Volatility-of-Volatility Risk in Asset Pricing

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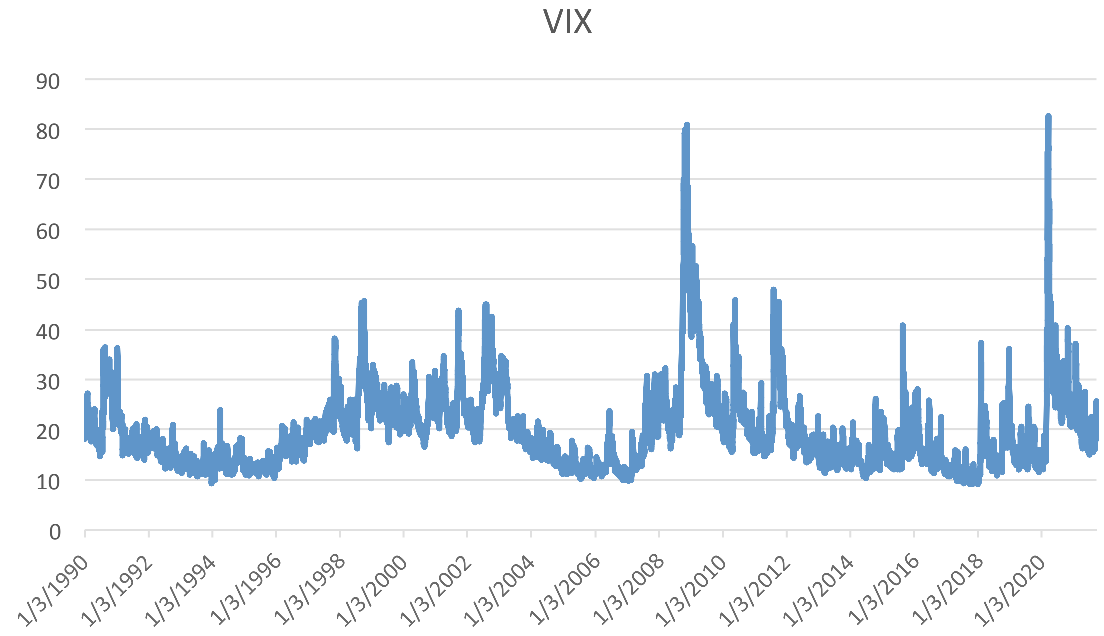
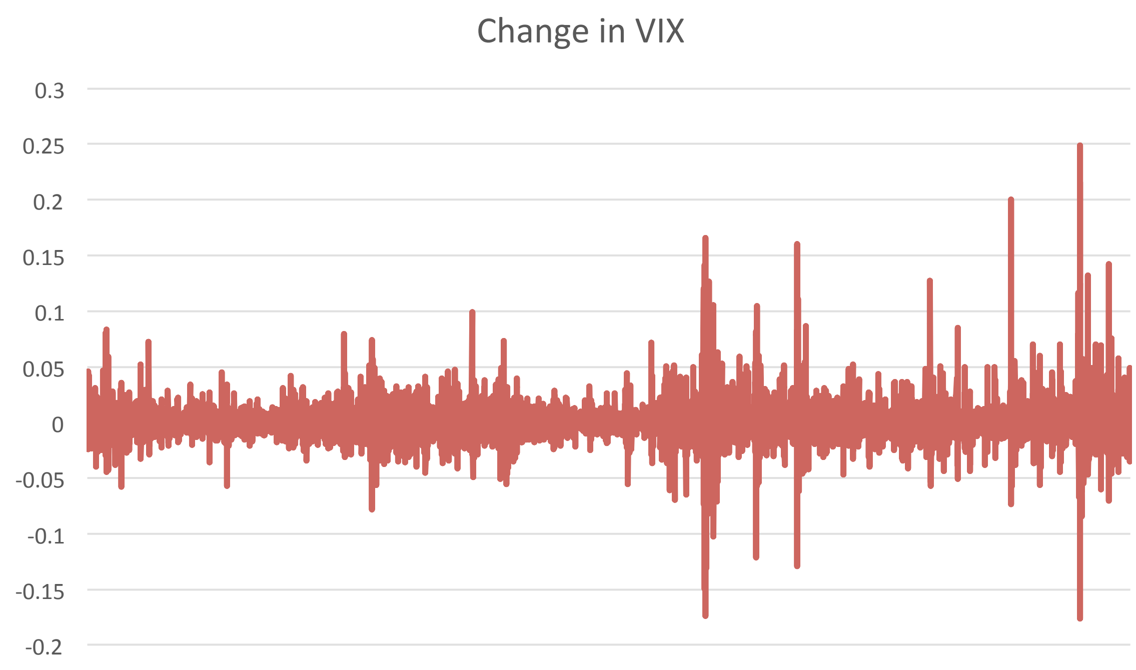
Motivation

- Large literature on the relation between market volatility and returns
  - Campbell & Hentschell, Glosten (1992), Jagannathan, Runkle (1993)
- Time-varying market volatility changes investment opportunity set by
  - Changing expectations of market returns
  - Changing risk-return trade-off
- Market Volatility is a systematic risk factor
  - Ang, Hodrick, Xing, Zhang (2006)

Market Volatility is Volatile





Market Volatility is Volatile

- Market crashes seem to be occurring quite frequently
  - 1987 - Black Monday
  - 1997 – Asian financial crisis
  - 1998 – Russian financial crisis
  - 2000 – Dot.com crash
  - 2001 – 9/11
  - 2008 – Financial crisis
  - 2010 – Euro crisis
  - 2020 – Covid-19

- If market volatility is volatile
  - Is Volatility-of-Volatility, VOV, a priced risk factor?
  - Is VOV related to crashes?

Model

- General equilibrium, representative agent model with Epstein-Zin preferences. Discrete time exchange economy.
  - Extension of long-run risk model of Bansal and Yaron (2004) and variance-of-variance model of Bollerslev, Tauchen and Zhou (2009)

Consumption growth $\quad g_{t+1} = \mu_g + x_t + \sigma_t z_{g,t+1}$

Long run risk $\quad x_{t+1} = \rho_x x_t + \varphi_x \sigma_t z_{x,t+1}$

Economic uncertainty $\quad \sigma_{t+1}^2 = \mu_\sigma + \rho_\sigma \sigma_t^2 + q_t z_{\sigma,t+1}$

Economic Var of Var $\quad q_{t+1}^2 = \mu_q + \rho_q q_t^2 + \varphi_q z_{q,t+1}$

Model Results

- Market return is negatively correlated with market volatility and VOV

$$CVRV_{mt} = Cov_t[r_{m,t+1}, V_{m,t+1}] < 0$$

$$CVRQ_m = Cov_t[r_{m,t+1}, Q_{m,t+1}] < 0$$

- Regression coefficient of $r_{m,t+1}$ on $CVRV_{mt}$ is negative while that on Q_{mt} is positive

$$r_{m,t+1} - r_{ft} = \alpha + \beta_{CVRV}^{pred} CVRV_{mt} + \epsilon_{t+1} ;\ \beta_{CVRV}^{pred} < 0$$

$$r_{m,t+1} - r_{ft} = \alpha + \beta_{Q}^{pred} Q_{mt} + \epsilon_{t+1} ;\ \beta_{Q}^{pred} > 0$$

- Risk premium on asset *i*

$$E_t[r_{i,t+1}] - r_{ft} \approx \lambda_m COV_t[r_{i,t+1}, r_{m,t+1}] + \lambda_V COV_t[r_{i,t+1}, V_{m,t+1}]$$

Estimation

- Recall $E_t[r_{i,t+1}] - r_{ft} \approx \lambda_m \; COV_t[r_{i,t+1}, r_{m,t+1}] + \lambda_V \; COV_t[r_{i,t+1}, V_{m,t+1}] + \lambda_Q \; COV_t[r_{i,t+1}, Q_{m,t+1}]$
  - r_m proxied by VWRETD, V_m proxied by ΔVIX
  - Q_m estimated using five-minute SPX data as per the following steps

1. Intraday stock returns $r_{t+1,j} \equiv \log[S_{t+j}/S_{t+j-1}]$, $j=1,2,\ldots M$ (# of 5-min intervals)
2. Barndoff-Nielsen and Shephard (2004) show

$$Realized\ Variance\ \ RV(r_{t+1}) = \sum_{j=1}^{M} r_{t+1,j}^2 \xrightarrow{M\to\infty} \int_t^{t+1} \sigma^2(s)ds + \sum_{j=1}^{N_{t+1}} J_{t+1,j}^2$$

$$Bipower\ Variance\ \ BV(r_{t+1}) = \frac{\pi}{2}\frac{M}{(M-1)} \sum_{j=2}^{M} |r_{t+1,j}|\,|r_{t+1,j-1}| \xrightarrow{M\to\infty} \int_t^{t+1} \sigma^2(s)ds$$

3. Following Bakshi, Kapadia, Madan (2003), estimate intraday model-free annualized implied volatilities using 30-day out-of-money SPX calls and puts

Data

- Tick-by-tick quotes for SPX from CBOE's Market Data Report (MDR)
  - January 1996 – December 2015
  - Linearly interpolate for maturity τ=30 days
  - Linearly interpolate moneyness levels
- OptionMetrics
  - Daily data for stock options and SPX
  - Zero Coupon interest rate curve and index dividend yield
- Daily and monthly returns from CRSP
- Compustat for accounting variables
- Intraday transactions data from TAQ
- $\Delta VIX_{t+1} = VIX_{t+1} - VIX_t$
- $\Delta VOV, \Delta CVRV, \Delta SKEW, and\ \Delta KURT$ as residuals from ARMA(1,1)

Preliminary Results

$$CVRV_t = -0.053 - 0.452\, VOV_t,\quad R^2 = 0.293,$$

(-2.70)

$$\Delta CVRV_t = -0.000 - 0.399\, \Delta VOV_t,\quad R^2 = 0.406,$$

$$(-2.54)$$

$$VRP_t = 0.174 + 0.145\, VOV_t,\quad R^2 = 0.069,$$

(2.53)

$$VRP_t = 0.164 - 0.277\, CVRV_t,\quad R^2 = 0.146,$$

(-2.07)

$$\Delta CVRV_t = -0.000 - 0.050\, \Delta VIX_t,\quad R^2 = 0.039,$$

$$(-1.90)$$

$$\Delta CVRV_t = -0.000 - 0.845\, \Delta VRP_t,\quad R^2 = 0.044,$$

$$(-1.91)$$

$$\Delta CVRV_t = 0.000 - 0.389 \Delta VOV_t - 0.019 \Delta VIX_t - 0.259 \Delta VRP_t,\quad R^2 = 0.424,$$

(-2.54) (-1.51) (-1.18)

	A. Dependent variable = daily rSPX-rf (t)									
	(1)		(2)		(3)		(4)		(5)	
Intercept	-0.457	(-0.29)	-0.729	(-0.46)	0.843	(0.36)	2.807	(0.99)	-0.229	(-0.16)
VIX (t-1)	0.024	(0.29)	-0.059	(-0.63)	-0.062	(-0.61)	-0.145	(-1.12)	0.012	(0.16)
VOV (t-1)	**5.009**	**(2.10)**	**4.770**	**(2.05)**	**4.767**	**(2.05)**			**4.739**	**(1.66)**
CVRV (t-1)							**-4.864**	**(-1.91)**	-0.674	(-0.28)
VRP (t-1)			**13.136**	**(3.75)**	**13.307**	**(3.83)**	**14.847**	**(3.72)**		
SKEW (t-1)					2.076	(1.56)	**2.300**	**(1.74)**		
KURT (t-1)					0.213	(1.43)	0.201	(1.33)		
JUMP (t-1)					0.074	(0.70)	0.064	(0.62)		
rSPX-rf (t-1)	**-0.061**	**(-3.29)**	**-0.039**	**(-2.14)**	**-0.037**	**(-2.05)**	**-0.033**	**(-1.77)**	**-0.061**	**(-3.26)**
Adj. R^2	.012		.019		.019		.016		.012	

	B. Dependent variable = monthly rSPX-rf (t)									
	(1)		(2)		(3)		(4)		(5)	
Intercept	0.058	(0.06)	-0.087	(-0.12)	0.418	(0.33)	0.825	(0.67)	0.029	(0.04)
VIX (t-1)	0.006	(0.13)	-0.022	(-0.70)	-0.029	(-0.87)	-0.048	(-1.39)	0.008	(0.20)
VOV (t-1)	**1.671**	**(2.73)**	**1.142**	**(2.02)**	**1.141**	**(1.92)**			1.753	(1.52)
CVRV (t-1)							**-0.478**	**(-2.25)**	0.050	(0.10)
VRP (t-1)			**4.747**	**(4.14)**	**4.591**	**(3.83)**	**5.061**	**(4.97)**		
SKEW (t-1)					-0.230	(-0.19)	-0.098	(-0.08)		
KURT (t-1)					-0.069	(-0.56)	-0.059	(-0.48)		
JUMP (t-1)					0.014	(0.29)	0.006	(0.12)		
rSPX-rf (t-1)	0.104	(1.39)	0.069	(1.02)	0.080	(1.17)	0.100	(1.35)	0.101	(1.34)
Adj. R^2	.008		.059		.050		.052		.003	

Predictive Regressions

	1-month horizon					
	[1]		[2]		[3]	
Intercept	0.358	(1.15)	-0.387	(-1.39)	-0.402	(-1.44)
VOV (t-1)	**1.620**	**(2.85)**			**1.003**	**(1.98)**
VRP (t-1)			**4.994**	**(5.01)**	**4.773**	**(4.92)**
Adj. R^2	.007		.061		.061	
OOS-R^2	.018 *		.096 *		.083 *	
	6-month horizon					
	[1]		[2]		[3]	
Intercept	2.462	(1.20)	0.310	(0.18)	0.261	(0.15)
VOV (t-1)	**5.048**	**(2.37)**			**3.261**	**(1.80)**
VRP (t-1)			**14.533**	**(6.77)**	**13.816**	**(7.76)**
Adj. R^2	.011		.071		.073	
OOS-R^2	-.053		.011		-.011	

Asset Pricing Tests

- Portfolio formation
  - Pre-ranking betas from stock-level time-series regressions at daily frequency

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT}\, MKT_{t+1} + \beta_{i,VIX}\, \Delta VIX_{t+1} + \beta_{i,VOV}\, \Delta VOV_{t+1} + \varepsilon_{i,t+1}$$

  - Independently sort into 5x5 portfolios based on $\beta_{i,VIX}$ and $\beta_{i,VOV}$
  - Form value-weight portfolios and then obtain post-ranking betas

- Conditional model

$$r_{p,t+1} - r_{f,t+1} = \alpha_p + \beta_{p,MKT,t}\, MKT_{t+1} + \beta_{p,VIX,t}\, \Delta VIX_{t+1} + \beta_{p,VOV}\, \Delta VOV_{t+1} + \varepsilon_{p,t+1}$$

$$= \alpha_p + (\beta_{p,MKT} + \beta_{p,MKT,V}\, VIX_t + $$

Asset Pricing Tests

- Other models

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,SKEW} \Delta SKEW_{t+1} + \beta_{i,KURT} \Delta KURT_{t+1} + \varepsilon_{i,t+1}$$

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,STR} STR_{t+1} + \varepsilon_{i,t+1}$$

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VOL} VOL_{t+1} + \varepsilon_{i,t+1}$$

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,JUMP} JUMP_{t+1} + \varepsilon_{i,t+1}$$

Control variables	Portfolios ranking 1	2	3	4	5	α-FFC4 5-1	t-stat	α-FF5 5-1	t-stat	α-HXZ 5-1	t-stat
	A. Performance of $\beta_{i,VoV}$-sorted portfolio, controlling for $\beta_{i,VIX}$										
$\beta_{i,VIX}$	0.92	0.71	0.50	0.51	0.20	**-0.86**	**(-2.79)**	**-0.62**	**(-2.50)**	**-0.98**	**(-3.41)**
	B. Performance of $\beta_{i,VIX}$-sorted portfolio, controlling for $\beta_{i,VoV}$										
$\beta_{i,VoV}$	0.68	0.65	0.68	0.57	0.26	**-0.63**	**(-2.57)**	-0.31	(-1.41)	**-0.44**	**(-1.79)**
	C. Performance of $\beta_{i,VoV}$-sorted portfolio, controlling for each control variable										
Size	0.24	0.29	0.28	0.21	-0.16	**-0.40**	**(-1.94)**	**-0.36**	**(-2.24)**	**-0.56**	**(-3.01)**
B/M	0.42	0.14	0.01	0.03	-0.26	**-0.68**	**(-2.48)**	**-0.52**	**(-2.21)**	**-0.84**	**(-3.06)**
RET_2_12	0.07	0.05	0.08	-0.06	-0.44	**-0.51**	**(-1.96)**	**-0.36**	**(-1.92)**	**-0.60**	**(-2.68)**
RET_1	0.15	0.11	-0.05	0.02	-0.47	**-0.61**	**(-2.16)**	**-0.41**	**(-1.81)**	**-0.74**	**(-2.75)**
ILLIQ	0.10	0.25	0.23	0.13	-0.32	**-0.42**	**(-1.98)**	**-0.36**	**(-2.01)**	**-0.60**	**(-3.10)**
$\beta_{i,SKEW}$	0.23	0.20	0.00	0.00	-0.41	**-0.64**	**(-2.37)**	**-0.47**	**(-2.24)**	**-0.79**	**(-3.19)**
$\beta_{i,KURT}$	0.15	0.09	0.06	-0.06	-0.43	**-0.59**	**(-2.14)**	**-0.43**	**(-1.96)**	**-0.75**	**(-3.00)**
$\beta_{i,STR}$	0.21	0.09	-0.12	-0.04	-0.53	**-0.75**	**(-2.63)**	**-0.51**	**(-2.21)**	**-0.88**	**(-3.31)**
$\beta_{i,VOL}$	0.19	0.07	-0.09	-0.10	-0.50	**-0.70**	**(-2.29)**	**-0.45**	**(-1.88)**	**-0.86**	**(-3.03)**
$\beta_{i,JUMP}$	0.19	0.15	-0.10	-0.05	-0.51	**-0.70**	**(-2.49)**	**-0.46**	**(-2.00)**	**-0.83**	**(-3.12)**
$\beta_{i,TAIL}$	0.37	0.23	0.01	-0.09	-0.32	**-0.69**	**(-2.37)**	**-0.49**	**(-1.91)**	**-0.82**	**(-2.95)**
	D. Performance of each alternative variable sorted portfolio, controlling for $\beta_{i,VIX}$										
$\beta_{i,VVIX}$	-0.36	-0.14	-0.09	0.17	-0.16	0.20	(0.75)	0.04	(0.16)	0.24	(0.92)
$\beta_{i,VRP}$	-0.17	0.01	0.05	-0.06	-0.39	-0.23	(-1.08)	0.00	(-0.00)	-0.01	(-0.06)
$\beta_{i,JOV}$	-0.18	0.06	-0.06	-0.02	-0.30	-0.12	(-0.47)	-0.14	(-0.54)	-0.17	(-0.63)

	Fama-MacBeth cross-sectional regressions									
	A. Constant beta models									
Test portfolios	$\beta_{i,VOV} \times \beta_{i,VIX}$ [1]		$\beta_{i,VOV} \times \beta_{i,VIX}$ [2]		$\beta_{i,VOV} \times \beta_{i,VIX}$ [3]		$\beta_{i,VOV} \times \beta_{i,VIX}$ [4]		100 portfolios [5]	
Intercept	-1.23	(-1.08)	-0.37	(-0.35)	0.32	(0.25)	-1.56	(-0.88)	-0.51	(-0.58)
$\beta_{p,MKT}$	1.84	(1.59)	0.99	(0.88)	0.30	(0.22)	2.20	(1.26)	1.06	(1.16)
$\beta_{p,VIX}$	**-7.09**	**(-1.83)**	-3.82	(-1.08)	-4.17	(-0.95)	-5.84	(-1.18)	-2.30	(-1.15)
$\beta_{p,VOV}$	**-3.35**	**(-2.51)**	**-3.05**	**(-2.33)**	**-3.01**	**(-2.84)**	**-2.59**	**(-2.42)**	**-1.49**	**(-2.17)**
$\beta_{p,VVIX}$			-0.96	(-0.07)						
$\beta_{p,SMB}$					-0.46	(-0.57)	-0.19	(-0.21)	0.55	(1.19)
$\beta_{p,HML}$					-0.14	(-0.26)	-0.84	(-1.17)	-0.50	(-1.13)
$\beta_{p,UMD}$					-0.86	(-0.96)				
$\beta_{p,RMW}$							1.09	(1.60)	**1.19**	**(2.52)**
$\beta_{p,CMA}$							-0.62	(-1.06)	-0.52	(-1.08)
$\beta_{p,SKEW}$					1.81	(1.11)	1.36	(0.85)	0.04	(0.07)
$\beta_{p,KURT}$					-8.03	(-0.71)	3.43	(0.33)	-7.22	(-1.16)
$\beta_{p,JUMP}$					-1.66	(-0.09)	-16.12	(-0.74)	-14.73	(-1.11)
Adj. R^2	.70		.63		.67		.70		.40	

	Fama-MacBeth cross-sectional regressions									
	B. Time-varying beta models									
Test portfolios	$\beta_{i,VOV} \times \beta_{i,VIX}$		$\beta_{i,VOV} \times \beta_{i,VIX}$		100 portfolios		100 portfolios		100 portfolios	
	[6]		[7]		[8]		[9]		[10]	
Intercept	1.04	(1.40)	-0.62	(-0.53)	1.21	(2.48)	0.95	(1.17)	0.30	(0.33)
$\beta_{p,MKT}$	-0.36	(-0.44)	1.21	(1.01)	-0.65	(-1.39)	-0.41	(-0.45)	0.24	(0.25)
$\beta_{p,VIX}$	1.31	(0.41)	-4.44	(-1.11)	-2.42	(-1.15)	-3.53	(-1.44)	-1.87	(-0.94)
$\beta_{p,VOV}$	**-3.27**	**(-2.54)**	**-2.73**	**(-2.60)**	**-1.64**	**(-2.25)**	**-1.09**	**(-1.65)**	**-1.10**	**(-1.66)**
$\beta_{p,MKT,V}$										
$\beta_{p,MKT,Q}$			**0.90**	**(1.81)**			0.12	(0.34)	0.14	(0.40)
$\beta_{p,VIX,Q}$			**0.13**	**(1.83)**			**0.09**	**(2.59)**	**0.06**	**(1.80)**
$\beta_{p,SMB}$			**-0.31**	**(-1.85)**			**-0.21**	**(-2.71)**	**-0.16**	**(-1.95)**
$\beta_{p,HML}$					-0.11	(-0.25)	0.13	(0.30)	0.75	(1.40)
$\beta_{p,UMD}$					0.13	(0.32)	0.14	(0.33)	-0.44	(-1.03)
$\beta_{p,RMW}$					-1.10	(-1.54)	-0.47	(-0.74)		
$\beta_{p,CMA}$									**1.14**	**(2.44)**
$\beta_{p,SKEW}$									-0.58	(-1.26)
$\beta_{p,KURT}$					0.25	(0.36)	0.36	(0.52)	0.09	(0.14)
$\beta_{p,JUMP}$					-1.17	(-0.22)	0.18	(0.03)	-5.37	(-0.88)
Adj. R^2					-15.52	(-1.25)	-15.26	(-1.19)	-15.19	(-1.11)

	C. Fama-MacBeth regressions: Individual stocks							
	Constant beta models				Time-varying beta models			
	[1]		[2]		[3]		[4]	
Intercept	-1.11	(-0.97)	0.50	(0.39)	-0.07	(-0.06)	1.38	(1.07)
log(Size($b))	**-0.28**	**(-4.95)**	**-0.19**	**(-2.95)**	**-0.27**	**(-4.92)**	**-0.18**	**(-2.92)**
log(B/M)	0.12	(1.04)	0.10	(0.81)	0.12	(1.03)	0.10	(0.82)
RET_2_12	0.33	(0.95)	0.19	(0.51)	0.34	(0.98)	0.19	(0.52)
RET_1	**-1.66**	**(-2.70)**	**-1.54**	**(-2.15)**	**-1.65**	**(-2.68)**	**-1.54**	**(-2.15)**
ILLIQ(10^6)	**0.50**	**(2.50)**	**2.99**	**(3.11)**	**0.50**	**(2.54)**	**3.01**	**(3.12)**
$\beta_{ip,MKT}$	**2.01**	**(1.67)**	0.42	(0.31)	1.02	(0.82)	-0.40	(-0.30)
$\beta_{ip,VIX}$	-0.02	(-0.00)	4.00	(0.85)	2.38	(0.59)	5.59	(1.20)
$\beta_{ip,VOV}$	**-3.05**	**(-4.29)**	**-3.10**	**(-4.20)**	**-1.18**	**(-1.98)**	**-1.29**	**(-2.03)**
$\beta_{ip,MKT,V}$					0.25	(0.53)	-0.35	(-0.67)
$\beta_{ip,MKT,Q}$					**0.20**	**(4.11)**	**0.16**	**(2.62)**
$\beta_{ip,VIX,Q}$					**-0.48**	**(-4.42)**	**-0.40**	**(-2.88)**
IVOL-TVOL			**0.78**	**(3.87)**			**0.77**	**(3.86)**
CIVOL-PIVOL			**4.10**	**(6.66)**			**4.06**	**(6.68)**
Adj. R^2	.07		.08		.07		.08	
No. obs	566,205		380,235		566,205		380,235	

Crash Periods

	Normal periods				Crash periods			
	(1)		(2)		(3)		(4)	
*A. Crash(t) =1 if MKT(t) <-13.10% (Mean-3*SD of MKT)*								
Intercept	-1.22	(-1.07)	-0.65	(-0.56)	-2.67	(-6.70)	3.68	(1.13)
$\beta_{p,MKT}$	**1.98**	**(1.69)**	1.39	(1.17)	**-14.03**	**(-64.19)**	**-20.52**	**(-5.43)**
$\beta_{p,VIX}$	**-7.35**	**(-1.87)**	-4.89	(-1.21)	**23.65**	**(2.70)**	**48.45**	**(7.37)**
$\beta_{p,VOV}$	**-3.29**	**(-2.47)**	**-2.71**	**(-2.58)**	**-10.44**	**(-1.96)**	**-5.30**	**(-4.15)**
$\beta_{p,MKT,V}$			**0.94**	**(1.86)**			**-3.87**	**(-4.58)**
$\beta_{p,MKT,Q}$			**0.13**	**(1.81)**			0.42	(0.53)
$\beta_{p,VIX,Q}$			**-0.30**	**(-1.82)**			-1.00	(-0.63)
Adj. R^2	.69		.70		.27		.18	

Barillas and Shanken

M1	M2	loglike (M1)	#param (M1)	AIC (M1)	loglike (M2)	loglike (M2*)	#param (M2)	AIC (M2)	ΔAIC (M1-M2)
FFC4+*rVIX*	FFC4	2,179.7	20	-4,319.5	1,740.0	436.5	19	-4,315.0	-4.49
FFC4+*rVOV*	FFC4	2,174.9	20	-4,309.8	1,740.0	429.2	19	-4,300.4	-9.36
FFC4+*rVIX* +*rVOV*	FFC4	2,616.1	27	-5,178.2	1,740.0	866.1	25	-5,162.2	-15.99
FF5+*rVIX*	FF5	3,071.3	27	-6,088.6	2,626.9	443.3	26	-6,088.4	-0.17
FF5+*rVOV*	FF5	3,059.0	27	-6,064.1	2,626.9	429.4	26	-6,060.7	-3.40
FF5+*rVIX* +*rVOV*	FF5	3,504.4	35	-6,938.8	2,626.9	873.3	33	-6,934.5	-4.26

Conclusion

- VOV is an important determinant of market risk premium
  - Exacerbates the negative covariance between market return and volatility
  - Predicts future market returns
  - Adds to the Sharpe Ratio of standard factors (FFC4 and FF5)
- VOV risk is priced in the cross-section
  - Stocks with higher VOV betas have lower returns
- Unconditionally, VIX beta and market beta are not priced
  - Consistent with theory, conditional component of VIX and market beta is priced
- During crashes
  - Premium on VOV risk increases during crashes
  - Premium on VIX and Mkt have wrong sign during crashes