X)$, with mean μ, variance σ^2, and entropy $\xi = h(x)$.
$N(\mu, \sigma^2)$ is the normal distribution with the same mean and variance as $F(x)$.
Normal-Divergence (χ_h) *is defined as the KL-divergence between N and F:*

$$\chi_h = D_{KL}(N||F)$$

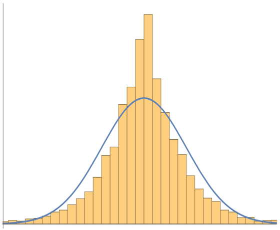
- χ_h measures the "information distance" between F and N, keeping the variance constant
- our models use N to approximate F: missing information due to this approximation?

◂ definitions

Information and Price Jump



$\Rightarrow \chi_h \sim 0$ $\Rightarrow$ **large** price jump



$\Rightarrow \chi_h \uparrow$ $\Rightarrow$ **small** price jump

Outline

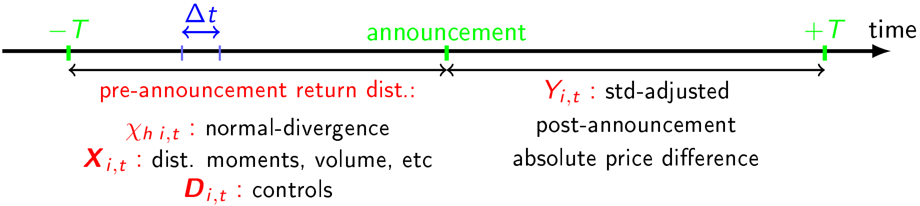
Data and Specification

stock price and characteristics data: Thomson Reuters I/B/E/S and the NYSE TAQ database. *S*&*P*500 firms, Jan 05 - Mar 16.

- 710 stocks, in 4750 stock-announcement pairs

Specification

$$Y_{i,t} = \alpha + \beta\chi_{h\ i,t} + \gamma\boldsymbol{X}_{i,t} + \kappa\boldsymbol{D}_{i,t} + \epsilon_{i,t}$$

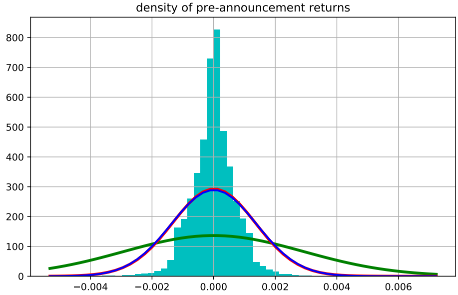


$$Y_{i,t} = \alpha + \beta \chi_{h\,i,t} + \gamma \boldsymbol{X}_{i,t} + \kappa \boldsymbol{D}_{i,t} + \epsilon_{i,t}$$

- time interval before and after announcement: T
  data of frequency: Δt
- $Y_{i,t} = \frac{|\mathbb{E}[p_{t+1}] - \mathbb{E}[p_t]|}{\sigma_t}$
  absolute value of the difference in price within interval T after and before the announcement, divided by the standard deviation of the return dist. before the announcement
- $\chi_{h\,i,t}$
  Normal-Divergence
- $X_{i,t}$
  earning surprise, std/skewness/kurtosis of pre-announcement return dist., market cap, average price/total traded volume pre-announcement
  - *earning surprise:* log absolute value of difference between weighted average analyst predictions, and the announced earning per share
- $D_{i,t}$
  stock, sector, month+year of announcement, quarter fixed effects

Kernel Density Estimation

- χ_h requires return distribution for each stock-announcement pair before and after the announcement
- Kernel Density Estimation (kde)
- 10-fold cross validation to pick the optimal bandwidth
- 5000 randomly chosen observations per dist



Test of Unimodality

- focus on observations with unimodal pre-announcement return distributions
- caveat: well-known test (e.g. DIP) hardly classify any dist as unimodal
- heuristic approach:
  - kde estimation
  - find the maximum peak.
  - ratio of the density of the two largest peaks $< \bar{f} = \frac{1}{100} \Rightarrow$ unimodal

Data. Summary Statistics

$\Delta t = 1s$, on $T = 4$ hour interval around announcement

	mean	std	Q1	median	Q3	$\hat{x}$
χ_h	0.022	0.028	0.001	0.010	0.031	0.766
annualized return std	1.112	1.071	0.595	0.820	1.216	1.039
earning surprise (log)	-3.133	1.988	-4.033	-3.041	-2.129	-1.576
market cap (million)	20.922	37.752	4.465	9.285	19.924	0.554
average price	48.695	43.663	25.057	40.247	60.784	1.115
volume	264.612	222.848	154.997	192.330	276.089	1.187
return kurtosis	9.402	34.813	3.415	4.509	6.205	0.270
return skewness	0.018	1.048	-0.076	-0.003	0.064	0.018
option implied jump	0.050	0.033	0.029	0.042	0.060	1.485
std adjusted announcement return	0.021	0.026	0.006	0.013	0.025	0.812

Announcement Return: Effect of Normal-Divergence χ_h

$\Delta t = 1s, T = 4$ hour interval

	(1)	(2)	(3)	(4)	(5)
constant	0.539 (47.74***)	0.562 (43.60***)	0.538 (32.10***)	0.530 (21.09***)	0.562 (20.88***)
χ_h	-0.048 (-5.31***)			-0.049 (-5.19***)	-0.055 (-5.62***)
return std		-0.058 (-6.46***)		-0.032 (-3.41***)	-0.033 (-3.39***)
earning surprise (log)			0.023 (2.51*)	0.027 (3.02**)	0.027 (3.05**)
market cap				0.080 (8.68***)	0.084 (8.84***)
average price				0.037 (3.71***)	0.032 (3.20**)
volume					-0.024 (-2.61**)
return kurtosis					0.022 (2.25*)
return skewness					0.020 (2.21*)
N	4742	4742	4742	4742	4742
Adj. R^2	0.57%	0.85%	0.11%	3.68%	4.03%

Announcement Return: Effect of Normal-Divergence χ_h

$\Delta t = 1s, T = 4$ hour interval; stock and time fixed effects

		(6)	(7)	(8)	(9)	(10)
constant		0.470	0.501	0.479	0.625	0.289
		(19.70***)	(21.50***)	(8.30***)	(5.08***)	(2.11*)
χ_h		-0.038	-0.041	-0.026	-0.038	-0.043
		(-4.08***)	(-3.92***)	(-2.31*)	(-3.49***)	(-3.67***)
return std		-0.046	-0.039	-0.042	-0.042	-0.043
		(-4.95***)	(-3.98***)	(-3.62***)	(-4.11***)	(-3.50***)
earning surprise (log)		0.056	0.031	0.045	0.031	0.046
		(6.23***)	(3.44***)	(4.71***)	(3.45***)	(4.89***)
market cap		0.059	0.083	0.029	0.084	0.019
		(5.91***)	(8.86***)	(1.19)	(8.91***)	(0.79)
average price		0.025	0.039	-0.001	0.029	0.018
		(2.50*)	(3.76***)	(-0.08)	(2.03*)	(1.11)
volume						-0.042
						(-3.21**)
return kurtosis						0.021
						(2.27*)
return skewness						0.021
						(2.43*)
controls	stock			✓		✓
	sectors	✓				✓
	quarter	✓	✓	✓	✓	✓
	year month		✓	✓	✓	✓
	$<$ 5$ stocks				✓	✓
N		4742	4742	4742	4742	4742
Adj R^2		20.77%	5.65%	29.48%	5.65%	29.85%

Option-Implied-Jump: Option Prices

- option prices: natural instrument of financial markets to learn about uncertainty/volatility surrounding the announcements
- Johannes and Dubinsky (2006): use option prices to do inference about uncertainty embedded in earnings announcements
  - **jump volatility risk premium**
  - Merton jump-diffusion model
- **time-series estimator,** $\sigma^{\mathbb{Q}}_{t,time}$
  uses changes in implied volatility before and after announcement

$$\begin{aligned} \sigma^{BS}_{t,T} &= \text{Black-Scholes implied vol. at date } t \text{ of option expiring in } T \text{ days} \\ (\sigma^{\mathbb{Q}}_{t,time})^2 &= T\left((\sigma^{BS}_{t,T})^2 - (\sigma^{BS}_{t+1,T-1})^2\right) \\ J_{t+1} &= \frac{r_{t+1}}{\sqrt{(\sigma^{\mathbb{Q}}_{t,time})^2 + \sigma^2/252}} \end{aligned}$$

- r_{t+1}: return at announcement
- use 90-day call options

Announcement Return: Control for option-Implied-Jump

		(11)	(12)	(13)	(14)	(15)
constant		0.358	0.395	0.322	0.492	0.303
		(22.56***)	(22.71***)	(11.01***)	(6.85***)	(2.82**)
χ_h			-0.045	-0.054	-0.022	-0.037
			(-5.07***)	(-5.69***)	(-2.01*)	(-3.26**)
return std				-0.128	-0.062	-0.068
				(-11.82***)	(-5.22***)	(-5.36***)
earning surprise (log)				0.011	0.040	0.042
				(1.25)	(4.22***)	(4.41***)
market cap				0.090	0.039	0.035
				(9.82***)	(1.65)	(1.48)
average price				0.043		
				(4.41***)		
volume				-0.008		-0.044
				(-0.82)		(-3.37***)
return kurtosis				0.038		0.024
				(4.03***)		(2.65**)
return skewness				0.020		0.020
				(2.24*)		(2.37*)
option implied jump		0.097	0.096	0.183	0.068	0.072
		(10.92***)	(10.81***)	(18.09***)	(5.91***)	(6.20***)
controls	stock				✓	✓
	sectors					✓
	quarter					✓
	year month				✓	✓
	$<$ 5$ stocks					✓
N		4742	4742	4742	4742	4742
Adj R^2		2.43%	2.94%	10.22%	30.09%	30.51%

Future Work: Other Controls

- put-call ratio: *sentiment*
  - number of traded put options divided by number of traded call options
  - rising put-call ratio
    - demand to sell $>$ demand to buy
    - bearish sentiment building in the market
- Sortino ratio:
  - modified Sharpe ratio: only penalizes returns below required rate of return T
  - *semi-deviation:*

    average of the squared deviations of values less than the mean (DR)

$$\frac{R - T}{DR}$$

Outline

How to Think Through the Empirical Finding?

- **observation.** true return distribution is not normal
- we use normal distribution to approximate the return distribution
- how to theoretically measure the "information distance"?
- use a **change-of-measure**
  - same technique as Black-Scholes formula!

◂ result

Model. Environment

- three periods, $t = 0, 1, 2$
- one risky asset, measure $\bar{x}$ held by the investors

Investors

- unit measure, who live in a large economy
- consume only at $t = 2$
- choose how much asset to hold at $t = 0$ (p_0)
- have to sell (cover) z fraction of his portfolio at $t = 1$ ($\tilde{p}_1$), and complementary fraction at $t = 2$ (p_2)

Model. Environment

Prices

- *interesting object is date* 1 *price:* $\tilde{p}_1 = \hat{p}_1 + r$
- $r \sim F(r)$: symmetric, determined in the larger economy to reflect the existing information, invariant to investor decision
- investor demand determines mean of the price distribution at $t = 1$: $\hat{p}_1$

jump in the expected price: $r_2 = p_2 - \mathbb{E}_F[\tilde{p}_1] = p_2 - \hat{p}_1$

Model. Generalized Portfolio Optimization

$$\max_{c,x} \mathbb{E}_F[1 - e^{S(c)}]$$
$$\text{s.t. } c = \rho x$$

- $S'(c) < 0$ and $S'^2(c) + S''(c) > 0$, and return $\rho \sim F(.)$ with (μ, σ^2)
- use Radon-Nikodym derivative $\frac{dF}{dN} \rightarrow$ change of measure on investor objective

$$\min_x \; \mathbb{E}_N \left[\log\left(u(S(c))\right) + \log\left(\frac{dF}{dN}\right) \right] + \frac{1}{2} Var_N \left[\log\left(u(S(c))\right) + \log\left(\frac{dF}{dN}\right) \right]$$

- investor optimal portfolio solves

$$\mathbb{E}_N \left[\frac{dS\big(c(x;\rho)\big)}{dx} \right] (1 + D_{KL}(N||F)) + \frac{1}{2}\frac{d}{dx} Var_N \left[S\big(c(x;\rho)\big) \right] - \mathbb{E}_N \left[\frac{dS\big(c(x;\rho)\big)}{dx} \log\left(\frac{dN}{dF}\right) \right] = 0$$

Price Jump at Announcement

$$S(x; r) = -\left(\alpha - (\alpha - \gamma)\mathbb{I}_{x(\bar{r} - z(r_2 - r)) > 0}\right)(\bar{r} - z(r_2 - r))\, x$$

- a notion of negative sentiments towards low outcomes
- higher absolute risk aversion at low levels of consumption
- FOC simplifies to

$$\mathbb{E}_N\left[S'(x)\right]\left(1 + D_{KL}(N||F)\right) + x\left(\mathbb{E}_N\left[S'^2(x)\right] - \mathbb{E}_N^2\left[S'(x)\right]\right)$$
$$-\,\mathbb{E}_N\left[S'(x)\log\left(\frac{dN}{dF}\right)\right] = 0$$

- impose market clearing

Predictive Power of Normal-Divergence

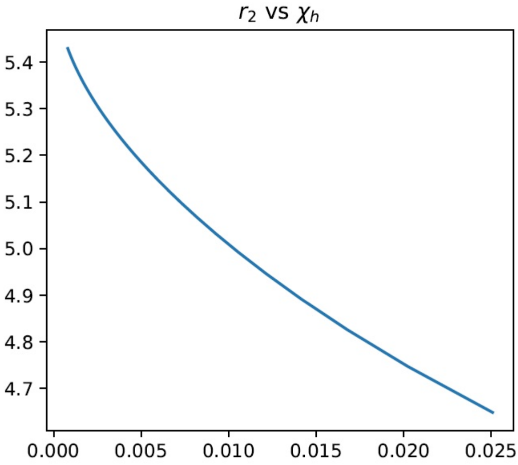
Proposition (Price Jump at Announcement)

In equilibrium expected jump at announcement, and the information in ex-ante return distribution, χ_h, are negatively correlated. Price jump r_2^* is characterized by

$$r_2^* = -\frac{a_0}{z(\alpha+\gamma)}\bar{x} - b_0(\chi_h + 1) + c_0$$

Model-Implied Price Jump at Announcement

$p_2 - \mathbb{E}_F[\tilde{p}_1]$ as a function of χ_h



- solve the model for the family of student-t distributions with degree of freedom varying from $5 - 80$

Outline

Conclusions

- **normal-divergence**: novel measure of information in return distribution
  - information distance between the return distribution prior to earning announcement, and normal distribution with the same standard deviation
  - *intuition:* gradual information revelation prior to earnings announcements
- significant (negative) explanatory power for price jump at announcement
- robust to many controls
  - higher moments of return distribution, earning surprise, option-implied-vol

What does this suggest?

- return distribution prior to announcement reflects investor information
- not captured by the commonly used moments of the return distribution
- not fully prices in options either!
- **but**, internalized by the investors $\rightarrow$ affects the price jump at announcement

Post Announcement Standard Deviation: Effect of χ_h

$\Delta t = 1$, $T = 4$ hour interval

		(1)	(2)	(3)	(4)	(5)
constant		1.000	1.585	0.550	0.937	0.965
		(22.59***)	(6.26***)	(9.57***)	(3.73***)	(3.80***)
χ_h		-0.046	-0.059	0.188	0.148	0.114
		(-2.83**)	(-3.29**)	(7.73***)	(5.44***)	(4.22***)
return std (σ_t)		0.853	0.568	0.988	1.100	0.819
		(52.38***)	(23.05***)	(31.04***)	(34.54***)	(18.54***)
average price		-0.088	-0.131		-0.115	-0.081
		(-5.41***)	(-4.21***)		(-3.82***)	(-2.61**)
$\chi_h \times \sigma_t$				-0.268	-0.450	-0.211
				(-7.67***)	(-12.04***)	(-5.32***)
option implied jump				0.214		0.181
				(10.72***)		(7.78***)
market cap				-0.113		-0.026
				(-7.32***)		(-0.59)
volume				-0.018		0.001
				(-1.18)		(0.06)
return kurtosis				-0.266		-0.213
				(-14.70***)		(-10.36***)
controls	sectors		✓		✓	✓
	year month		✓			✓
	quarter		✓		✓	✓
	stock		✓		✓	✓
	$<$ 5$ stocks		✓		✓	✓
N		4290	4290	4290	4290	4290

Related Literature

- this paper: jump variation can be significantly different from predictions solely based on market volatility
- Andersen, Fusari, and Todorov (2017): variation in negative jump tail risk
- contrast to
  - no-arbitrage asset pricing models: jump intensity proportional to volatility or its factors (Bates (1991), Pan (2002)))
  - consumption-based equilibrium models: tight affine connection between the variation in jump risk and volatility Drechsler and Yaron (2010))

- this paper: quantify the information in the return distribution
- role of information-based trading in stock returns
  - PIN: probability of information-based trading (Easley, Hvidkjaer, and O'hara (2002,2010), Vega (2006), Sadka (2006), Duarte and Young (2009))

Preliminaries

Definition (Differential Entropy)

For a continuous random variable X *with pdf* $f(X)$,

$$h(X) = -\int_S f(x) \log f(x) dx$$

Remark: $X \sim N(\mu, \sigma^2)$ *achieves the maximum entropy fixing the second moment.*

Moreover, $\xi = h(X) = \ln(\sigma\sqrt{2\pi e})$.

Definition (Kullback- Leibler (KL-) Divergence)

For probability distributions $f(x)$ *and* $g(x)$ *of a continuous random variable* X

$$D_{KL}(G||F) = \int_{-\infty}^{+\infty} g(x) \log \frac{g(x)}{f(x)} dx.$$

back